PI-22

Study on particle pinch mechanism for DEMO

M. Yagi

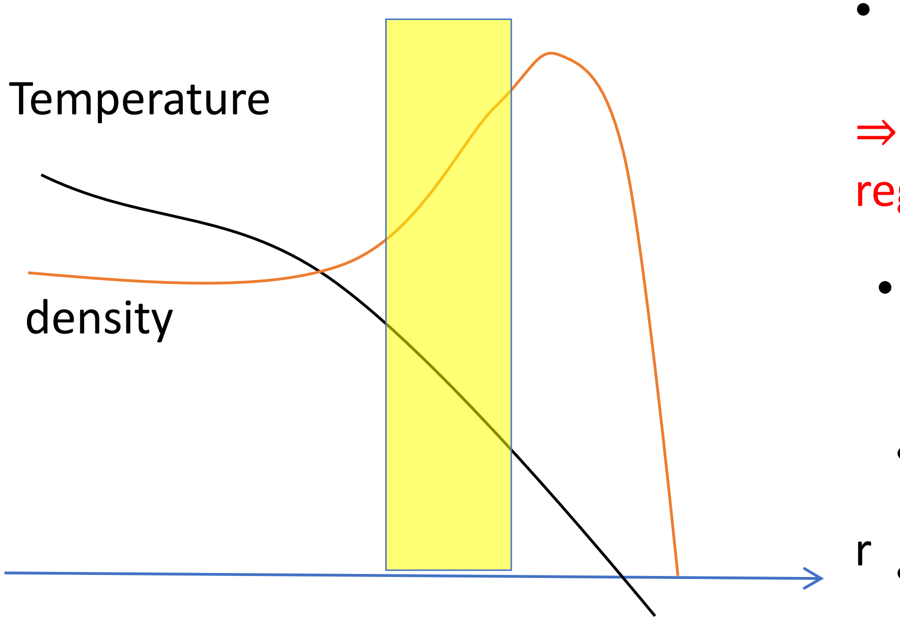
QST, Rokkasho Fusion Institute

yagi.masatoshi@qst.go.jp

16th IAEA Technical Meeting on Energetic Particles in Magnetic Confinement Systems – Theory of Plasma Instabilities (EPPI)
4 Sep. 2019, Shizuoka, Japan

Introduction

Particle transport in the inversed-density-gradient, in fueling



- Hollow density profile is often seen after pellet injection or gas-puff.

⇒inversed density gradient appears in the edge region.

- Relevant to
  - What mechanism of the particle pinch?

- Fueling physics contributes ITER/DEMO development.
- Works on validation of particle transport physics. [Angioni '09][Wan '10][Tangered '16][Angioni '17]

We here undergo edge and local first-principle simulation to investigate particle transport in the region of the negative density gradient.

Linear calculation exhibits two modes with ion and electron direction rotation

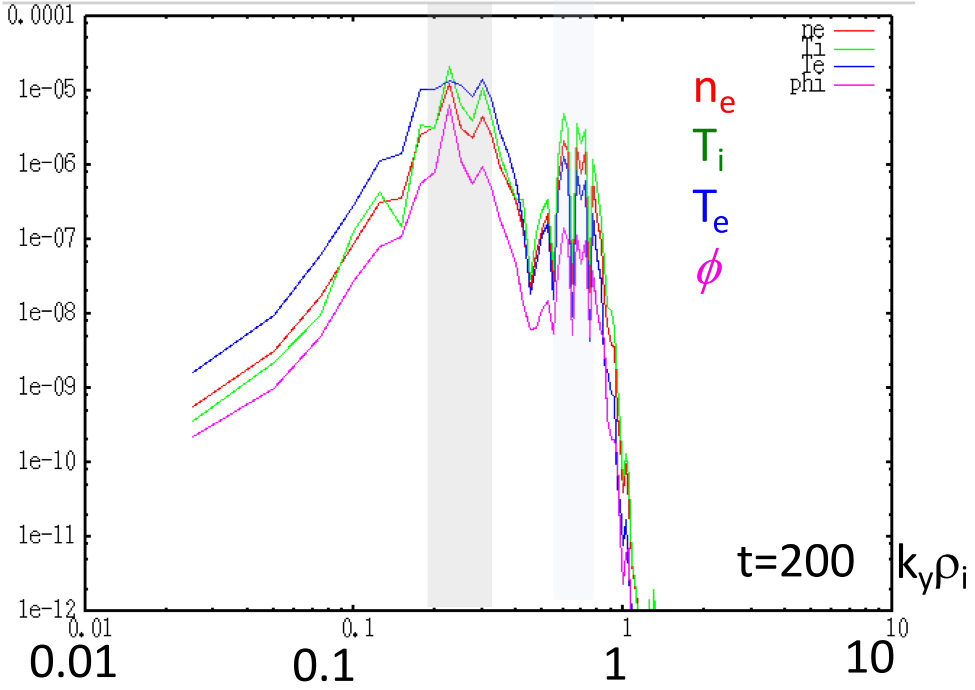
Edge plasma parameters: (R/LT=47, R/Ln=23)

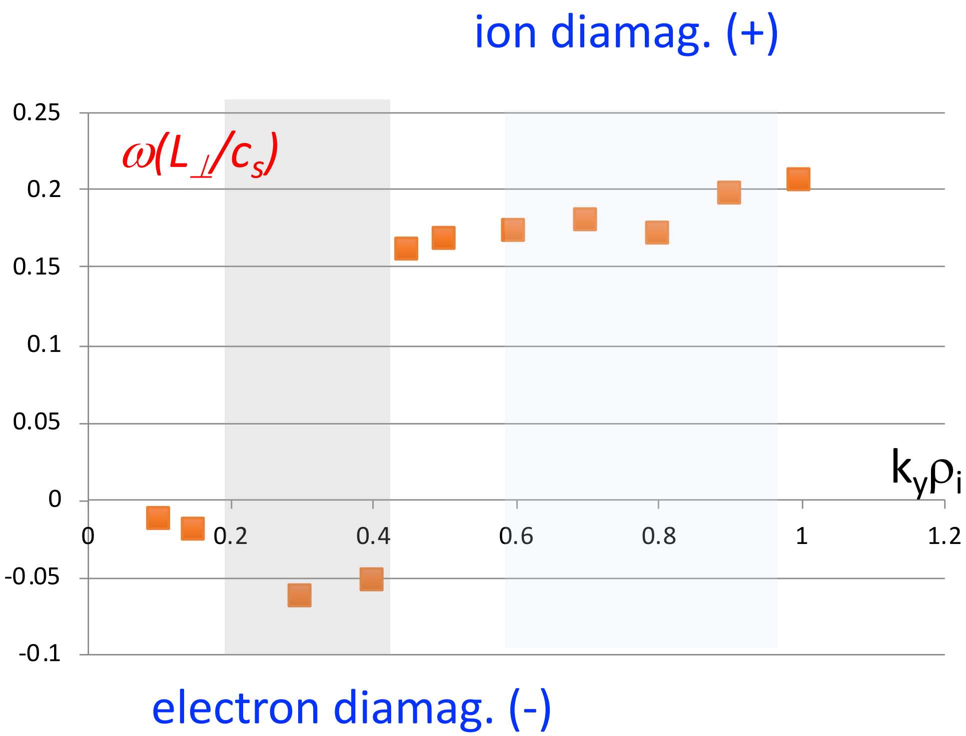
R_0=165cm, n_e=2.0x10^{13} cm^{-3}, Te=100 eV, L_n=-7cm, L_{Ti}=L_{Te}=3.5cm, B=2.5 T, T_i/T_e=1, a/R=0.303,

$L_\perp/R$= 0.0212, q=3.5, Normalized beta=6.44x10^{-5}, $\nu_i(L_\perp/c_s)$=000956, $\nu_e(L_\perp/c_s)$=0.823

(nx,ny,ns,nz,nw)=(32,128,32,32,16), Lx/Ly=1.0, Δt=0.005

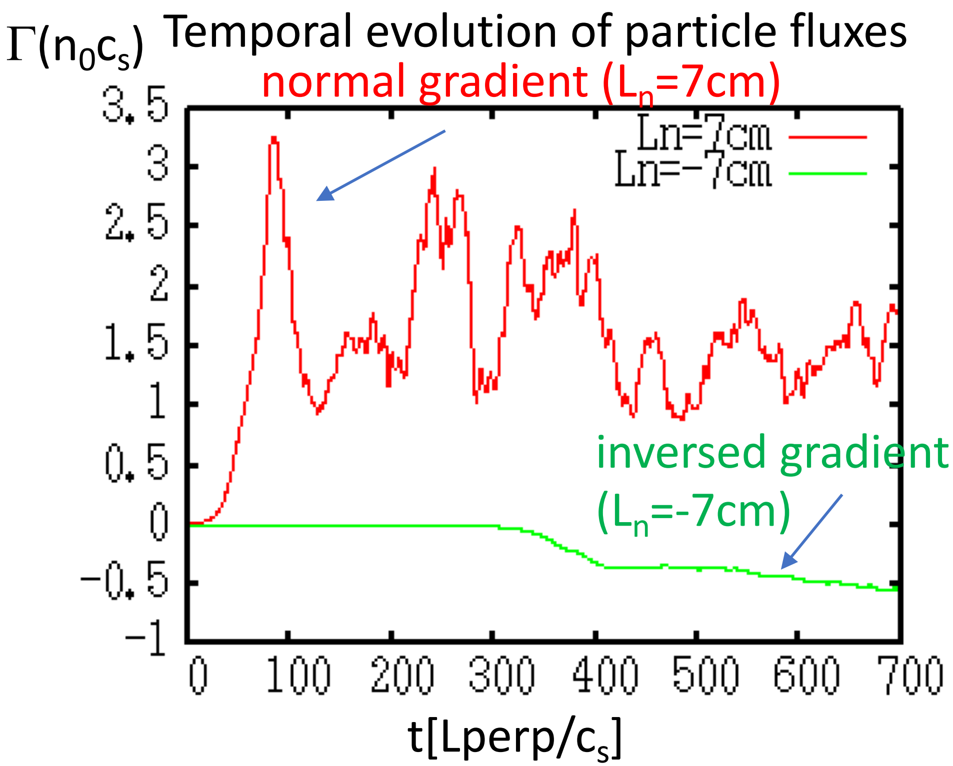
Power spectra in linear phase

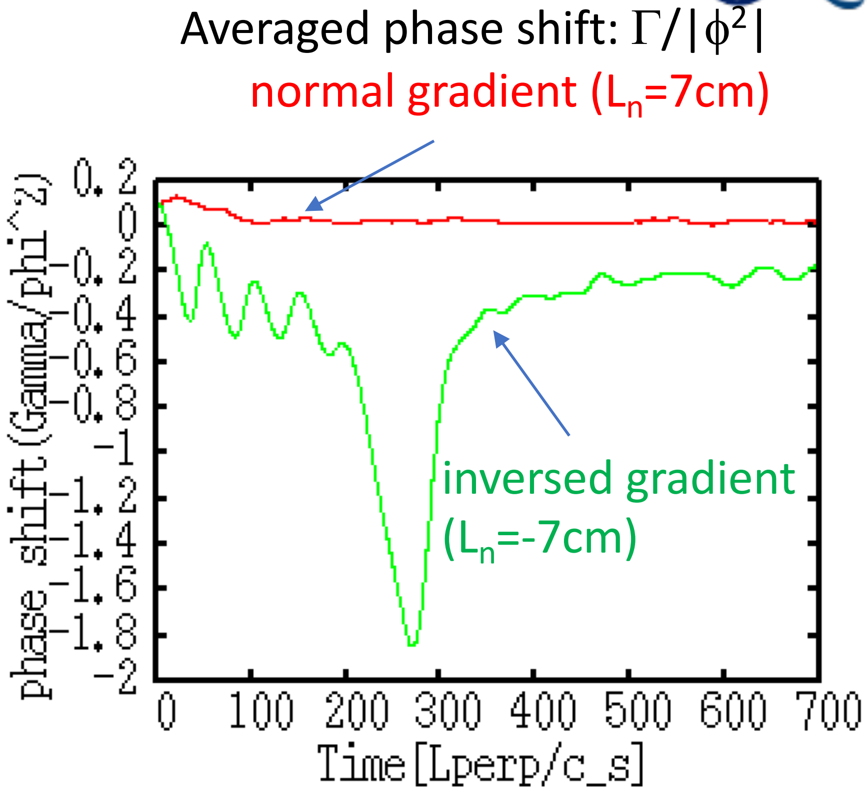




K. Miki et al., APTWG 2017

Comparison of normal/inversed gradient cases





- We observe **inward particle flux** with the inversed density gradient.
- Instead, For a normal gradient, we observe positive particle flux, compared to the case with the inversed gradient.
- Taking an averaged phase shift $\Gamma/|\phi^2|$, the observed inward particle flux is relatively large.

For the inversed density gradient, we expect a distinct mode dynamics of the particle transport!

Simulation Parameters

$$L_T k_\parallel \sim 0.024, \eta = \frac{L_n}{L_T} = -2, \omega_* \frac{L_T}{c_s} \sim -0.15, \quad k_y \rho_s = 0.3, \ \omega_\chi \frac{L_T}{c_s} \sim 1.27$$

$$\eta_i = -2, \hat{\eta}_e \equiv \frac{3}{2}\left(\eta_e - \frac{2}{3}\right) = -4, b_s = k_\theta^2 \rho_s^2 = 0.09, \bar{\omega}_\chi = \frac{\omega_\chi}{\alpha_T' \omega_*} \sim -4.8,$$

$$\xi = \frac{k_\parallel c_s}{\omega_*} \sim -0.16, \eta_i \xi^2 = -5.12 \times 10^{-2}$$

$$b_s = 0.09, \tau = \frac{T_e}{T_i} = 1, \boxed{K = \frac{1 + \eta_i}{\tau} = -1}, G = 2\epsilon_n K \sim 8.48 \times 10^{-2},$$

$$\epsilon_n = \frac{L_n}{R} \sim -4.24 \times 10^{-2}, \hat{\eta}_i = K\xi^2 \sim -2.56 \times 10^{-2},$$

Local Analysis (Slab)

[B. Coppi and C. Spight, PRL 41 1978]

$$\frac{\tilde{n}_e}{n} = \frac{e\phi}{T_e}(1+i\delta), \boxed{\delta = \frac{\alpha_T'}{\omega_\chi}\left(\omega - \omega_* + \frac{3}{2}\omega_*\eta_e\right)}, \alpha_T' = 1.76, \omega_\chi = \frac{k_\parallel^2 v_{th}^2}{\nu_e}$$

Dispersion Relation

$$\Omega^4 - \Omega^3\left(i\bar{\omega}_\chi - \hat{\eta}_e\right) + i\bar{\omega}_\chi\Omega^2 + i\eta_i\xi^2\bar{\omega}_\chi = 0, \quad \text{where} \quad \boxed{\Omega \equiv \frac{\omega}{\omega_*}}, \bar{\omega}_\chi = \frac{\omega_\chi}{\alpha_T'\omega_*},$$

$$\hat{\eta}_e \equiv \frac{3}{2}\left(\eta_e - \frac{2}{3}\right), \xi = \frac{k_\parallel c_s}{\omega_*}$$

$$\Omega = 3.3 - 4.3i, 0.66 - 0.6i$$

Electron Drift Wave in the case of $\eta_i = 0$

$$\Omega^2 - \Omega\left(i\bar{\omega}_\chi - \hat{\eta}_e\right) + i\bar{\omega}_\chi = 0$$

For $\hat{\eta}_e \gg |\omega_\chi|$, $\quad \omega \approx \omega_* < 0, \gamma \approx (\hat{\eta}_e\omega_*)(-\omega_*)\dfrac{\alpha_T'}{\omega_\chi} > 0$

$\Omega = \boxed{3.3 - 4.3i}\ 0.71 - 0.55i$ Note that low frequency mode is coupled with slab ITG

$$\delta \approx 3.3\omega_* - \omega_* + \frac{3}{2}\omega_*\eta_e \approx -0.7\omega_* > 0$$

$$\Gamma = \langle n v_{ExB}\rangle = -\frac{c}{B}\frac{en}{T_e}\sum_k \delta k_y |\phi_k|^2 < 0$$

Particle Pinch

Ion Mixing Mode **with** η_i<0

$$\Omega^3\left(i\overline{\omega}_\chi - \hat{\eta}_e\right) - i\overline{\omega}_\chi\Omega^2 - i\eta_i\frac{k_\parallel^2 c_s^2}{\omega_*^2}\overline{\omega}_\chi = 0 \quad \boxed{\Omega = 0.55 - 0.54i}$$

$$\delta \approx 0.55\omega_* - \omega_* + \frac{3}{2}\omega_*\eta_e \approx -3.45\omega_* > 0$$

Low frequency mode

Growth rate of Electron Drift Wave is larger than Ion Mixing Mode for η <0

Particle pinch is likely produced by Electron Drift Wave

Nonlocal Analysis

Strong Ballooning Limit $2(\cos\chi + s\chi\sin\chi) \approx 2 + (2s-1)\chi^2$

$$\frac{\partial^2}{\partial\chi^2}\left[\frac{c_s^2}{\omega_*^2\Omega^2 q^2 R^2}\frac{\partial^2\phi}{\partial\chi^2} + B\phi + C\chi^2\phi\right] - iD\phi = 0$$

$$B = \left(1 - \frac{1}{\Omega}\right)\frac{1}{A} + b_s + \frac{2\epsilon_n}{\Omega} \qquad A = 1 + \frac{K}{\Omega} \qquad C = b_s s^2 + \frac{2\epsilon_n}{\Omega}\left(s - \frac{1}{2}\right)$$

$$D = \frac{\alpha_T'}{A}\frac{\nu_e\omega_*}{v_e^2}q^2R^2(\hat{\eta}_e + \Omega)$$

$$\chi = \alpha\theta$$

$$\frac{\partial^2}{\partial\theta^2}\left[\frac{\partial^2\phi}{\partial\theta^2} + E\phi - \frac{1}{4}\theta^2\phi\right] - iF\phi = 0$$

$$E = \frac{\omega_*^2\Omega^2 q^2 R^2}{c_s^2}\alpha^2 B \qquad F = \frac{\omega_*^2\Omega^2 q^2 R^2}{c_s^2}\alpha^4 D \qquad \frac{\omega_*^2\Omega^2 q^2 R^2}{c_s^2}\alpha^4 C = -\frac{1}{4}$$

Noting $\phi \propto \exp\left(-\frac{\alpha^2\theta^2}{4}\right)$ $\quad \alpha_\pm^2 \sim \frac{1}{2}\frac{c_s}{qR\omega_* b_s^{1/2} s}\frac{\pm i\Omega_r \pm \Omega_i}{\Omega_r^2 + \Omega_i^2}$ $\quad \begin{cases}\alpha_+^2 & \text{for } \Omega_i > 0\\ \alpha_-^2 & \text{for } \Omega_i < 0\end{cases}$

$$\phi = \sum_n a_n u_n \qquad \frac{\partial^2 u_n}{\partial \theta^2} + \left(n + \frac{1}{2} - \frac{\theta^2}{4}\right) u_n = 0$$

$$\sum_n a_n \left\{E - \left(n + \frac{1}{2}\right)\right\} \left\{\left(n + \frac{1}{2}\right) - \frac{\theta^2}{4}\right\} u_n + iF \sum_n a_n u_n = 0$$

$$\int_{-\infty}^{+\infty} d\theta u_n^2(\theta) = \sqrt{2\pi}\, n! \qquad \int_{-\infty}^{+\infty} d\theta \theta^2 u_n^2(\theta) = \sqrt{2\pi}\,(2n+1) n!$$

$$E = \left(n + \frac{1}{2}\right) - \frac{2iF}{n + 1/2}$$

Dispersion Relation

$$\boxed{\begin{aligned} & b_s \left(\frac{q}{\epsilon_n}\right)^2 \Omega(b_s s^2 \Omega + \epsilon_n (2s-1)) \left\{(\Omega - 1)\,\Omega + (b_s \Omega + 2\epsilon_n)(\Omega + K)\right\}^2 \\ & + \left\{(2n+1)(b_s s^2 \Omega + \epsilon_n (2s-1))(\Omega + K) + i\overline{\nu}_e \Omega^2 (\hat{\eta}_e + \Omega)\right\}^2 = 0 \end{aligned}}$$

$$\bar{\nu}_e = \frac{2\alpha'_T}{2n+1} \frac{\nu_e \omega_*}{v_e^2} q^2 R^2$$

ITG Mode

For the opposite case with $b_s\Omega \ll 2\epsilon_n$ and $\boxed{K = -1}$

$$\Omega(\Omega + 2\epsilon_n)^2 + \overline{s}\left(2n + 1 + i\frac{\overline{\nu}}{2n+1}\frac{\Omega^2(\Omega + \hat{\eta}_e)}{\Omega - 1}\right)^2 = 0$$

where

$$\begin{aligned} \overline{s} &= \frac{\epsilon_n^3}{b_s q^2}(2s - 1) \\ \overline{\nu} &= \frac{2k}{\epsilon_n(2s-1)}\frac{\nu_e \omega_{*e}}{v_e^2} q^2 R^2 \end{aligned}$$

For $\overline{\nu} = 0$ and $b_s \ll 1$,

$$\boxed{\Omega^3 + 4\epsilon_n \Omega^2 + 4\epsilon_n^2 \Omega + \overline{s}^* = 0}$$

where $\overline{s}^* = \overline{s}(2n+1)^2$.

Electron Drift Mode

For special case with $K = -1$

$$b_s\left(\frac{q}{\epsilon_n}\right)^2 \Omega(b_s s^2\Omega + \epsilon_n(2s-1))\left\{(\Omega-1)\,\Omega + (b_s\Omega + 2\epsilon_n)(\Omega + K)\right\}^2$$

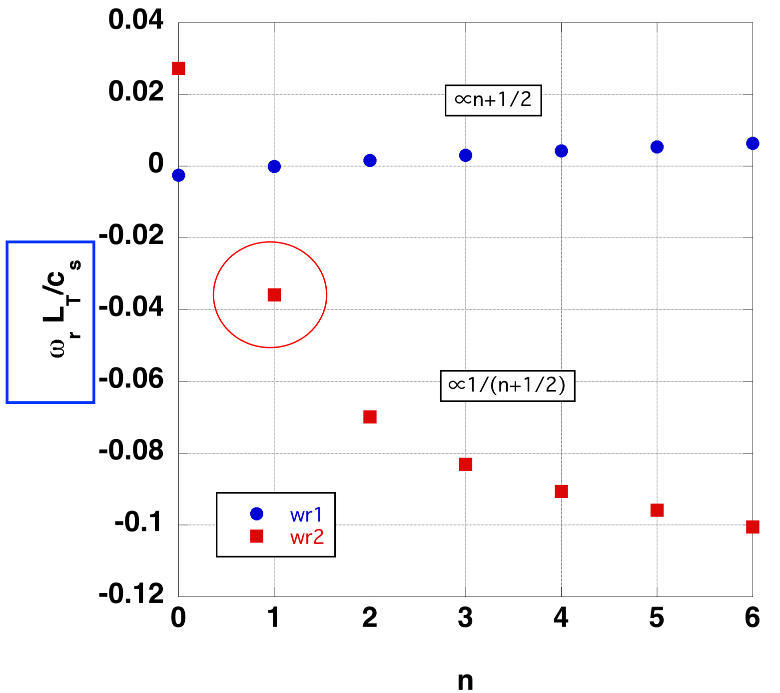
$$+\left\{\cancel{(2n+1)(b_s s^2\Omega + \epsilon_n(2s-1))(\Omega+K)} + i\overline{\nu}_e\Omega^2(\hat{\eta}_e + \Omega)\right\}^2 = 0$$

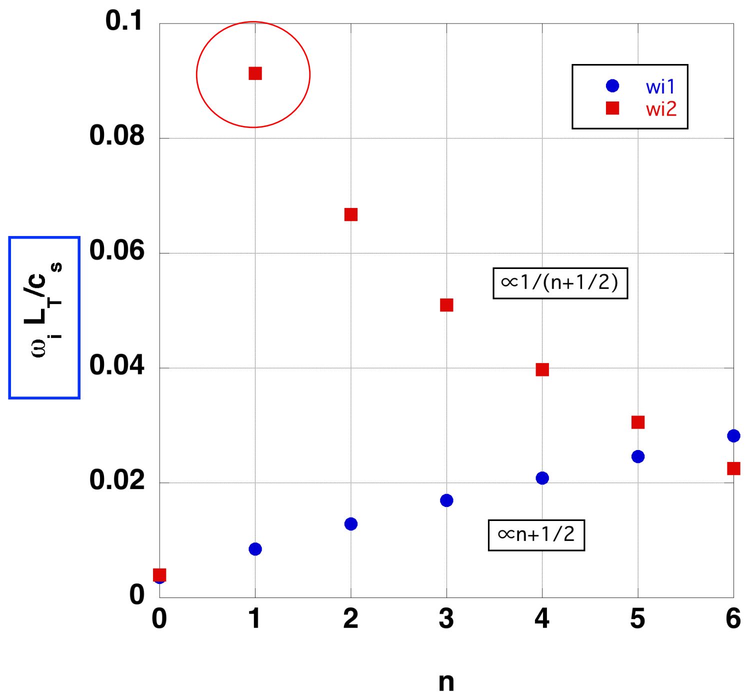
$$(\Omega + \epsilon_n^*)(\Omega-1)^2\big((1+b_s)\Omega + 2\epsilon_n\big)^2 - {\nu_e^*}^2\Omega^3(\hat{\eta}_e + \Omega)^2 = 0$$

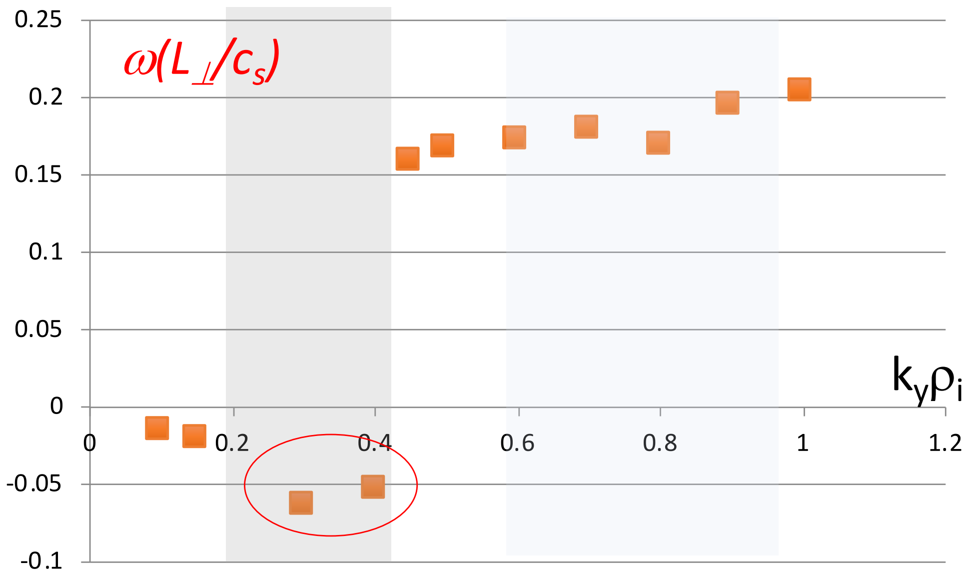
$$\epsilon_n^* \equiv \frac{\epsilon_n(2s-1)}{b_s s^2} \qquad \nu_e^* \equiv \frac{\bar{\nu}_e\epsilon_n}{b_s q s}$$

Assuming $\Omega \gg 2\epsilon_n\,, b_s \ll 1$

$$\boxed{(\Omega + \epsilon_n^*)(\Omega-1)^2 - {\nu_e^*}^2\Omega(\hat{\eta}_e + \Omega)^2 = 0}$$







Difference of frequency (20~30%) may come from ion collision frequency which is not taken into account in present analysis.

Non-adiabatic response via Trapped Electron

Collisionless limit

Destabilization of trapped electron mode by grad B drift resonance

Inverted density profiles effectively suppress it and lead to a considerable reduction in the energy transport. They also result in an inward flux of both energy and particles in the energy transport.

W. M. Tang, et al., PRL 1975, J. C. Adam, et al., PF 1976

$$\Gamma = \Gamma^p + \Gamma^t$$

$$\Gamma^p \equiv -\frac{2c}{B}\frac{en}{T_e}\sum_{k>0}\delta_\chi|\phi_k|^2$$

In semi-collisional regime

Dissipative trapped electron mode may play a role for particle pinch.

For DTEM, $\delta^t = Im\sqrt{\varepsilon}\left\langle\frac{\omega_e^\dagger - \omega}{\omega + i\nu_{eff}}\right\rangle \approx -\frac{\varepsilon^{3/2}}{\nu_{ei}}\eta_e\omega_{*e}$

In the limit of $\nu_{eff} \gg |\omega| \sim |\omega_*|$ $\gamma \approx -\delta\omega_*$ so that if $\omega_* < 0$ DTE is stabilized.

In this case, saturation level should be determined by electron drift wave.

$$\Gamma^t \sim -\frac{c}{B}\frac{en}{T_e}\sum_k k_y|\phi_k|^2\delta^t$$

$$\sim +\frac{c}{B}\frac{en}{T_e}\sum_k k_y|\phi_k|^2\frac{\varepsilon^{3/2}}{\nu_{ei}}\eta_e\omega_{*e} > 0 \qquad \text{(outward flux)}$$

$$\Gamma^p \sim -\frac{c}{B}\frac{en}{T_e}\sum_k k_y|\phi_k|^2\frac{\alpha_T'}{\omega_\chi}\frac{3}{2}\eta_e\omega_{*e} < 0 \qquad \text{(inward flux)}$$

If $\nu_{ei}^2 \geq \varepsilon^{3/2}k_\parallel^2 v_e^2$ then $\Gamma^p \geq \Gamma^t$ $k_\parallel = \dfrac{m-nq}{qR} \sim \dfrac{1}{qR}$ $\nu_{ei}^2 \geq \nu_p\nu_b \gg \nu_b^2$

in plateau regime

DTE produces the outward particle flux which weakens particle pinch, however, this effect could be weak in plateau regime.

For normal density gradient case, ion-mixing mode and CTEM could be candidates for particle pinch.

Summary

We found there exist two unstable modes in inverted density profile, namely, ion mixing mode and electron drift wave in semi-collisional regime

Ion mixing mode is driven by negative compression (slab ITG) and non-adiabatic electron response.

Electron drift wave with **inverted** density gradient $L_n \equiv -\frac{d}{dr}\ln n < 0$ is dominant in this regime which produces particle pinch.

DTE produces the outward particle flux so that it weakens the inward pinch.

The simulation result can be understood based on electron drift wave in inverted density profile.

Electron drift wave, ion mixing mode and CTEM are candidates for particle pinch which could be used to develop scenario for fuel supply in DEMO.