Kinetic Aspects of High-Z Pellet Modeling for Disruption Mitigation

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A. K. Fontanilla and B. N. Breizman, *Heating and Ablation of High-Z Cryogenic Pellets in High Temperature Plasmas,* Nucl. Fusion **59**, 096033 (2019).

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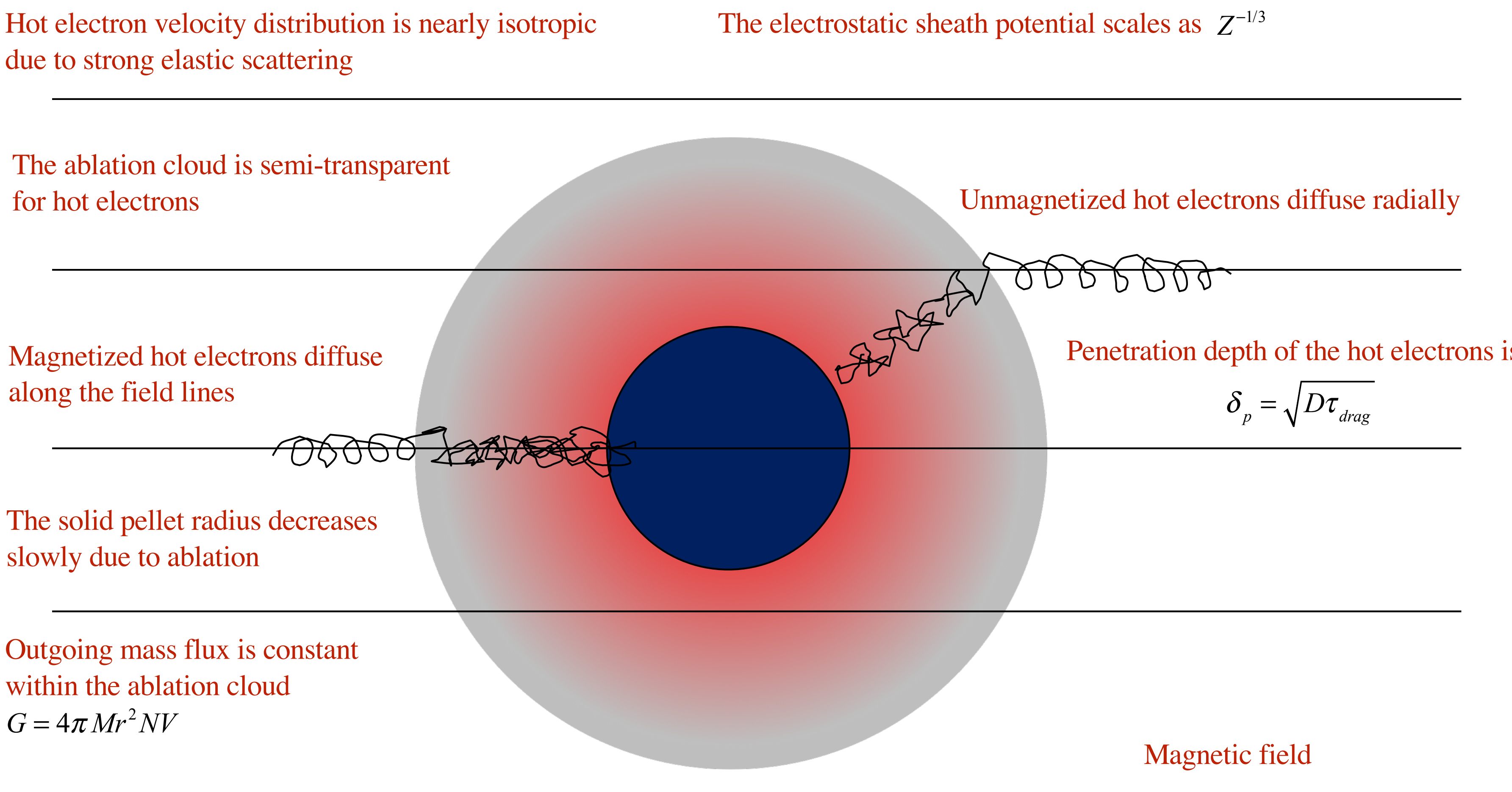
Characteristic features of the problem

- Pellets offer more controllable penetration of the injected material than MGI
- Hot plasma electrons heat the pellet surface. The ablated gas throttles the electron heat flux to the surface until 3D expansion makes the gas shield semi-transparent
- Collisional scattering and slowing down of the hot electrons in the pellet material need a kinetic description with the electron gyromotion included
- Strong backscattering of the hot electrons reduces the electrostatic sheath potential
- Elastic scattering reduces the hot electron penetration depth and the resulting ablation rate

Schematic of high-Z pellet ablation



Strong backscattering reduces electrostatic sheath

- Hot electron flux into the pellet scales as $j_{hot} \sim \frac{1}{\sqrt{Z}} n_\infty \sqrt{\frac{T_\infty}{m}}$

- Return flux of the emitted cold electrons satisfies the Child-Langmuir law: $j_{cold} \sim \left(\frac{e\varphi}{m}\right)^{3/2} \frac{m}{e^2 d^2}$

- The sheath width is roughly the Debye length: $d^2 \sim \frac{T_\infty}{n_\infty e^2}$

- The return cold flux balanced the hot particle flux, which gives

$$e\varphi \sim \frac{T_\infty}{Z^{1/3}} << T_\infty$$

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Ablation scenario revision for high-Z pellets

- Hot electron diffuse into the pellet until they slow down due to electron drag. Their penetration depth is $\delta_p \sim \frac{1}{Z\sqrt{Z}} \frac{1}{N_p \sigma_{ee}} \sqrt{\ln\Lambda_{ee} / \ln\Lambda_{ei}}$. Strong elastic scattering reduces the penetration depth and the resulting ablation rate.

- The heated surface layer of the pellet expands radially and becomes semi-transparent when it broadens to pellet radius. The flow is roughly sonic at this point ($MV_*^2 \sim T_*$).

- When the flow becomes sonic, the semi-transparent cloud is heated to a temperature

$$T_* \sim ZT_\infty n_\infty \sqrt{\frac{T_\infty}{m}} \sigma_{ee} \frac{R_p}{V_*} \qquad \Rightarrow \qquad V_*^3 \sim \frac{R_p}{M} \frac{Zn_\infty \ln\Lambda_{ee}}{T_\infty^{1/2}}$$

- Half of the incoming heat flux is absorbed in the cloud : $NR_p \sim N_p \delta_p$

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Flow velocity, cloud density and ablation rate

- Flow velocity for sonic expansion:

$$V_* \sim \left(Z T_\infty n_\infty \sqrt{\frac{T_\infty}{m}} \sigma_{ee} \frac{R_p}{M} \right)^{1/3}$$

- Density of semi-transparent ablation cloud:

$$N_* \sim N_p \frac{\delta_p}{R_p} \sim N_p \frac{\delta_p}{R_p} \sim \frac{1}{Z\sqrt{Z}} \sqrt{\frac{\ln \Lambda_{ee}}{\ln \Lambda_{ei}}} \frac{1}{R_p \sigma_{ee}}$$

- Ablation rate estimate:

$$G \sim 4\pi M R_p^2 N_* V_* \sim 4\pi M R_p \frac{1}{Z\sqrt{Z}} \sqrt{\frac{\ln \Lambda_{ee}}{\ln \Lambda_{ei}}} \frac{1}{\sigma_{ee}} \left(\frac{R_p}{M} Z T_\infty n_\infty \sigma_{ee} \sqrt{\frac{T_\infty}{m}} \right)^{1/3}$$

$$G \sim \frac{4\pi M}{\left(\pi e^4\right)^{2/3}} \left(\frac{1}{m^{1/2} M} \right)^{1/3} R_p^{4/3} \frac{n_\infty^{1/3} T_\infty^{11/6}}{Z^{7/6} \left(\ln \Lambda_{ei}\right)^{1/2} \left(\ln \Lambda_{ee}\right)^{1/6}}$$

Kinetic heating calculation

- Kinetic equation for hot electrons:

$$div\left(\mathbf{u}f\right)-\frac{1}{u^2}\frac{\partial}{\partial u}u^3\nu_{ee}f+\omega_c\frac{\partial f}{\partial\psi}=\frac{\nu_{ei}}{2\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial f}{\partial\theta}+\frac{\nu_{ei}}{2\sin^2\theta}\frac{\partial^2 f}{\partial\psi^2} \qquad \nu_{ei}=\nu_{ee}\frac{Z\ln\Lambda_{ei}}{\ln\Lambda_{ee}}$$

- The hot electron distribution is nearly isotropic in the high-Z case, i.e. $f\left(\mathbf{r},\mathbf{u}\right)=F\left(\mathbf{r},u\right)+O(1/Z)$.

- The kinetic equation reduces to an axisymmetric diffusion- type equation for the isotropic distribution:

$$-\frac{1}{3}\frac{\partial}{\partial z}\frac{u^2}{\nu_{ei}}\frac{\partial F}{\partial z}-\frac{1}{3\rho}\frac{\partial}{\partial\rho}\rho\frac{\nu_{ei}}{\omega_c^2+\nu_{ei}^2}\frac{\partial F}{\partial\rho}-\frac{1}{u^2}\frac{\partial}{\partial u}u^3\nu_{ee}F=0$$

- Power deposition by hot electrons per unit volume:

$$Q=\frac{16\pi^2e^4ZN\ln\Lambda_{ee}}{m}\int F\,du^2$$

Two reductions of the kinetic equation

- Strongly magnetized hot electrons: $\omega_c >> \nu_{ei}$

$$\frac{\partial F}{\partial \tau} + \frac{\partial^2 F}{\partial \xi^2} = 0$$

- Unmagnetized hot electrons: $\omega_c << \nu_{ei}$

$$\frac{\partial F}{\partial \tau} + \frac{1}{r^2}\frac{\partial}{\partial \xi} r^2 \frac{\partial F}{\partial \xi} = 0$$

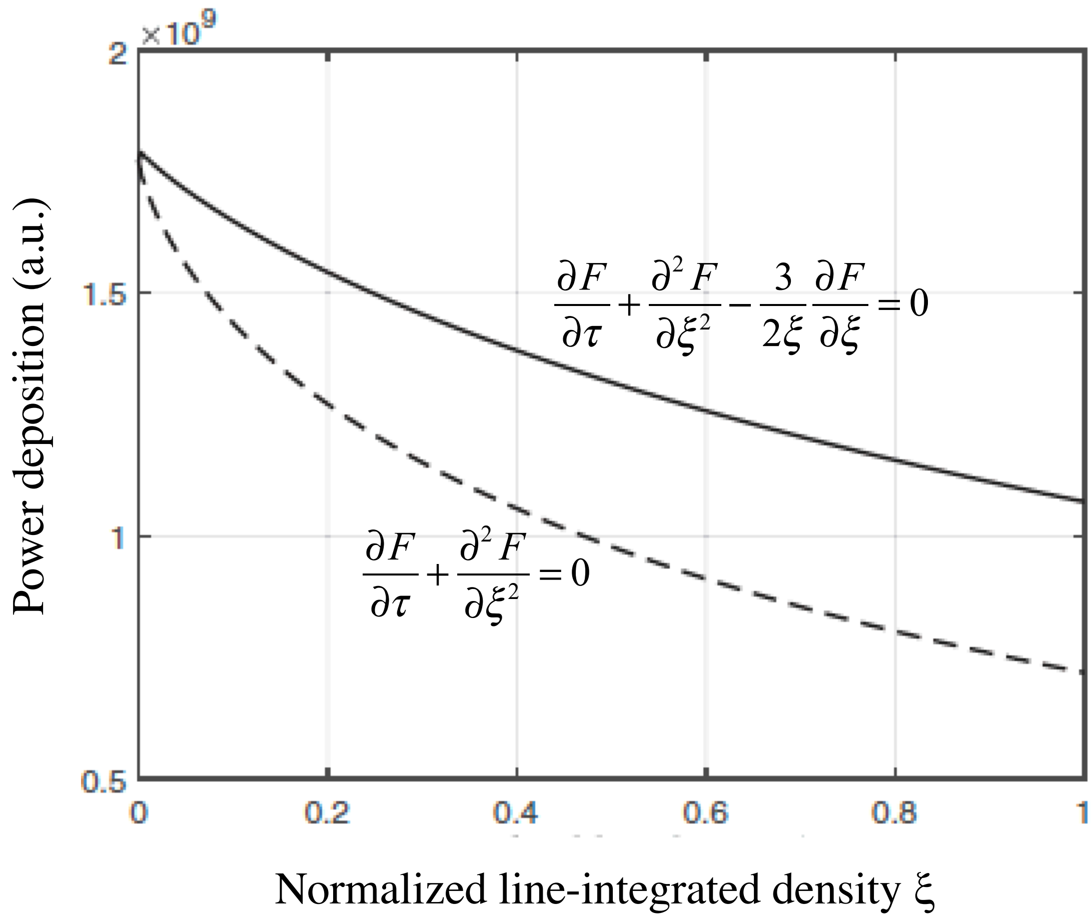
- Normalized line-integrated density:

$$\xi = \frac{8\pi Z e^4}{T_\infty^2}\left(3Z \ln\Lambda_{ee} \ln\Lambda_{ei}\right)^{1/2} \int_z^\infty N(z')\,dz'$$

- Normalized time-like velocity variable:

$$\tau = \frac{1}{8}\left(u\sqrt{\frac{m}{T_\infty}}\right)^8$$

Comparison of power deposition profiles for unmagnetized and magnetized hot electrons



We use an asymptotic expression for the cloud density to express r in terms of ξ in the diffusion equation:

$$N \sim r^{-7/3} \quad \Rightarrow \quad r^2 \sim \xi^{-3/2}$$

Gas flow modeling

- We use the fluid model from [Parks and Turnbull 1978], but with a kinetic calculation of the power deposition Q
- Flow velocity at subsonic-supersonic transition ($\mu = 1$): $V_*^2 = 2\pi R_*^3 \dfrac{(\gamma-1)Q_*}{G}$

$$G = 4\pi MR^2NV = const$$
$$MNV\frac{dV}{dR} + \frac{d}{dR}NT = 0$$
$$\frac{G}{4\pi R^2}\frac{d}{dR}\left(\frac{\gamma T/M}{\gamma-1} + \frac{V^2}{2}\right) = Q$$

$$N = G/(4\pi R^2 V)$$
$$\left(\mu^2-1\right)\frac{dV}{dR} = \frac{2V}{R}\left[1 - 2\pi R^3\frac{(\gamma-1)\mu^2}{2V^2G}Q\right]$$
$$\left(\mu^2-1\right)\frac{d\mu}{dR} = \frac{\mu\left(\gamma\mu^2-1\right)}{R}\left[1 - 2\pi R^3\frac{(\gamma-1)\mu^2}{V^2G}Q\right] - \frac{\mu\left(\mu^2-1\right)}{R}$$

- Spherical expansion
- Cloud remains neutral with a constant adiabatic index γ
- Surface boundary condition implies negligible sublimation energy
- Radiative losses not accounted for
- Ignore the electrostatic sheath which scales as $Z^{-1/3}$

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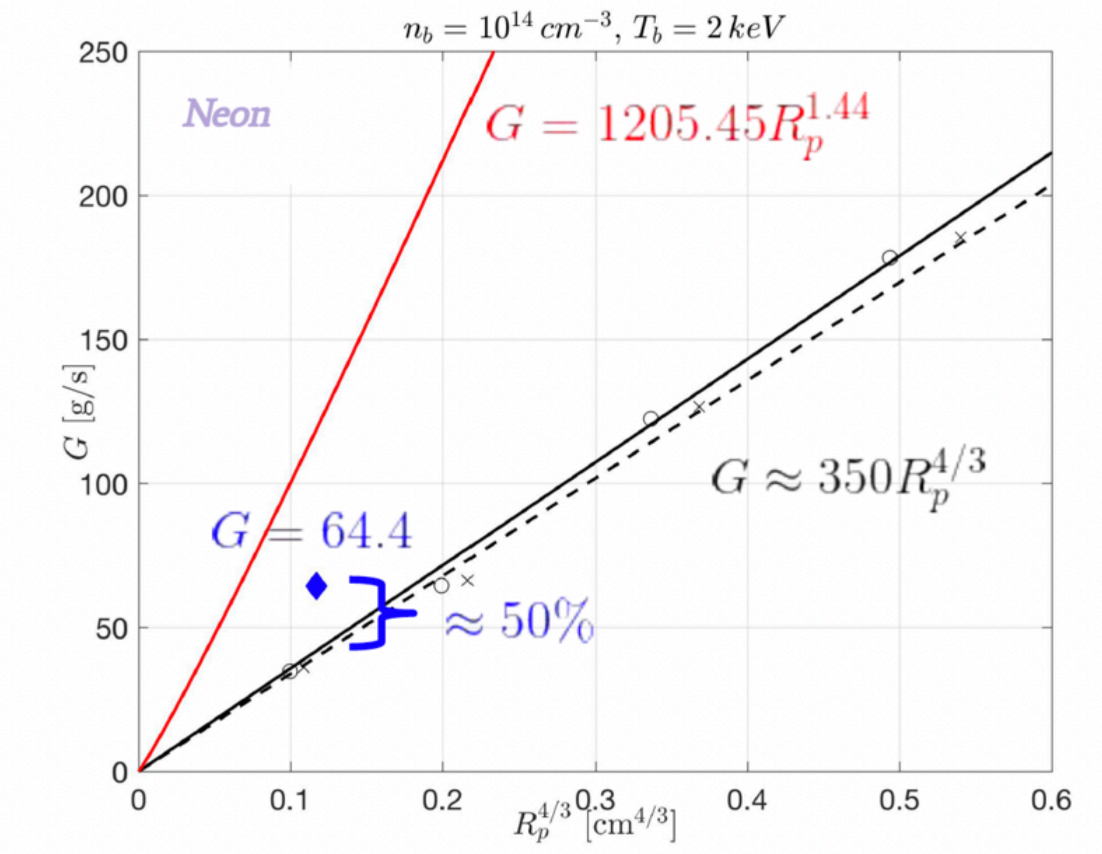
Reduction of ablation rate due to elastic scattering



X's and dashed: $\frac{\partial F}{\partial \tau} + \frac{\partial^2 F}{\partial \xi^2} = 0$

O's and solid: $\frac{\partial F}{\partial \tau} + \frac{\partial^2 F}{\partial \xi^2} - \frac{3}{2\xi}\frac{\partial F}{\partial \xi} = 0$

[R. Samulyak and P. Parks 2019]

[Sergeev et al 2006]

- Predictions from [Sergeev et al. 2006] are too high.
- Difference between our results and [Parks & Samulyak] shows significant sensitivity of the heat deposition to elastic scattering.
- All ablation rates agree well in pellet radius scaling.

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Summary

- The first principle kinetic calculation of the heat deposition gives a noticeably lower ablation rate for the high-Z pellets than the pre-existing estimates
- Strong elastic scattering of the incident electrons reduces the role of electrostatic shielding
- Magnetization of the incident electrons can modify the heat deposition geometry significantly
- Kinetic calculations of the heat deposition provide an updated input for fluid simulations of the pellet ablation process