Australian National University

Long range Alfvenic frequency chirping in tokamaks

Hooman Hezaveh[1], Zhisong Qu[1], Boris Breizman[2], Matthew Hole[1,3]

[1]The Australian National University, [2] Institute for Fusion Studies, The University of Texas at Austin, [3]Australian Nuclear Science and Technology Organisation

Hooman.hezaveh@anu.edu.au

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ABSTRACT

- **We have developed a theoretical framework to model the adiabatic hard nonlinear evolution of a Global Alfven eigenmode (GAE), which is destabilized by energetic particles (EPs).**
- It is shown that the peak of the radial profile is shifted and also broadens due to frequency chirping. The time rate of frequency change is calculated using the energy balance.
- The model illustrates the theoretical treatment of long range adiabatic frequency sweeping observed for Alfven gap modes in real experiments.

BACKGROUND

Why is this study important to magnetic confinement ?

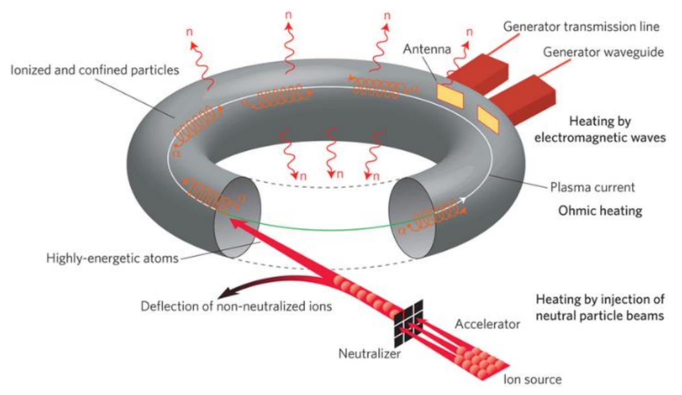

Fig 1. Existence of EPs due to RF heating and neutral beam injection

Excitation of Alfvén Gap modes (Eigenmodes) during the slowing-down process [1] ⇒ Enhanced particle diffusion in radial direction

- Degrade the heat confinement
- Subject the material of the containment vessel to increased erosion

An ability to model and control these kinetically driven instabilities is crucial!

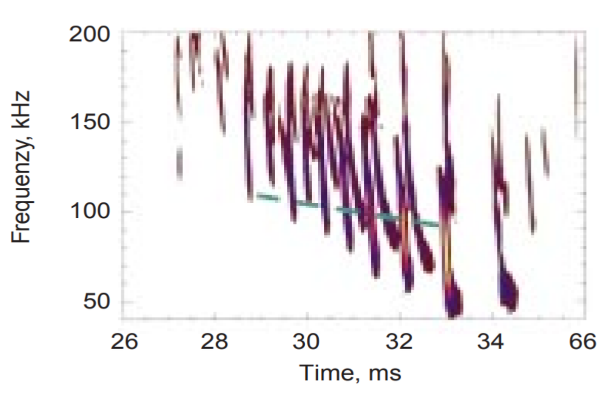

Fig 2. The wave-particle interaction may result in long range frequency sweeping [2]

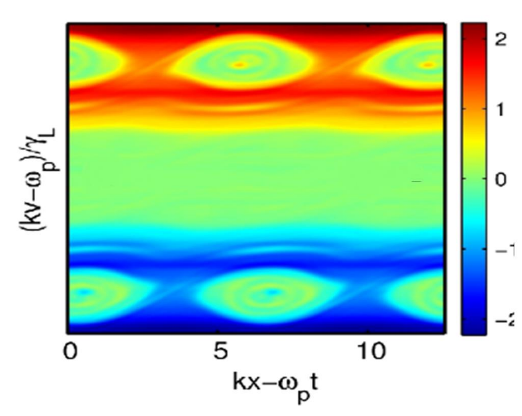

Fig 3. Frequency sweeping is attributed to the motion of phase–space structures, holes and clumps [3]

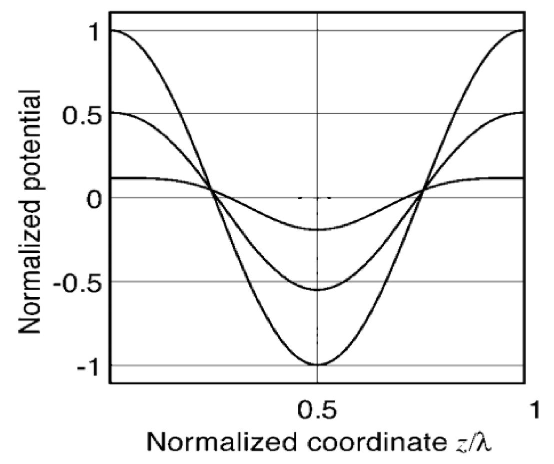

Fig 4. Notable change in the spatial profile of a Langmuir wave due to long range chirping in the adiabatic limit [4]

The model

We have generalized the model introduced in Ref. 4 to study the long range frequency chirping of a GAE for a near-threshold instability

- ✓ We focus on the evolution of the radial structure of the eigenfunction.
- ✓ A single poloidal (m) and toroidal (n) mode number!
- ✓ Toroidal effects on EP dynamics are retained in a high aspect ratio tokamak limit

The total Lagrangian of the system: $L = L_{\text{wave}} + \sum_{\text{fast particles}} L_{\text{particles}} + \sum_{\text{fast particles}} L_{\text{int}}.$

Particle Lagrangian (littleJohn, gyro-averaged)

$$L_{\text{particles}} = P_\theta\dot\theta + P_\varphi\dot\varphi + P_\Omega\dot\Omega - H_0(P_\theta, P_\varphi, P_\Omega, \theta)$$

$$P_\theta = e\psi(r,\theta) + m_i v_\parallel b_\theta(r,\theta),\quad P_\varphi = -e\chi(r) + m_i v_\parallel b_\varphi(r,\theta),\quad P_\Omega = \frac{m_i}{e}\mu,$$

Canonical transformation to action-angle variables

$$(\tilde\theta, \tilde\varphi, \tilde\Omega, P_{\tilde\theta}, P_{\tilde\varphi}, P_{\tilde\Omega})$$

$$\dot{\tilde\theta} = \omega_{\tilde\theta} \approx \frac{V_\parallel(P_{\tilde\theta}, P_{\tilde\varphi}, P_{\tilde\Omega})}{q(r_0)R_0}$$

$$\dot{\tilde\varphi} = \omega_{\tilde\varphi} \approx \frac{V_\parallel(P_{\tilde\theta}, P_{\tilde\varphi}, P_{\tilde\Omega})}{R_0},$$

❖Highly passing limit!
❖NBI scenario!
❖Small orbit-width!

Perturbed wave, interaction Lagrangian, wave Lagrangian

We represent the radial profile of the GAE using finite elements (FEs) and expand it in AA variables of the unperturbed motion:

$$\Phi(\boldsymbol{r}, t) = \sum_{l=1}^{s} \lambda_l(t) Y_l(r) e^{im\theta + in\varphi - i\alpha(t)} + c.c,$$

The FE amplitude (λ) and α are real quantities and $\frac{d\ln\lambda_l}{dt}$

Base functions (Y) represent a smooth radial profile

$$L_{\text{int}} = \sum_l \sum_p \lambda_l(t) V_{p,n,l} e^{ip\tilde\theta + in\tilde\varphi - i\alpha(t)} + c.c$$

Coupling strength (orbit-averaged mode amplitude)

$$L_{\text{wave}} = 2\dot\alpha^2[\boldsymbol{\lambda}^\intercal\cdot\mathsf{M}\cdot\boldsymbol{\lambda}] - 2[\boldsymbol{\lambda}^\intercal\cdot\mathsf{N}\cdot\boldsymbol{\lambda}]$$

Matrices of discrete MHD equations

Varying the total Lagrangian of the system with respect to the FE amplitude, gives the nonlinear equation describing the evolution of the radial profile of the mode:

$$(\dot\alpha^2\mathsf{M} - \mathsf{N})\cdot\boldsymbol{\lambda} = -\frac{1}{4}\int d^3p\, d^3q\, \delta f(\boldsymbol{q}, \boldsymbol{p}, t)\sum_p \begin{bmatrix} V_{p,n,l=1} \\ \vdots \\ V_{p,n,l=s} \end{bmatrix} e^{ip\tilde\theta + in\tilde\varphi - i\alpha(t)} + c.c,$$

The perturbed phase-space density should be found from the kinetic equation

Krook collisions inside the bulk plasma provide damping mechanism (γ_d), which is implicitly included!

EP dynamics in Phase-space

Total Hamiltonian

$$H = H_0(P_{\tilde\theta}, P_{\tilde\varphi}, P_{\tilde\Omega}) + H_{\text{int}}$$
$$H_{\text{int}} = -L_{\text{int}}$$

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New Hamiltonian

$$K \approx \frac{1}{2}\frac{\partial^2 H_0}{\partial P_1^2}(\Pi, P_2, P_3)[P_1 - \Pi]^2 - \sum_l \lambda_l V_{p,n,l}(\Pi, P_2, P_3)e^{i\zeta_p} + c.c.$$

→ Wave-particle interaction is essentially 1D

Another canonical transformation to simplify the dynamics

$$P_1 = \frac{1}{p}P_{\tilde\theta} \qquad Q_1 = \zeta = p\tilde\theta + n\tilde\varphi - \alpha(t)$$
$$P_2 = P_{\tilde\varphi} - \frac{n}{p}P_{\tilde\theta} \qquad Q_2 = \tilde\varphi$$
$$P_3 = P_{\tilde\Omega} \qquad Q_3 = \tilde\Omega$$

Adiabatic limit allows bounce averaging the kinetic equation and finding δf analytically in the nonlinear case! [5,6]

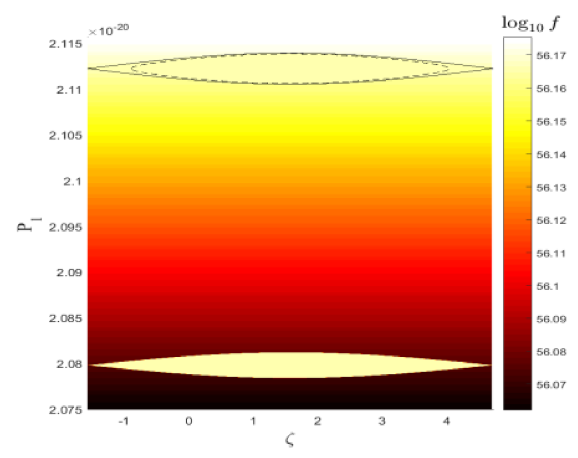

Fig 5. Evolution of a shrinking separatrix during chirping

Frequency chirping rate

The dissipated power into the bulk plasma = power released by the phase–space structures energy →

$$\frac{\partial(\dot\alpha - \dot\alpha_{t=0})^2}{\partial t} = \frac{-8\gamma_d[\dot\alpha^2\boldsymbol{\lambda}^\intercal\cdot\mathsf{M}\cdot\boldsymbol{\lambda} + \boldsymbol{\lambda}^\intercal\cdot\mathsf{N}\cdot\boldsymbol{\lambda}](\dot\alpha - \dot\alpha_{t=0})}{\sum_{P_2}\left[\iint \delta f\, dP_1 d\zeta \left.\frac{\dot\alpha}{\frac{\partial^2 H_0}{\partial P_1^2}}\right|_{P_2,P_3}\right]}$$

Results

Physical parameters: $B_\varphi(r=0) = 2T$ $R_o = 3.5m$ $n_{Bulk} = 5\times10^{20}m^{-3}$ $\gamma_l \sim 10^4 s^{-1}$ Linear growth rate

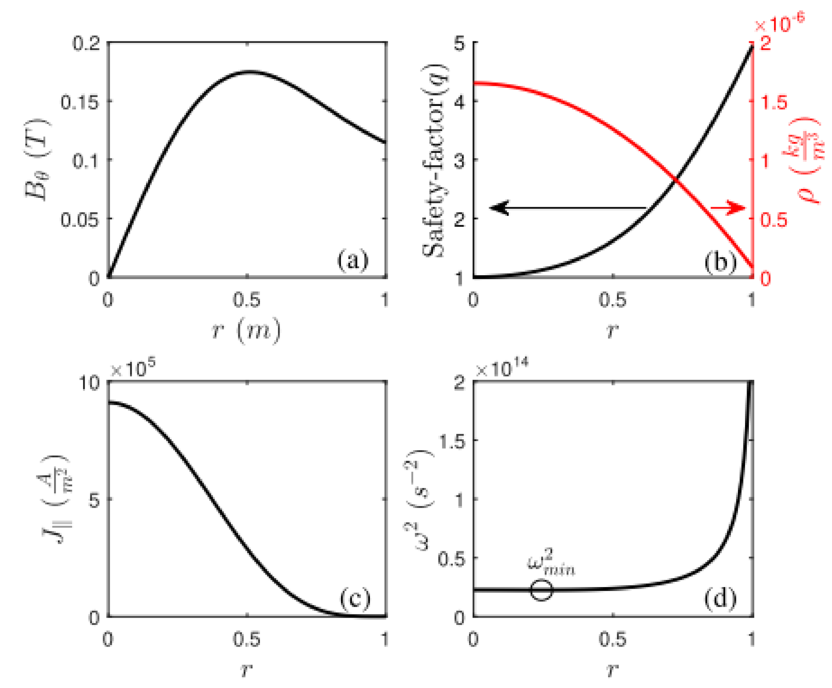

Fig 6. Equilibrium parameters. The GAE lies just below the shear Alfven continuum

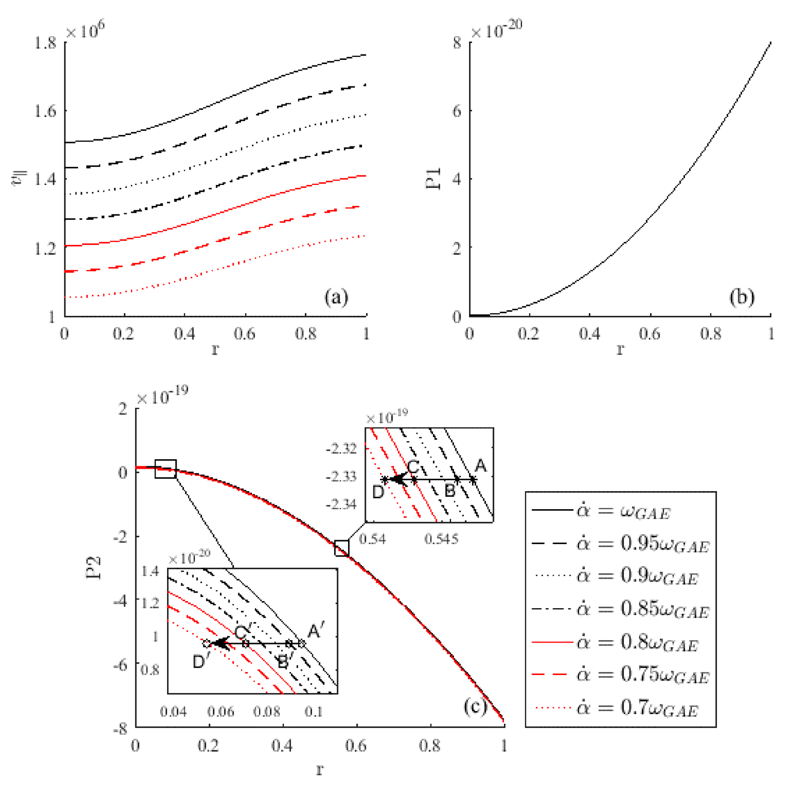

Fig 7. Equilibrium parameters. The GAE lies just below the shear Alfven continuum

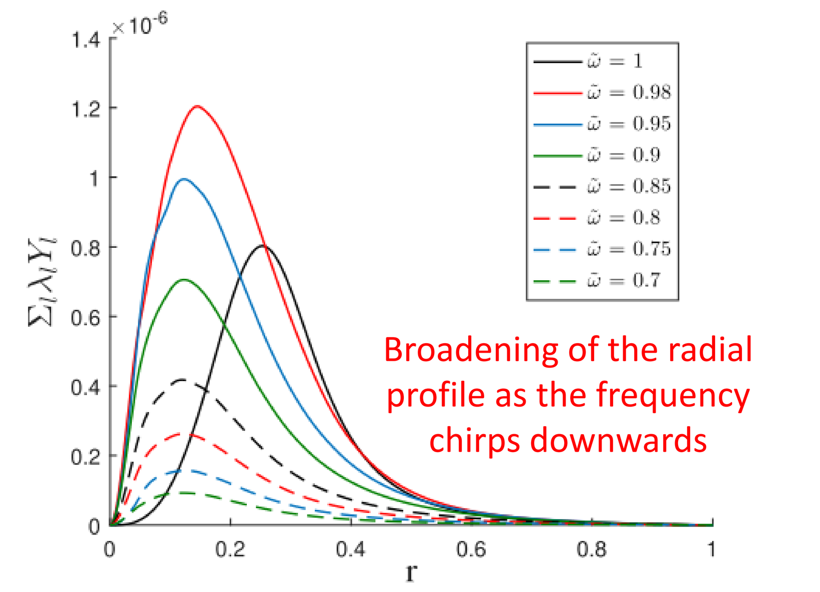

Fig 8. Evolution of the radial profile of the GAE during frequency chirping.

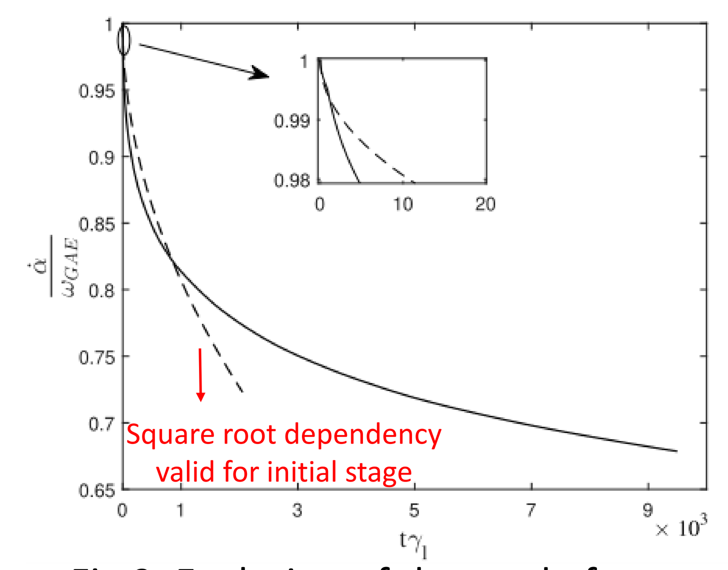

Fig 9. Evolution of the mode frequency

Slowing down distribution function

$$F_0 = \frac{n_0 A}{v^3 + v_c^3} e^{\frac{P_\varphi}{\Delta P_\varphi}} \delta(P_3 - 0^+)$$

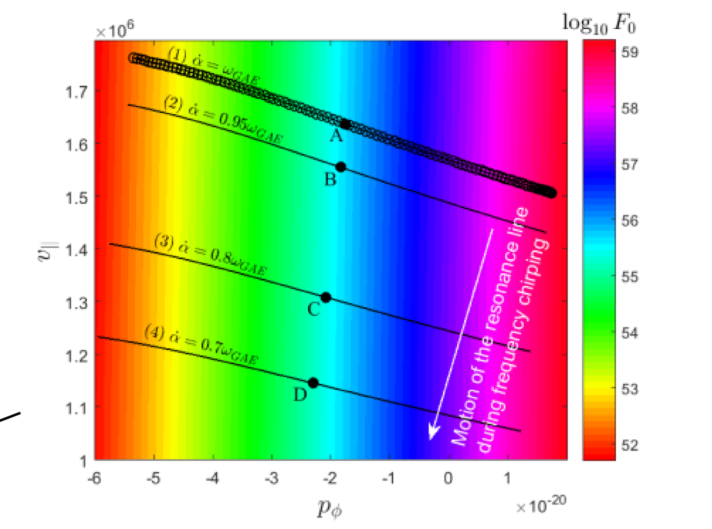

Fig 10. A slice of phase-space with resonance line at different frequencies.

CONCLUSION

- A theoretical description has been developed to study the hard nonlinear evolution of a GAE in resonance with co-passing EPs.
- A full description of the problem is aimed in our research plan. This includes adding the toroidal effects to the bulk description and allowing the EPs nonlinearity to update all the components of the mode structure.

ACKNOWLEDGEMENTS / REFERENCES

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