Accurate Estimation of Tearing Mode Stability Parameters in the KSTAR using High-resolution 2D ECEI diagnostic

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Evolution of the tearing mode island size ($w$) is described with

$$a_1 \frac{\tau_r}{r_s} \frac{dw}{dt} = r_s \Delta' + a_2 \sqrt{\epsilon} \frac{\beta_\theta}{w} \frac{L_q}{L_p} \frac{w}{w^2 + (w_c)^2} + \cdots$$

- $\tau_r = \mu_0 r_s^2 / \eta$ : the current diffusion time
- $\eta$ : the plasma resistivity
- $\epsilon = r_s / R$ : inverse aspect ratio
- $\beta_\theta$ : the plasma poloidal beta
- $L_q = q/(dq/dr)$ and $L_p = p/(dp/dr)$ where $q$ is the safety factor and $p$ is the plasma pressure

$a_1$ and $a_2$ : coefficients related to flux geometry of the magnetic island

$\Delta'(w) \sim \tilde{B}_\theta (r_s + w) - \tilde{B}_\theta (r_s - w)$ classical stability index

$w_c = \sqrt{R q L_q} \left( \frac{k_\perp}{k_\parallel} \right)^{1/4}$ critical width for pressure flattening

- Parameters such as $a_1$, $a_2$, $\Delta'$, and $w_c$ can not be “measured”
  
  Method to estimate those parameters is required!
Method to estimate $\Delta'$ and $w_c$

- Electron temperature profile near the magnetic island (away from heat sink or source) by heat flow equation $\nabla \cdot (\kappa_\parallel \nabla T - \kappa_\perp \nabla T) = 0$

$$[\kappa_\parallel \nabla_\parallel^2 + \kappa_\perp \nabla_\perp^2]T_e \approx \left[\frac{\kappa_\parallel}{\kappa_\perp} \nabla_\parallel^2 + \nabla^2\right]T_e = 0$$

$$w_c = \sqrt{\frac{RqL_q}{m} \left(\frac{\kappa_\perp}{\kappa_\parallel}\right)^{1/4}}$$

where magnetic field is represented with the helical flux function $\psi = \psi_0 + \psi_1$

$$B = \nabla \psi \times \hat{\eta} + B_\eta \hat{\eta}$$

then,

$$\nabla_\parallel = \hat{b} \cdot \nabla \approx \frac{1}{|B|} \left( -\frac{m}{r_s} \psi_1 \sin \zeta \right) \left( \frac{\partial}{\partial r} - \frac{m}{r_s} \frac{\partial \psi_1}{\partial \zeta} \cos \zeta \right) \approx \frac{m}{r_s|B|} \left( \psi_1 \sin \zeta \frac{\partial}{\partial r} + \psi_0 \frac{\partial}{\partial \zeta} + \psi_1 \cos \zeta \frac{\partial}{\partial \zeta} \right)$$

with helical angle $\zeta = m\theta - n\phi$ and

$$\psi_0(r) = \frac{\mu_0 I_0}{8\pi} \left( \frac{r}{a} \right)^2 \left( \frac{r_s}{a} \right)^2$$

$$\psi_1(r) = \frac{\mu_0 I_0 \alpha}{8\pi} \left( \frac{r}{r_s} \right)^m \left( 1 - \beta \frac{r}{r_s} \right)$$

for $r \leq r_s$

$$\psi_1(r) = \frac{\mu_0 I_0 \alpha (1-\beta - \gamma + \gamma r/r_s)}{(r/r_s)^{m+1}}$$

for $r > r_s$

$$\Delta' \equiv \left[ \frac{d\psi_1}{dr} + \frac{d\psi_1}{dr} \right] / \psi_1$$

- Electron temperature profile solution $T_e(r, \zeta)$ becomes a function of parameters $\alpha, \beta, \gamma$, and $\frac{\kappa_\parallel}{\kappa_\perp} \rightarrow$ a function of $\Delta'(\alpha, \beta, \gamma)$ and $w_c = \sqrt{\frac{RqL_q}{m} \left(\frac{\kappa_\perp}{\kappa_\parallel}\right)^{1/4}}$

J.P. Meskat et al., PPCF (2001)
- Fine 2D electron temperature fluctuation ($\delta T_e / \langle T_e \rangle_t$) measurement near the island by the KSTAR ECEI diagnostic.

- ECEI measurement reveals detail $T_e$ structure of tearing mode on $(r, \zeta)$ space → can be compared with the $T_e$ model to estimate $\Delta'$ and $w_c$.

- Local 24 (vertical) X 8 (radial) = 192 measurement points

- Spatial resolution in $r$ direction < 1 cm
The $T_e(r, \zeta)$ model $\rightarrow$ synthetic $\delta T_e/\langle T_e \rangle_t$ for the direct comparison with the measured $\delta T_e/\langle T_e \rangle_t$ images by the ECEI diagnostic

$F_{\text{inst}}$ is instrumental 2D response function of each channel

$$T_{e,\text{syn}} = \frac{\int T_e F_{\text{inst}}(R,z)\,dr\,dz}{\int F_{\text{inst}}(R,z)\,dr\,dz}$$

$$\frac{\delta T_{e,\text{syn}}}{\langle T_{e,\text{syn}} \rangle_t} = \frac{T_{e,\text{syn}} - \langle T_{e,\text{syn}} \rangle_t}{\langle T_{e,\text{syn}} \rangle_t}$$

Normalization with the time average

Find best matching $(\alpha, \beta, \gamma, \kappa_\perp/\kappa_\parallel)$ between $\delta T_{e,\text{syn}}/\langle T_{e,\text{syn}} \rangle_t$ and $\delta T_{e,\text{ECEI}}/\langle T_{e,\text{ECEI}} \rangle_t$

Monte-Carlo Method for initial values

Is difference smaller enough?

Yes

Matching parameters

$p_{\text{fin}} = [\alpha, \beta, \gamma, \kappa_\perp/\kappa_\parallel]$

Estimate $\Delta'$ and $w_c$

No

Update the parameters with Levenberg-Marquardt Algorithm
\[ r_s \Delta' = -1.633 \pm 1.265 \]
\[ w_c = 0.612 \pm 0.0726 \text{ cm} \]

\[ r_s \Delta' \text{ is found to be negative (classically stable)} \]
\[ w_c < w \text{ implies that the pressure profile inside the island is flat and the lost bootstrap current is destabilizing} \]
Method to estimate $a_1$ and $a_2$

- Unknown parameters in modified Rutherford equation (MRE) for the KSTAR

$$a_1 \frac{\tau_r}{r_s} \frac{dw}{dt} = r_s \Delta' + a_2 \sqrt{\varepsilon} \frac{\beta_\theta}{w} \frac{L_q}{L_p} \frac{w}{w^2 + (w_c)^2} + \ldots$$

can be estimated by the ECE images

$a_1$ and $a_2$ are integration coefficients which depend on magnetic geometry of the island, and they can be determined by fitting the measured island size evolution with the MRE

- Stepwise approach to estimate $a_1$ and $a_2$ for more accuracy

First, consider the plasma such that $a_2 \sqrt{\varepsilon} \frac{\beta_\theta}{w} \frac{L_q}{L_p} \frac{w}{w^2 + (w_c)^2} \ll r_s \Delta'$, then the equation returns to the original Rutherford equation

$$\mu_0 r_s^2 / \eta \text{ where } \eta \text{ is Spitzer resistivity}$$

estimated for the KSTAR plasma geometry

$$a_1 \frac{\tau_r}{r_s} \frac{dw}{dt} = r_s \Delta'$$

estimated by the ECE images

estimated by the magnetic fluctuation measurement (Mirnov coil)
Estimation of $a_1$ for the KSTAR plasma

- The low $\beta_\theta$ plasma with the constant $m/n = 2/1$ island growth rate

**KSTAR plasma # 7318**
The L-mode typical diverted plasma with neutral beam injection of 0.6 MW and electron cyclotron resonance heating of 0.3 MW
It has a constant growth rate of island size ($dw/dt = \text{const}$) from 0.6—0.8 s

\[ a_1 \frac{\tau_r}{r_s} \frac{dw}{dt} \approx r_s \Delta' \]

ECE image of $m=2$ island

- at 0.76 s
- at 0.8 s

Island size is estimated by Mirnov coil and calibrated with the ECE image
- The \( \Delta' \) estimation in the KSTAR \# 7318

\[ r_s \Delta' = 0.52 \pm 0.37 \]

Parameter sets whose \( \chi^2 < 0.1665 \) are selected for the estimation (below the dashed line)

- \( \frac{dw}{dt} \) in the KSTAR \# 7318

Island size was estimated by magnetic fluctuation amplitude measured by Mirnov coil and calibrated with the ECE image within \( \pm 1 \) cm accuracy

Linear fitting provides \( \frac{dw}{dt} = 0.322 \pm 0.012 \)

- \( a_1 \) in the KSTAR \# 7318

From the Rutherford equation \( a_1 \frac{\tau_r}{r_s} \frac{dw}{dt} = r_s \Delta', \) \( a_1 = 0.26 \pm 0.16 \) is obtained.

Spitzer transverse resistivity \( \eta_\perp = 1.03 \times 10^{-4} Z \ln \Lambda T^{-3/2} [\Omega m] \) is used for \( \tau_r = \frac{\mu_0 r_s^2}{\eta_\perp} \sim 1.56. \)

This coefficient \( a_1 \) can be applied to the plasma whose magnetic geometry is similar to \# 7318. Theoretical \( a_1 \) estimation with the cylindrical plasma assumption is 0.82
Summary and Discussion

- Method to estimate parameters $\Delta'$, $w_c$, and $a_1$ of modified Rutherford equation is developed

- Obtain the form of the Rutherford equation for the KSTAR plasma

\[
0.26 \frac{\tau r}{r_s} \frac{dw}{dt} = r_s \Delta'
\]

The coefficient $a_1 = 0.26$ can be used for $a_2$ determination in the modified Rutherford equation

\[
0.26 \frac{\tau r}{r_s} \frac{dw}{dt} = r_s \Delta' + a_2 \sqrt{\epsilon} \frac{\beta \theta L_q}{w L_p w^2 + (w_c)^2} + \cdots
\]

- The obtained Rutherford equation will be checked with the M3D–C1 simulation
data points \((t_i, y_i)\)
model function values with given parameters \(p\)
goodness-of-fit (chi-squared error)

\[
\begin{align*}
\text{goal: find } p \text{ which minimizes } \chi^2(p) \\
\text{initial } p \downarrow \\
1. \text{ gradient decent method} \\
\text{update } p \text{ by } p = p + \epsilon(-\nabla \chi^2) \\
1. \text{ Gauss-Newton method} \\
\text{update } p \text{ by } p = p + h \text{ where } \frac{\partial \chi^2}{\partial h} = 0 \\
\downarrow \\
\text{final } p
\end{align*}
\]

\[
\chi^2(p) = \frac{1}{2} \sum_{i=1}^{m} \left[ \frac{y(t_i) - \hat{y}(t_i; p)}{\omega_i} \right]^2
\]

Levenberg–Marquardt Algorithm (LMA): the most standard multi-parameter fit algorithm
EC Emission profile

Radial Natural line width: relativistic broadening or Doppler broadening, \( \cdots \) + re-absorption process

Relativistic broadening + re-absorption: \( \frac{1}{N^2} \frac{j_\omega}{\alpha_\omega} = \frac{\omega^2}{8\pi^3 c^2 kT_e} \)

Emission profile for \( f_0 = 2 f_{ce} \approx 90 \) GHz
Re-absorption reduces the width
\( w_{rel} \approx 0.5 \) cm

Instrument broadening:
Frequency bandwidth of each channel = 0.7 GHz \( \rightarrow w_{instr} \approx 1.5 \) cm

Doppler effect due to finite beam size:
\[ \Delta \omega_D \approx \frac{2\sqrt{2}\log_2}{w_{beam}} \nu_T \rightarrow w_D < 0.5 \) cm

Vertical: Gaussian-like response (designed by optics)

M. Bornatici et al., Nucl. Fusion, 23, 9 (1983)