Feasibility of using Orbit Tomography to infer the Runaway Electron Distribution Function from Bremsstrahlung Measurements

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During a disruption event a strong electric field is generated, causing supra-thermal electrons to reach relativistic speeds. Due to the severe damage the runaway electrons can inflict upon ITER's plasma facing components and cooling systems, developing strategies to both prevent the formation of and to safely dissipate the runaways is critically important to ITER's success. However, development of mitigation strategies is hindered by the difficulty of measuring the runaway electron's distribution function as most runaway-electron diagnostics can only provide partial information about the runaway-electron phase-space. Fortunately, using Orbit Tomography, a technique developed in the fast-ion community, multiple measurements can be combined to infer the runaway electron distribution function to unprecedented dimensionality.

DIII-D's Gamma Ray Imager (GRI) provides multiple spatially and energy resolved bremsstrahlung measurements of the runaway electron distribution. Calculations of the GRI's orbit weight functions i.e. phase-space sensitivities shows favorable conditions for doing Orbit Tomography. In this work we will explore the feasibility of doing Orbit Tomography with GRI measurements. Orbit weight functions for the GRI calculated by the SOFT code will be presented along with reconstructions of the runaway electron distribution function from synthetic measurements.

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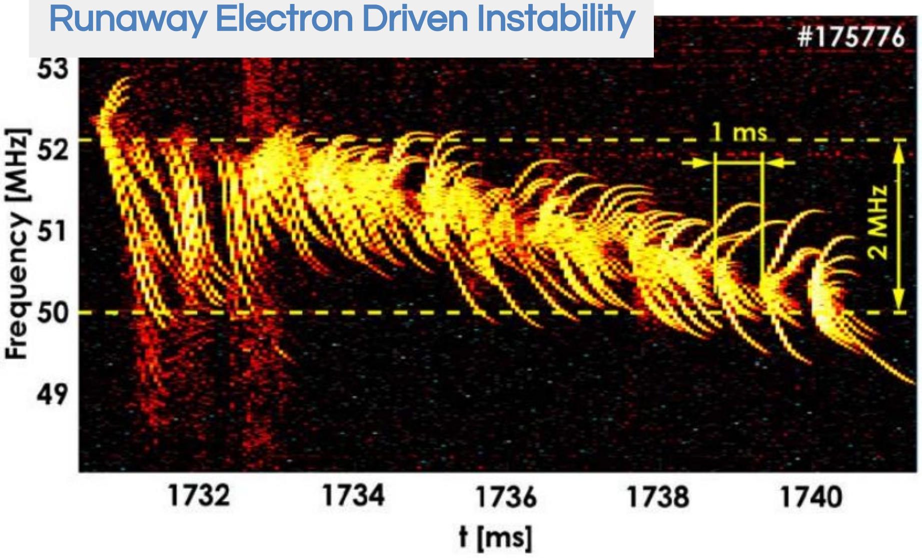
Runaway Electrons can severely damage critical vessel components and drive instabilities

Understanding of the phase-space dynamics is needed to mitigate and control runaway electrons

Wall damage caused by runaway electrons at JET

- Created during a thermal quench/disruption
- MeV energy electrons
- Can severely damage vessel components upon impact
- Can drive instabilities

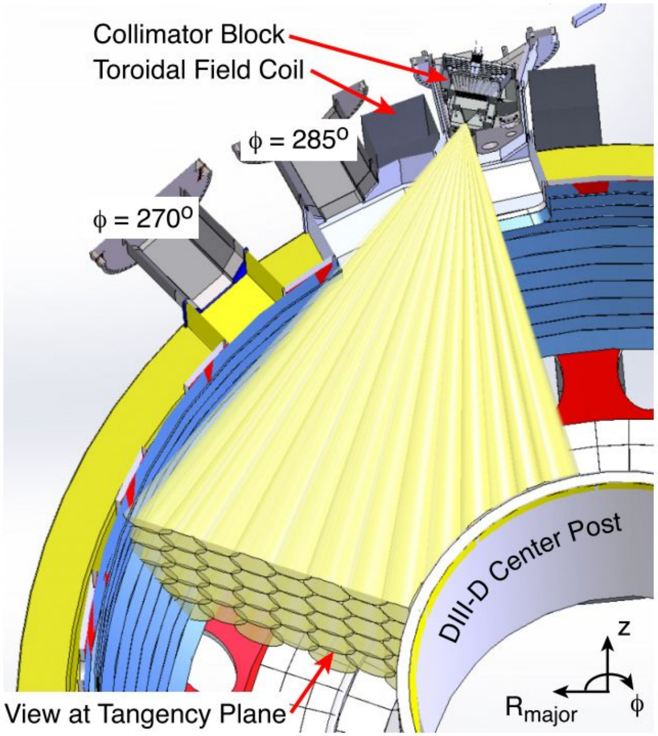
Runaway Electron Driven Instability

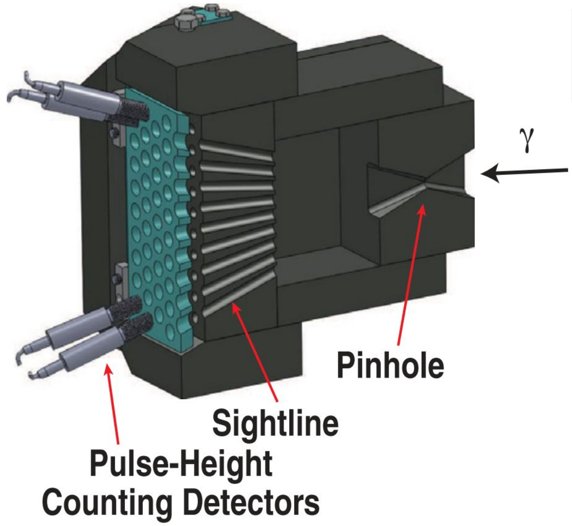


DIII-D's Gamma Ray Imager (GRI) measures a distribution of photon energies emitted via Bremsstrahlung

The Gamma Ray Imager is a pinhole camera with multiple pulse height counting detectors that are used to build up a histogram of photon energies over an integration time ~100ms

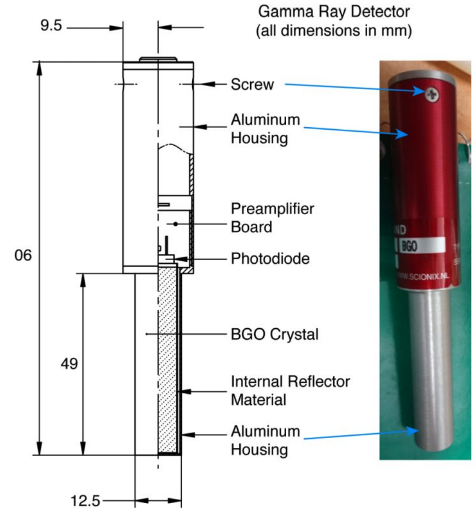
CAD View of GRI Midplane





Gamma Ray Imager (GRI)

Bismuth-Germanate Detector

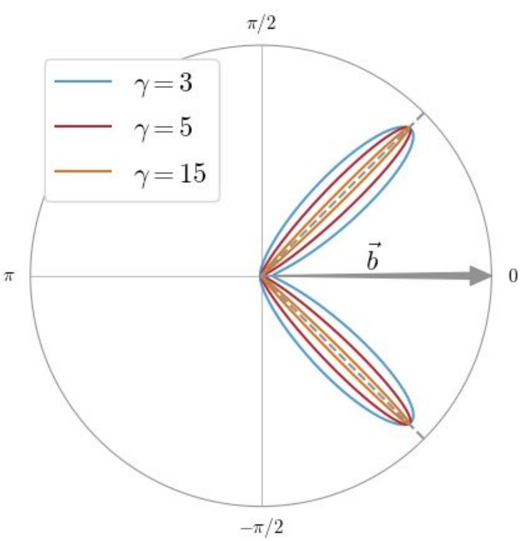


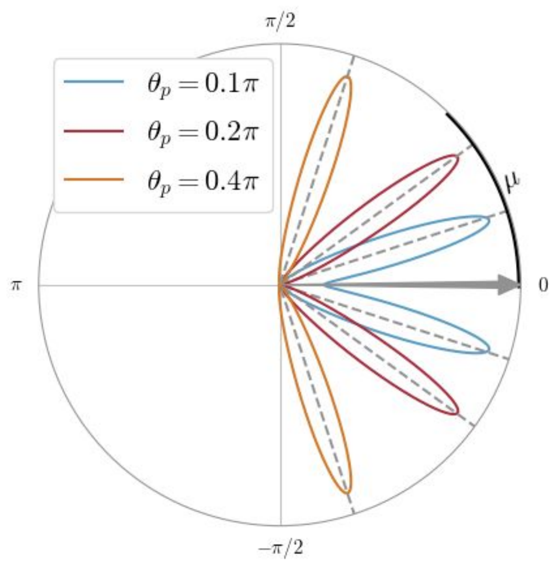
Interpreting a GRI chord's photon energy distribution requires understanding its phase-space sensitivities

D.C. Pace Rev. Sci. Instr. 87 (2016)
C.M. Cooper Rev. Sci. Instr. 87 (2016)

Rudimentary spatially localized phase-space sensitivity trends can be understood from geometric arguments

Bremsstrahlung is emitted into a cone with a width ~ 1/γ and opening angle equal to the electrons pitch angle θ_p

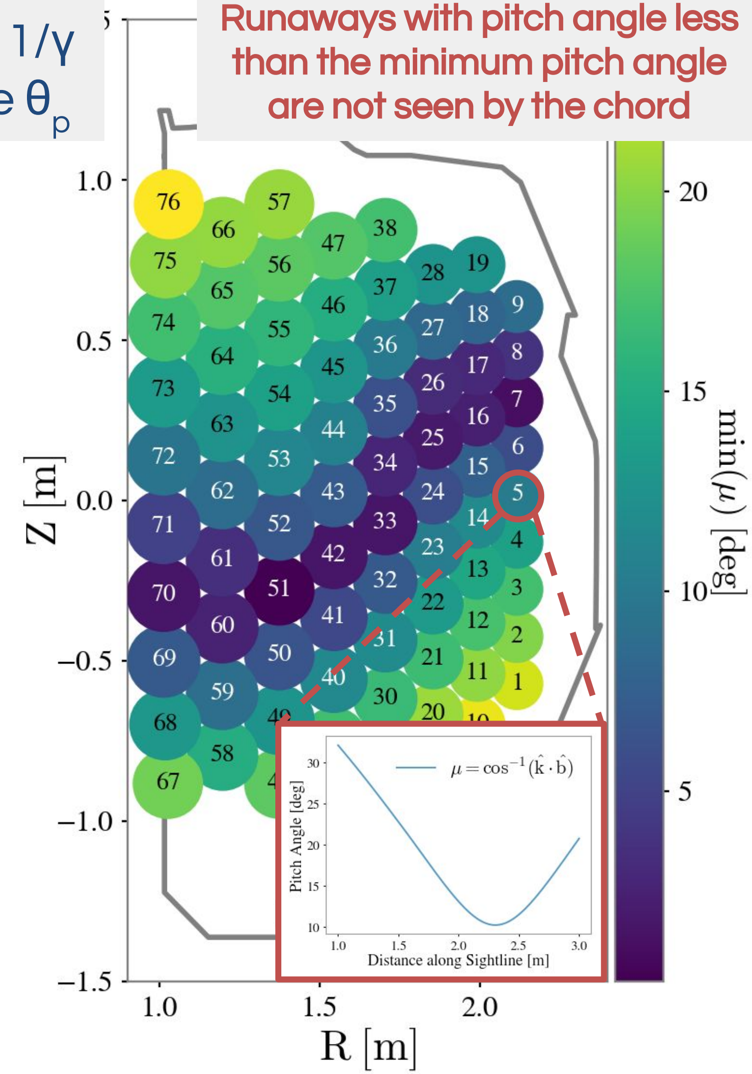




Detector is more sensitive to large Lorentz factors due to angular concentration

$$\left\langle \frac{d^2 N_\gamma}{dk d\Omega} \right\rangle \approx \frac{1}{2\pi} \frac{dN_\gamma}{dk} \delta(\cos(\mu) - \cos(\theta_p)) \quad \gamma \gg 1$$

Runaways with pitch angle less than the minimum pitch angle are not seen by the chord



Minimum pitch angle along chord

Orbit Weight Functions gives us a complete phase-space sensitivity

$$S_i = \int W_i(\mathbf{x})\, F(\mathbf{x})\, d\mathbf{x}$$

A diagnostic signal (S_i) is modeled as a 6D integration over the energetic particle phase-space where W(x) is the expected diagnostic signal and F(x) is the distribution function.

$$W_i(\mathbf{J}) = \prod_j \frac{1}{\tau_j} \int W_i(\mathbf{J}, \mathbf{\Theta})\, d\mathbf{\Theta}$$

The dimensionality of the weight function can be reduced by averaging over "unneeded" coordinates

$$f(x) = \int g(x, y) p(y) dy$$

$$S_i = \int W_i(\mathbf{J})\, F(\mathbf{J})\, d\mathbf{J}$$

Action-angle coordinates (J,Θ) allow us to average over the angle coordinates without losing model fidelity

Three Action Coordinates (**J**)

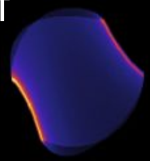
- Lorentz Factor (γ)
- Maximal R along orbit (R_m)
- Pitch at R_m (ξ)

L. Stagner Physics of Plasmas 24 (2017)

SOFT

The Synchrotron-detecting Orbit Following Toolkit

SOFT code uses a variant of Orbit Weight Functions to calculate Synchrotron and Bremsstrahlung signals



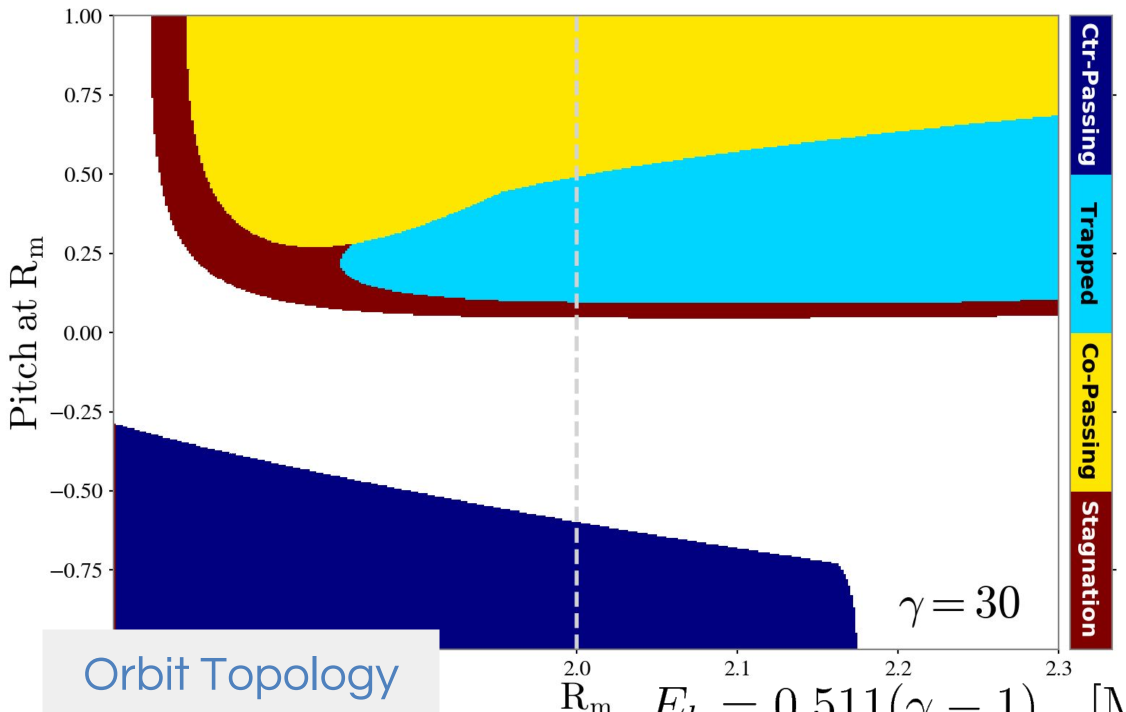
M. Hoppe Nuclear Fusion 58 (2018)

For Guiding-center motion Action Coordinates define an orbit

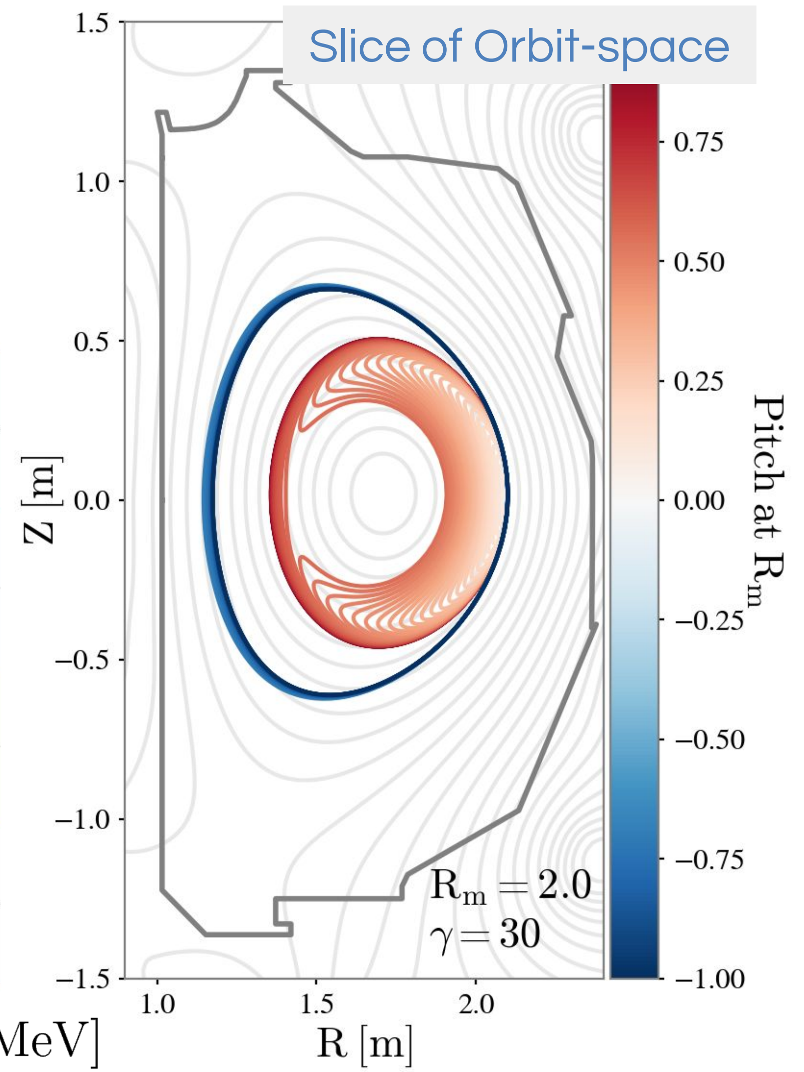
Orbit Coordinate System

- Intuitive
- Easy to enumerate all possible orbits
- Unique labeling of orbits
  - One-to-one correspondence between points in space and an orbit
- Similar to coordinate system used by Rome, CQL3D, and others

Only orbits with $\xi > 0.8$ are expected to exist in experiment

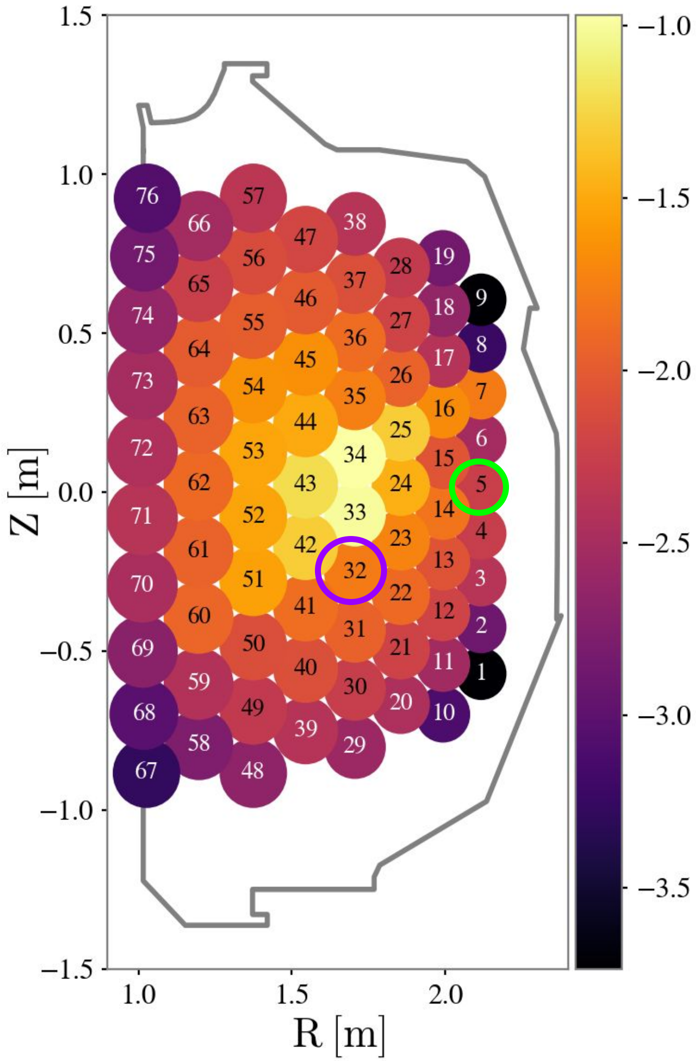


$$E_k = 0.511(\gamma - 1) \quad [\text{MeV}]$$

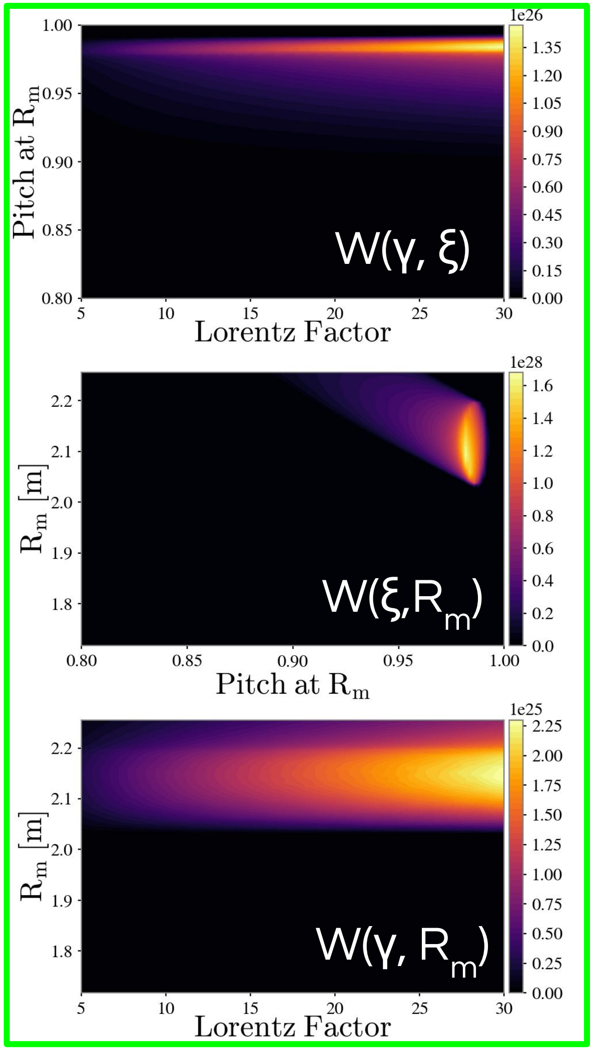


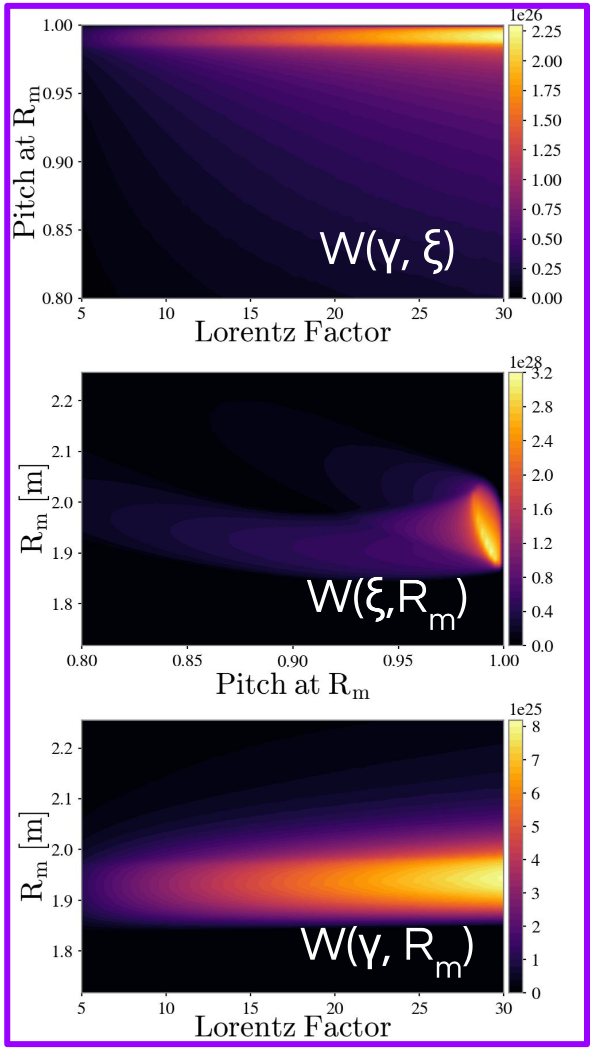
Orbit Weight Functions show pitch angle cutoff and increased sensitivity at large Lorentz Factors

Log Integrated Sensitivity



Orbit Weight Function Projections

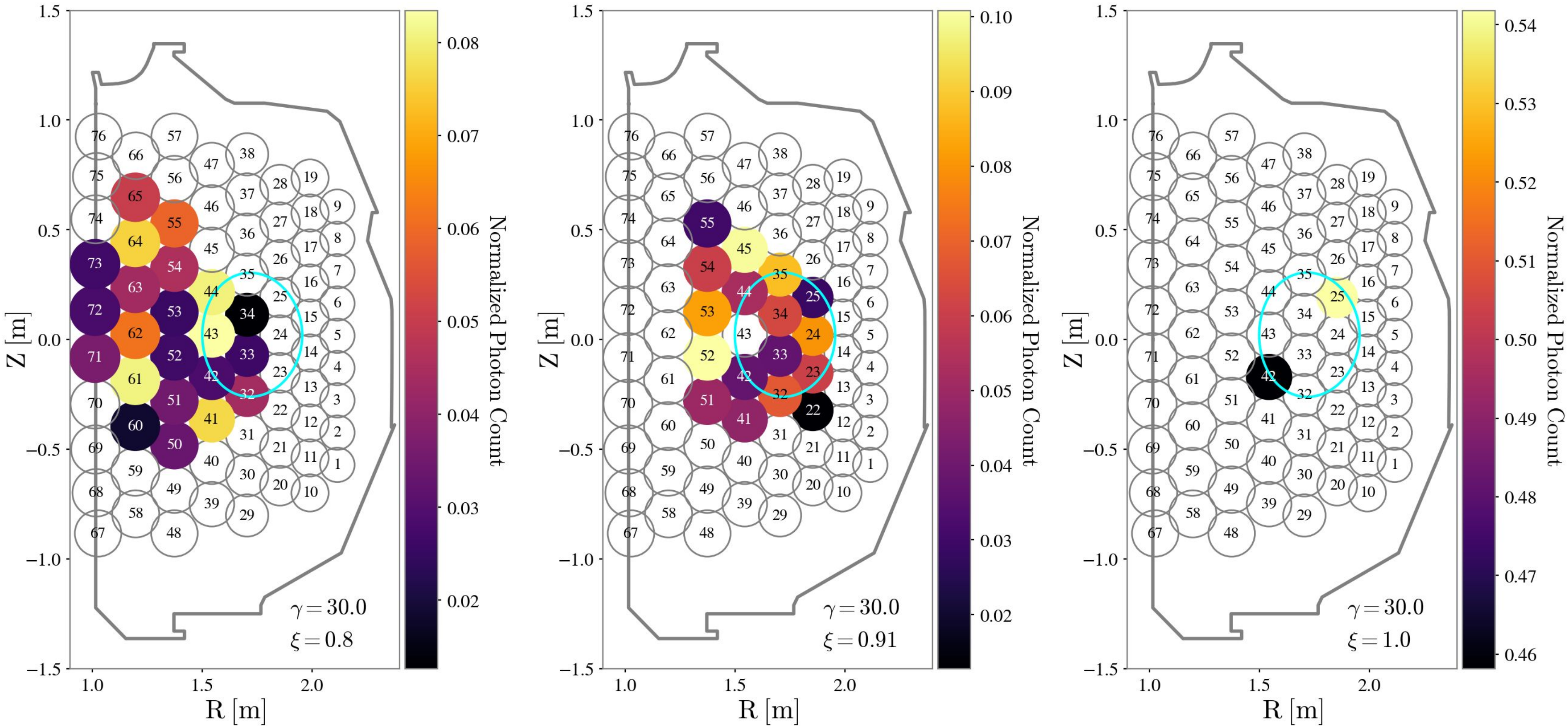




Increasing the orbit's initial pitch decreases the number of GRI chords that are sensitive to the orbit

Increasing Pitch

Orbits near the magnetic axis are small and can only be seen by a few GRI chords



It will be harder to measure centrally peaked distributions at pitch close to one due to fewer sensitive chords and smaller orbits

Orbit weight functions can be used to infer the entire runaway electron distribution

Orbit Tomography is a way to infer an energetic particle distribution without assuming a function form.

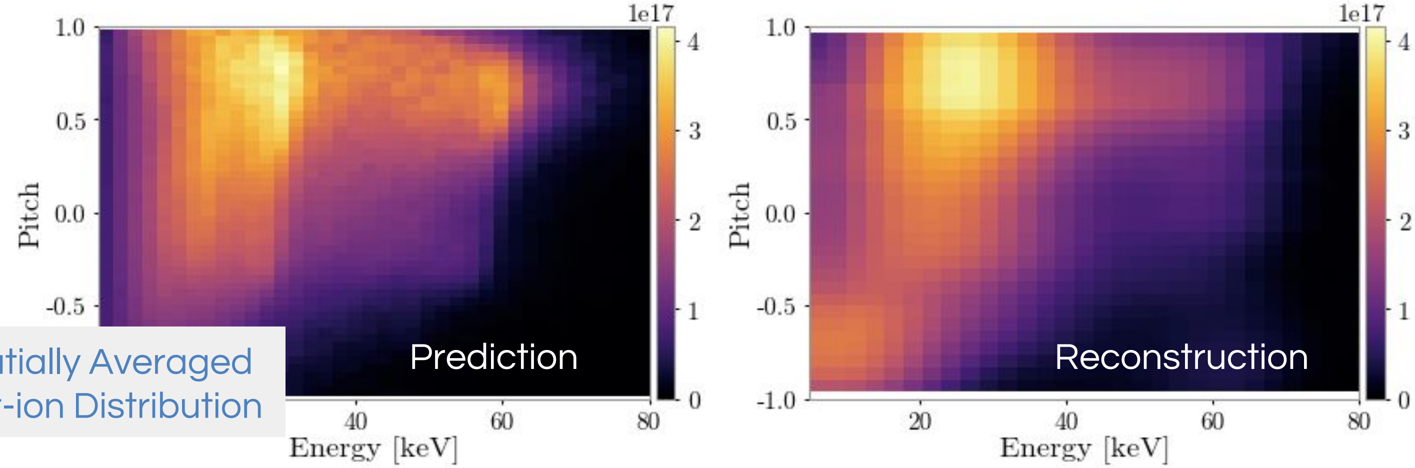
$$S_i = \int W_i(\mathbf{J})\, F(\mathbf{J})\, d\mathbf{J} \quad \xrightarrow{\text{DISCRETIZE}} \quad \mathbf{s} = \mathbf{W} \cdot \mathbf{f}$$

$$\text{minimize}(\, ||\mathbf{W} \cdot \mathbf{f} - \mathbf{s}||_2 + \alpha ||\mathbf{\Gamma}||_2 \,) \qquad \mathbf{f} \geq 0$$

System of Linear equations is solved using Tikhonov regularization with a non-negative constraint

α is chosen via evidence optimization

Reconstruction of the fast-ion distribution function before a sawtooth crash from FIDA measurements taken at ASDEX Upgrade

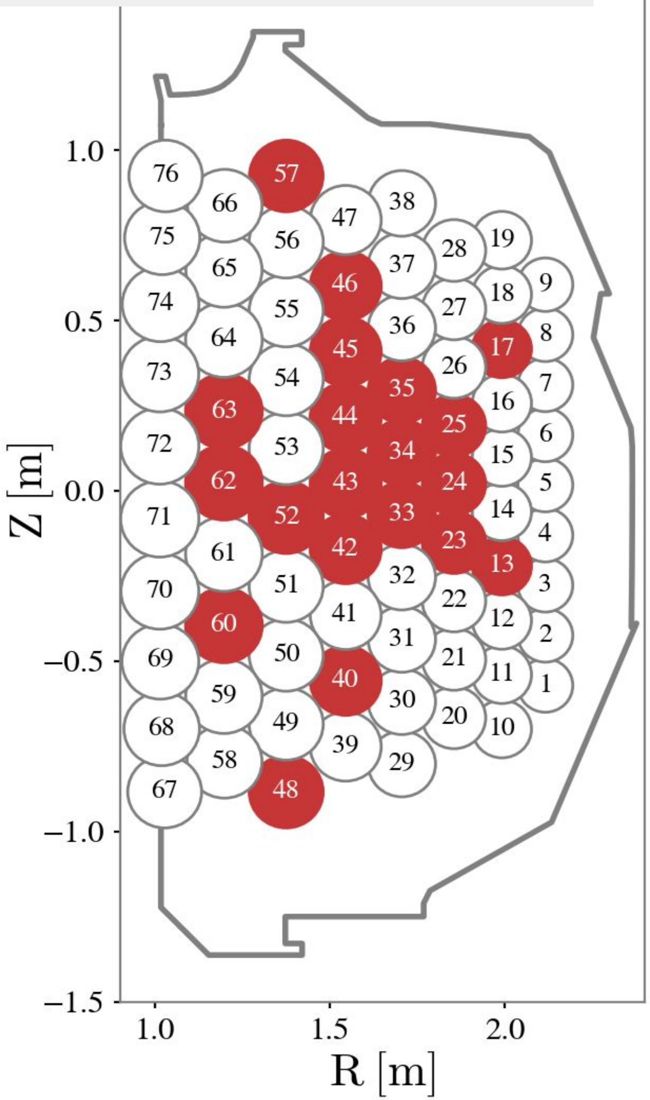


Spatially Averaged Fast-ion Distribution

ASDEX Upgrade

Orbit-space is well coveraged but few measurements near the magnetic axis and near pitches close to one hinder tomographic reconstruction

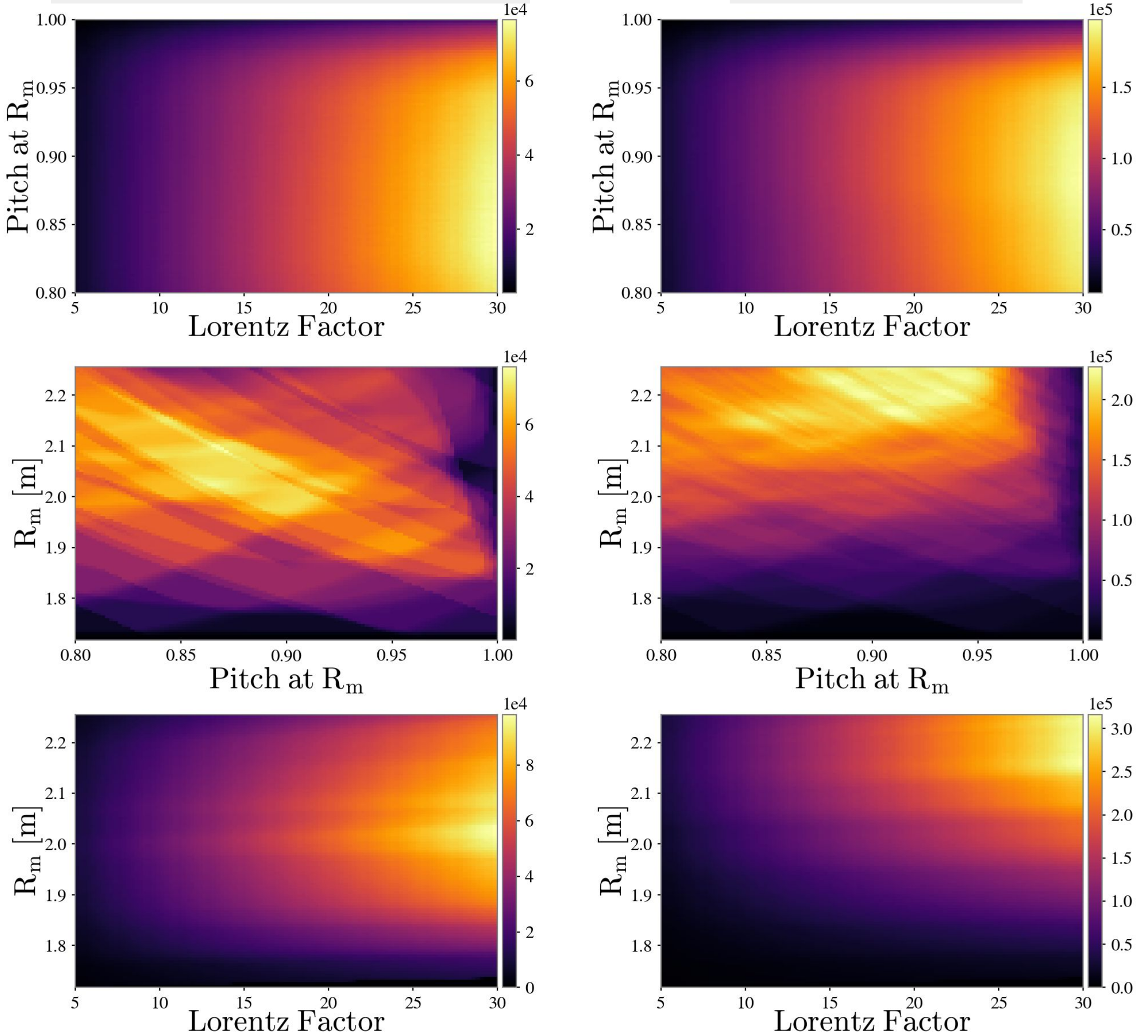
2016 Experimental Setup



Number of measurements seen for each orbit

2016 Experimental Chords

All Chords



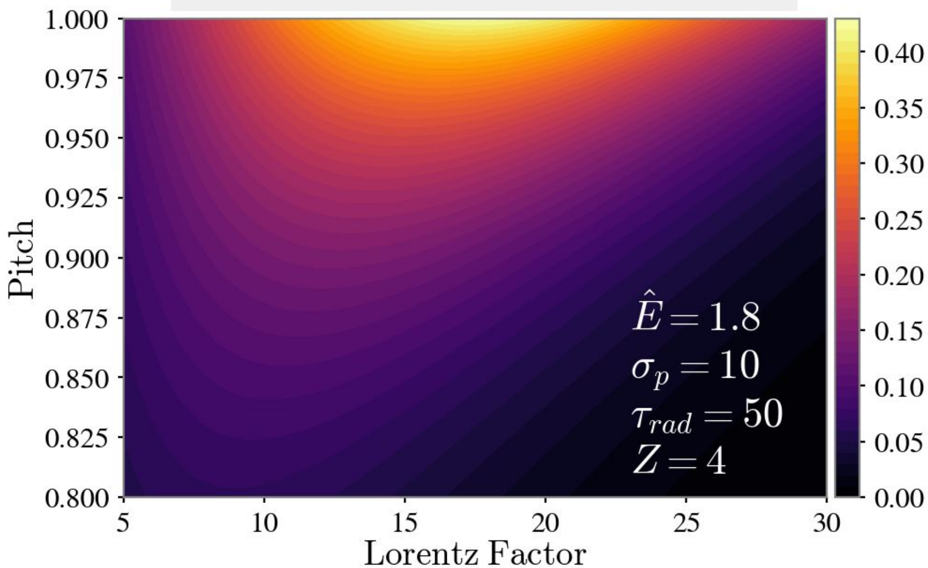
A theoretical runaway electron distribution function is used for benchmarking

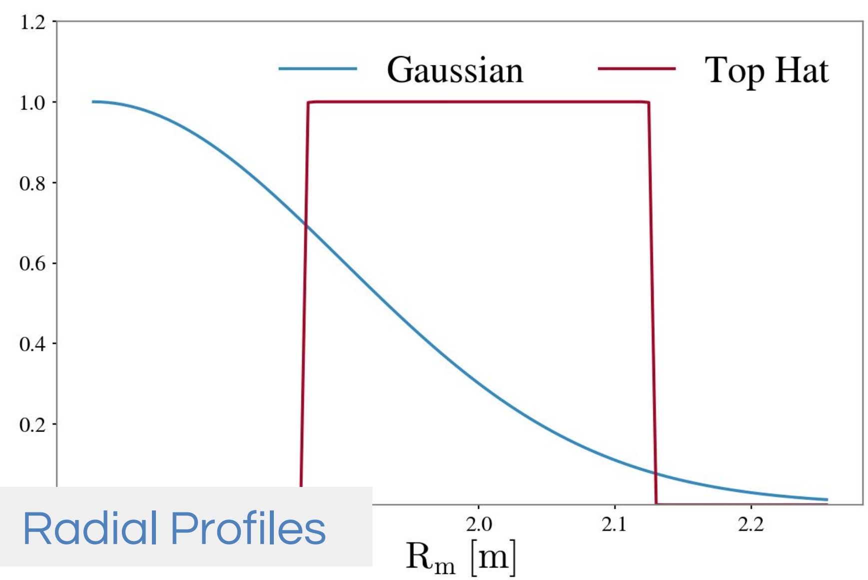
Near threshold solution for the relativistic Fokker-Planck equation when pitch-angle equilibration occurs on a faster time scale then the momentum evolution time

$$U(p) = \left[\frac{1}{A(p)} - \frac{1}{\tanh A(p)}\right]\left(\frac{Z+1}{\hat{E}\tau_{rad}}\frac{p^2+1}{p} - \hat{E}\right) - 1 - \frac{1}{p^2} \qquad A(p) = \frac{2\hat{E}}{Z+1}\frac{p^2}{\sqrt{p^2+1}}$$

$$F(p,\theta) = \mathcal{N}(p_{max}, \sigma_p^2)\frac{A(p)}{2\sinh A(p)}\exp(A(p)\cos\theta) \qquad U(p_{max}) = 0$$

Velocity-space Distribution



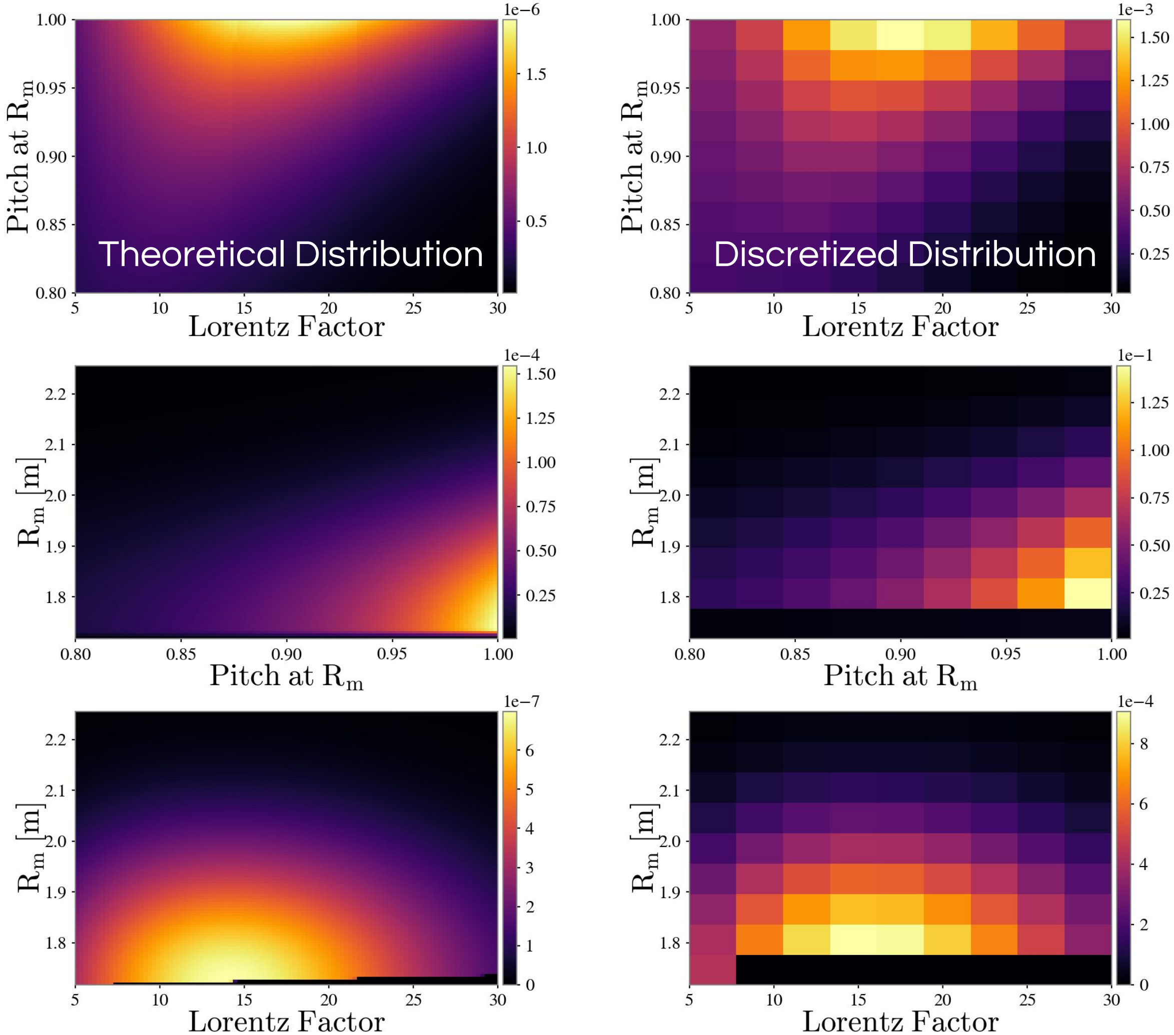


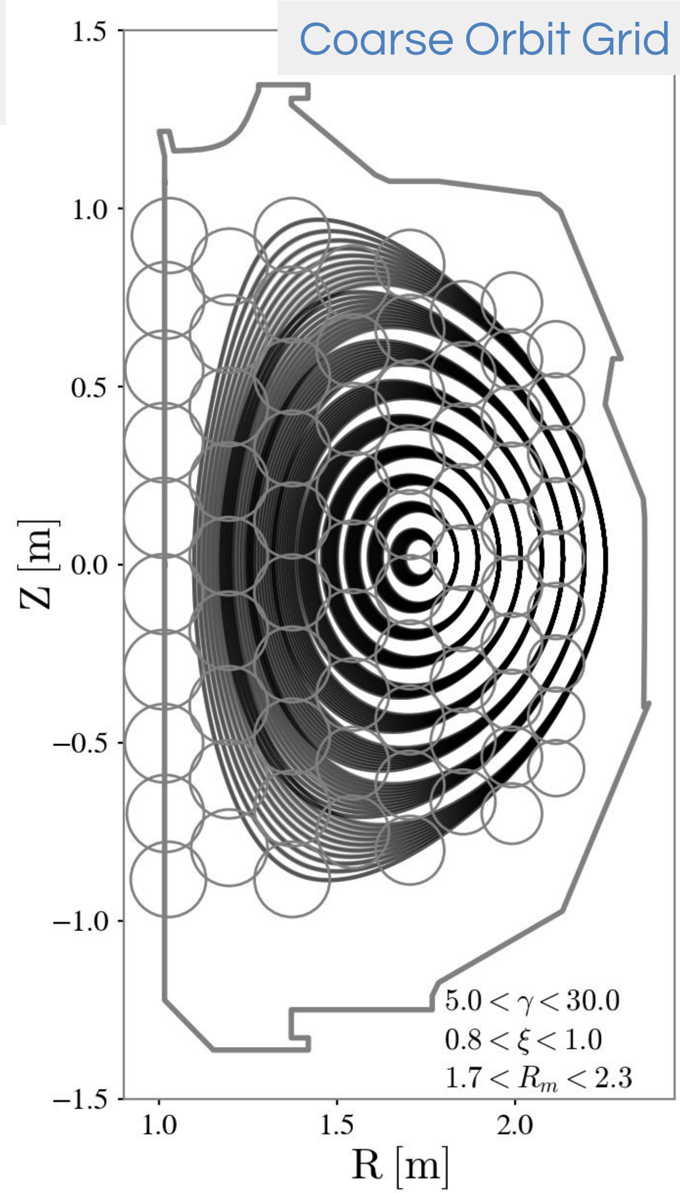
Radial Profiles

P. Aleynikov, B. Breizman Physics Review Letters **114** (2015)

Runaway electron distribution is inferred on a coarse grid

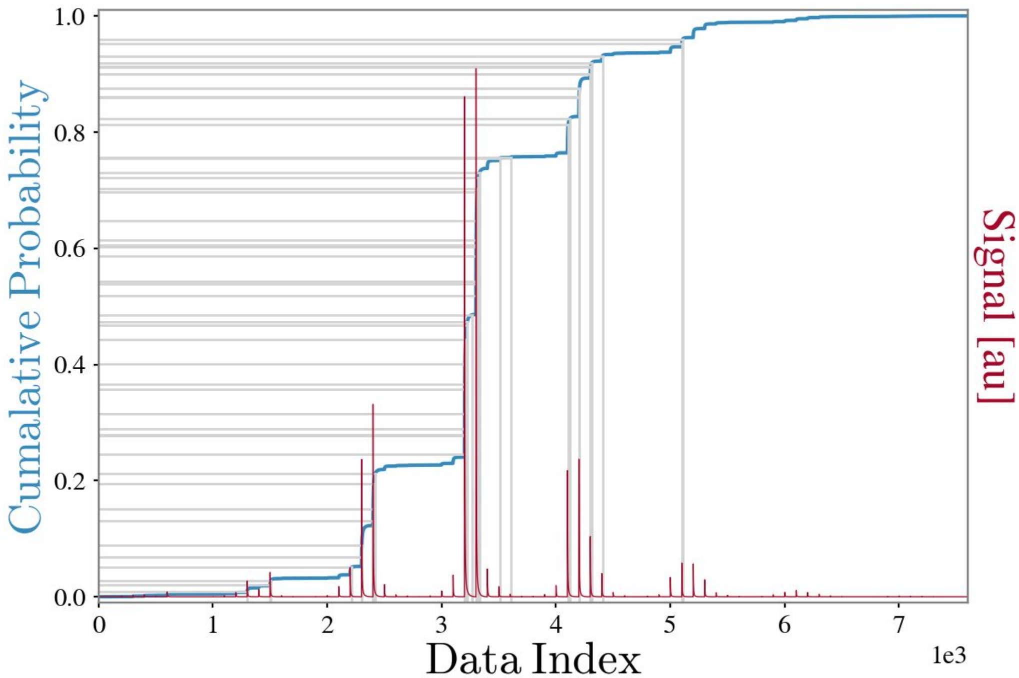
The runaway electron distribution is discretized to create an overdetermined system of linear equations





Synthetic data is generated using a realistic noise model

Inverse CDF sampling of concatenated GRI energy spectrum

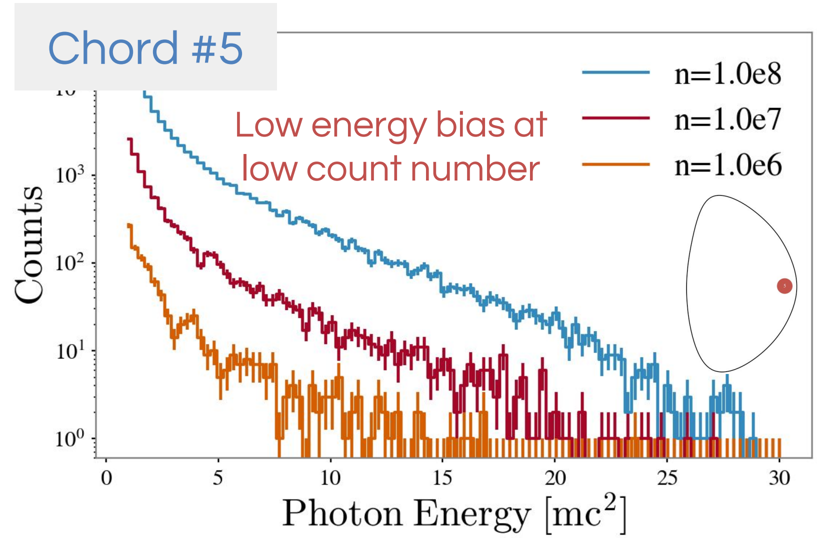


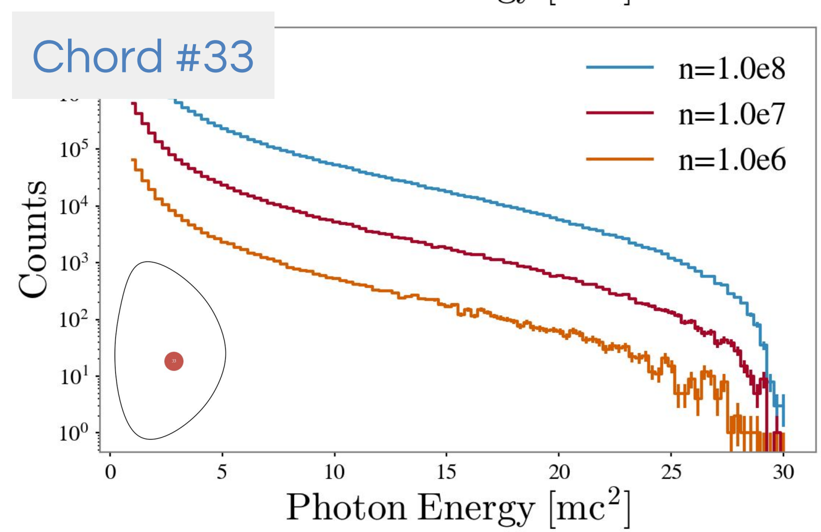
The GRI uses pulse height counting over a fixed time period to generate a distribution of photon energies

This process can be simulated via inverse CDF sampling of the synthetic data

Changing the number of pulses detected changes the noise level

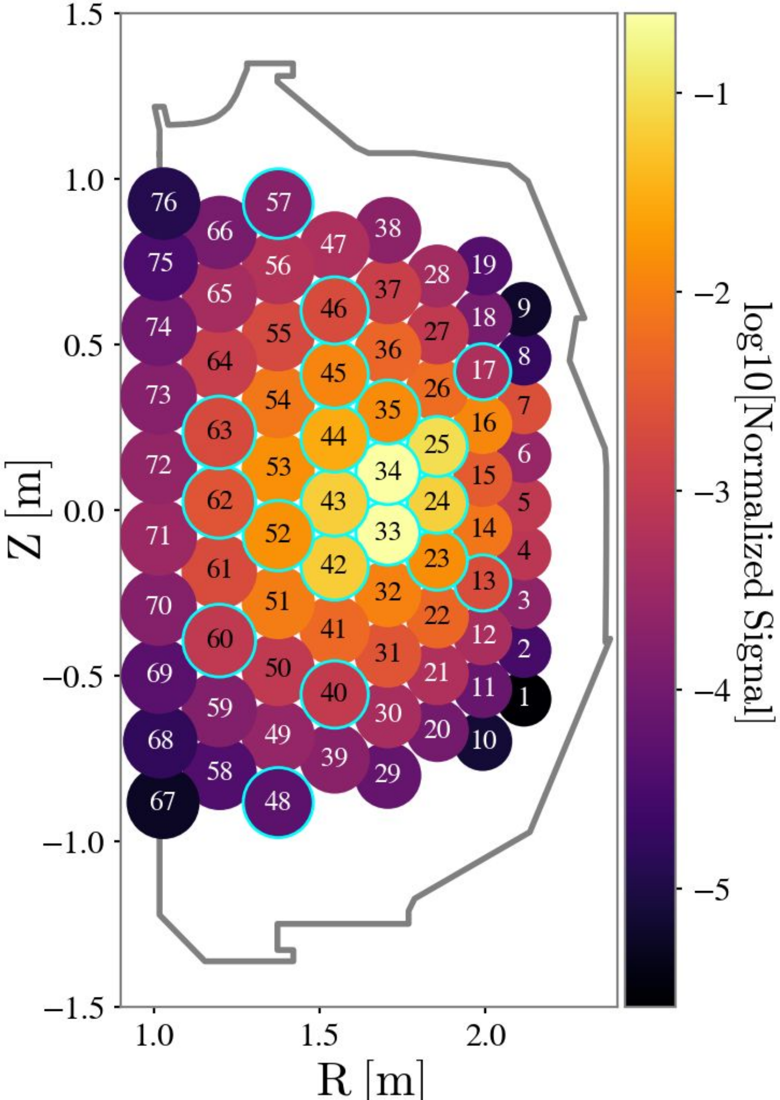
Synthetic Energy Spectrum





Distribution can be successfully reconstructed

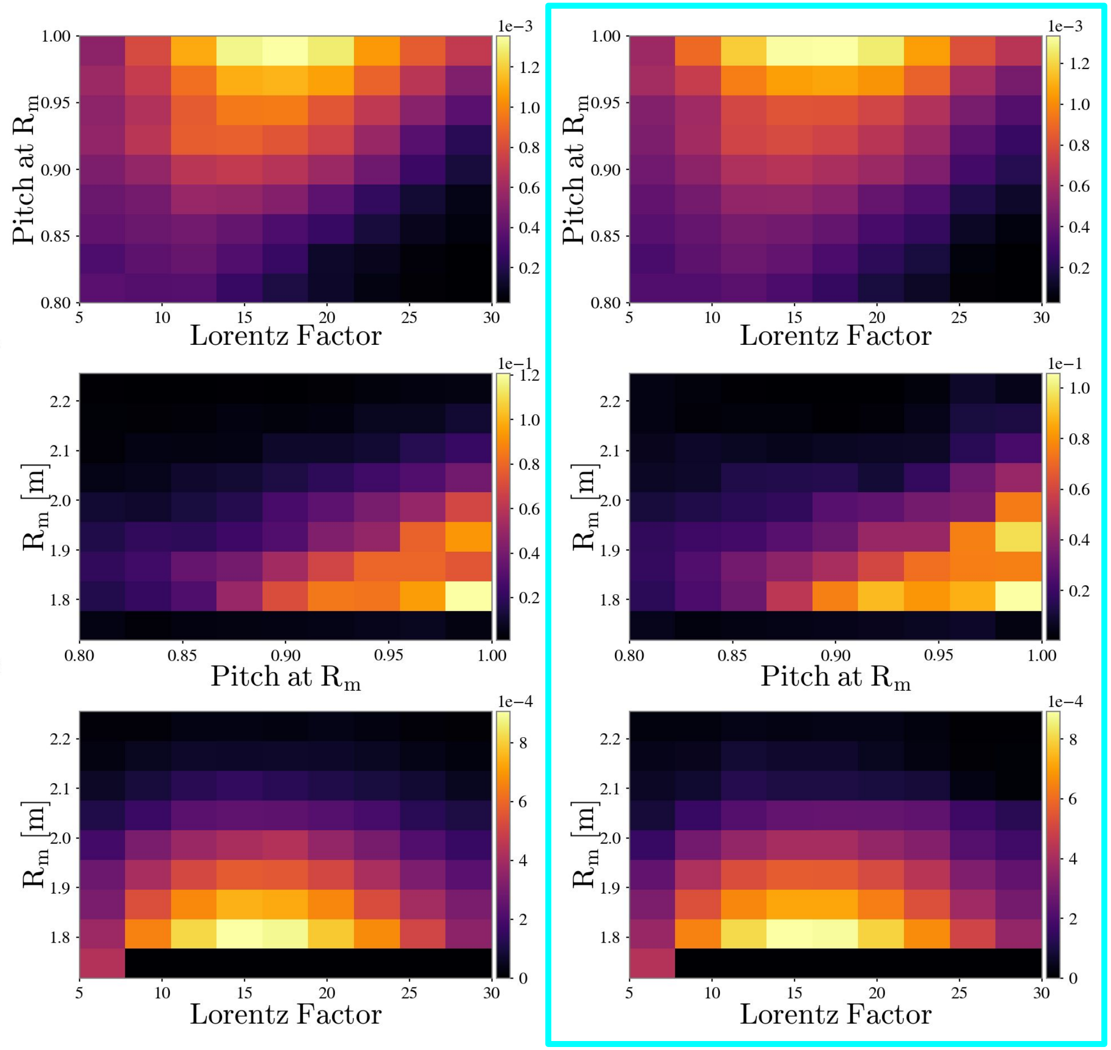
Integrated Signal



Gaussian Radial Dependency

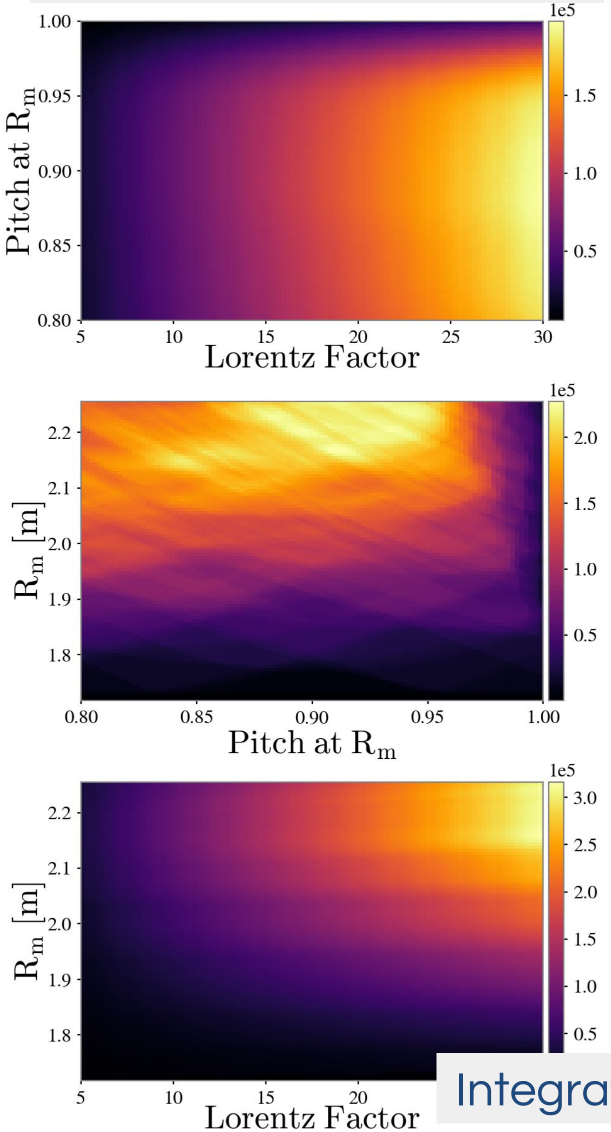
All Chords

2016 Experimental Chords



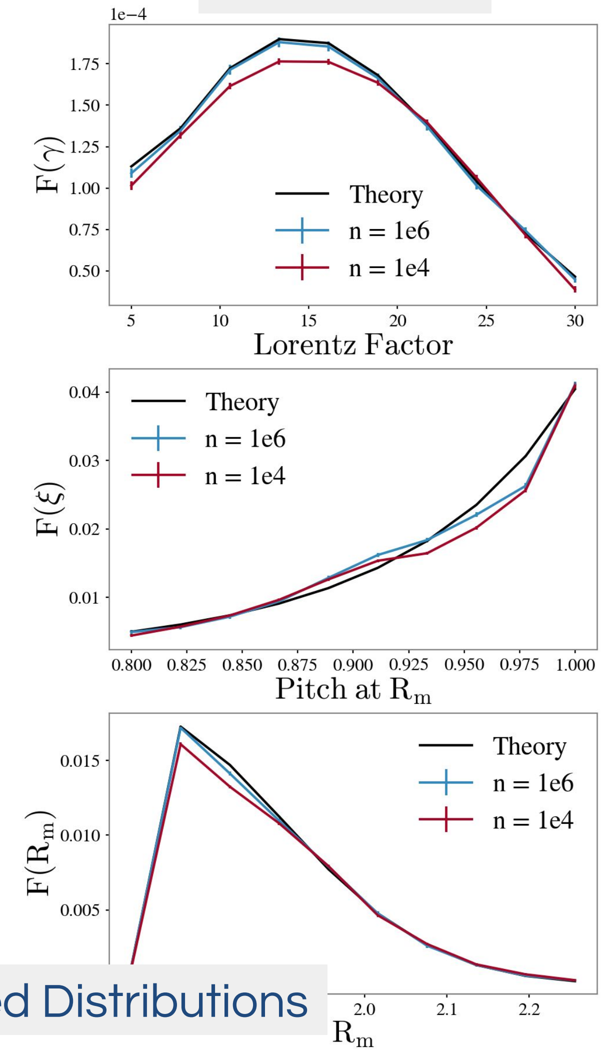
Bias in reconstructions roughly correspond to orbits with fewer measurements

All GRI Chords Coverage

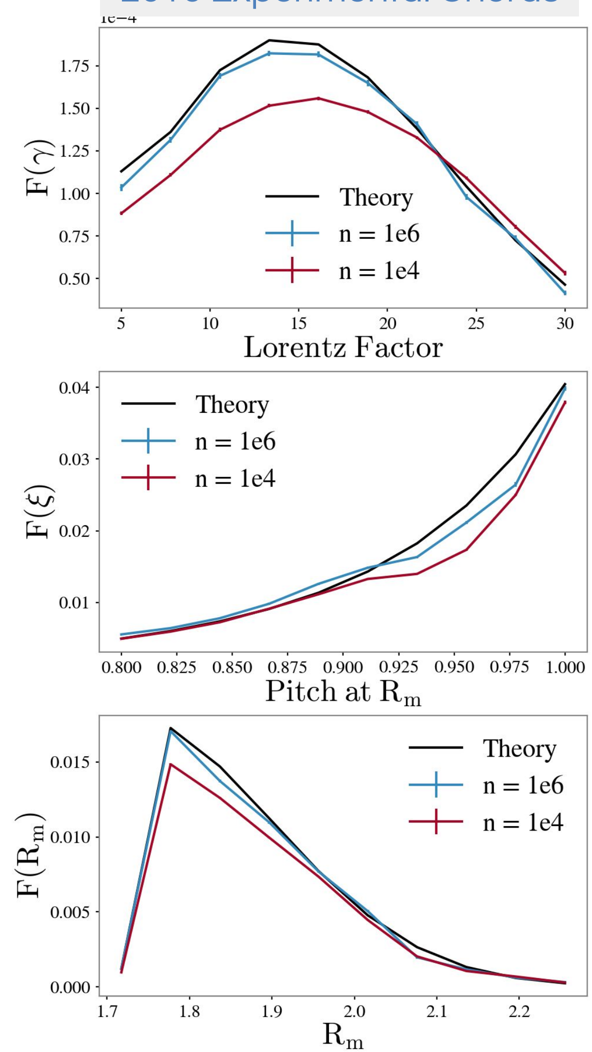


Increased noise exacerbates biases

All Chords



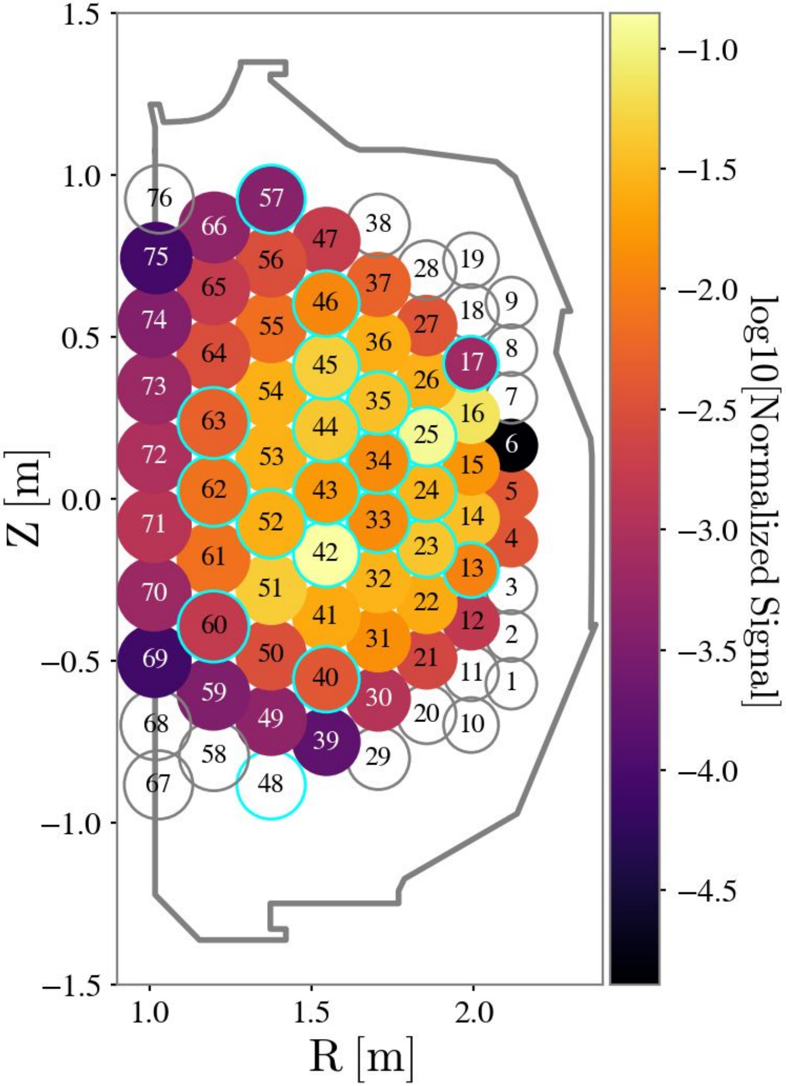
2016 Experimental Chords



Integrated Distributions

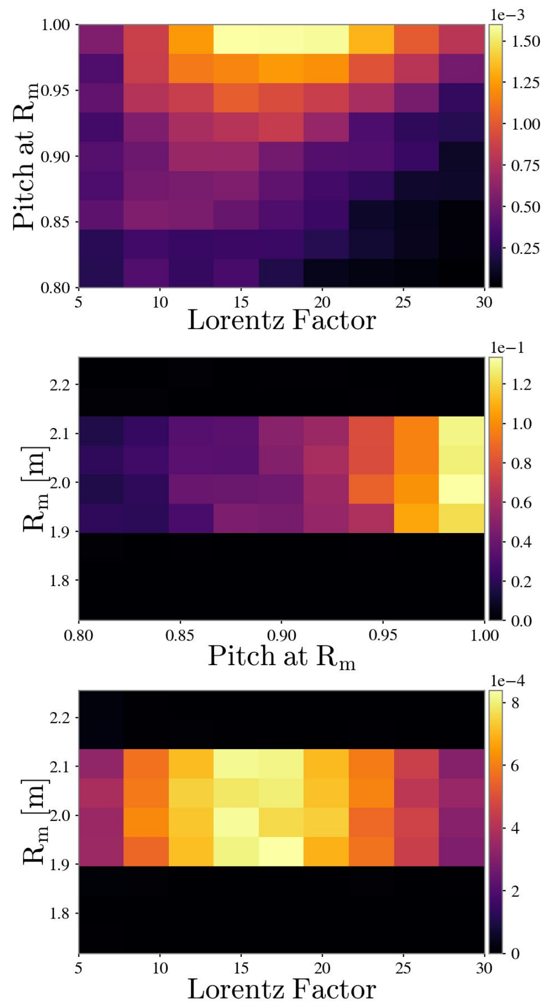
Reconstruction bias depends on runaway electron distribution function

Integrated Signal

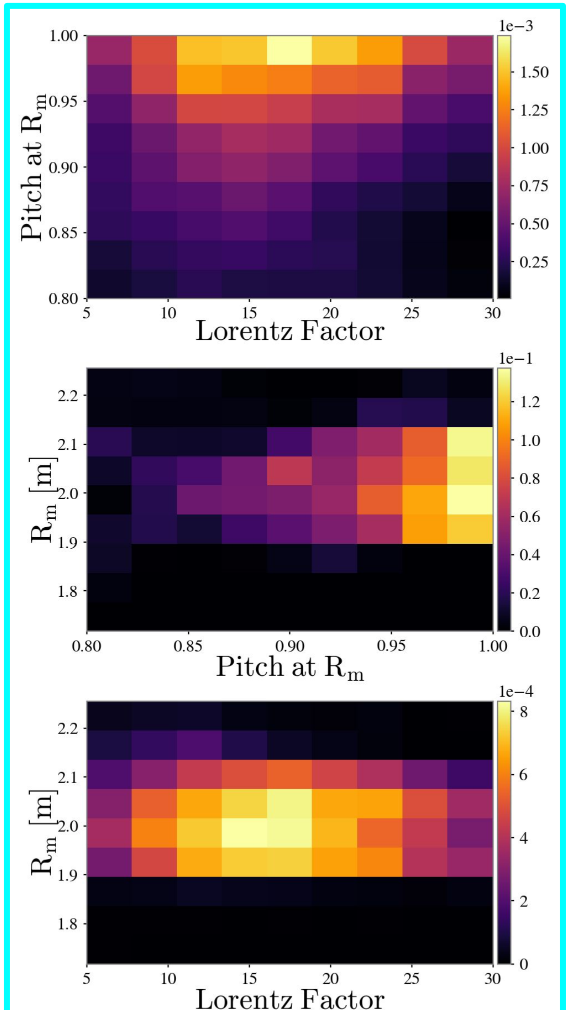


"Top Hat" Radial Dependency

All Chords



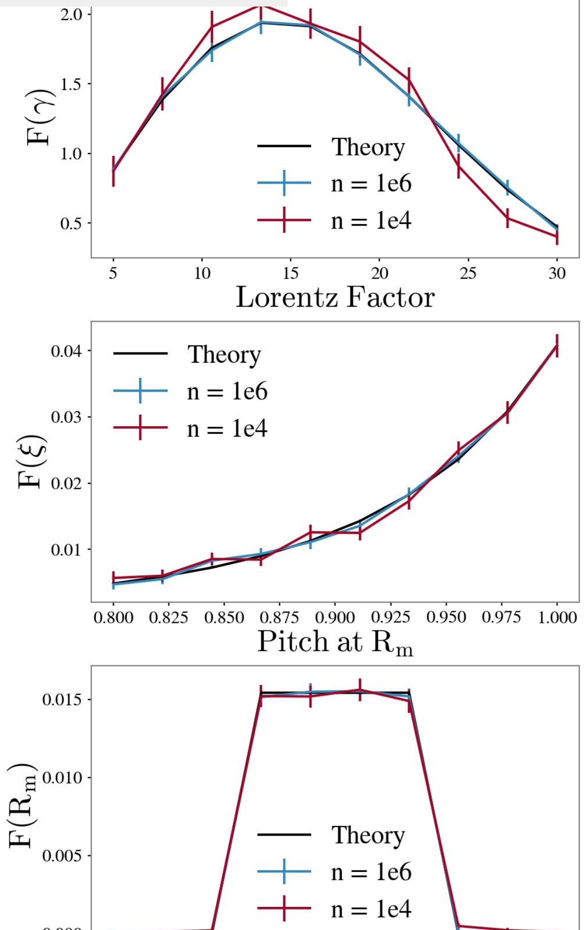
2016 Experimental Chords



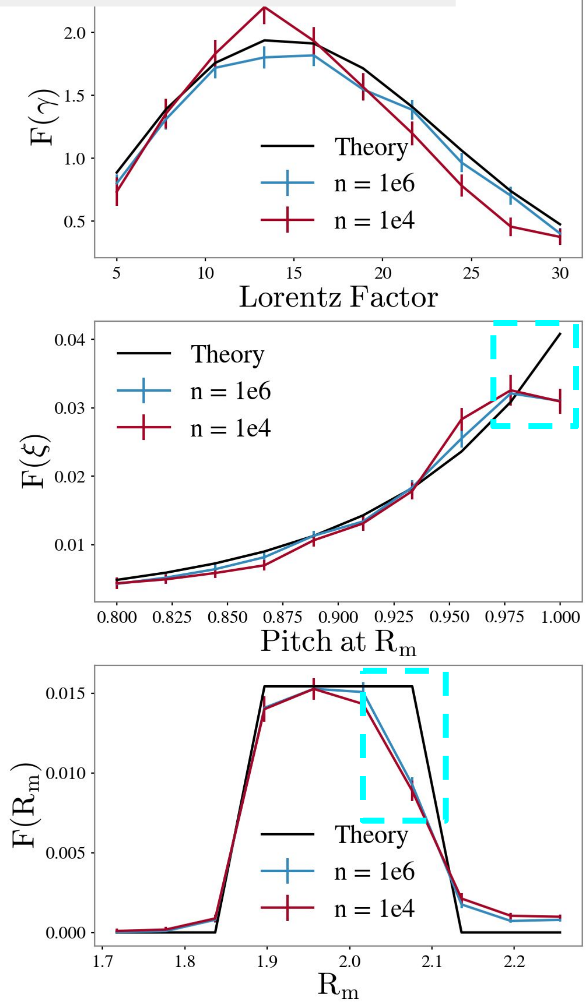
Reconstruction bias depends on runaway electron distribution function

A hole only present in the experimental coverage can be easily seen

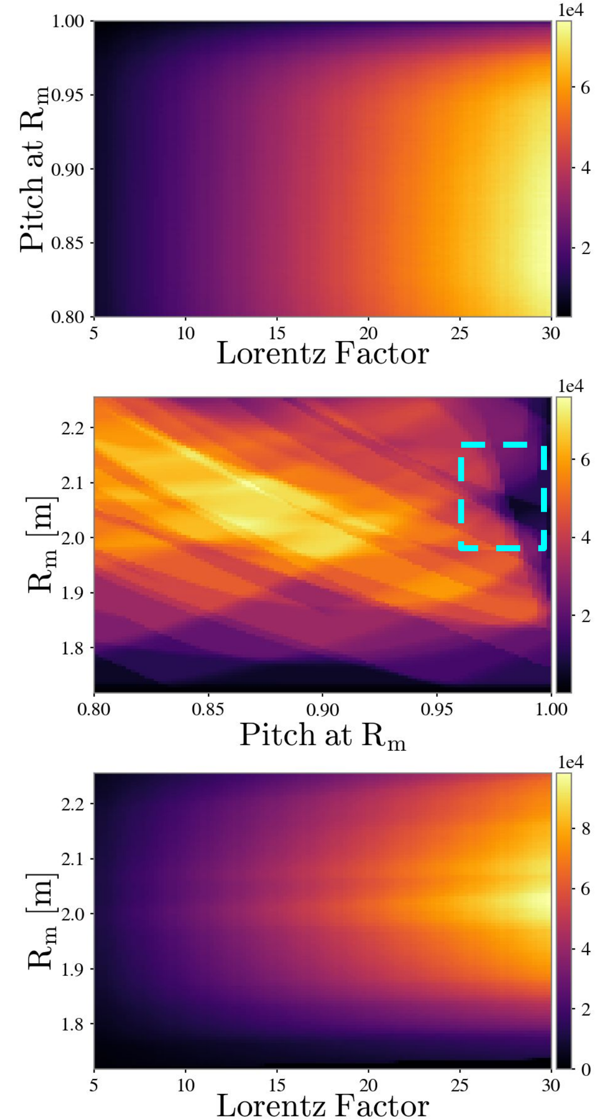
All Chords



2016 Experimental Chords



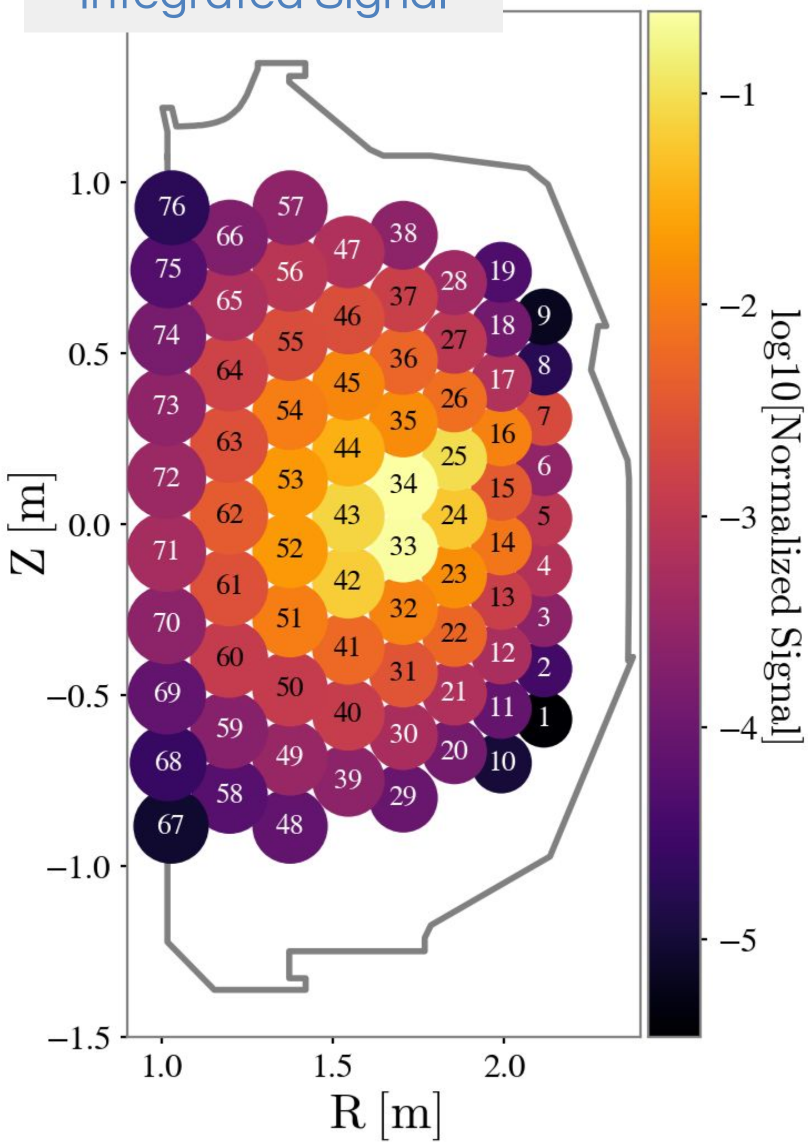
2016 Experimental Coverage



Integrated Distributions

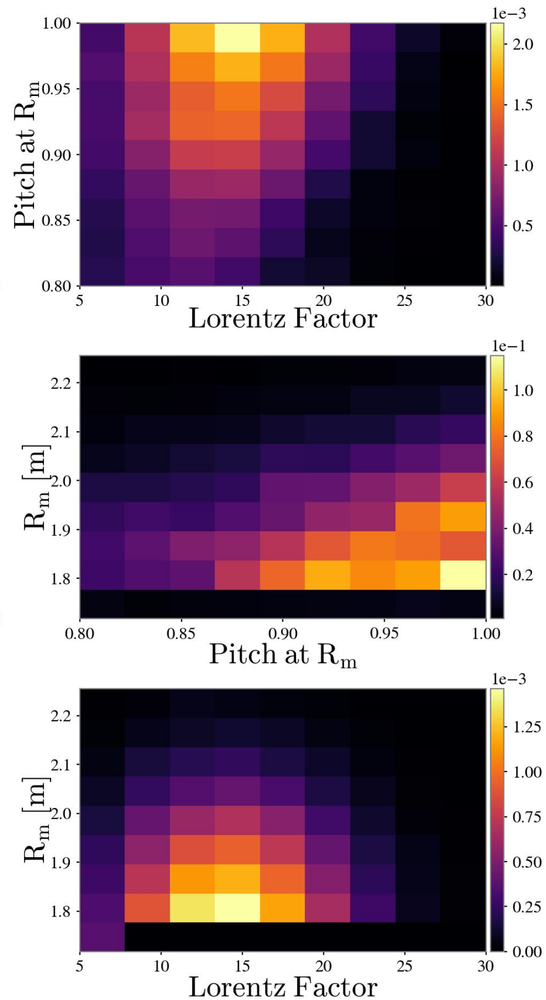
Bias in Lorentz Factor distribution primarily caused by sampling rather than poor phase-space coverage

Integrated Signal

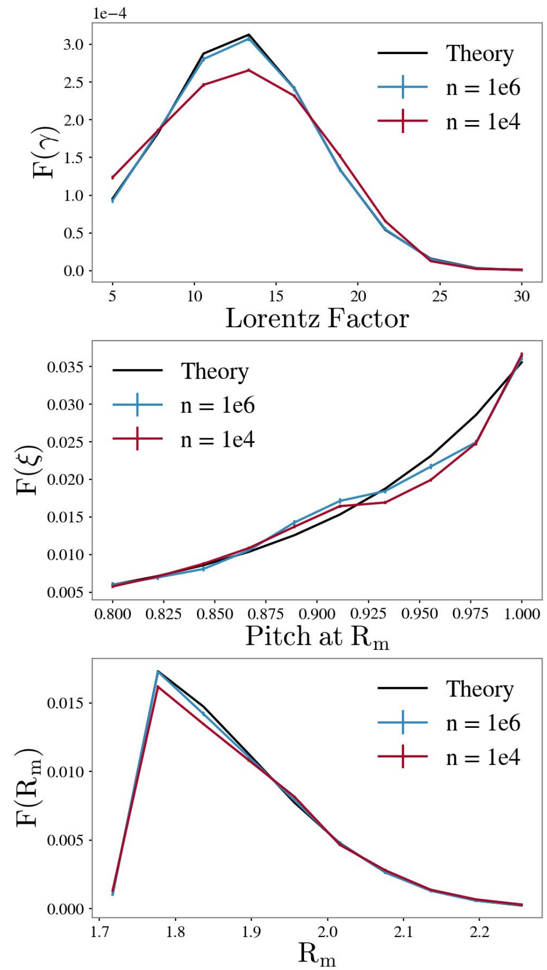


Shifting the distribution to lower Lorentz factor has little effect on bias on despite decreased coverage

All Chords

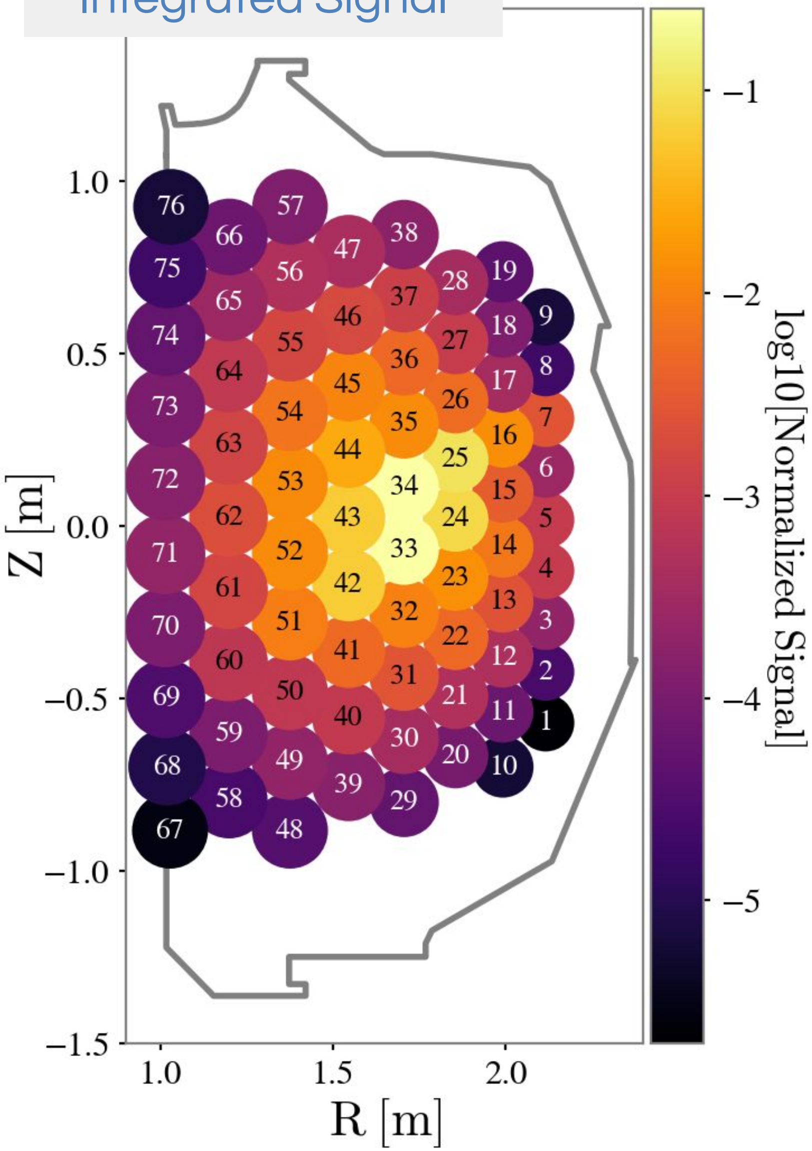


Integrated Distributions



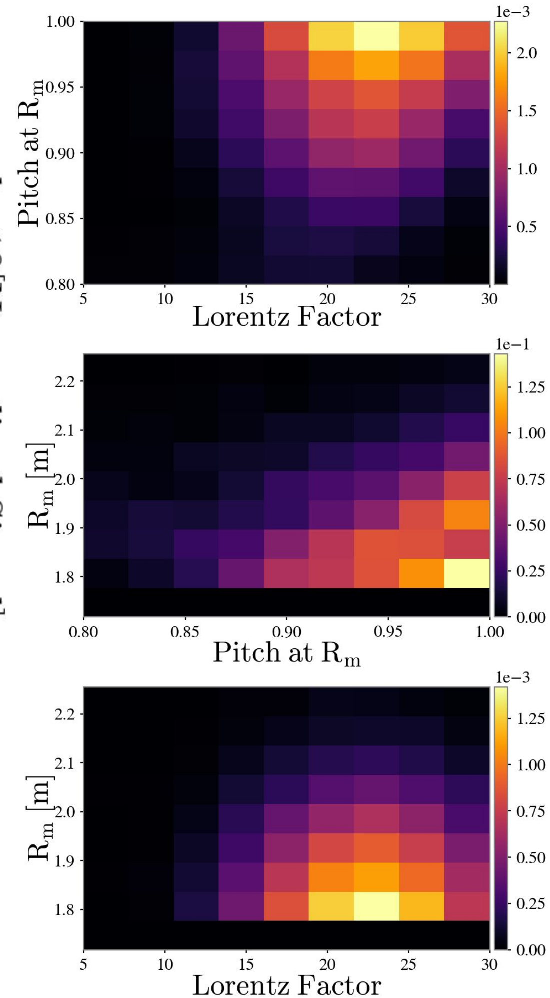
Distributions with large Lorentz Factors are more sensitive to sampling bias

Integrated Signal

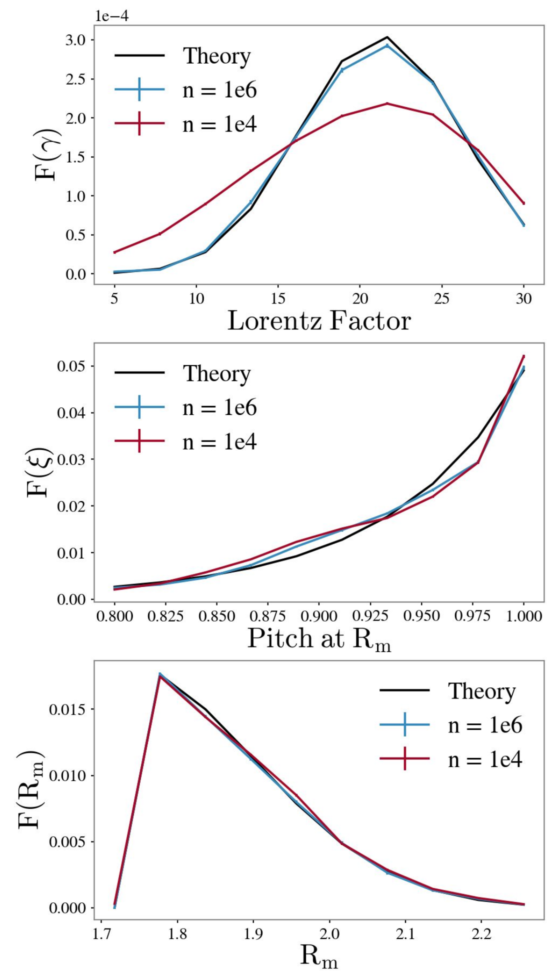


Shifting the distribution to larger Lorentz factor increases effect of sampling bias

All Chords



Integrated Distributions



Conclusions & Future Directions

Conclusions:

- ➢ Orbit Weight Functions can be used to infer the runaway electron distribution function
- ➢ Existing Gamma Ray Imager chords have sufficient coverage of the runaway electron orbit-space for Orbit Tomography
- ➢ Orbit Tomography can infer the total runaway electron distribution function
- ➢ Bias is the main source of error in the reconstructions

Future Directions:

- ➢ Doing Orbit Tomography with experimental data is forthcoming
- ➢ Apply similar analysis to the study of new Stacked Scintillator Detectors