

Quasi-periodic frequency sweeping in electron cyclotron emission of mirror-confined plasma sustained by high-power microwaves

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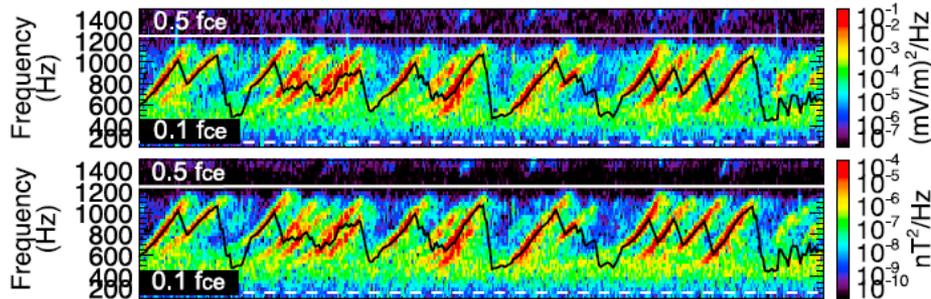
16th Technical Meeting on Energetic Particles in Magnetic Confinement Systems —
Theory of Plasma Instabilities, 3-6 September 2019, Shizuoka City, Japan

Outline

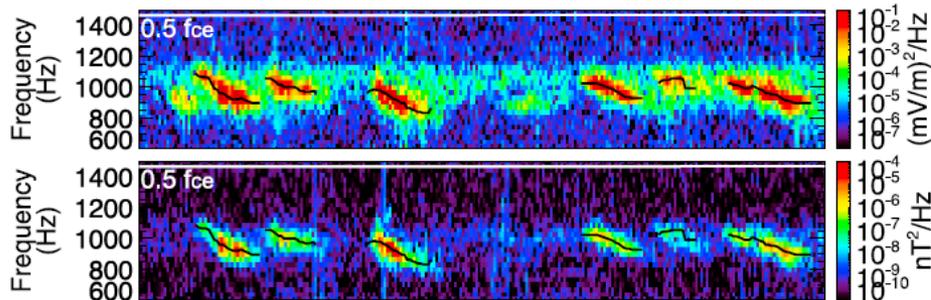
- Motivation
- Dedicated experiments on ECRH-driven instabilities
- Application of quasi-linear model to many observed features of stimulated emission in ECR plasma
- Beyond the quasi-linear theory
- SMIS-37 experimental facility
- Chirping microwave emission during plasma decay stage
- Application of the Berk-Breizman model to the experimental data
- Summary

Plasma instabilities due to fast particles

Chorus emissions in the magnetosphere of the Earth THEMIS D spacecraft (raising tone)



THEMIS E spacecraft (falling tone)

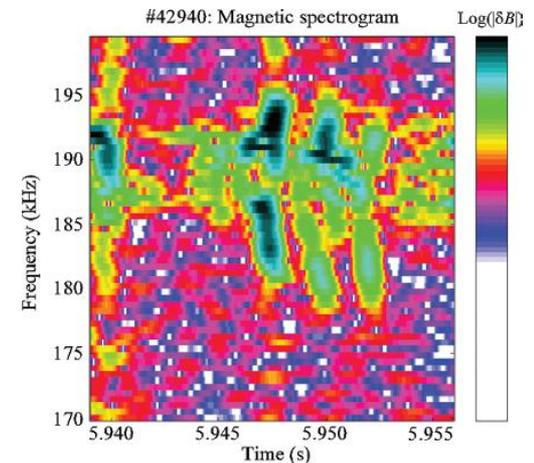
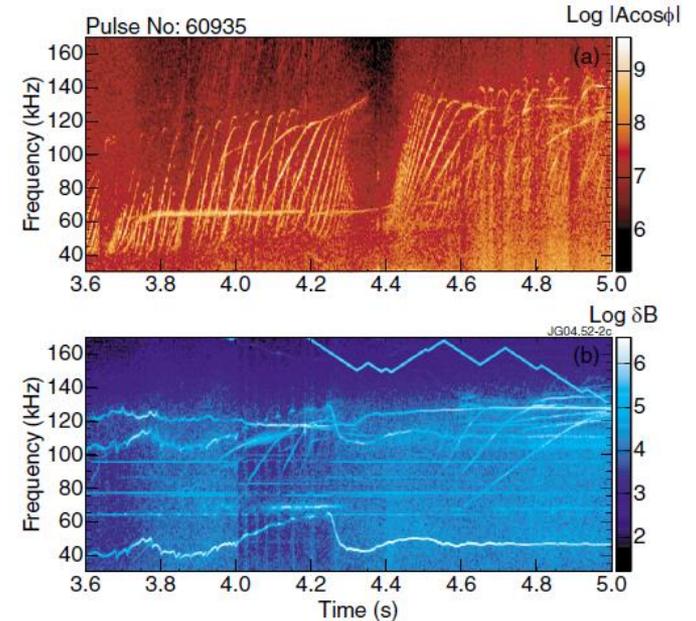


W. Li et al., Geophys. Res. Lett., **38**, L14103, 2011

S.D. Pinches et al., Plasma Phys. Control. Fusion, **46**, S47, 2004

S.E. Sharapov et al., Nucl. Fusion, **53**, 104022, 2013

Alfven eigenmodes



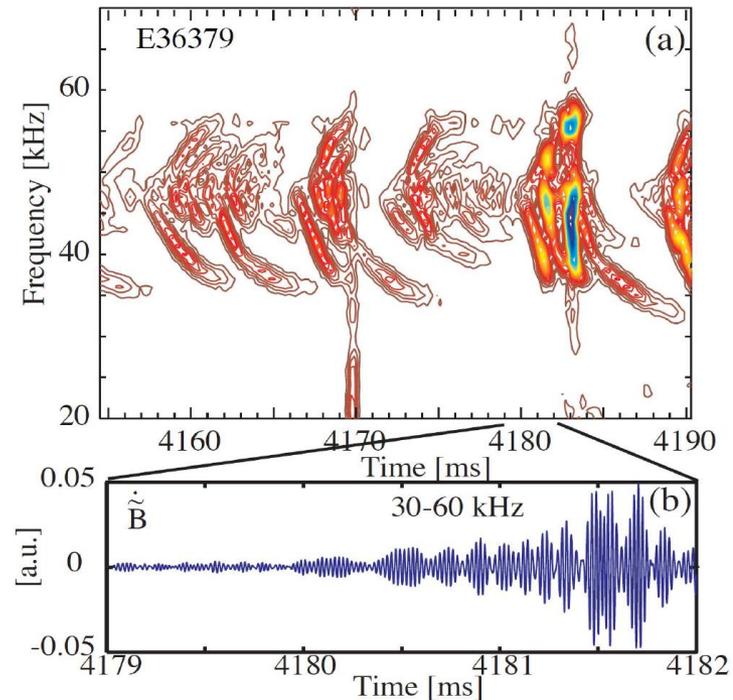
Frequency sweeping Toroidal Alfvén Eigenmodes

Tokamaks:

- ASDEX-Upgrade
- MAST
- JET
- DIII-D
- NSTX
- JT-60U

Stellarator TJ-II

Frequency range:
30-300 kHz

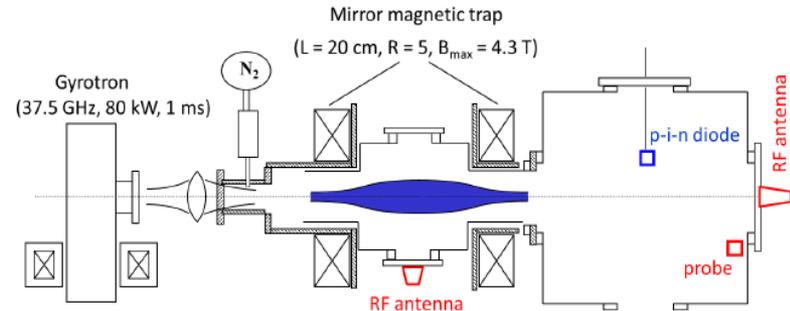


- (a) Magnetic spectrogram of NBI-driven Alfvén instabilities in JT-60U discharge E36379;
(b) Mirnov coil signal

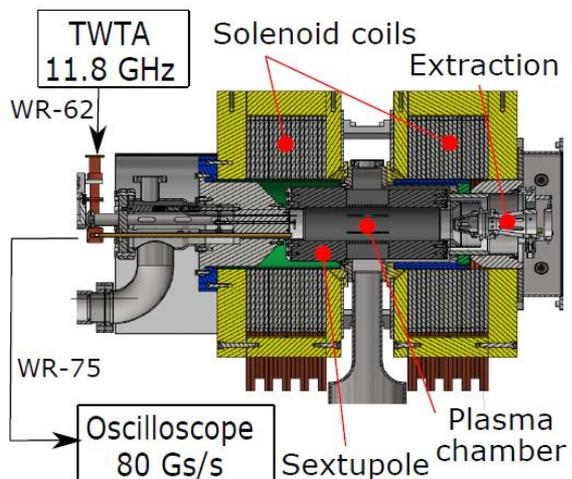
Dedicated experiments on ECRH-driven instabilities

- Broadband oscilloscopes (up to 60 GHz / 160 GSa/s) allow direct recording of $E(t)$

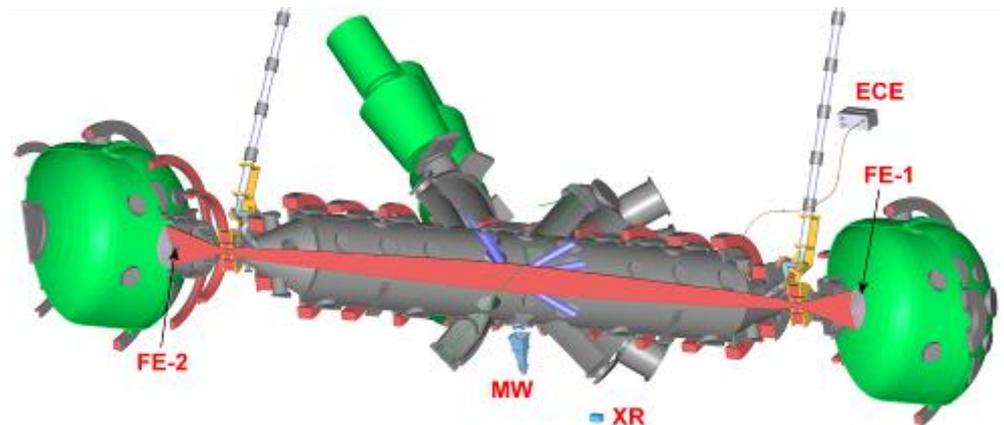
SMIS-37 (IAP)
100 kW @ 37 GHz



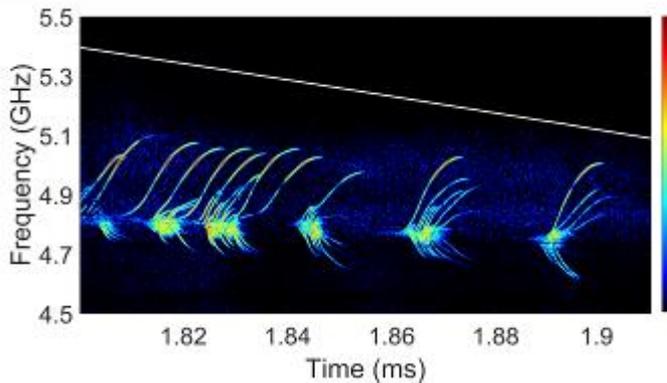
JYFL Ion Source (Univ Jyvaskyla)
250 W @ 11-14 GHz



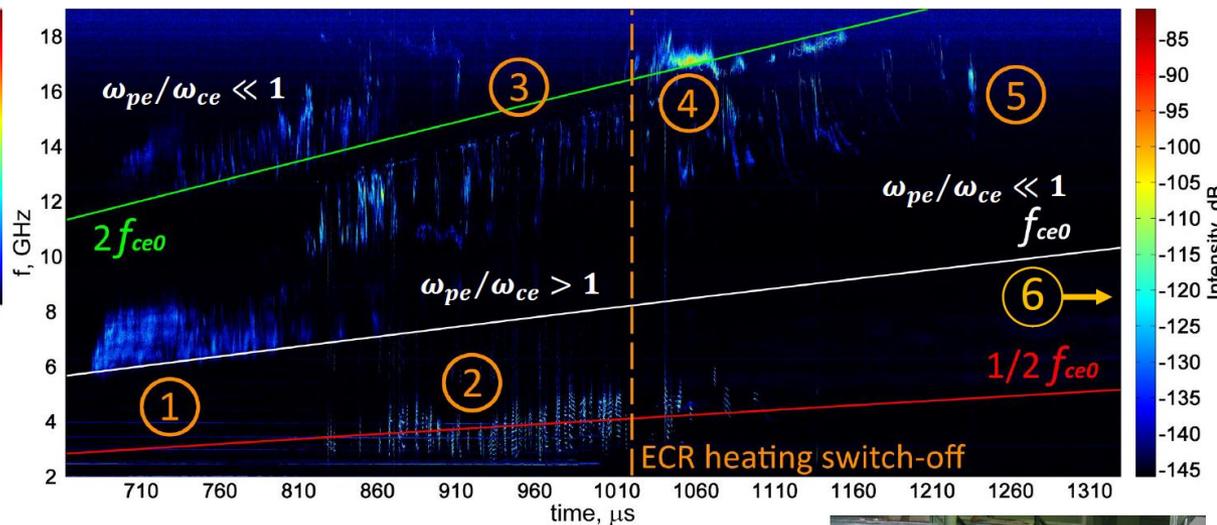
GDT (Budker Inst)
800 kW @ 54.5 GHz



“Zoo” in dynamical spectrum of mw emission

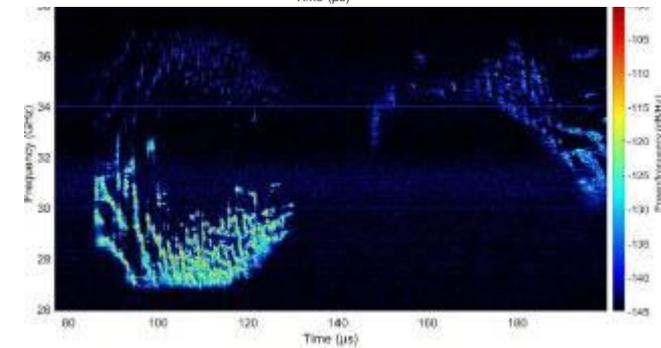
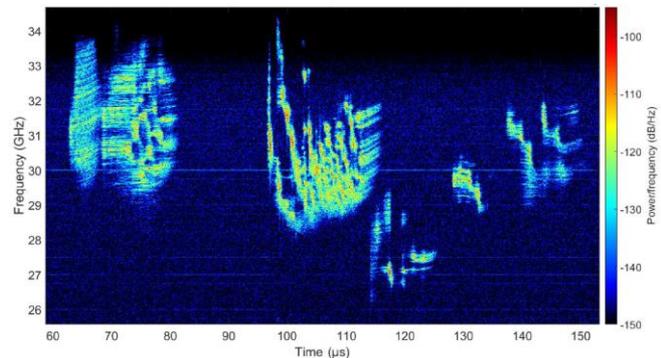
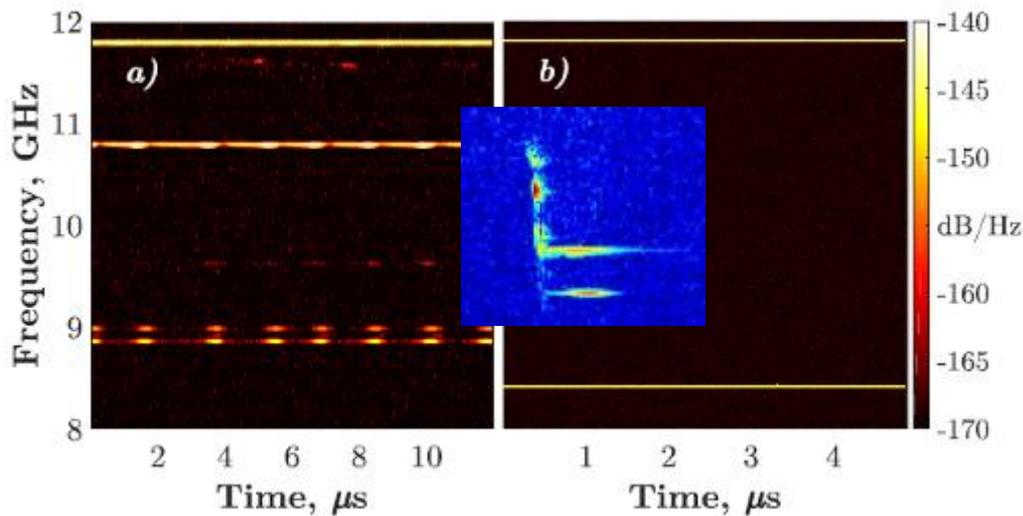


SMIS-37 (IAP)



GDT (Budker Inst)

JYFL Ion Source (Uni Jyvaskyla)



Two lessons that we have learned

- Lesson 1: Quasi-linear theory for maser instability
- Lesson 2: Hole & clump dynamics in ECE

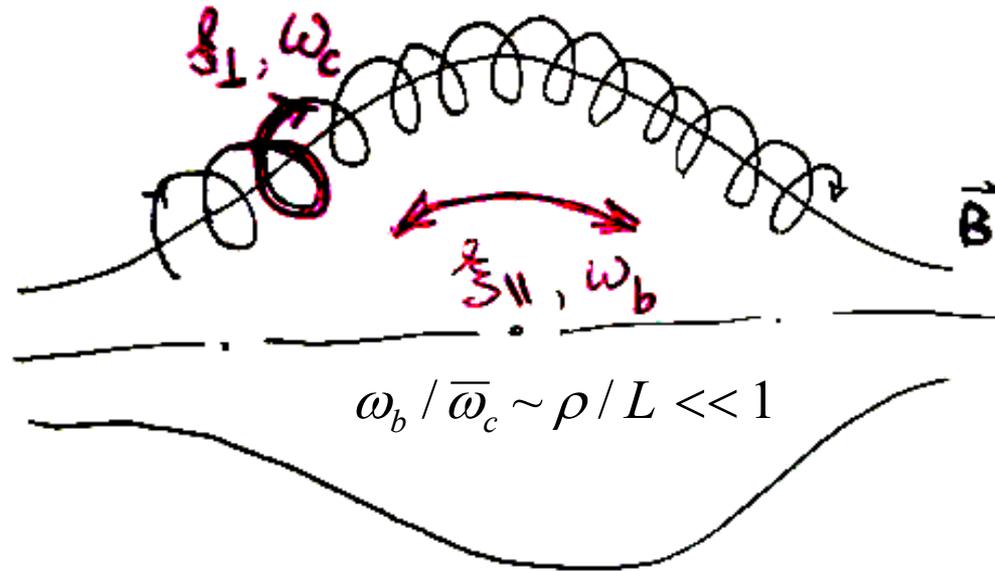
Wave-particle resonance in a magnetic mirror

Action-angle formalism

$$H_0 = I_{\perp} \bar{\omega}_c (I_{\perp}, I_{\parallel}) + \frac{1}{2} I_{\parallel} \omega_b (I_{\perp}, I_{\parallel})$$

$$\begin{cases} d\xi_{\perp} / dt = \bar{\omega}_c, & I_{\perp} = m v_{\perp}^2 / 2B(l) \\ d\xi_{\parallel} / dt = \omega_b, & I_{\parallel} = \oint v_{\parallel} dl \end{cases}$$

$$\bar{\omega}_c = \frac{1}{\omega_b} \oint \frac{\omega_c}{v_{\parallel}} dl, \quad \omega_b = \oint \frac{dl}{v_{\parallel}}$$



$$H = H_0 + \frac{e}{mc} \mathbf{A} \cdot \mathbf{p} = H_0 + \text{Re} \left[C(t) \sum_{s,n} V_{sn} \exp(is\xi_{\perp} + in\xi_{\parallel} - i\omega_0 t) \right]$$

$$s \bar{\omega}_c + n \omega_b = \omega_0$$

Cyclotron harmonics

Bounce resonances, for electrons are usually overlapped resulting in global diffusion in phase-space (quasilinear theory)

Quasi-linear diffusion in a magnetic mirror

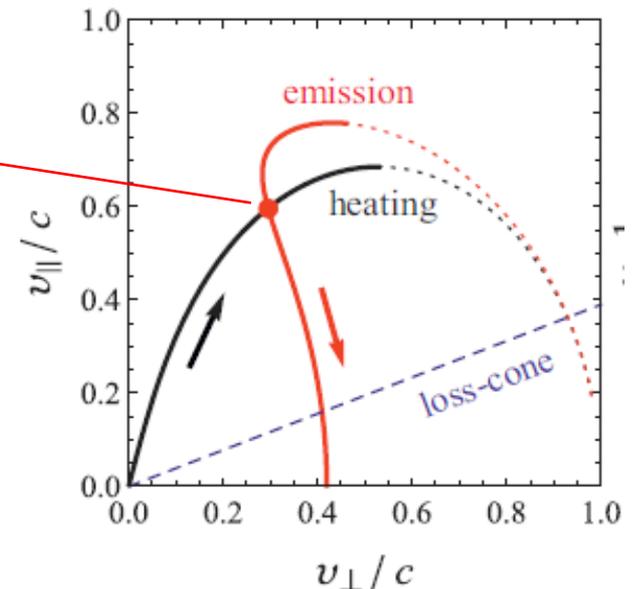
$$H = H_0 + \frac{e}{mc} \mathbf{A} \cdot \mathbf{p} = H_0 + \text{Re} \left[C(t) \sum_{s,n} V_{sn} \exp(is\xi_{\perp} + in\xi_{\parallel} - i\omega_0 t) \right]$$

1) Interaction with waves conserves $K = mc^2(\gamma - 1) - I_{\perp} \omega_0 = \text{const}$
 then consider $(I_{\perp}, I_{\parallel}) \rightarrow (K, \kappa)$, i.e. one-dimensional distribution function $F(t, \kappa)$

2) Self-consistent electromagnetic field (fixed mode) $|C(t)|^2 \rightarrow E(t)$, $V_{sm} \rightarrow D, K$

$$\frac{\partial F}{\partial t} = \frac{\partial}{\partial \kappa} \left((ED + D_0) \frac{\partial F}{\partial \kappa} \right) + J,$$

$$\frac{\partial E}{\partial t} = E \int_0^{\infty} \int_{\kappa_c}^1 K \frac{\partial F}{\partial \kappa} d\kappa dv - \nu E,$$



3) Describes both ECR heating and maser instability!

Quasi-linear model describes many observed features of stimulated emission in simple terms

- X-mode emission at the start-up phase
- Z-mode emission in rarefied decaying plasma
- Whistler waves during the stationary ECR

SMIS37

Reviewed in Shalashov et al. Phys. Plasmas 24 032111 (2017)

- Excitation of plasma waves under the double-plasma-resonance

Mansfeld et al. Planet. Space Sci. 164158 (2018)

- Stochastic grouping of ECE bursts in a decaying plasma

Shalashov et al. PPCF 54 085023 (2012)

- Fast electron losses at GDT

Shalashov et al. Phys. Plasmas 24 082506 (2017)

- Controlled transition between periodic and CW regimes

Shalashov et al. PRL 120 155001 (2018)

JYFL ECRIS

Shalashov et al. EPL. 24 35001 (2018)

- Stabilization of burst activity by two-frequency ECRH

Tarvainen et al. Rev.Sci. Instr. 86 023301 (2015) - experiment

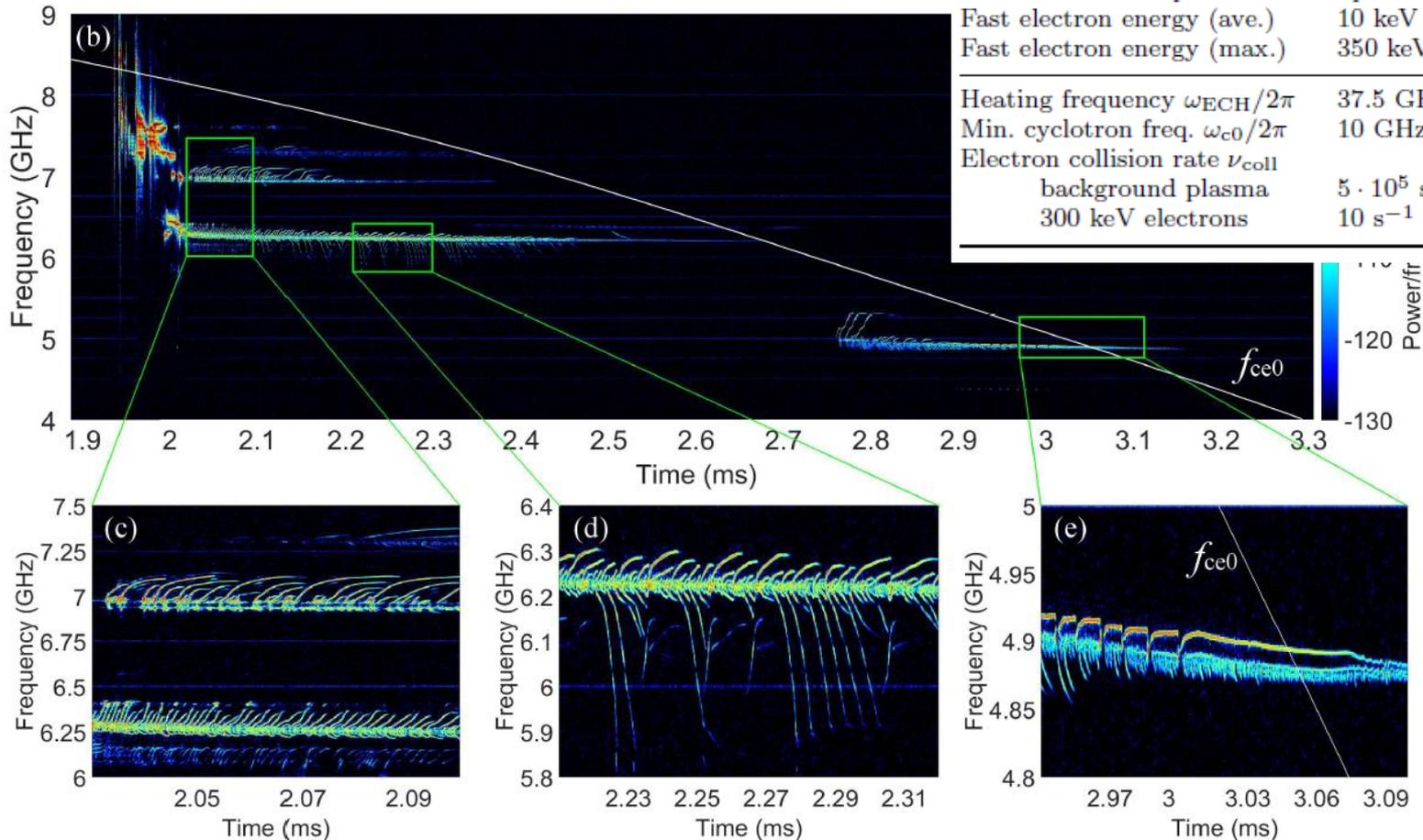
theory is still not finished!

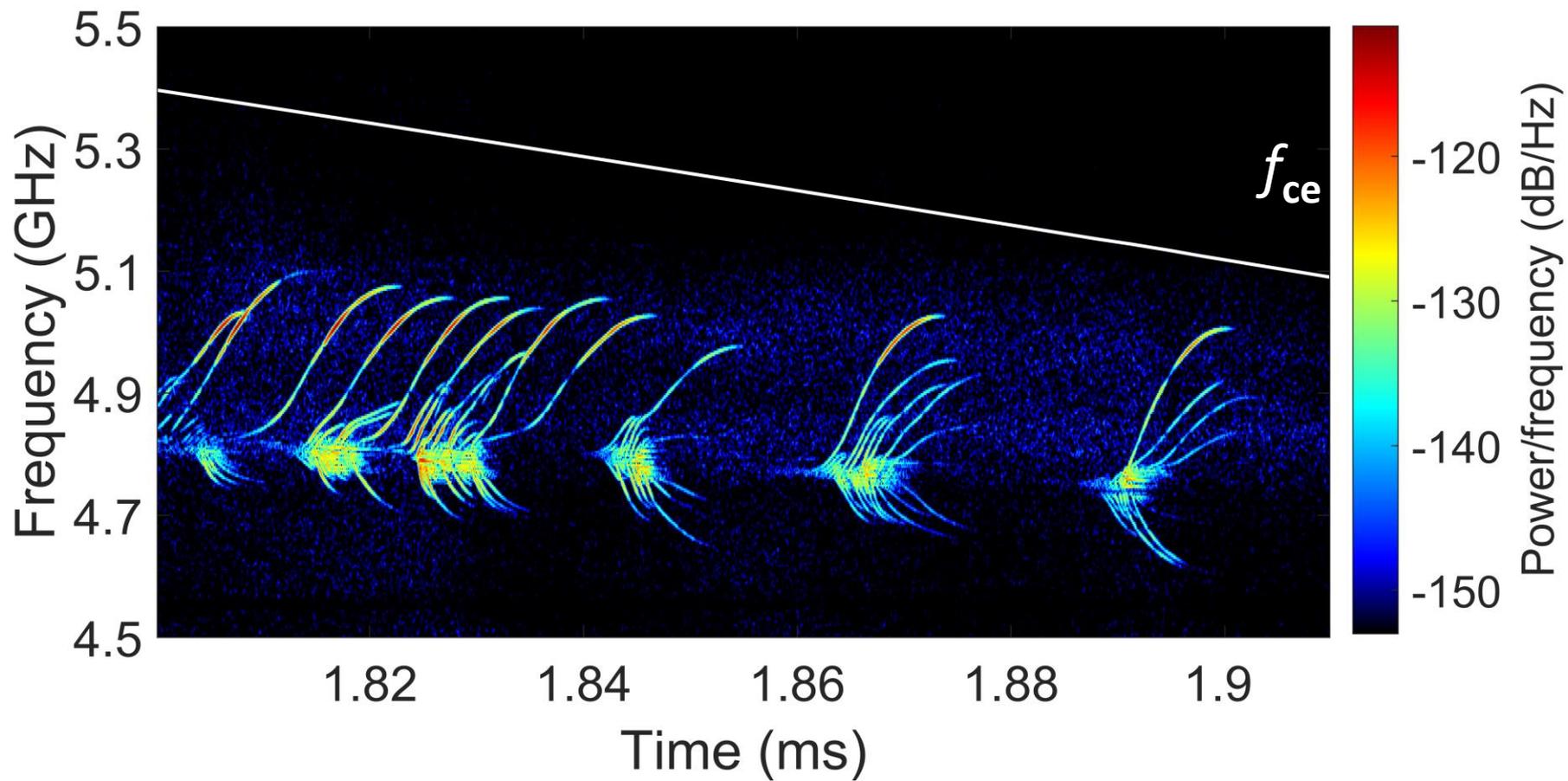
Beyond the quasi-linear theory

- Fast periodic frequency sweeps in ECE discovered at SMIS-37
- Observed at fixed lines in rare plasma after ECRH switch-off
- No precipitations of fast electrons
- Low power compared to other inst.

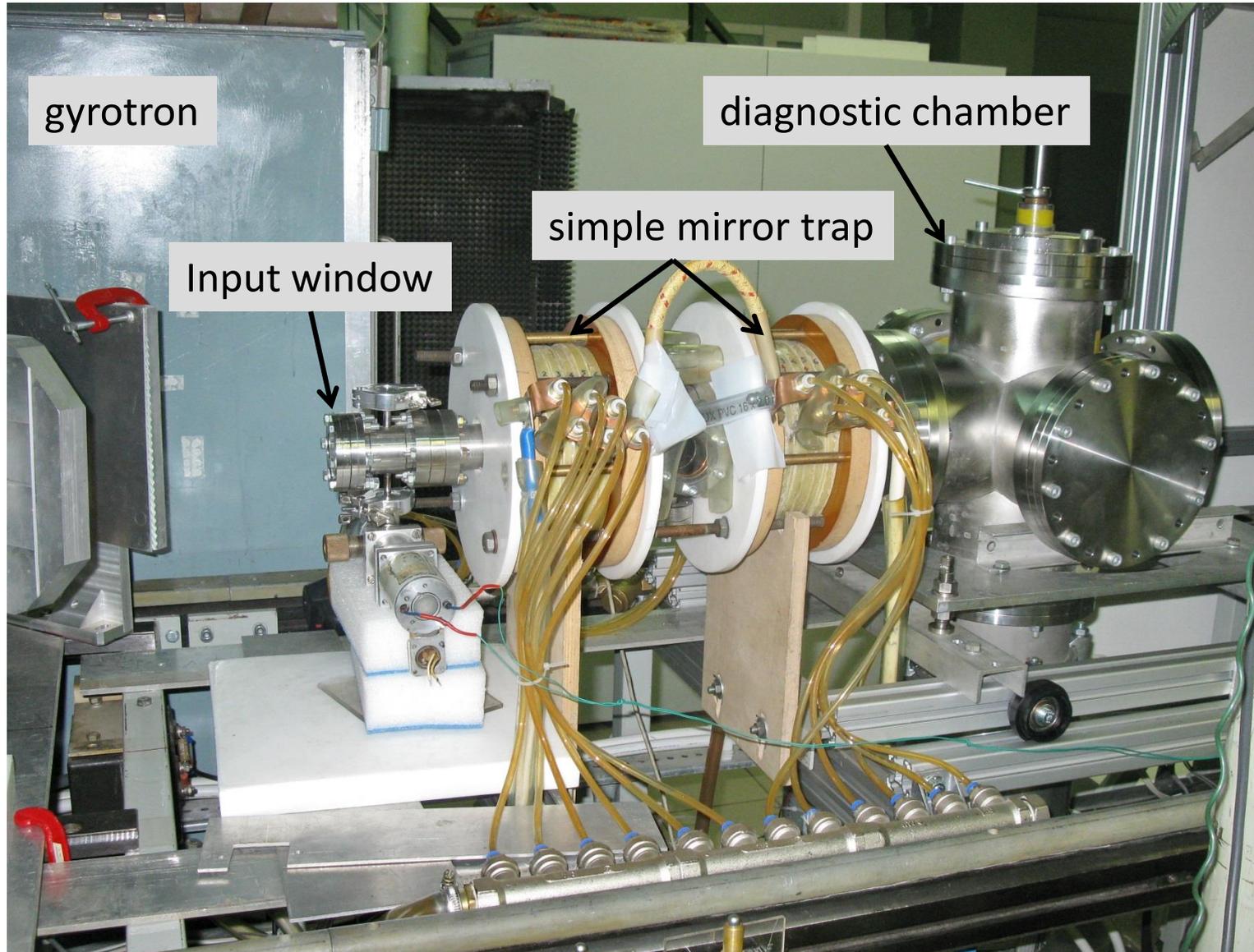
Viktorov et al. *Eur. Phys. Lett.* 116 55001 (2016)

	ECRH	Decay
Background plasma density	$\sim 10^{13} \text{ cm}^{-3}$	$\lesssim 10^{11} \text{ cm}^{-3}$
Fast el. density (1–30 keV)	10^{11} cm^{-3}	10^{11} cm^{-3}
Fast el. density (>100 keV)	10^9 cm^{-3}	10^9 cm^{-3}
Bulk electron temperature	up to 300 eV	$\sim 1 \text{ eV}$
Fast electron energy (ave.)	10 keV	10 keV
Fast electron energy (max.)	350 keV	300 keV
Heating frequency $\omega_{\text{ECH}}/2\pi$	37.5 GHz	–
Min. cyclotron freq. $\omega_{c0}/2\pi$	10 GHz	8 GHz
Electron collision rate ν_{coll}		
background plasma	$5 \cdot 10^5 \text{ s}^{-1}$	10^7 s^{-1}
300 keV electrons	10 s^{-1}	0.1 s^{-1}

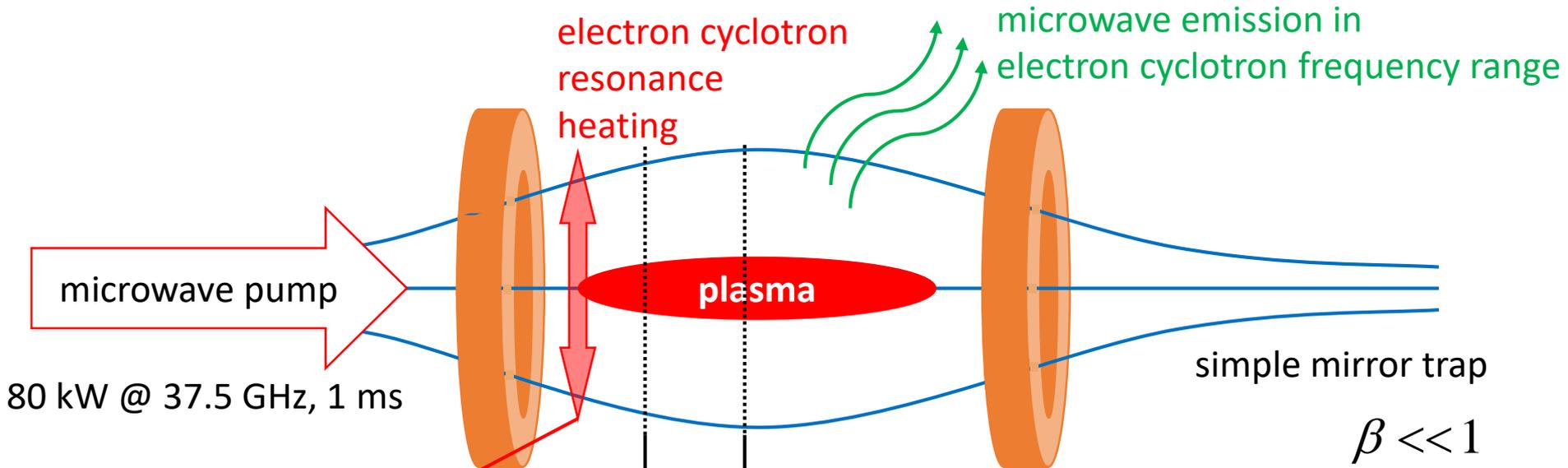




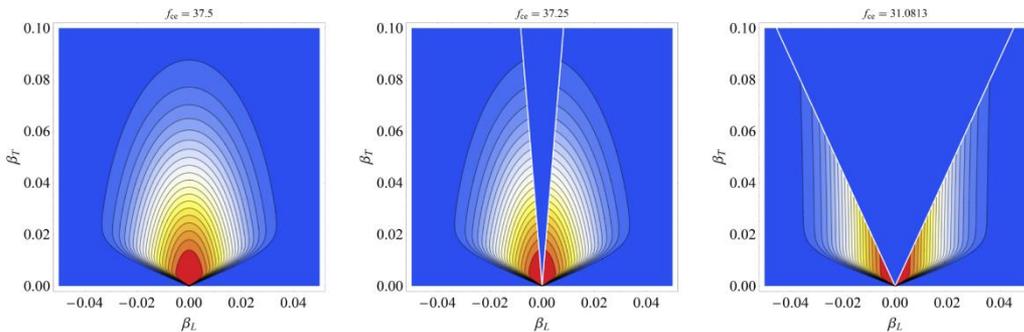
Experimental setup



Experimental setup



mirror ratio $R = 4 \div 10$
trap length ~ 25 cm
magnetic field up to 4.3 T



Unstable distributions of hot electrons

1) Background plasma

$T_e \sim 300$ eV

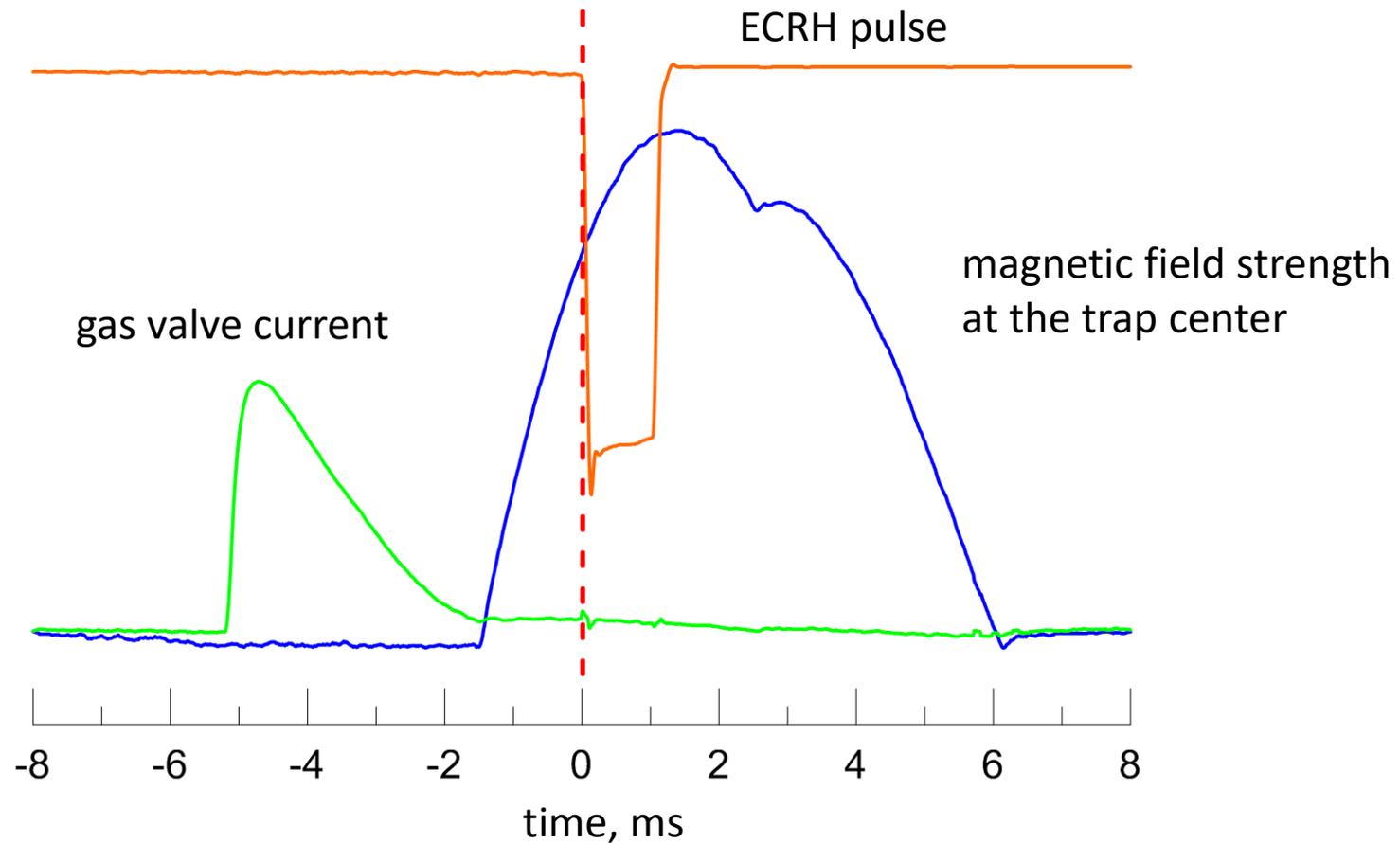
$N_e \sim 10^{13} - 10^{14}$ cm $^{-3}$

2) Hot electrons

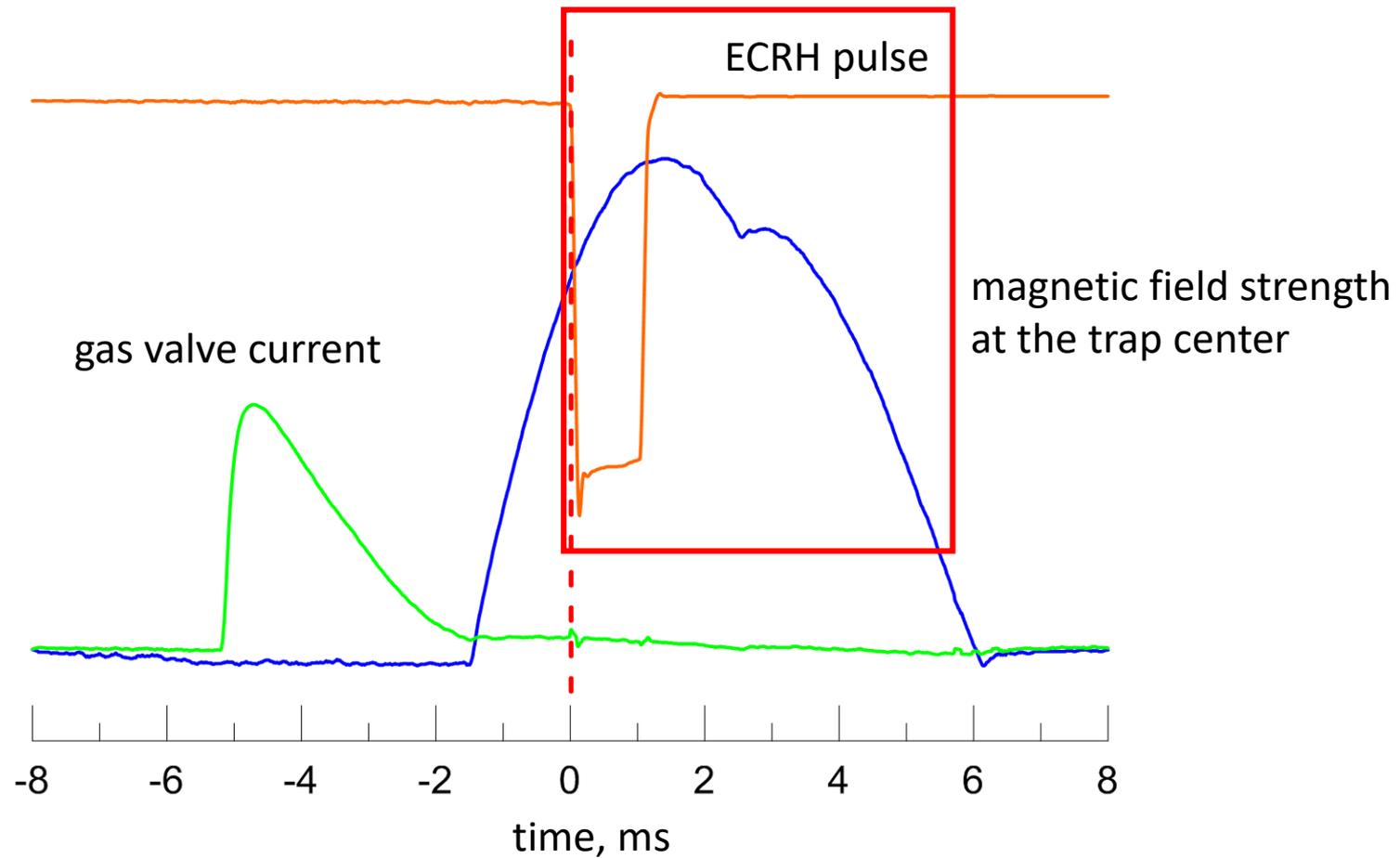
$T_h \sim 10$ keV

$N_h \sim 10^{10} - 10^{11}$ cm $^{-3}$

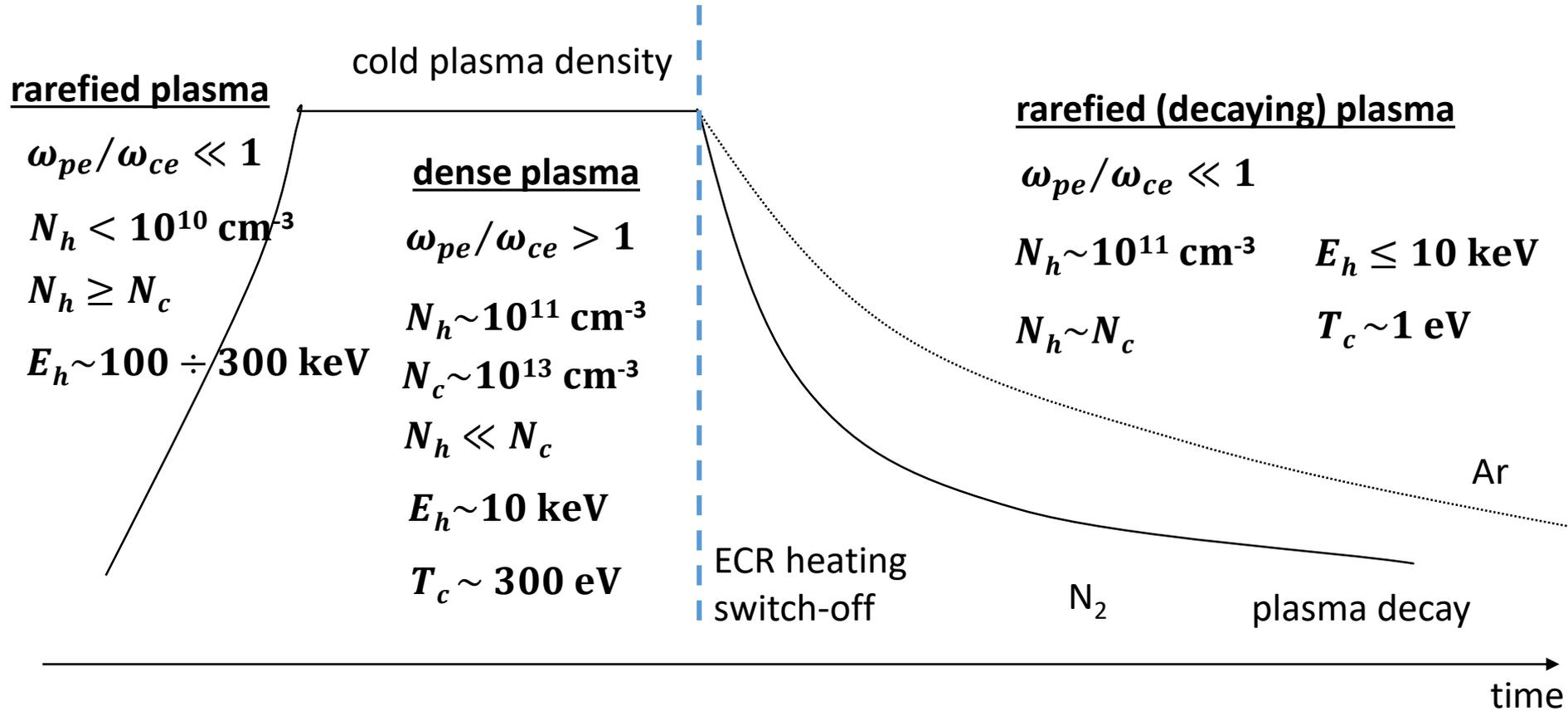
The synchronization scheme of the experimental setup



The synchronization scheme of the experimental setup

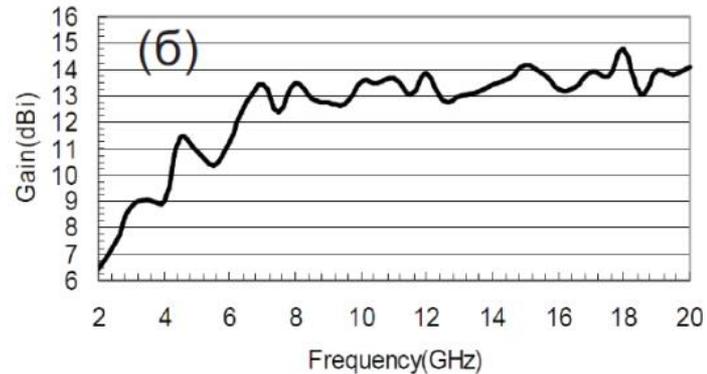


Plasma parameters

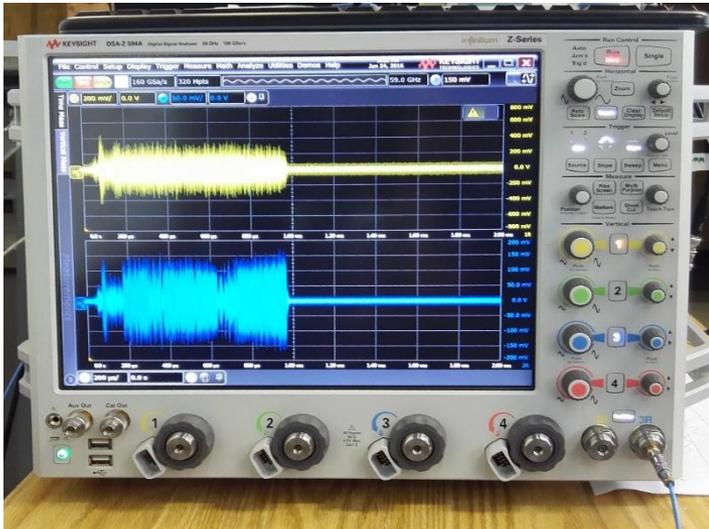


Microwave diagnostics of plasma emission

Broadband horn antenna 2-20 GHz, input aperture 104x78 mm² +
low-pass filter 24.660 GHz (30dB rejection frequency)

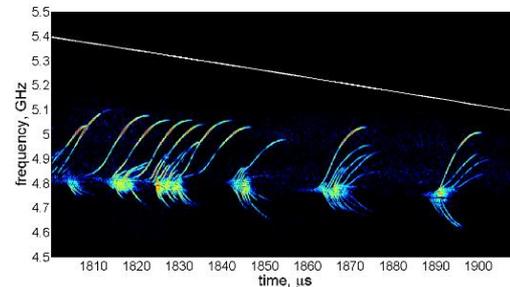
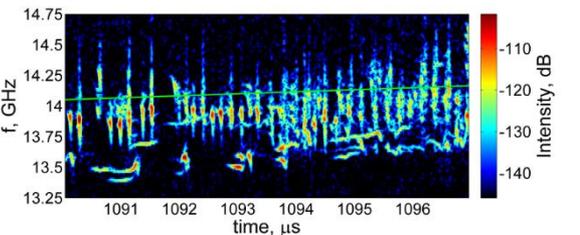
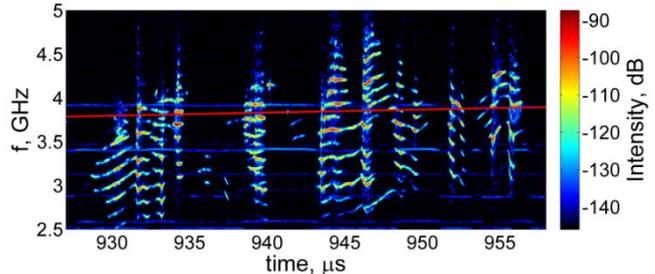
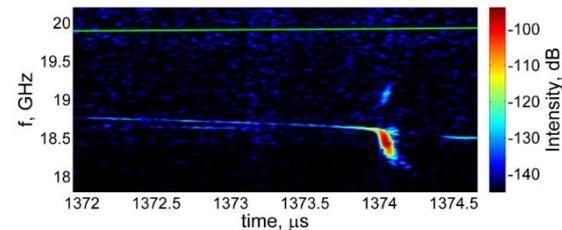
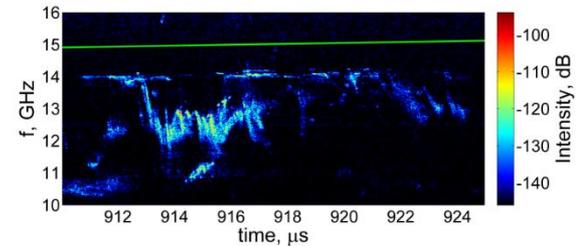
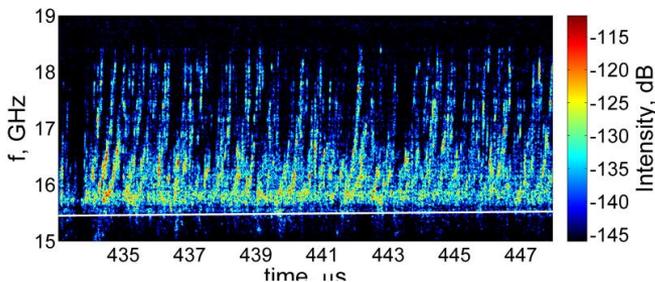
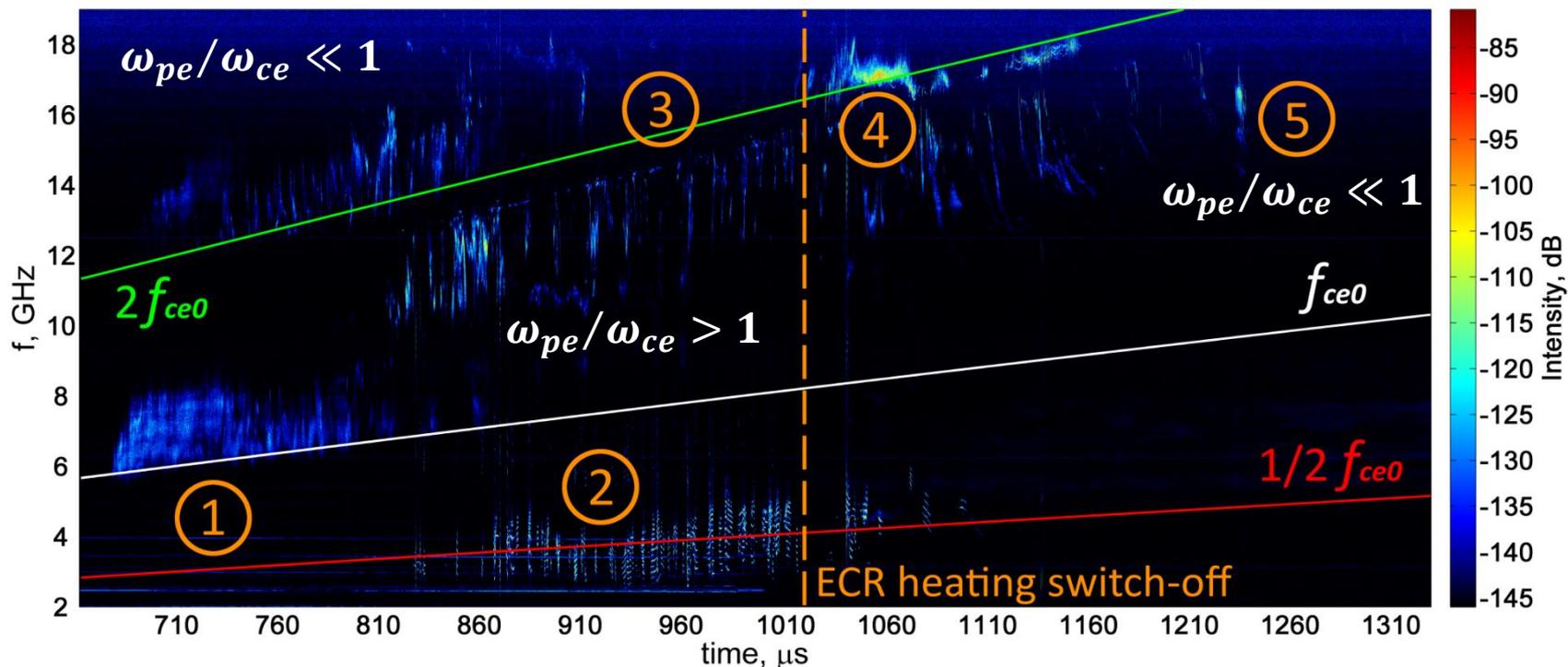


Broadband oscilloscope KeySight DSA-Z 594A

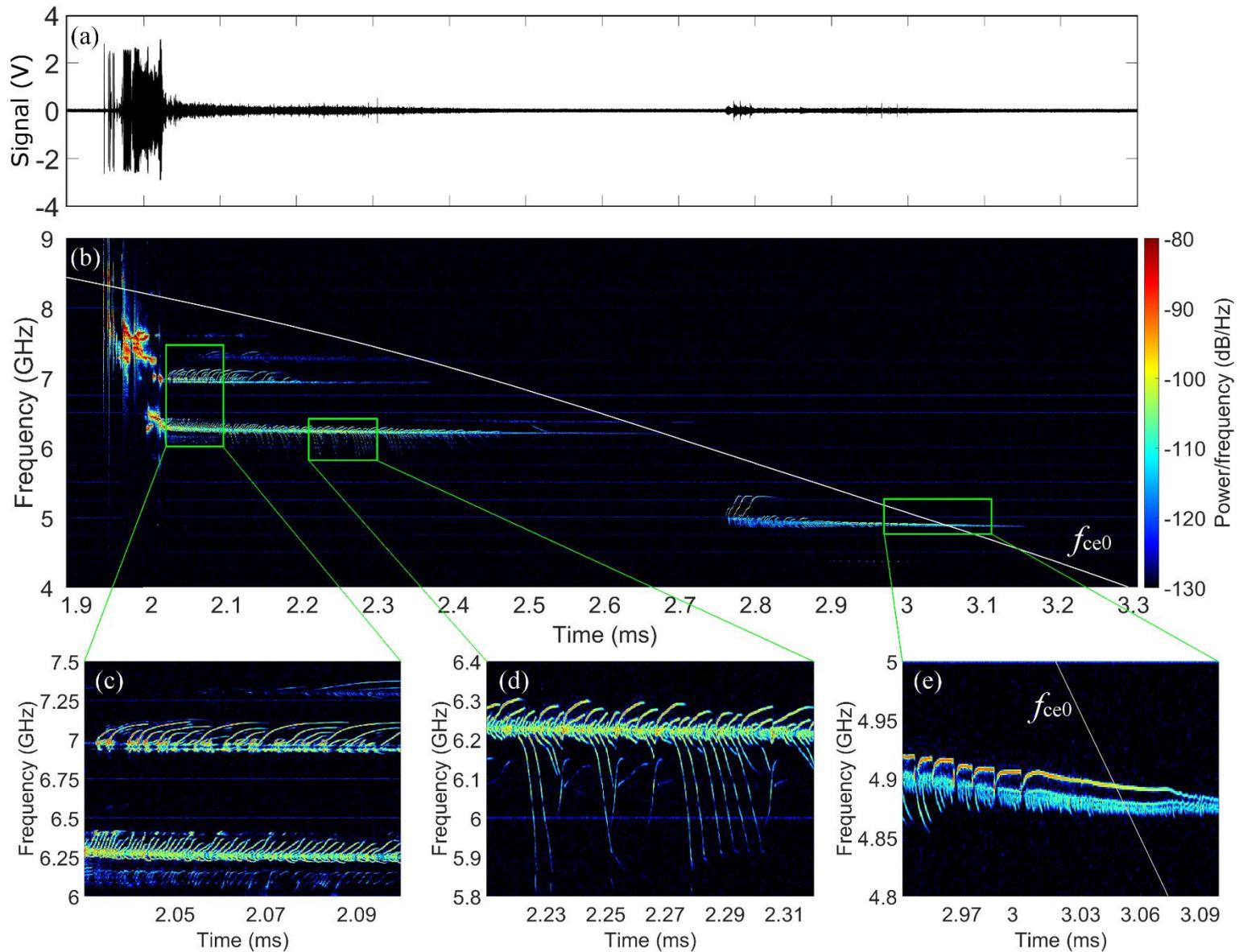


- 4 channels, analog bandwidth 33 GHz, sampling rate 80 GSa/s (up to 25 ms)
- **2 channels, analog bandwidth 59 GHz, sampling rate 160 GSa/s (up to 12.5 ms)**
- Up to 2 billions samples per channel
- Maximum temporal resolution 6.25 ps (160 GSa/s)

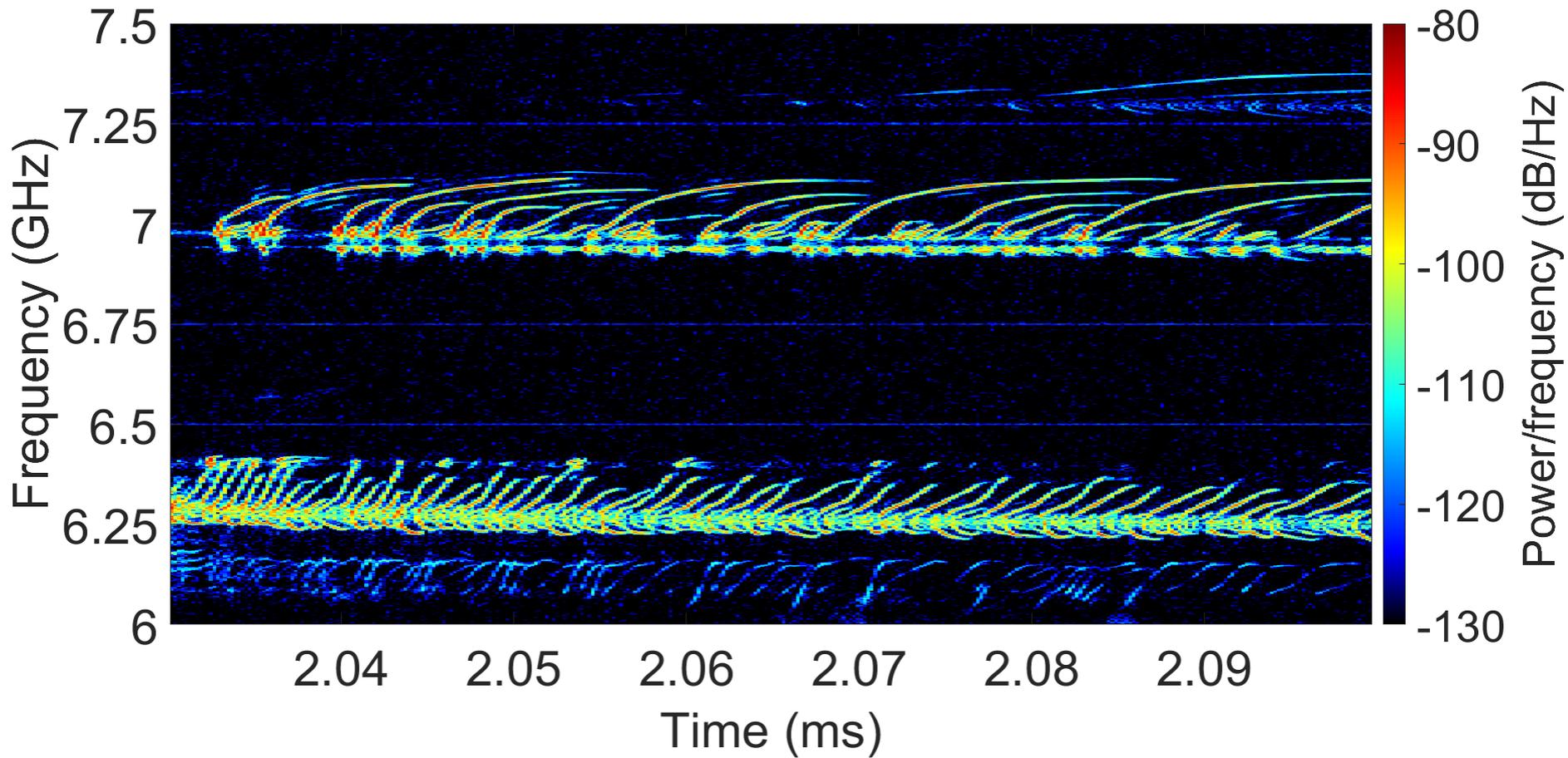
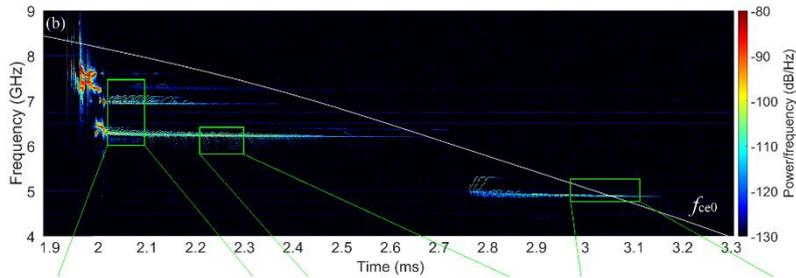
Overview of plasma microwave emission



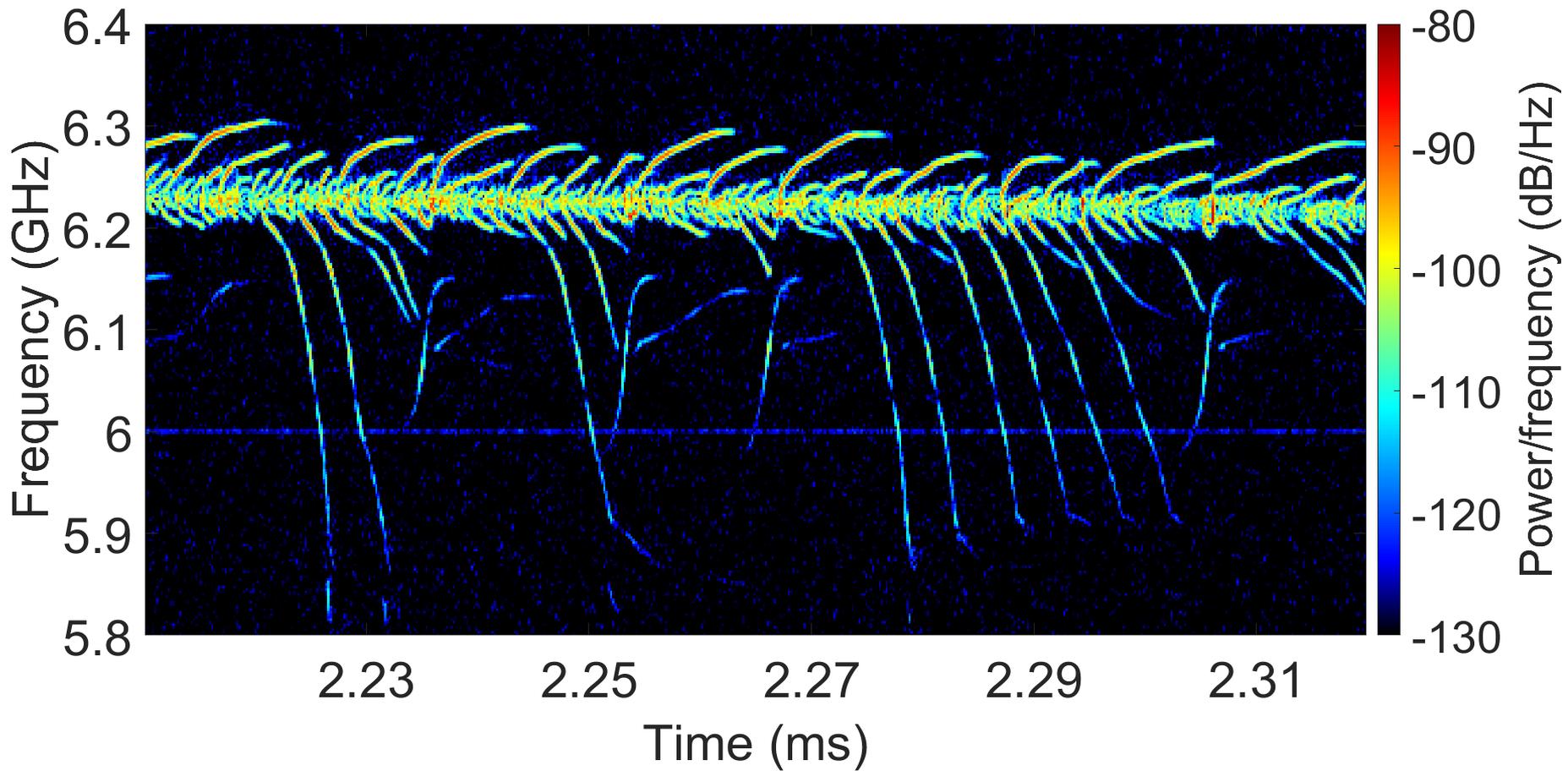
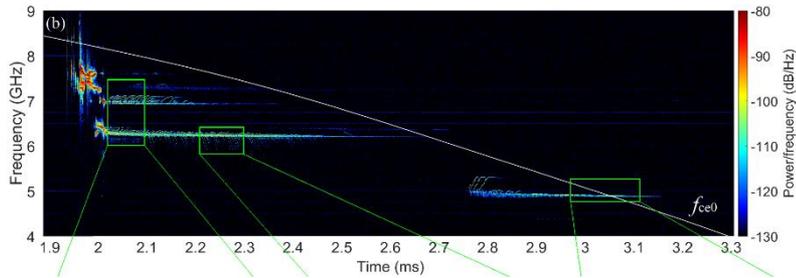
Plasma microwave emission during decay stage



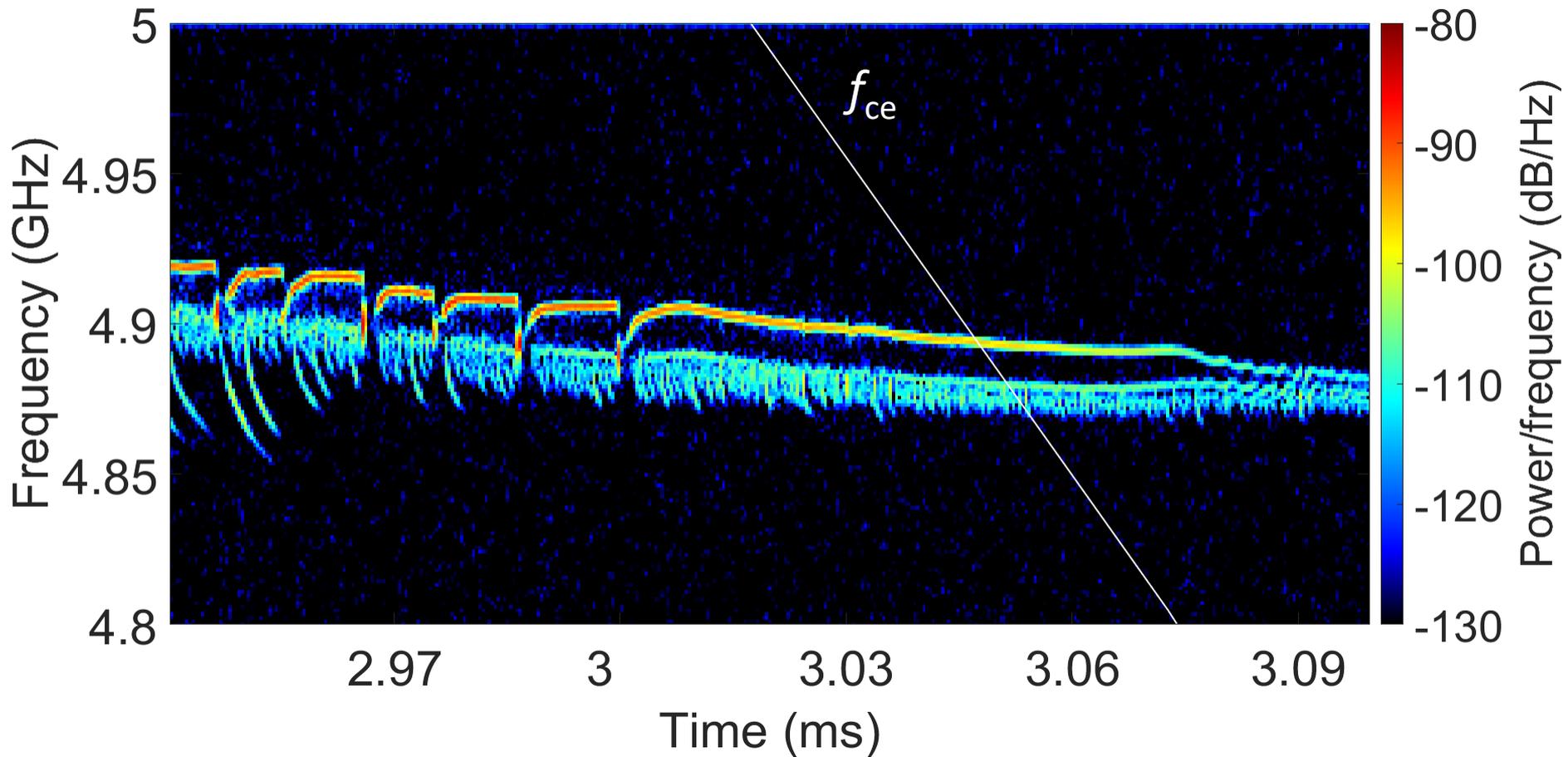
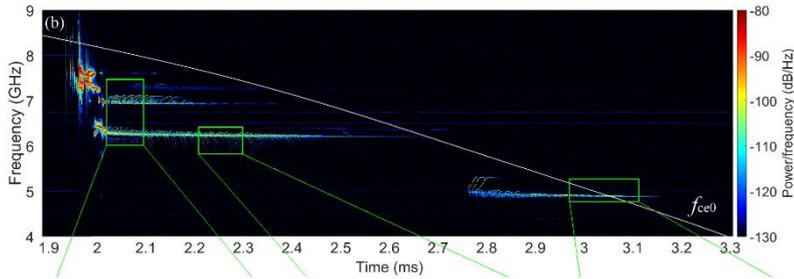
Fine structure of the microwave emission spectrum



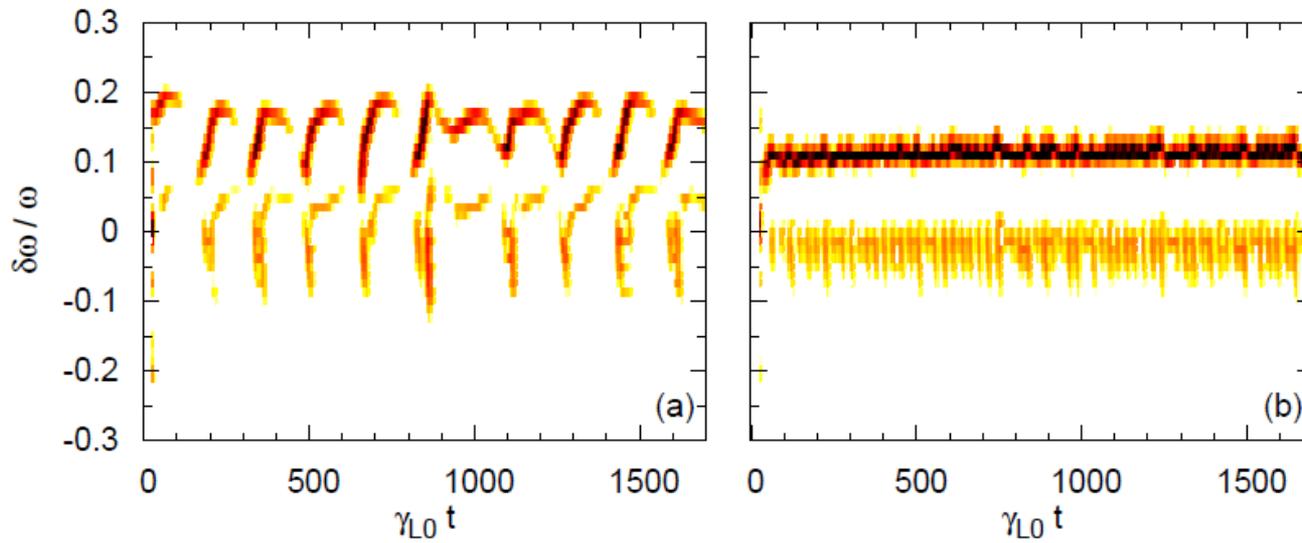
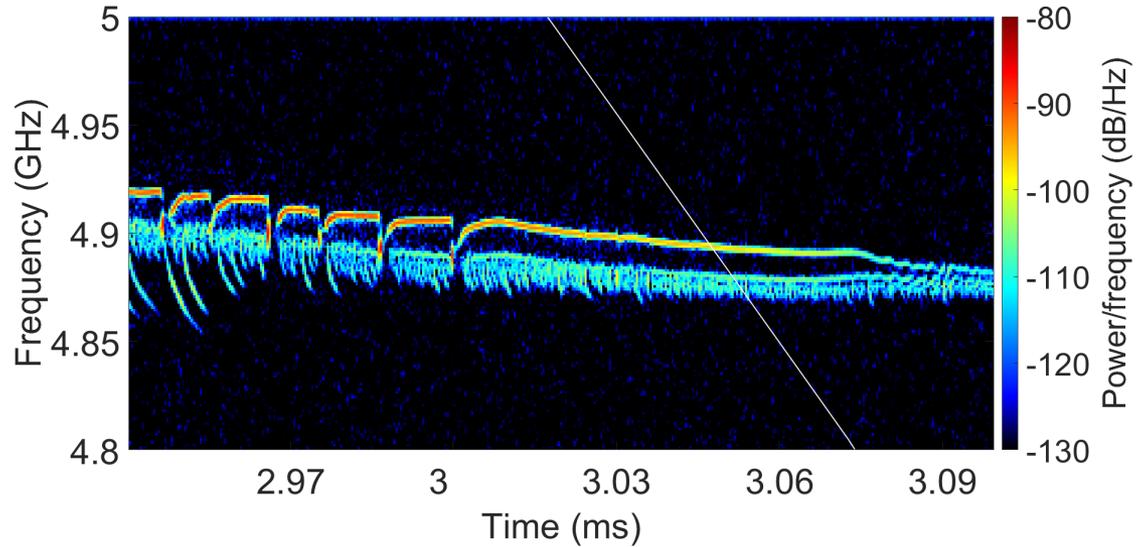
Fine structure of the microwave emission spectrum (2)



Fine structure of the microwave emission spectrum (3)

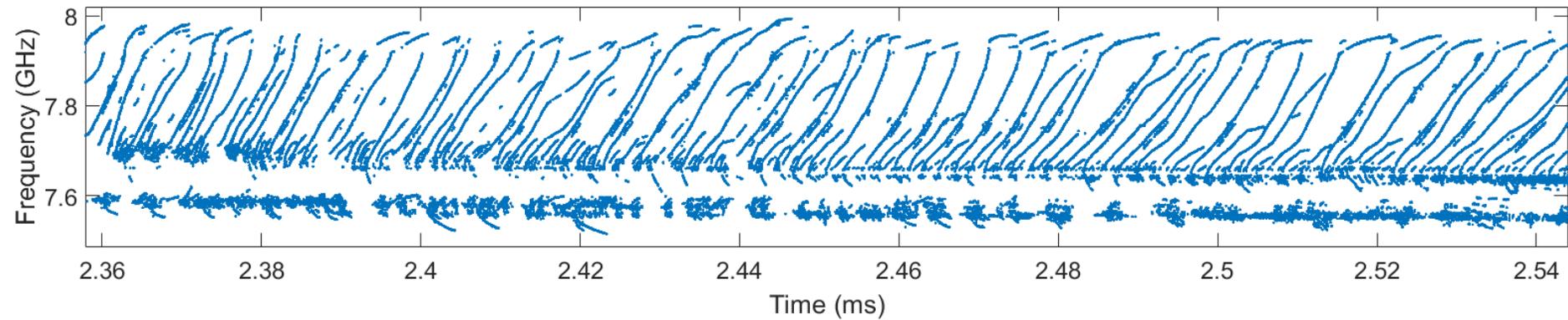
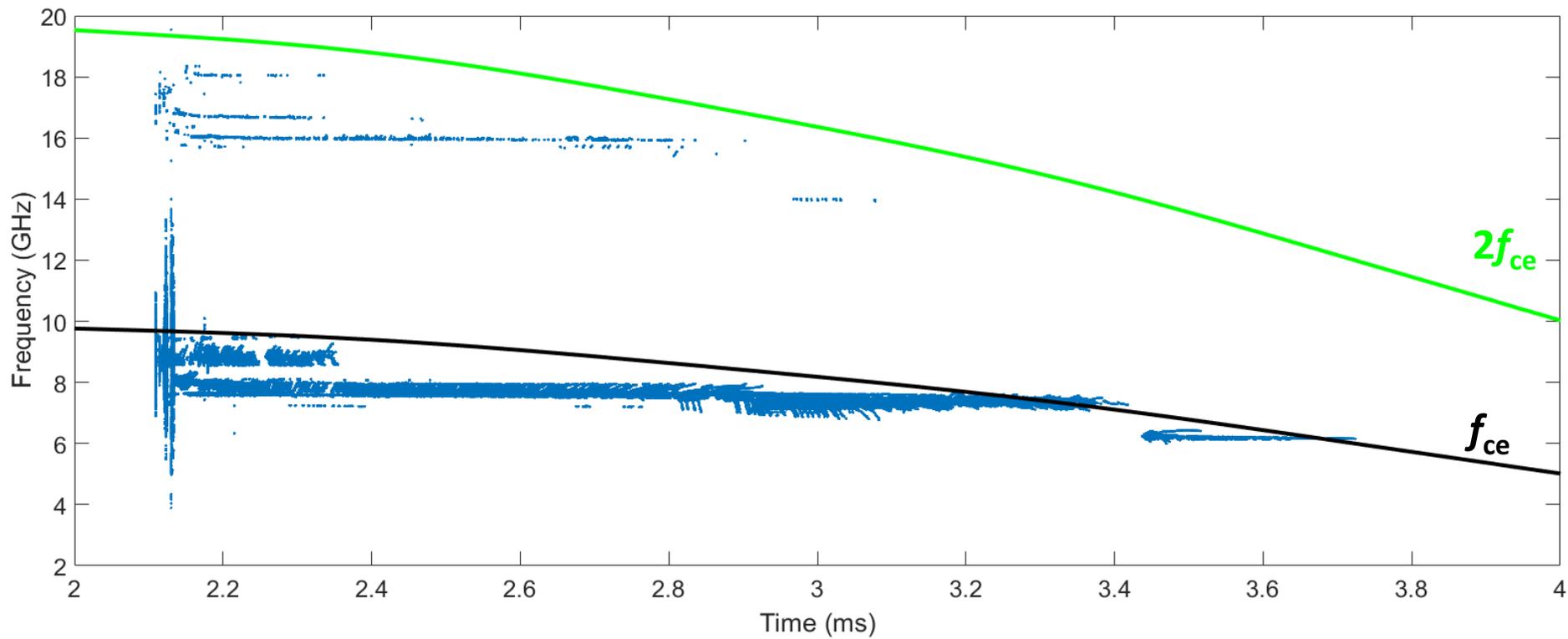


Transition from chirping to steady state emission

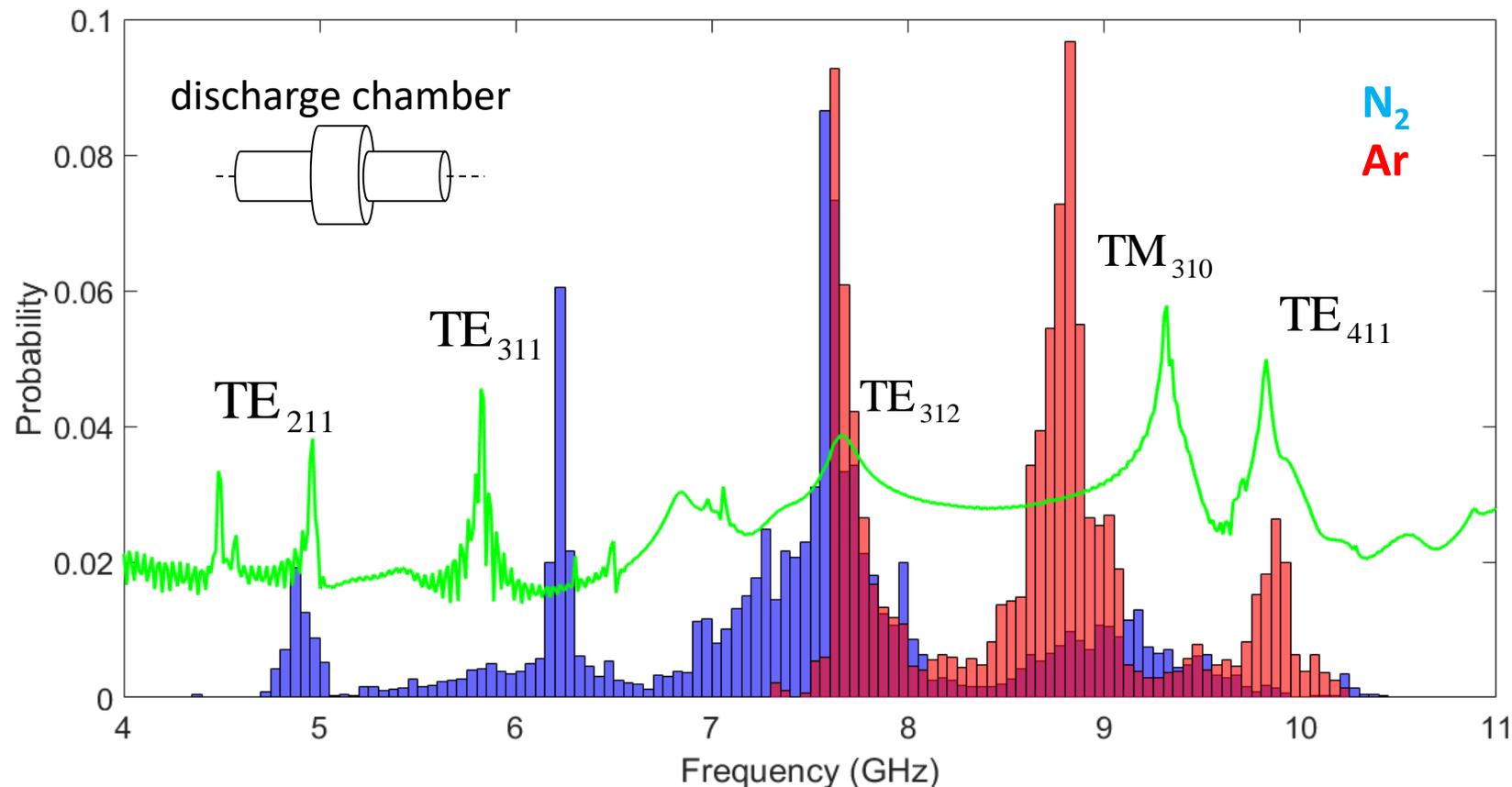


less collisions

more collisions

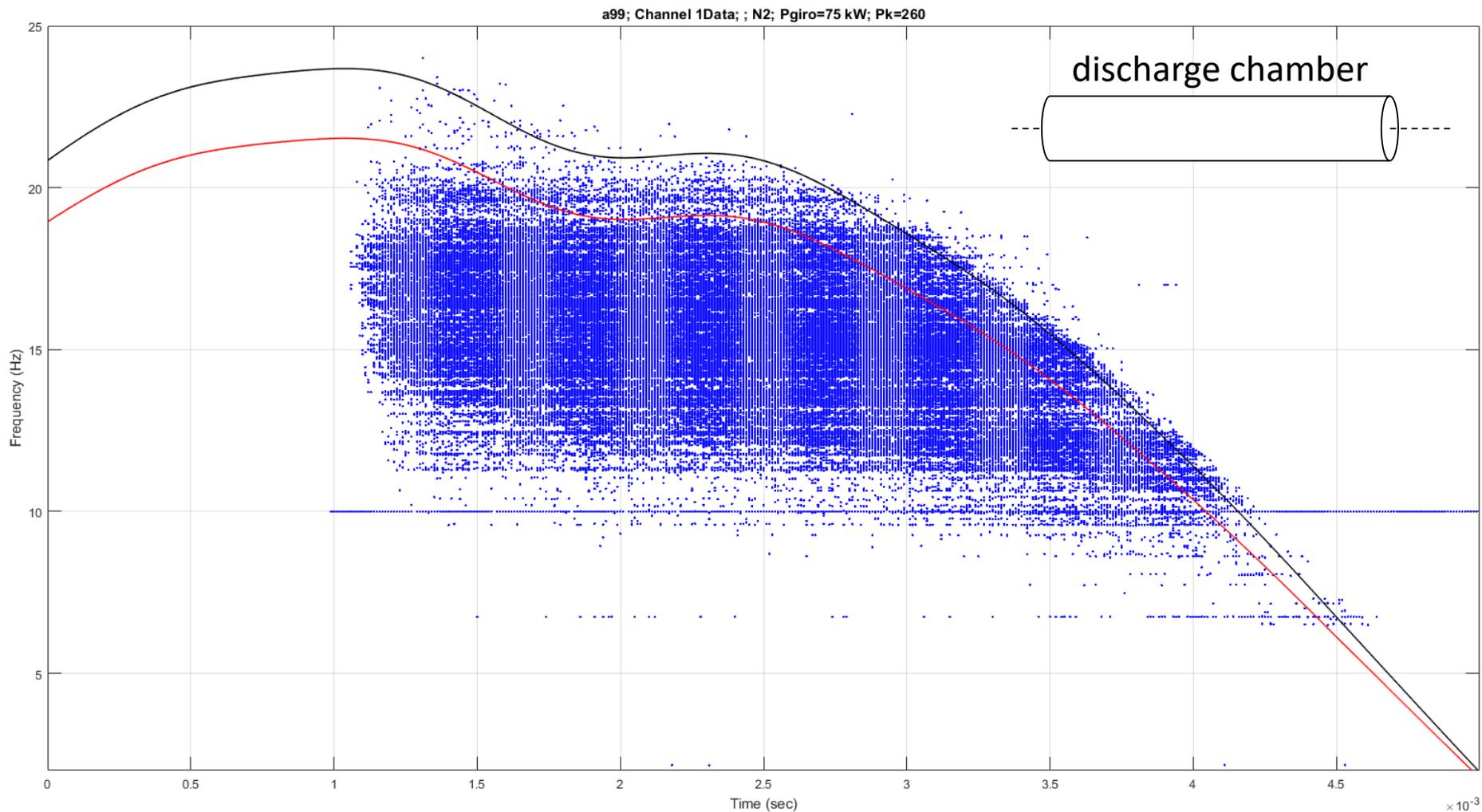


The microwave emission frequency bands

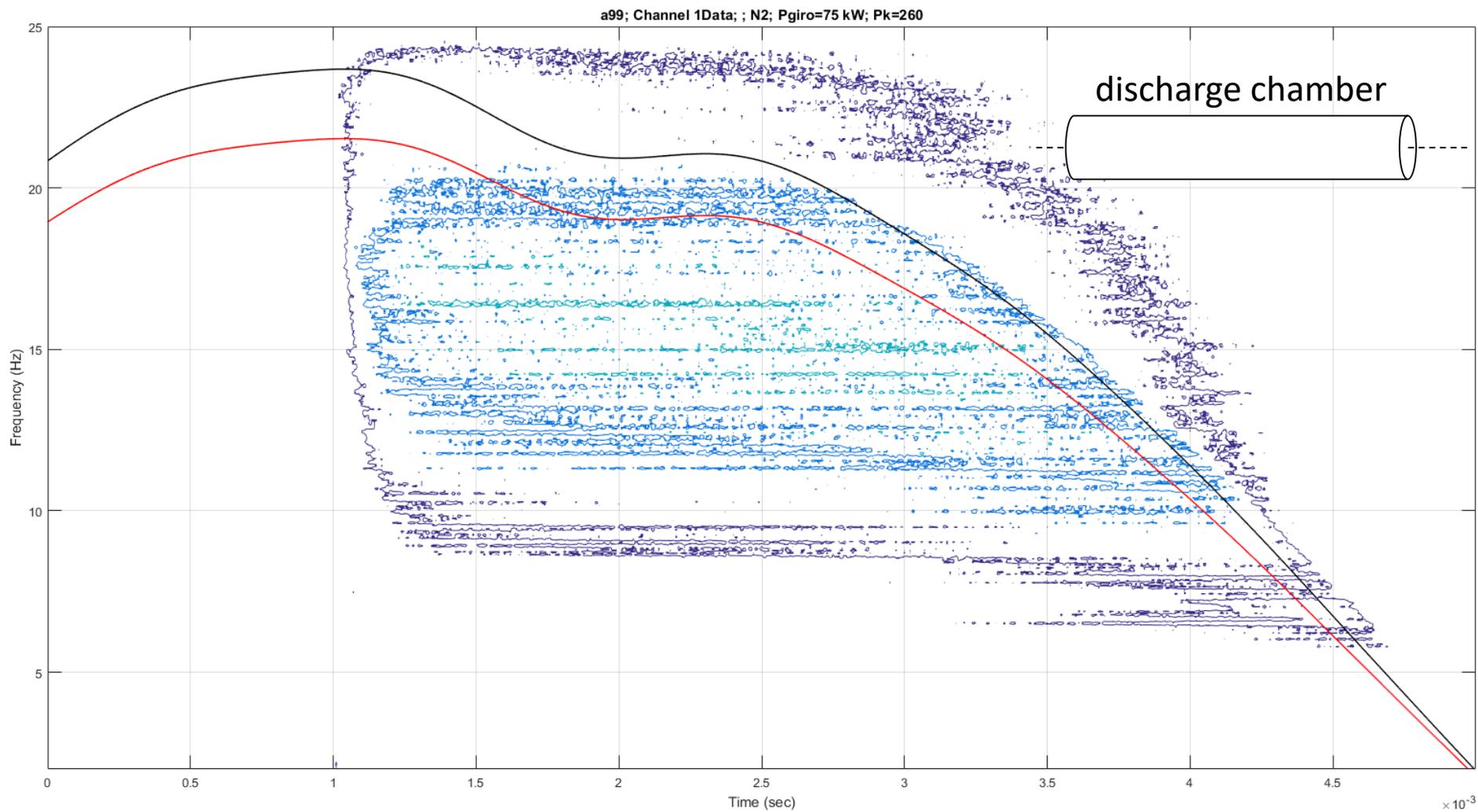


- Open cylindrical inserts lead to efficient mode selection of the central discharge chamber.
- Only those modes for which the cutoff frequencies of the cylindrical inserts modes with the same azimuthal number are higher than the fundamental frequency of the central section remain high-Q.

Noise-like emission during plasma decay stage

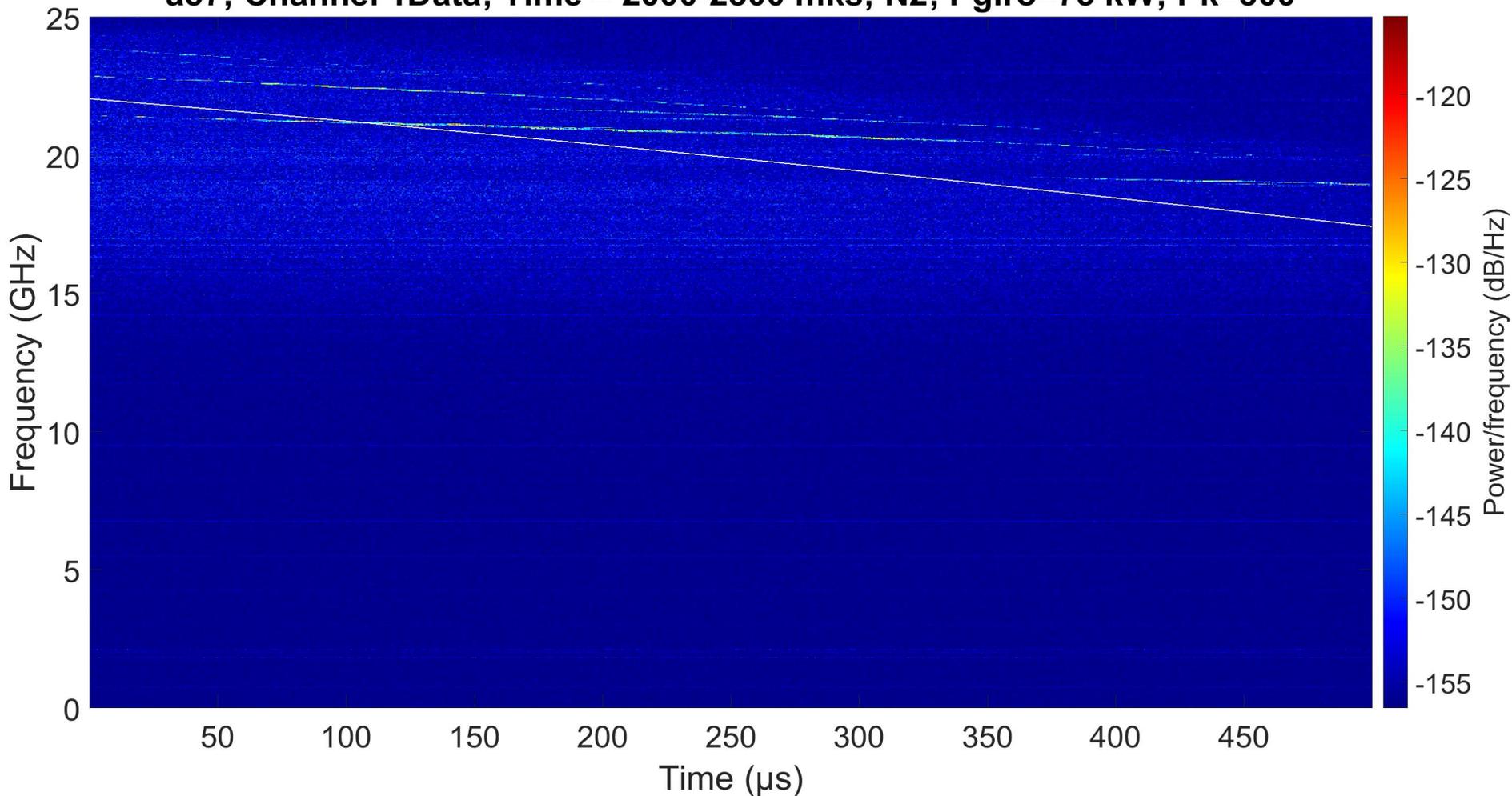


Noise-like emission during plasma decay stage



Noise-like emission during plasma decay stage: seeds of narrow-band mw-emission

a57; Channel 1Data; Time = 2000-2500 mks; N2; Pgiro=75 kW; Pk=300



Return to basic wave-particle resonance in a magnetic mirror (at the fundamental ECR)

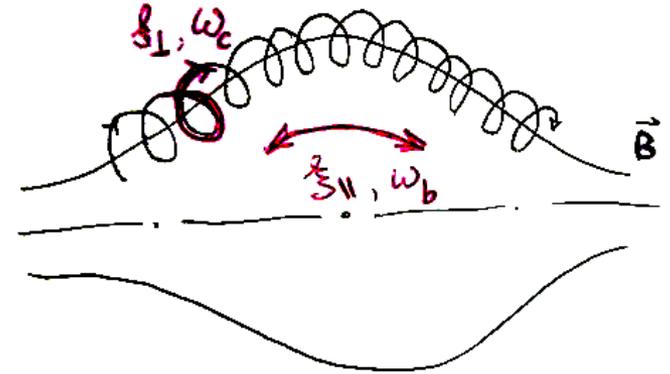
$$\Omega(I_{\perp}, I_{\parallel}) = \bar{\omega}_c + n\omega_b = \omega_0$$

For very fast electrons bounce resonances do not overlap,

$$\omega_b / \bar{\omega}_c \sim 1/30$$

$$\omega_b \gg v_{\text{eff}}, \Delta\omega$$

then we consider one separate resonance n



$$H = H_0 + \frac{e}{mc} \mathbf{A} \cdot \mathbf{p} = H_0 + \text{Re} \left[C(t) \sum_n V_n \exp(i\xi_{\perp} + in\xi_{\parallel} - i\omega_0 t) \right]$$

$i\xi$

$$\frac{\partial f}{\partial t} + \Omega \frac{\partial f}{\partial \xi} - \text{Re} [iV_{\bar{n}} C(t) e^{i\xi - i\omega_0 t}] \frac{\partial \Omega}{\partial I_{\perp}} \frac{\partial f}{\partial \Omega} = \text{St} f.$$

Effective one-dimensional non-linear problem, ξ is the interaction phase with the field, canonical momentum Ω has the same meaning as κ in the quasi-linear theory.

Self-consistent description of waves and particles

$$E = -\frac{1}{c} \frac{\partial A}{\partial t} \approx \frac{\omega_0}{c} \operatorname{Re} [i A_0(z, r_\perp) C(t) \exp(-i\omega_0 t)] \quad A_0 = \text{mode of a vacuum chamber}$$

$$\frac{dC}{dt} = -\frac{2\pi i \omega_0}{c} e^{i\omega_0 t} \int \mathbf{A}_0^\dagger \cdot \mathbf{j} d^3 r - \gamma_d C. \quad \leftarrow \text{Maxwell equations}$$

described by the same V_n

$$H = H_0 + \frac{e}{mc} \mathbf{A} \cdot \mathbf{p} = H_0 + \operatorname{Re} \left[C(t) \sum_n V_n \exp(\underbrace{i\xi_\perp + in\xi_\parallel}_{i\xi} - i\omega_0 t) \right]$$

$$\frac{\partial f}{\partial t} + \Omega \frac{\partial f}{\partial \xi} - \operatorname{Re} [i V_{\bar{n}} C(t) e^{i\xi - i\omega_0 t}] \frac{\partial \Omega}{\partial I_\perp} \frac{\partial f}{\partial \Omega} = \operatorname{St} f.$$

$$\frac{dC}{dt} = -4\pi^2 i \omega_0 \int V_{\bar{n}}^\dagger f_1 dI_\perp dI_\parallel - \gamma_d C.$$

Formally it is equivalent to the electrostatic Berk-Breizman problem \rightarrow saves our efforts!

Simplest Berk-Breizman problem

Inverse non-linear Landau damping changes to “hole & clump” instability when $St f$ and γ_d become essential

$$\frac{\partial f}{\partial t} + v \frac{\partial f}{\partial x} + \frac{e}{m} E_x \cos(kx - \omega_0 t) \frac{\partial f}{\partial v} = St f$$

$$\frac{\partial E_x}{\partial t} = -4\pi e \int (f - f_0) v dv - \gamma_d E_x$$

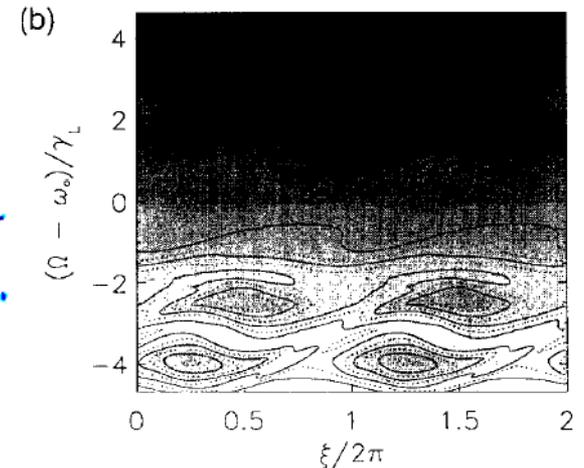
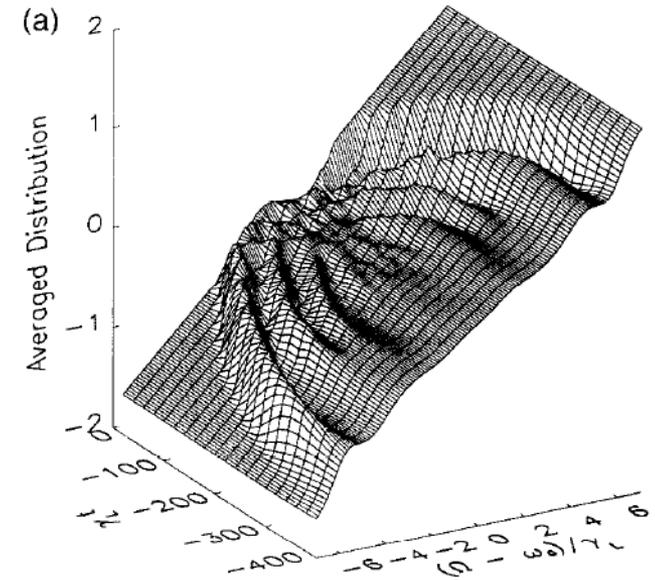
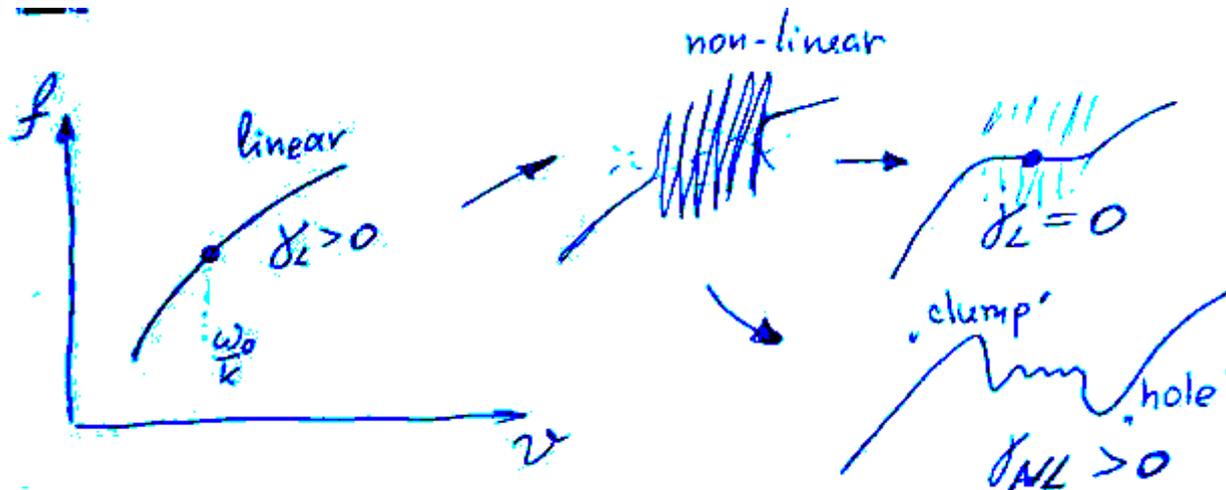


Fig. 3. Particle distribution function with holes and clumps. (a) The spatially averaged distribution as a function of time and $\Omega - \omega_0$. (b) A gray-scale image of the distribution function in phase space at $\gamma_L t = 120$. White corresponds to the smallest values of f and black the largest values. The original resonance is located in the gray area at the mid-line. The islands, corresponding to holes and clumps, are also gray although they are surrounded by other shades of the ambient phase space fluid.

Berk, Breizman, Pekker, PRL 76 1256 (1996)

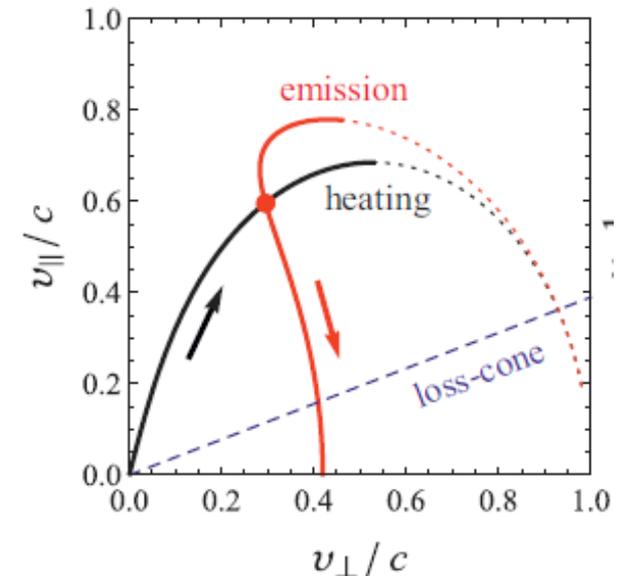
Berk, Breizman, Petviashvili, Phys. Lett. A 234 213 (1997)

How to apply the model to real experiments

- 1) Initial unstable distribution function →
quasi-linear plateau formed during ECRH

$$F = F_0 \exp\left(-\frac{\mathcal{K}(I_{\perp}, I_{\parallel})}{T_e}\right) \begin{cases} 1 & \text{for } \gamma < \gamma^* \\ 0 & \text{for } \gamma > \gamma^* \end{cases}$$

$$\gamma_L = 4\pi^3 \sigma \omega_0 \int \delta(\Omega - \omega_0) |V_{\bar{n}}|^2 \frac{\partial F}{\partial \mathcal{K}} d\mathcal{K} d\bar{I}_{\parallel}$$



- 2) Bounce-resonance harmonic

$$\Omega(I_{\perp}, I_{\parallel}) = \omega_0$$

$$\mathcal{K}(I_{\perp}, I_{\parallel}) = 0$$

$$\varepsilon < \varepsilon^*$$

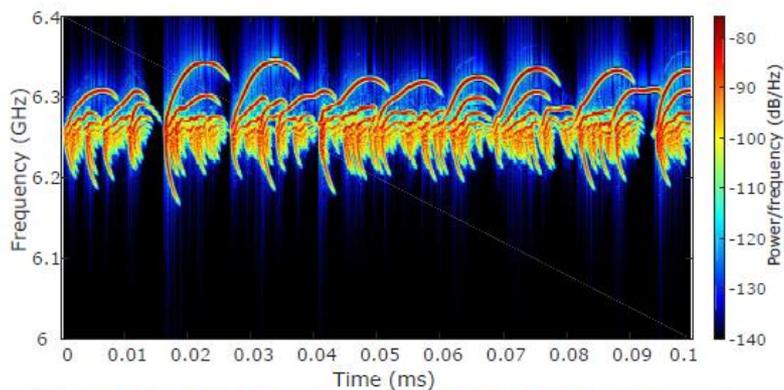
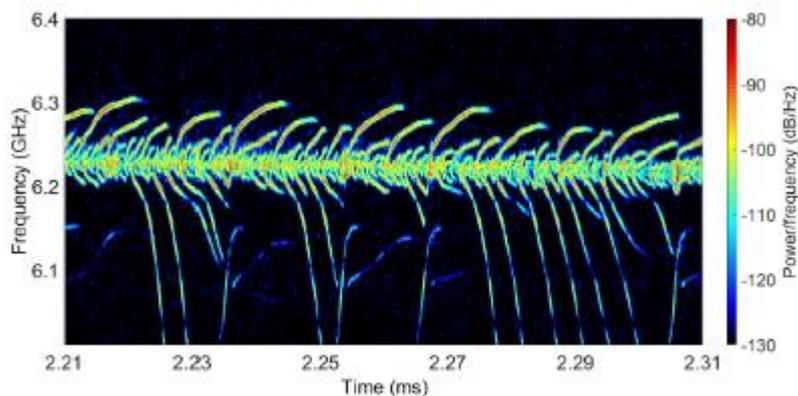
$$\gamma_L > 0$$

n	-30	-32	-34	-36	-38	-40	-42
ϵ , keV	380	320	270	235	205	180	160
γ_L , $\times 10^7 \text{s}^{-1}$	12	7.2	3.6	1.5	0.3	-0.3	-0.5
$\tilde{\omega}_b/2\pi$, GHz	0.54	0.50	0.46	0.43	0.40	0.38	0.36
$\omega_c/2\pi$, GHz	10.8	10.1	9.5	9.1	8.7	8.4	8.2
z_{loc} , cm	2.8	2.4	2.0	1.8	1.5	1.0	0.8

- 3) Wave dissipation → Q -factor of the vacuum chamber (adjusted)
 4) Collision integral for fast electrons → background plasma (adjusted)
 5) Initial wave amplitude → thermal equilibrium with hot electrons

Modeling of experimental spectra with BOT code

Nitrogen



Argon

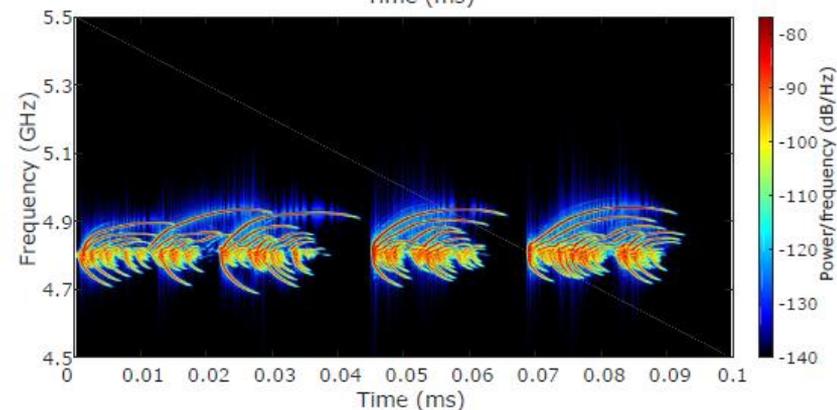
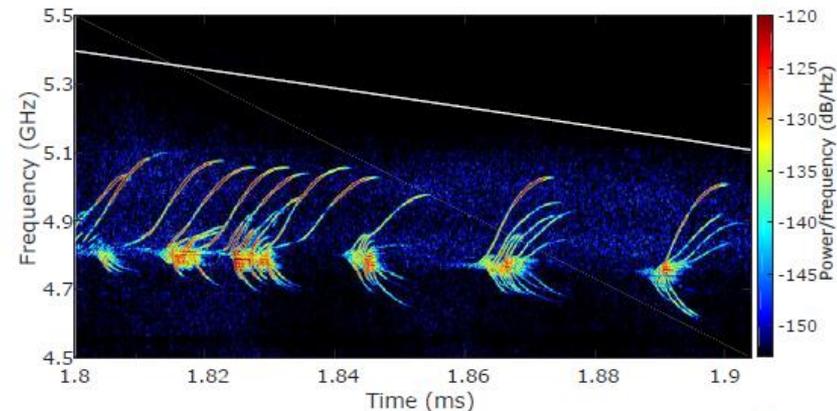


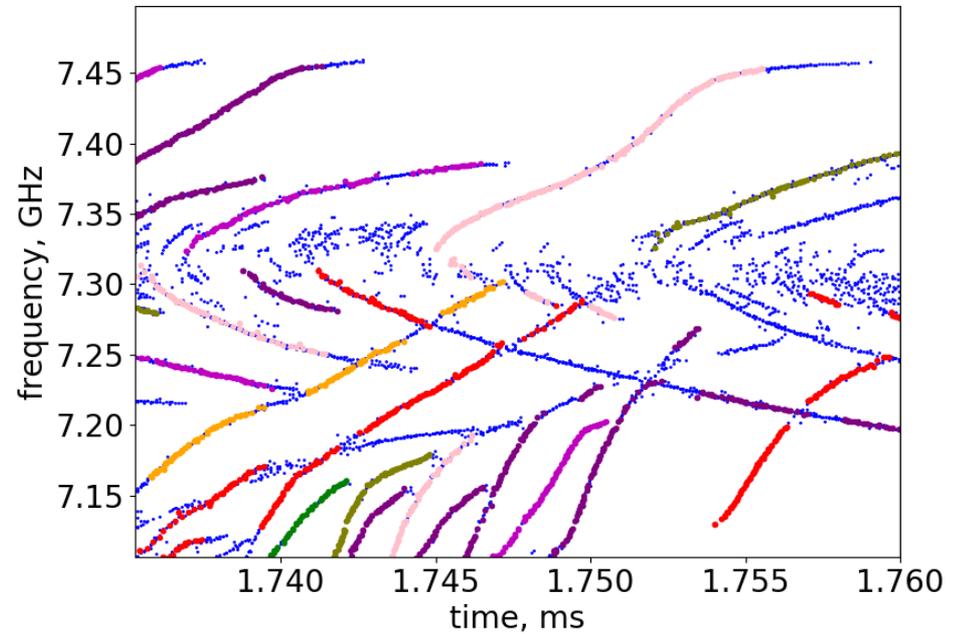
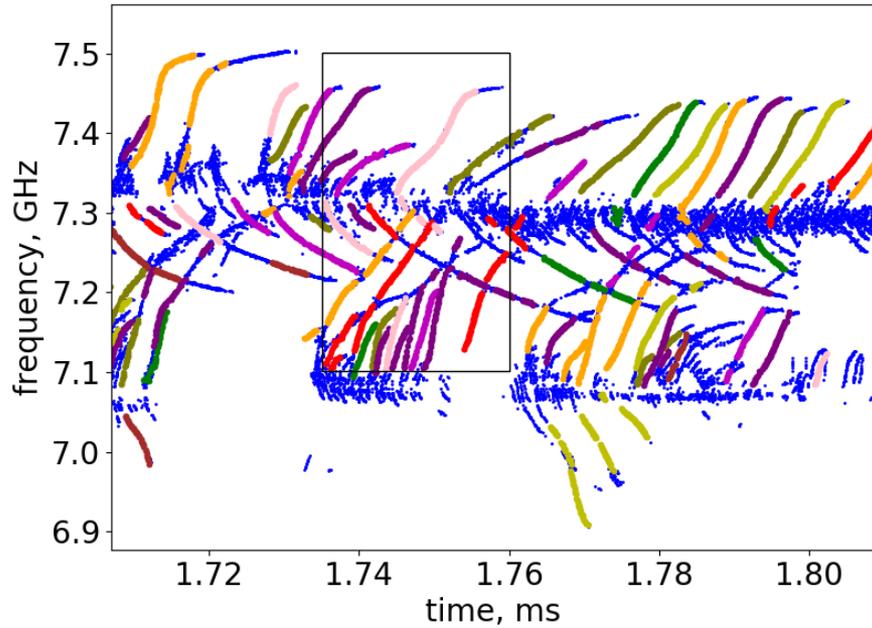
Table 3. Physical parameters for BOT simulations.

Parameter	Initial guess	After adjustment
γ_L	$3.6 \times 10^7 \text{ s}^{-1}$	$7.0 \times 10^7 \text{ s}^{-1}$
γ_d	$3.45 \times 10^7 \text{ s}^{-1}$	$6.44 \times 10^7 \text{ s}^{-1}$
ν_{diff}	$5.3 \times 10^6 \text{ s}^{-1}$	$7.8 \times 10^6 \text{ s}^{-1}$
ν_{drag}	$6.3 \times 10^4 \text{ s}^{-1}$	$9.0 \times 10^6 \text{ s}^{-1}$

BOT code: Lilley M K 2011,
code.google.com/p/bump-on-tail

A G Shalashov, E D Gospodchikov, M E Viktorov //
 Plasma Phys. Control. Fusion, V.61, N.8, P.085020 (2019)

Automatic selection of chirping events



Analysis of the experimental data: the Berk-Breizman model

Frequency change in the wave packet:

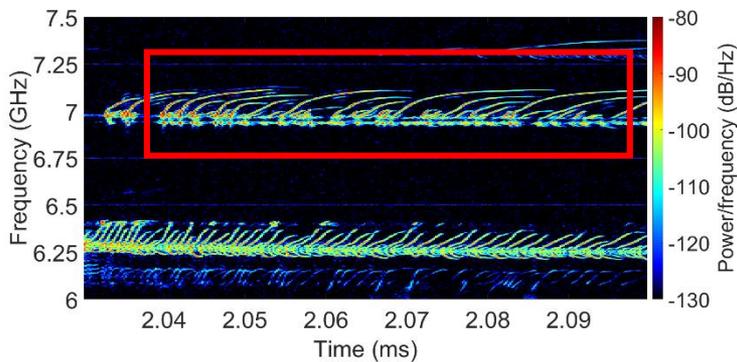
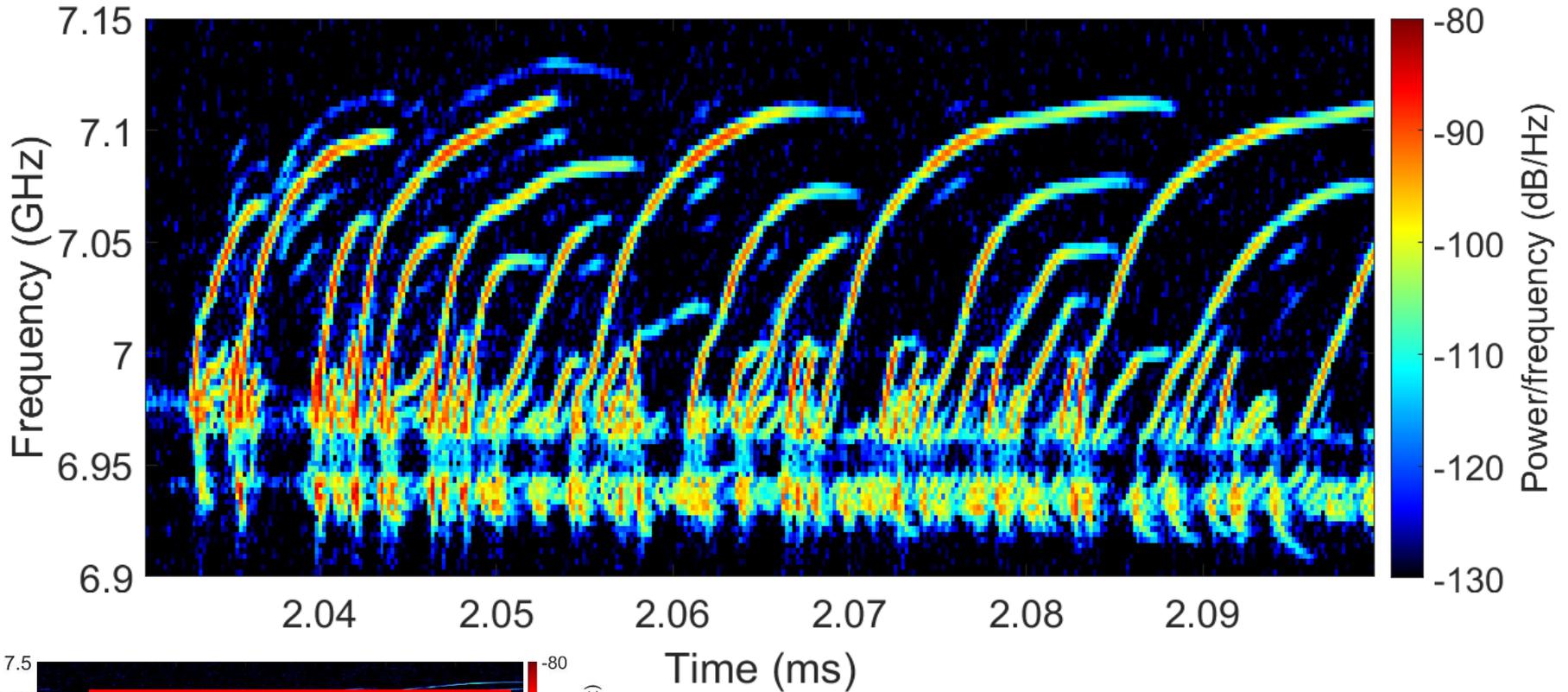
$$\delta\omega \approx \frac{16\sqrt{2}}{3\sqrt{3}\pi^2} \gamma_L \sqrt{\gamma_d t} \equiv \sqrt{At} \quad \longrightarrow \quad \gamma_L^2 \gamma_d \approx 5A$$

H. L. Berk, B. N. Breizman, N. V. Petviashvili, Phys. Lett. A **234**, 213 (1997).

Instability condition:

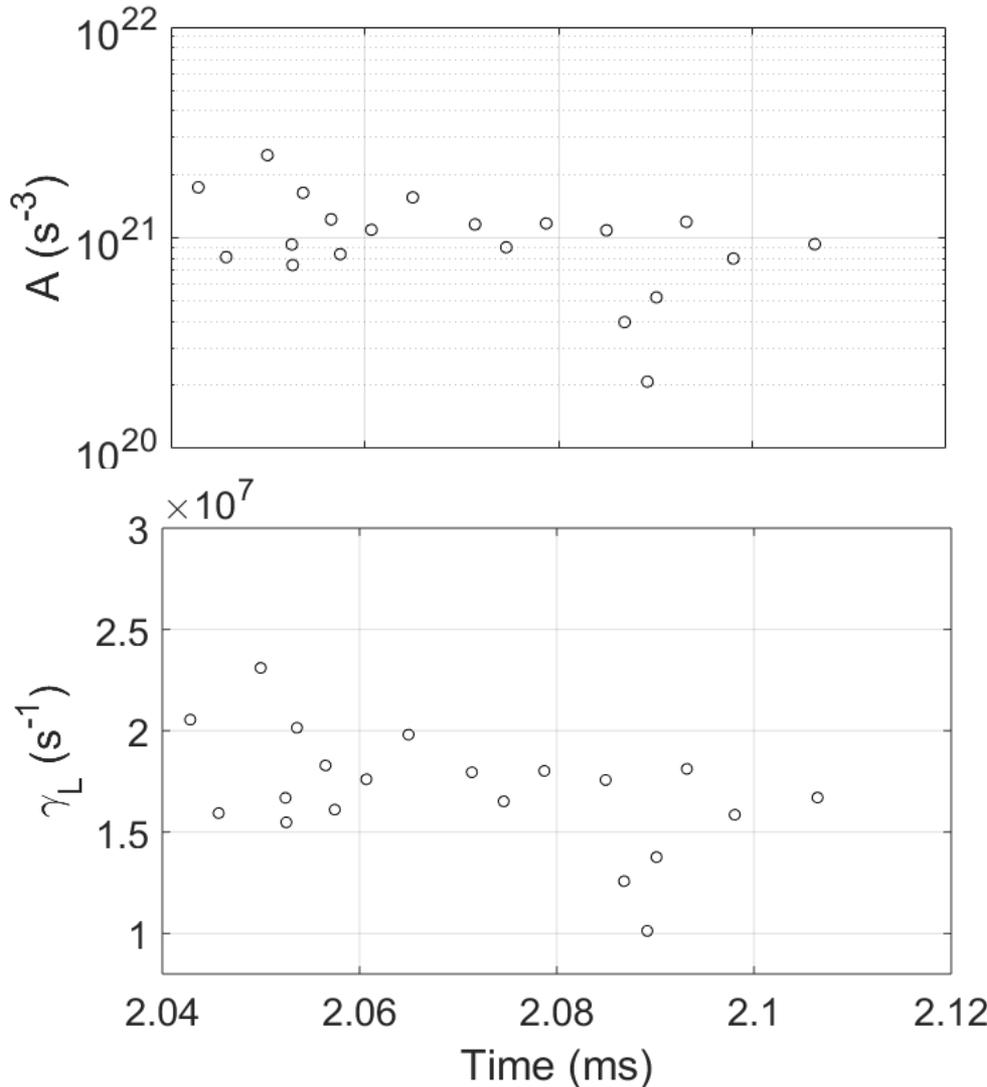
$$\gamma_L \geq \gamma_d \quad \longrightarrow \quad \gamma_L = \gamma_d \approx \sqrt[3]{5A}$$

Analysis of the experimental data: the Berk-Breizman model



$$\delta\omega \approx \sqrt{At}$$

Analysis of the experimental data: the Berk-Breizman model



$$\delta\omega \approx \sqrt{At}$$

$$A = (0.4 - 2) \times 10^{21} \text{ s}^{-3}$$

$$\gamma_L = (1 - 2.5) \times 10^7 \text{ s}^{-1}$$

Conclusions

- The chirping frequency patterns in the plasma EC emission, which are very similar to those predicted by the Berk-Breizman model, were observed during the plasma decay stage, characterized by a high relative density of the fast electrons
- To explain the features of plasma emission spectra we formulate a self-consistent kinetic equation for the distribution of resonant electrons and an equation for complex amplitudes of unstable modes excited under combined (cyclotron and bounce) resonance in an inhomogeneous plasma
- We show that the problem of disturbing electromagnetic modes in an inhomogeneous plasma with a mirror magnetic field configuration can be reduced to equations describing a much simpler situation of exciting electrostatic oscillations in a homogeneous plasma under conditions of nonlinear Landau damping, which are described in the well-known Berk-Breizman model
- Under the conditions of our experiment a case may occur where these combined resonances do not overlap
- Similarities to nature phenomena: Earth magnetosphere, solar flares

Thank you for your attention!

ご清聴ありがとうございました