Residual Zonal Flows for Non-Maxwellian Equilibrium Distribution Function

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Outline

- Introduction

- Residual Zonal Flows and Polarization Shielding

- $\alpha$-Particle Effects in ITER and Beyond

- Isotopic Dependence

- Conclusion
Zonal Flows in Magnetic Fusion Research

- Zonal flows regulate turbulence and transport.
- Turbulence in most cases produces zonal flows.
- Characteristics predicted by simulation and theory, have been confirmed from experiments.

T.S. Hahm et al., PPCF (2000) from GTC Simulation
D.K. Gupta et al., PRL (2006) from Tokamak (DIII-D)
A. Fujisawa et al., PRL (2004) from Stellarator (CHS)
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Residual Zonal Flows in Toroidal Geometry

- Based on gyro-Landau-fluid closure (up to mid 90’s), ZF is completely damped even in collisionless plasmas

- Rosenbluth-Hinton [PRL ‘98] ZF undamped from Gyrokinetic theory

\[ \text{RH residual level} : \frac{1}{1 + 1.63q^2 / \sqrt{\epsilon}} \]

- Gyrokinetic codes are now benchmarked against the analytic results!
- Most transport models don’t explicitly include zonal flows yet. (exceptions : M. Nunami et al, PoP (2013), E. Narita et al., PPCF (2018))
Nonlinear Gyrokinetics

[Frieman and Chen, Phys. Fluids (1982)]

Guiding Center Motion $\rightarrow$ (Drift kinetics) $B$

Gyro Center Motion $\rightarrow$ (Gyrokinetics) $B$

Point particle with magnetic moment $\mu$

Gyro Center Motion

Ring with radius $\rho$
magnetic moment $\mu$

• Drift wave turbulence
  – (Wave vector $k$) (gyroradius) $\sim O(1)$
  – Wave frequency $\omega \ll$ gyrofrequency $\Omega$

• Modern gyrokinetics via Lie-transform of phase-space Lagrangian keep intact the underlying symmetry and conservation laws $\rightarrow$ Gyrokinetics in firmer theoretical foundations.

• Gyrokinetic ion (+ drift kinetic or adiabatic electron), and **GK Maxwell’s equation**

\[ f = f(\vec{x}, \mu, v_\parallel) \]
Bounce Kinetics from the 2nd Adiabatic Invariant

* For $\omega << \Omega_\sigma$, and $\rho << L_B$ ,

$$\frac{d\mu}{dt} = 0 \quad \Rightarrow \quad \text{Gyrokinetics can be derived.}$$

"1st "

* For $\omega << \omega_b \sigma << \Omega_\sigma$, and $\Delta_{\text{Banana}} << L_B$ ,

The 2nd adiabatic invariant, $J = \oint d\ell v_\parallel$ is conserved.

Bounce(-averaged) Kinetic Equation can be derived.

* Evolution of "Banana-Center"

$$F_{B.C.}(\alpha, \beta; J, t)$$

- bounce-angle ignorable in addition to gyro-angle
- $J$ as a parameter in addition to $\mu$
Disparate Temporal Scales in Residual Zonal Flow Problem

Ⅰ. Quasi-neutrality: \(0 = n_e(\vec{x}) - n_i(\vec{x})\)

Ⅱ. Polarization Shielding (Finite Larmor Radius effect): from Gyrokinetics,

\[
t \gg \Omega_{ci}^{-1} \\
(\omega \ll \Omega_{ci})
\]

\[
\chi_{cl} \frac{e\Phi_{ZF}(0)}{T_i} = \frac{n_{i,gc}(\vec{x}) - n_e(\vec{x})}{n_0}
\]

Ⅲ. Neoclassical enhancement of polarization shielding (Finite Banana Orbit Width effect): from Bouncekinetics,

\[
t \gg \omega_{bi}^{-1} \\
(\omega \ll \omega_{bi})
\]

\[
(\chi_{Neo} + \chi_{cl}) \frac{e\Phi_{ZF}(\infty)}{T_i} = \frac{n_{i,bc}(\vec{x}) - n_e(\vec{x})}{n_0}
\]

Neoclassical Polarization Density from Bounce-kinetic Approach [Fong & Hahm, PoP (1999)]

- Generalized Polarization Shielding:

\[
\chi_{total} = \chi_{Neo} + \chi_{cl} \quad \text{(Polarizability; Susceptibility)}
\]
Residual Zonal Flows determined by the Polarization Shielding

- \( R_{ZF} = \frac{\phi_{ZF}(\infty)}{\phi_{ZF}(0)} = \frac{\chi_{cl}}{\chi_{Neo} + \chi_{cl}} \) - Rosenbluth and Hinton PRL (1998)

- Modern GK/BK provides a systematic procedure of its calculation

[\text{L. Wang & T.S. Hahm PoP 16 062309 (2009)}]
Generalized Polarization Shielding depends on $k_r$ of ZF and Orbit Width.

$$\left\langle \delta n \right\rangle_v = -\frac{q}{T} \sum_k \delta \phi_k e^{iS(r)} \chi_k^{\text{total}}$$

$$\chi_k^{\text{total}} = \chi_k^{\text{cl}} + \chi_k^{\text{nc}}, \quad \chi_k^{\text{cl}} = 1 - \Gamma_0(b)$$

- For trapped particles:

$$\chi_{k,nc}^{tr} \approx \left( \frac{2}{\pi} \right)^{3/2} \sqrt{\epsilon} \int_0^\infty dt^2 e^{-t^2} \frac{J_0^2(\sqrt{2k_r\rho_Tt})}{FLR} \int_0^1 d\kappa^2 K(\kappa) \left[ 1 - \frac{J_0^2(a\kappa)}{FOW} \right]$$

- For passing particles:

$$\chi_{k,nc}^{p} \approx \left( \frac{2}{\pi} \right)^{3/2} \sqrt{\epsilon} \int_0^\infty dt^2 e^{-t^2} \frac{J_0^2(\sqrt{2k_r\rho_Tt})}{FLR} \int_1^{\frac{\sqrt{1+\pi}}{2\kappa}} d\kappa K(\kappa^{-1}) \sum_{\sigma = \pm} \left( 1 - \left| \left\langle e^{i\sigma a \sqrt{\kappa^2 - \sin^2(\xi/2)}} \right\rangle_t \right|^2 \right)$$

(expressions for Maxwellian $F_0$)
Zonal Flow Spectra: ITG turbulence vs TEM turbulence

• Both cases exhibit broad $k_r$ spectra
• It extends further to higher $k_r$ region for TEM

from T.S. Hahm et al.
GTC simulation of **ITG**

from J.M. Kwon et al.
GKPSP simulation of **TEM**
Further Research Progress on Residual Zonal Flows

- Rosenbluth and Hinton dealt with long wavelength zonal flows only with $k_r \rho_{\theta i} \ll 1$

- Extensions to shorter wavelength regime

  P. Monreal et al., PPCF (2016)

- Modern gyrokinetic/bouncekinetic approach for all wavelength regime

- More accurate procedure outlined in

- Applications to
  - Isotopic dependence of confinement
    T.S. Hahm et al, NF (2013)
  - Impurity Effects
    W.X. Guo, L. Wang et al., NF(2017)
  - Effects of RMP on H-mode transition
    G.J. Choi and T.S. Hahm, NF(2018)

All considered Maxwellian $F_0$. 
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α-particle Effects on Confinement of Burning Plasmas

• α-particle effects on Alfvenic energetic particle modes:

• α-particle transport due to micro turbulence
  C. Angioni and A. Peeters, PoP (2008)

• α-particle effects on mean plasma rotation:
  (eg., α-particle’s large orbit loss) found to be insignificant for ITER:

⚠️ No previous works on α-particle effects on Residual Zonal flows
General Formulas of Polarizabilities for arbitrary \( F(Z) \)

Duthoit, Brizard and Hahm, PoP (2014)

- **Polarization Density**;
  \[
  n_{cl} = \frac{Ze|\delta\phi|}{m} \int_{0}^{\infty} \int_{-\infty}^{\infty} 2\pi d\nu_{\parallel} d\mu \left( 1 - J_{0}^{2}(k_{r}\rho_{i}) \right) \left( -\frac{\partial}{\partial \mu} \right) F(Z_{gy})
  \]

- **Neoclassical Polarization Density**;
  \[
  [n_{Neo}]_{\psi} = \frac{Ze|\delta\phi|}{m} B \int_{k_{i}}^{\kappa_{f}} \int_{0}^{\infty} 4\pi R_{\parallel} \omega_{\parallel} d\mu d\kappa \frac{\omega_{\parallel}}{\omega_{b,t}}
  \]
  \[
  \times J_{0}^{2}(k_{r}\rho_{i}) \left( 1 - \left| \langle e^{i\Delta\zeta} \rangle \right|_{b,t}^{2} \right) \left( -\frac{1}{\omega_{b,t} \partial J} \right) F(Z_{bg\gamma})
  \]

\( \rho_{i} \): particle’s Larmor radius \( \kappa \): pitch angle parameter

\( \kappa \in [0,1) \) for trapped particles,

\( \in \left[ 1, \frac{1+\epsilon}{2\epsilon} \right] \) for passing particles

\( R_{\parallel} = qR \): connection length \( \omega_{\parallel} = \frac{p_{\parallel}e}{2\sqrt{\kappa m R_{\parallel}}} \): characteristic parallel frequency

\( \omega_{b} \): bounce frequency \( \omega_{t} \): transit frequency
Asymptotic Analyses for Different Wavelength Regimes

Ⅰ. Long wavelength regime: \( k_r \rho_{\theta i} \ll 1 \)

Ⅱ. Intermediate wavelength regime: \( k_r \rho_i \lesssim 1 \lesssim k_r \rho_{\theta i} \)

Ⅲ. Short wavelength regime: \( 1 \ll k_r \rho_i \)

Integrations are possible in terms of various special functions for
Classical Polarization in the long wavelength limit is independent of $F(Z)$

- \[ \frac{n_{cl}}{n_0} = \chi_{cl} \frac{Z|e|\delta\phi}{T} \]

- \[ n_{cl}^{long} \simeq \frac{Z|e|\delta\phi}{T} \int_0^\infty \int_{-\infty}^\infty 2\pi d\nu_{\parallel} \frac{Bd\mu}{m} \left( \frac{1}{2} k_r \rho_i^2 \right) \left( -\frac{T}{B} \frac{\partial}{\partial \mu} \right) F(Z) \]

\[ = \left( k_r \rho_i^T \right)^2 n_0 \frac{Z|e|\delta\phi}{T} \]

→ Familiar expression for polarization density in gyrokinetic-Possion equation

- \[ \chi_{cl} = \left( k_r \rho_i^T \right)^2, \rho_i^T : \text{Larmor radius at thermal velocity} \]
Neo-Polarization in the long wavelength limit is identical for any isotropic $F(Z)$

- \[ [n_{Neo,b}]_\psi = Z|e|\delta \phi \frac{T^{1/2}}{m^{3/2}} \int_{0}^{\infty} \int_{0}^{1} 8\sqrt{\epsilon y}dkdy \alpha^2 (E(\kappa) - (1 - \kappa)K(\kappa)) \left(-\frac{\partial}{\partial y}\right)F(y) \]

\[ \approx 1.20\epsilon^{3/2} (k_r \rho_{\theta i}^T)^2 n_0 \frac{Z|e|\delta \phi}{T} \]

- \[ [n_{Neo,t}]_\psi = Z|e|\delta \phi \frac{T^{1/2}}{m^{3/2}} \int_{0}^{\infty} \int_{1}^{1+\epsilon/2\epsilon} 8\sqrt{\epsilon y}dkdy \alpha^2 \sqrt{\kappa} \left(E(\kappa^{-1}) - \frac{\pi^2}{4K(\kappa^{-1})}\right) \left(-\frac{\partial}{\partial y}\right)F(y) \]

\[ \approx 0.43\epsilon^{3/2} (k_r \rho_{\theta i}^T)^2 n_0 \frac{Z|e|\delta \phi}{T} \]

Here, \( y = \frac{E}{T} \approx \frac{\mu B_0}{T}, \quad \alpha = \sqrt{2\epsilon} k_r \rho_{\theta i} \)

- \[ \frac{n_{Neo}}{n_0} = \frac{n_{Neo,b} + n_{Neo,t}}{n_0} = 1.63\epsilon^{3/2} (k_r \rho_{\theta i}^T)^2 n_0 \frac{Z|e|\delta \phi}{T} \quad \text{(for any isotropic distribution!)} \]

R : Residual Zonal Flow;

\[ R_{ZF} = \frac{V_{EB}(t \to \infty)}{V_{EB}(t \to 0)} = \frac{n_{cl}}{n_{cl} + n_{Neo}} = \frac{\chi_{cl}}{\chi_{cl} + \chi_{Neo}} \approx \left(1 + 1.63 \frac{q^2}{\sqrt{\epsilon}}\right)^{-1}, \text{ the same as RH 98} \]

which has been derived using Maxwellian $F_0$. 

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We consider

1) Slowing Down distribution of $\alpha$-particles born at 3.5 MeV and background electrons with $T_e \sim 10\, keV$
2) Equivalent Maxwellian distribution of $\alpha$-particles with the same average kinetic energy
3) Reference case without $\alpha$-particles ($T_i = T_e \sim 10\, keV$)
4) Finally, 10% $\alpha$-particle concentration with background consisting of equal amount of D and T ($T_i = T_e \sim 10\, keV$)

Highlights of Results with Practical Interests

- $\alpha$-particles enhance residual zonal flows, effect is maximum at $k_r \rho_{i,eff} \sim 10^{-1}$ (for D 50%, T 50% mixture, $\rho_{i,eff} = \sum_a c m_a v_{Ta} / Z_a |e|B$).
- For 10% concentration, $\sim 10\%$ enhancement at $k_r \rho_{i,eff} \sim 10^{-1}$ is expected.
- Effects can be considerable for ITER, and significant for DEMO and reactors.
- $R_{ZF}(k_r \rho_{i,eff}) = \frac{\chi_{cl}}{\chi_{Neo} + \chi_{cl}}$
  i) $\chi_{cl}$ is a monotonically increasing in $k_r$, ($\sim \tanh$-like shape)
  Transition occurs at lower $k_r$ in the presence of energetic $\alpha$’s
  ($k_r \rho_{i,eff} \sim 10^{-1}, k_r \rho_{\alpha} \sim 1$)
  ii) $\chi_{nc}$ peaks at similar $k_r$ value and decreases as a function of $k_r$ for higher $k_r$.
  i, ii) $\Rightarrow$ $R_{ZF}$ is enhanced for $k_r \rho_{i,eff} \sim 10^{-1}$
Slowing Down Distribution Function

• \( F_{SD}(v) = \frac{n_\alpha}{4\pi v_c^3 A_2} \frac{H(v_\alpha - v)}{1 + (v/v_c)^3} \)

\( \frac{1}{2} m_\alpha v_\alpha^2 = 3.5 \text{MeV} \)

\( v_c^3 = 3 \sqrt{\frac{\pi m_e}{2 m_\alpha}} Z_{eff} v_{th,e}^3 \) : slowing down critical velocity

• \( n_\alpha \bar{E}_{SD} = \frac{1}{2} \int m_\alpha v^2 F_{SD}(v) d^3v \equiv \frac{3}{2} n_\alpha T_{SD} = \frac{A_4}{2A_2} n_\alpha T_c \)

\( A_n \left( \frac{v_\alpha}{v_c} \right) = \int_0^{v_\alpha/v_c} \frac{x^n}{1 + x^3} dx \) : appear in various expressions.

\( T_c = m_\alpha v_c^2 \)
Classical Polarization in the short wavelength limit

\[ \frac{n_{cl}^{short}}{n_0} \approx \frac{Ze|\delta\phi|}{T} \frac{A_4}{3A_2} \left[ \frac{A_0}{A_2} - \frac{1}{2A_2 k_r \rho_c} \right] \]

- \( \rho_c \): Larmor radius with critical velocity \( v_c \)
- Asymptotic value > 1, for \( T_e \lesssim 16 \text{keV} \), < 1 for \( T_e \gtrsim 16 \text{keV} \)
  and, = 1 for Boltzmann (adiabatic) response for Maxwellian \( F_0 \propto \left( 1 - \Gamma_0(b_i) \right) \)
- Asymptotic behaviors are similar for cases considered, but transition occurs at different \( k_r \) value.

"1 - \( \Gamma_0(k_r^2 \rho_i^2) \) "

for Maxwellian

for \( T_e = 10 \text{keV} \)
$\chi_{Neo}$ peaks when Finite Orbit Width enhancement and FLR-reduction balance

\[ n_{Neo,b} = Z |e| \delta \phi \frac{\sqrt{eT}}{m^{3/2}} \int_0^\infty \int_1^1 2\sqrt{2\pi}dkdx \frac{\omega_\parallel}{\omega_b} xJ_0^2(k_r \rho_i^T x) \left\{ 1 - J_0^2(\alpha a_1(\kappa)) \right\} \left( -\frac{\partial}{\partial x} \right) F(x) \]
\[ n_{Neo,t} = Z |e| \delta \phi \frac{\sqrt{eT}}{m^{3/2}} \int_0^\infty \int_1^{(1+\epsilon)/2\epsilon} 2\sqrt{2\pi}dkdx \frac{\omega_\parallel}{\omega_t} xJ_0^2(k_r \rho_i^T x) \left( in\ green \right) : decreasing\ in\ k_r \]
\[ \times \left\{ 1 - J_0^2(\alpha b_2(\kappa)) \right\} \left( -\frac{\partial}{\partial x} \right) F(x) \left( in\ red \right) : increasing\ in\ k_r \]

where $a_1(\kappa) = 2 \frac{\omega_b}{\omega_\parallel} \text{sech} \left[ 0.5\pi \frac{K(1-\kappa)}{K(\kappa)} \right]$  

$b_2(\kappa) = \frac{\omega_t}{\omega_\parallel} \text{sech} \left[ \pi \frac{K(1-\kappa^{-1})}{K(\kappa^{-1})} \right]$  

$q = 2.0$  

$\epsilon = 0.1$  

$T_e = 10keV$
$\alpha$-Particle Effects can be considerable in ITER

- $10\%$ enhancement at $k_r \rho_{i,\text{eff}} \sim 10^{-1}$.

$q = 2.0$
$\epsilon = 0.1$
$T_e = 10\text{keV}$
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Isotopic Dependence of Fine-scale Residual Zonal Flow

[T.S. Hahm, L. Wang, W.X. Wang et al., NF 53, 072002 (2013)]

- For the same temperature, fast increasing of residual Zonal Flow for D plasmas starts at lower $q_r$ than that of H plasma.
Isotopic Dependence is appreciable at $q_r \rho_H \sim 0.5$

[T.S. Hahm, L. Wang, W.X. Wang et al., NF 53, 072002 (2013)]

- Residual Zonal Flow for D is higher than that for H for $q_r \rho_H$ around 0.5

---→ This mechanism works better for moderately short wavelength drift wave turbulence (eg. TEM)
Trapped Electron Mode (TEM)

- Nonlinear theory [Similon-Diamond, PF ‘84]  
  \[ \tau_E \sim n_e a R^2 \]

  Derived with no hint from gyrokinetic simulations, or movies

- But, C-Mod validation using TEM-ITG was initially unsatisfactory for electron thermal transport [Lin et al., Phys. Plasmas ‘09]

- Consideration of impurities improved the comparison, [M. Porkolab, private communications ‘12].

- Fine scale \((k_r \rho_i \sim 0.5)\) ZF can be strong

  [Lu Wang and Hahm, PoP ‘09; Xiao et al., PRL ‘09; W.X. Wang et al., PoP ‘10].
Isotopic Dependence of Zonal Flow from Gyrokinetic Simulations

- “Nonlinear Gyrokinetic Simulations exhibit isotopic dependence”
  [J. Garcia et al., Nucl. Fusion 57, 014007 (2017)]

- Zonal Flow structures
Experimental Evidence for Isotopic Dependence of ZF?


Long range correlation in toroidal direction decreases as H/D content increases from TEXTOR

Also, B. Liu et al., [Nucl. Fusion, 55 112002 (2015)] from TJ-II.
Conclusions I.

- $\alpha$-particles enhance residual zonal flows with $k_r \rho_{i,eff} \sim 10^{-1}$.
- For 10% concentration, $\sim 10\%$ enhancement at $k_r \rho_{i,eff} \sim 10^{-1}$ is expected.
- So effects can be considerable for ITER, and significant for DEMO and reactors.

- $R_{ZF}(k_r \rho_{i,eff}) = \frac{\chi_{cl}}{\chi_{Neo} + \chi_{cl}}$
  
i) $\chi_{cl}$ is a monotonically increasing in $k_r$, ($\sim tanh$-like shape)
  Transition occurs at lower $k_r$ in the presence of energetic $\alpha$’s
  
  $(k_r \rho_{i,eff} \sim 10^{-1}, k_r \rho_\alpha \sim 1)$
  
  ii) $\chi_{nc}$ peaks at similar $k_r$ value and decreases as a function of $k_r$ for
  higher $k_r$.
  
i, ii) $\Rightarrow R_{ZF}$ is enhanced for $k_r \rho_{i,eff} \sim 10^{-1}$
Conclusions II.

• Underlying physical mechanism behind the isotopic dependence of confinement is not known yet.

• We studied isotopic dependence of fine scale residual Zonal Flows by using our generalized polarization shielding formula.

• We found stronger residual ZF for D plasma than that for H plasma for $q_r \rho_H$ around 0.5

• This can possibly lead to lower turbulence and transport and better confinement of D plasmas than H plasmas, in qualitative agreement with experimental results. [T.S. Hahm et al., NF 53, 072002 (2013)]

• Fine-resolution nonlinear gyrokinetic simulations and fluctuation measurements from experiments addressing this mechanism are on-going.
Zonal Flow, an example of meso-scale structure, which is far from the system size, is self-sustained.

- New Review Paper on Meso-scale Physics: