Feedback of a neoclassical tearing mode on drift wave - Zonal Flow turbulence

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1. Introduction

2. Motivation: evidence of turbulence role in island evolution

3. Neoclassical Tearing Mode model with turbulence feedback
   - Basic Mechanism
   - Equations
   - 0D NTM predator-prey model
   - 1D NTM predator-prey model

4. Experimental signatures
   - Predictions of the model

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predicting Neoclassical Tearing Mode onset critical to ITER

What is the role of microturbulence and Zonal Flows?
onset & control of an NTM on JT60U [Isayama PPCF 2000]

- What sets the threshold island width?
  - Effect of turbulence on the bootstrap-current?
  - Effect of island on turbulence drive & ZFs?

- ad-hoc heat diffusivity $\chi_\perp$ in island $\leftrightarrow$ turbulence
- turbulence-ZF sets threshold island width

- neoclassical picture: trapped particles: $\nabla p$ $\leftrightarrow$ bootstrap current

But turbulence plays crucial role: $\nabla p$ affects island

Note: ZFs are very strong near the island separatrix
'polarization current' issue: not addressed here
Evidence for turbulence role in island evolution

- Evidence of turbulence role in island dynamics (HL-2A)
- K. Zao APTWG’13

- Island modulates turbulence
- $m=0,n=0$ mode (ZF) couples to the 3:1 vortex flow
associated to bootstrap current:

\[ \delta j_\parallel = \delta j_\parallel^{\text{induct}} - D_\parallel \nabla_\parallel (\delta \phi - \delta n) + \sqrt{\epsilon} \frac{\partial}{\partial x} \delta p \]

due to neoclassical damping of electron flow & enhanced by trapped-electrons (trapped-fraction \( \sqrt{\epsilon} \))

island growth and saturation if \( \Delta' < 0 \) [Carrera ‘86]

temperature flattened by island \( \rightarrow \) modified Rutherford Eq:

\[ \frac{dW}{dt} = \Delta' + \Delta_{bs}(W) \]

\( \Delta_{bs}(W) \sim \frac{\beta_p}{W} \), for large \( W \), with \( \beta_p \) : poloidal beta
What sets the threshold island width?

- **Key point**: competition between:
  - parallel heat transport along tilted field lines
  v.s. \( ⊥ \) heat transport across flux surfaces
- [Fitzpatrick 1995]

\[
\text{island growth if: } W \gg W_{turb0} = \left[ \frac{\chi_{turb}}{\chi||} \frac{L_s^2}{k_y^2} \right]^{1/4}, \text{ with } \chi_{turb} : \text{ad-hoc}
\]

- **but**
  - \( \chi_{turb} \text{ self-consistently} \) determined @ constant power \( Q \)
  - \( \leftrightarrow \) threshold island width = power threshold (onset \( \beta_p \))
  - \( \chi_{turb} \) is affected via:
    i) self-regulation by ZFs i.e. ZF \( \nearrow \) : turb. \( \searrow \)
    ii) depletion of turbulence drive i.e. \( \nabla T_e \searrow \) : turb. \( \searrow \)
    iii) island-induced ZF damping i.e. ZF \( \searrow \) : turb. \( \nearrow \)
  - \( \leftrightarrow \chi_{turb} = \chi_{turb}(W, Q) \)
Marginal stability of NTM coupled to marginal stability of DW-ZF → extended onset criterion

- parallel transport v.s. \( \perp \) transport (including effect of island)
- marginal stability \( (d/dt = 0) \) of DW-ZF predator-prey model perturbed by island
  \( \leftrightarrow \) turbulence energy \( \epsilon \) (and ZF energy \( V_{ZF} \)) as function of heat flux \( Q \) and island-width \( W \)
- inject in marginally stable Rutherford equation (below)

**Extended criterion** for island growth:

\[
\text{island growth if: } \frac{QW}{W^2 + W_{turb0}^2 \sqrt{\epsilon(W, Q)/\epsilon_0}} - |\Delta'| \geq 0
\]

Marginal stability of turbulence/ZF energy v.s. heat flux \( Q \) and island-width \( W \)
0D model: predator-prey model with NTM coupling

Equations

\[
\frac{d\epsilon}{dt} = \frac{Q\epsilon}{W_{\text{turb}0}^4 [\epsilon/\epsilon_0] + W^4} - \alpha \epsilon v_{ZF}^2 - \gamma_{NL} \epsilon^2
\]

\[
\frac{dv_{ZF}^2}{dt} = \alpha \epsilon v_{ZF}^2 - \left[1 + \frac{\mu_{\text{MHD}}}{\mu} W^4\right] \mu v_{ZF}^2, \text{ with } (\mu_{\text{MHD}} W^4 / \mu) \ll 1
\]

\[
\frac{dW}{dt} = -|\Delta'| + \frac{QW}{W_{\text{turb}0}^2 \sqrt{\epsilon/\epsilon_0} + W^2}
\]

- with fixed heat flux \(Q \leftrightarrow \beta_p\)
- \(\epsilon\): DW turbulence energy
- \(v_{ZF}^2\): Zonal Flow energy
- \(W\): island-chain width
- \(\alpha\): DW-ZF coupling parameter
- \(\mu \sim \nu_{ii}\): ZF neoclassical friction
- turb. driven by electron temperature gradient (TEM, ETG...)
Model curve $dW/dt$ v.s. $W$ (analytic) w/o ZFs

- modifications of **NTM onset** / no effect on **NTM saturation**
- no ZFs: threshold island **larger** than Fitzpatrick threshold.
- self-consistent model curve with Zonal Flows not tractable analytically (co-dimension 2 bifurcation)
  $\rightarrow$ numerical evaluation
At marginal stability
\[ \frac{dW}{dt} = 0, \frac{d\epsilon}{dt} = 0, \frac{dV_{ZF}^2}{dt} = 0 \]

<table>
<thead>
<tr>
<th>case</th>
<th>turb. energy</th>
<th>threshold island</th>
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</thead>
<tbody>
<tr>
<td>w/o ZF</td>
<td>( \epsilon = \epsilon_1(W, Q) \sim \sqrt{W^8 + 4Q/\gamma_{NL}} - W^4 )</td>
<td>graphically using 0D code</td>
</tr>
<tr>
<td>with ZF</td>
<td>( \epsilon ) set by ZFs and W</td>
<td></td>
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</table>

- With ZFs, turbulence at marginal stability exhibits a threshold in \( Q \) and \( W \)

\[
\epsilon = \epsilon_1(W, Q) - [\epsilon_1(W, Q) - \epsilon_2]H(\epsilon_1(W, Q)/\epsilon_2 - 1))
\]

- codimension-2 threshold represented by Heaviside function

\[
H(\epsilon_1(W, Q)/\epsilon_2 - 1))
\]
Numerical Results: Dynamics & stability of 0D model

with Zonal Flows: unstable

- turb. regulated by ZFs: seed-island succeeds in flattening $T_e$ profile:
  - turb $\searrow$ thus ZFs $\searrow$

w/o Zonal Flows: stable

- turb. regulated by self-damping: seed-island cannot flatten $T_e$ profile

Note 1: ZFs destabilize NTMs

Note 2: NTM seed-island modifies the DW-ZF dynamics
Zonal Flow impact on model curve $dW/dt = f(W)$ [numerical]

- modifications of **NTM onset** / no effect on NTM saturation
- with ZFs: threshold island & $\beta_{onset}$ smaller than w/o ZFs.
- note that larger $dW/dt$ corresponds to smaller $\beta_{onset}$
1D model: predator-prey model with NTM coupling

\[
\frac{\partial I}{\partial t} = \left[-\frac{\partial T_e}{\partial x} + \frac{\partial T_e}{\partial x}\right]_c I - \alpha I v_{ZF}^2 + \frac{\partial}{\partial x} \left[I \frac{\partial I}{\partial x}\right]
\]

\[
\frac{\partial v_{ZF}^2}{\partial t} = \alpha I v_{ZF}^2 - \left[1 + \frac{\mu_{MHD}(x, W)}{\mu} W^4\right] \mu v_{ZF}^2
\]

\[
dW = -|\Delta'| - \frac{c_1}{W} \frac{\partial T_e}{\partial x}\bigg|_{\text{sep}}
\]

- with Cst heat flux (heat source), based on Miki et al. PoP ’12

\[
\frac{\partial T_e}{\partial t} = \frac{\partial}{\partial x} \left[\chi_{QL} I \frac{\partial T_e}{\partial x}\right] + S_{\text{heat}}
\]

- island-induced $T_e$ flattening not implemented yet (in 1D)
- cannot address threshold physics $\rightarrow$ saturation physics
- $I$: DW turbulence intensity
- $v_{ZF}^2$: Zonal Flow intensity
NTM saturation physics: negative feedback on ZFs

- $T_e$ flattening not implemented: $\rightarrow$ ZFs $\downarrow$ and turbulence $\uparrow$ in island region

- island-width is modulated:
Predictions of the 0D NTM Predator-Prey model

- Threshold island-width

<table>
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<tr>
<th>Fitzpatrick ‘95</th>
<th>our model with ZFs</th>
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<tbody>
<tr>
<td>$W \geq W_{turb0} \sim \epsilon_0^{1/4} \chi_\parallel^{-1/4}$</td>
<td>$W \geq W_{turb} \sim \mu^{1/4} \chi_\parallel^{-1/4}$</td>
</tr>
<tr>
<td>with turb. energy $\epsilon_0$ ad-hoc</td>
<td>with $\mu \sim \nu_{ii}$</td>
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Model curve $dW/dt$ v.s. $W$ for different ZF damping $\mu$

- ZFs **destabilize** NTM
- neoclassical ZF damping: **stabilizes** NTM

**predictions**: scaling with
- ion-ion collision freq.: $\nu_{ii}$: suggests collisionality scan
Discussion

- Threshold island-width predicted to depend on ZF damping
  - predicts threshold island: ↑ with neo ZF damping
    ↓ with island-induced ZF damping

- key-points:
  - Zonal Flows regulate turbulence → smaller threshold-island
depletion of turbulence-drive due to $T_e$ flattening

- open question: turbulence spreading into the island?

- prediction of island-width modulation by the DW-ZF limit-cycle ↔ LCO in $(\epsilon, V_{ZF}, W)$ space

- experimental evidence? (K. Zhao unpublished)

- expression for the DW-ZF-island LCO frequency?
  without island: $\omega_{LCO} \sim \sqrt{Q\mu}$
Summary and conclusions

- Feedback of island on Zonal Flows
- threshold island-width @low collisionality
- key-point:
  - ZFs regulate turbulence, cross-field transport
- prediction of island-width modulation by the DW-ZF limit-cycle

Open Questions
  - Back-reaction of Zonal Flows on island: polarization current?
  - Coherent ZFs v.s. random ZFs?
  - flow direction @ resonance surface?
    (associated to screening/amplif.)
  - island effect on turbulence-driven toroidal rotation?
    (Toroidal Zonal Flows)