Physical Characteristics of Neoclassical Toroidal Viscosity in Tokamaks for Rotation Control and the Evaluation of Plasma Response


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The physical characteristics of NTV investigated in tokamaks for rotation control and the evaluation of plasma response

- **Motivation**
  - Low magnitude ($\delta B/B_0 \sim O(10^{-3})$) 3D magnetic fields are used favorably used in tokamaks (e.g. ELM suppression, MHD mode control)
  - 3D fields of this magnitude can produce neoclassical toroidal viscosity (NTV), which can:
    - Alter plasma rotation
    - Significantly reduce fusion gain, $Q$, by increased alpha particle transport ($\delta B/B_0 \sim O(10^{-4})$)
  - Therefore, it is important to understand NTV in tokamaks, backed by accurate (~$O(1)$) quantitative modeling

- **Outline**
  - NTV physical characteristics
  - NTV comparison of theory to experiment
  - NTV experiments and assessment of plasma response
  - Application of NTV to plasma rotation control for NSTX-U
Neoclassical Toroidal Viscosity (NTV) can be studied through the application of 3D fields in tokamaks

- Theory: NTV strength varies with plasma collisionality $\nu$, $\delta B^2$, rotation

$$\langle \hat{e}_i \nabla \bullet \Pi \rangle_{(1/\nu)} = B_i R \left( \frac{1}{B_i} \right) \left( \frac{1}{R^3} \right) \frac{\lambda_i p_i}{\pi^{3/2} \nu_i} \epsilon^{3/2} (\omega_\phi - \omega_{NC}) I_\lambda$$


plasma rotation $T_i^{5/2}$

KSTAR 3D coils

NSTX 3D coils
Non-resonant NTV characteristics (e.g. in NSTX and KSTAR)

- 3D field configurations with dominant toroidal mode number \( n > 1 \) can alter the plasma rotation profile, \( \omega_\phi \) without mode locking
- Experimentally, NTV torque is radially extended, with a relatively smooth profile
- NTV changes continuously as the applied 3D field is increased
- \( T_{NTV} \) is not simply an integrated torque applied at the plasma boundary, but a radial profile – e.g. \( \omega_\phi \) shear can be changed

These aspects are generally favorable for rotation control; give potential mode control

Questions remain

- e.g. Is there hysteresis when \( \omega_\phi \) is altered by NTV?
KSTAR experiments show essentially no hysteresis in steady-state $\omega_\phi$ profile vs. applied 3D field strength

- Experiment run to produce various steady-state $\omega_\phi$ with different 3D field evolution.

- The steady-state rotation profile reached is generally independent of the starting point of $\omega_\phi$.
  - depends just on the applied 3D field current level.
  - important for rotation control.

- Absence of hysteresis further confirmed in very recent experiments with 6 steps in 3D field current.
Neoclassical Toroidal Viscosity varies as $\delta B^2$, and $T_i^{2.27}$ in KSTAR experiments, expected by theory

- NTV torque $T_{NTV}$ expected to scale as $\delta B^2$ and $T_i^{2.5}$ in the “$1/\nu$ regime”

\[ \omega_\phi \text{ steady-state reached each } \delta B \text{ step} \]

Best fit: $\propto \delta B^2$

Best fit: $\propto T_i^{2.27}$

Y.S. Park, et al., IAEA FEC 2014: EX/P8-05 (Fri. PM)
3D field perturbation experiments conducted to measure the $T_{NTV}$ profile in NSTX

- High normalized beta plasma targets typically chosen
  - Typically near or above $n = 1$ no-wall limit (for higher $T_i$)

- Apply or otherwise change 3D field on a timescale significantly faster than the momentum diffusion time, $\tau_m$
  - Analysis before/after 3D field application isolates $T_{NTV}$ in the momentum diffusion equation; $-dL/dt = T_{NTV}$

- $dL/dt$ measured experimentally and compared to theoretically computed $T_{NTV}$ on this timescale
  - $dL/dt$ profile can change significantly on timescales $> \tau_m$ (diffuses radially, broadens, leads to significant error compared to $T_{NTV}$)

- Focus on non-resonant applied 3D field configurations
  - To avoid driving MHD modes
  - Resonant fields (e.g. $n = 1$) are more strongly screened by plasma
Theoretical NTV torque density profiles, $T_{NTV}$, are computed for NSTX using theory applicable to all collisionality regimes.

Non-axisymmetric coils fully modelled in 3D

3D field definition

$$\delta B = \vec{b} \cdot \left( \frac{\vec{B}}{B} \right) + \left( \vec{\xi} \cdot \nabla B \right)$$

plasma displacement

General considerations

- In tokamaks, $\xi$ not typically measured, can lead to large error
- “Fully-penetrated field constraint” used to define $\xi$
  $\left( B_{2D} \cdot \nabla \xi = \vec{b} \right)$
  - Singularities avoided by standard finite island width assumption
- For NSTX, $|\xi| \sim 0.3$ cm $<< \varepsilon^{0.5} \rho_i$,
  therefore, ion banana width-averaging is used for ion channel
  - Can explain why strong resonant peaks in NTV profile are not observed in experiment

NTV analysis of NSTX – data interfaced to NTVTOK

(Y. Sun, Liang, Shaing, et al., NF 51 (2011) 053015)

- Use Shaing’s “connected NTV model”, covers all $\nu$, superbanana plateau regimes
  (K.C. Shaing, Sabbagh, Chu, NF 50 (2010) 025022)

- Full 3D coil specification and $\delta B$ spectrum, ion and electron components computed, no aspect ratio assumptions
Measured NTV torque density profiles quantitatively compare well to computed $T_{NTV}$ using fully-penetrated 3D field

$T_{NTV}$ (theory) scaled to match peak value of measured $-dL/dt$

- Scale factor $((dL/dt)/T_{NTV}) = 1.7$ and $0.6$ (for cases shown above) – $O(1)$ agreement
- $O(1)$ agreement using “fully-penetrated 3D field” indicates that plasma response is not strongly amplified from this “vacuum field assumption” ($T_{NTV} \sim \delta B^2$)
Plasma response from fully-penetrated 3D field used in NTV experimental analysis matches M3D-C¹ single fluid model

- NTV experimental data is a strong quantitative constraint on plasma response of $\delta B$
  - Because the measured NTV scales as $T_{NTV} \propto \delta B^2$,

- Level of agreement varies along the profile
  - Good agreement between NTVTOK / M3D-C¹ single fluid models in strong NTV region
  - M3D-C¹ core $<\delta B>$ larger than NTVTOK
    - Core mode in M3D-C¹
  - M3D-C¹ edge $<\delta B>$ smaller
    - Experimental $T_{NTV}$ too small in this region to constraint $\delta B$
Non-resonant NTV and NBI used as actuators in state-space rotation feedback controller designed for NSTX-U

- Momentum force balance – \( \omega_\phi \) decomposed into Bessel function states
  \[
  \sum n_i m_i \left( R^2 \right) \frac{\partial \omega}{\partial t} = \left( \frac{\partial V}{\partial \rho} \right)^{-1} \frac{\partial}{\partial \rho} \left[ \sum n_i m_i \chi_\phi \left( \frac{\partial V}{\partial \rho} \right)^2 \frac{\partial \omega}{\partial \rho} \right] + T_{NBI} + T_{NTV}
  \]

- NTV torque:
  \[
  T_{NTV} \propto K \times f \left( n_{e,i} K_1 T_{e,i} K_2 \right) g \left( \delta B(\rho) \right) \left[ I_{coil}^2 \omega \right] \] (non-linear)

Rotation evolution and NBI and NTV torque profiles

3D coil current and NBI power (actuators)
When $T_i$ is included in NTV rotation controller model, 3D field current and NBI power can compensate for $T_i$ variations.

Rotation evolution and NBI and NTV torque profiles

3D coil current and NBI power (actuators)

$$T_{NTV} \propto K \times f\left(n_{e,i}^{K1}T_i^{K2}\right) g\left(\delta B(\rho)\right) I_{coil}^2 \omega$$

- NTV torque profile model for feedback dependent on ion temperature: $K1 = 0, K2 = 2.5$
Physical characteristics of NTV are investigated in tokamaks for rotation control and the evaluation of plasma response

- Experiments on NSTX and KSTAR show that non-resonant NTV torque $T_{NTV}$ from applied 3D field is a radially extended, relatively smooth profile

- Analysis of KSTAR shows $T_{NTV} \propto (\delta B_{3D})^2$; $T_{NTV} \propto T_i^{2.27}$; no hysteresis on the rotation profile when altered by non-resonant NTV (key for control)

- 3D field perturbation experiments in NSTX using both $n = 2$ and $n = 3$ field configurations measure the $T_{NTV}$ profile

- The measured $T_{NTV}$ profile quantitatively compares well between experiment and Shaing’s “connected NTV theory”

- Non-resonant $T_{NTV}$ profile in NSTX is quantitatively consistent with “fully-penetrated field” assumption of plasma response

- Surface-averaged 3D field profile from M3D-C$^1$ single fluid model consistent with field used for quantitative NTV agreement in experiment

- Rotation controller using NTV and NBI designed/tested for NSTX-U
Extra slides for poster
Non-resonant NTV and NBI used as actuators in state-space rotation feedback controller designed for NSTX-U

- Momentum force balance – $\omega_\phi$ decomposed into Bessel function states

$$\sum_i n_i m_i \langle R^2 \rangle \frac{\partial \omega}{\partial t} = \left( \frac{\partial V}{\partial \rho} \right)^{-1} \frac{\partial}{\partial \rho} \left[ \sum_i n_i m_i \chi_\phi \left\langle \left( R \nabla \rho \right)^2 \right\rangle \frac{\partial \omega}{\partial \rho} \right] + T_{NBI} + T_{NTV}$$

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Rotation evolution and NBI and NTV torque profiles

3D coil current and NBI power (actuators)

I. Goumiri (PU), S.A. Sabbagh (Columbia U.), D.A. Gates, S.P. Gerhardt (PPPL)
When $T_i$ is included in NTV rotation controller model, 3D field current and NBI power can compensate for $T_i$ variations.

Rotation evolution and NBI and NTV torque profiles

$$T_{NTV} \propto K \times f \left( n_{e,i}^{K1} T_i^{K2} \right) g \left( \delta B (\rho) \right) \left[ I_{\text{coil}}^2 \omega \right]$$

$K1 = 0$, $K2 = 2.5$

- NTV torque profile model for feedback dependent on ion temperature
Measured NTV torque density profiles quantitatively compare well to computed $T_{NTV}$ using fully-penetrated 3D field

- $T_{NTV}$ (theory) scaled to match peak value of measured $-dL/dt$
  - Scale factor $(dL/dt)/T_{NTV} = 1.7$ and 0.6 (for cases shown above) – $O(1)$ agreement
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Very recently, high beta plasmas transiently reached $\beta_N = 4$ in 2014 campaign

- Values obtained using fully converged KSTAR EFIT reconstructions
- High values reached transiently at lowered $B_T$
  - $B_T$ in range 0.9 - 1.2 T
  - $\beta_N$ up to 4 with $i_i \sim 0.8$ for duration longer than $\tau_E$ ~60 ms in these discharges
  - $\beta_N / l_i = 5$ is ~ 40% over the computed $n = 1$ ideal MHD no-wall limit
- Adding newly available 3rd neutral beam source may further increase the operating performance in the ongoing device campaign

KSTAR operating space containing ~11,500 equilibria

Y.S Park, et al., IAEA FEC 2014 paper EX/P8-05 (Fri. PM)
S.W. Yoon, et al., IAEA FEC 2014 paper OV/3-4 (Tues. AM)
Non-resonant Neoclassical Toroidal Viscosity (NTV) physics will be used for the first time in rotation feedback control

- Momentum force balance – $\omega_\phi$ decomposed into Bessel function states

$$\sum_i n_i m_i \left\langle R^2 \right\rangle \frac{\partial \omega}{\partial t} = \left( \frac{\partial V}{\partial \rho} \right)^{-1} \frac{\partial V}{\partial \rho} \sum_i n_i m_i \chi_\phi \left\langle (R \nabla \rho)^2 \right\rangle \frac{\partial \omega}{\partial \rho} + T_{NBI} + T_{NTV}$$

- NTV torque:

$$T_{NTV} \propto K \times f \left( n_{e,i} K_1 T_{e,i} K_2 \right) g \left( \delta B(\rho) \right) \left[ I_{coil}^2 \omega \right] \text{ (non-linear)}$$

Feedback using NTV: “n=3” $\delta B(\rho)$ spectrum

I. Goumiri (PU), S.A. Sabbagh (Columbia U.), D.A. Gates, S.P. Gerhardt (PPPL)
Plasma rotation control has been demonstrated for the first time with TRANSP using NBI and NTV actuators.

This case uses pre-programmed 3D coil current and NBI feedback.
Please sign-up for a poster copy
Several ordered publications by K.C. Shaing, et al. led to the “Combined” NTV Formulation

Publications (chronological order)

Topic
- Plateau transport
- Island NTV
- Collisional, $1/n$ regimes
- Banana, $1/n$ regimes
- Multiple trapping
- Orbit squeezing
- Coll. b’dary layer, $\nu^{0.5}$
- Low $\nu$ regimes
- Superbanana plateau
- Superbanana regime
- Bounce/transition/drift res.
- $J_{\text{bootstrap}}$ w/resonances
- Combined NTV formula
- $\nabla B$ drift in CBL analysis
- Flux/force gen. coords.
- SBP regime refinement
- NTV brief overview
EX/1-4: Physical Characteristics of Neoclassical Toroidal Viscosity in Tokamaks for Rotation Control and the Evaluation of Plasma Response

Highlights

- Experimental NTV characteristics
  - NTV experiments on NSTX and KSTAR
  - NTV torque $T_{NTV}$ from applied 3D field is a radially extended, relatively smooth profile
  - Perturbation experiments measure $T_{NTV}$ profile

- Aspects of NTV for rotation control
  - Varies as $\delta \mathbf{B}^2$: $T_{NTV} \propto T_i^{5/2}$ in primary collisionality regime for large tokamaks
  - No hysteresis on the rotation profile when altered by non-resonant NTV is key for control
  - Rotation controller using NTV and NBI tested for NSTX-U; model-based design saves power

- NTV analysis to assess plasma response
  - Non-resonant NTV quantitatively consistent with fully-penetrated field assumption
  - Surface-averaged 3D field profile from M3D-C1 single fluid model consistent with field used for quantitative NTV agreement in experiment

Perturbation experiments measure NTV torque profile and compare to theory

Rotation controller using NTV and NBI