ENERGY LOSS AND PITCH-ANGLE SCATTERING OF RUNAWAY ELECTRONS DUE TO KINETIC INSTABILITIES

CHANG LIU
Princeton Plasma Physics Laboratory
Princeton, NJ, United States
Email: cliu@pppl.gov

EERO HIRVIJOKI
Aalto University
Espoo, Finland

GUO-YONG FU
Zhejiang University
Hangzhou, Zhejiang, China

DYLAN P. BRENNAN
Princeton University
Princeton, NJ, United States

AMITAVA BHATTACHARJEE
Princeton Plasma Physics Laboratory
Princeton, NJ, United States

CARLOS PAZ-SOLDAN
General Atomics
San Diego, California, United States

Abstract

In recent study of tokamak physics, runaway electrons (REs) have attracted a lot of attention, mainly due to their deleterious effects on the device during disruption events. It is predicted that in ITER disruptions, a large amount of highly energetic runaway electrons can be produced through hot-tail generation and avalanche mechanism, which can cause severe damage to the plasma facing material. The theory of runaway electron avalanche has been studied, but the prediction of avalanche growth rate and threshold electric field are different from the experimental observations. In the paper, the effects of kinetic instabilities on the dynamics of runaway electrons in momentum space is investigated using a newly-developed simulation model, and the discrepancy is partly explained. The results show that the whistler waves excited through the kinetic instabilities can cause runaway electrons to be scattered to large pitch angles and form vortices in momentum space, creating a new energy loss channel, which explains the higher-than-expected critical electric field and the loss of the runaway electron population in the low energy regime identified experimentally. The effects of kinetic instabilities on the electron cyclotron emission (ECE) from runaway electrons is also investigated.

1. INTRODUCTION

In tokamak experiments, runaway electrons can be generated in both the flattop and the disruption scenarios. Although their generation is not significant in current tokamak experiments, it is predicted that for future tokamak device with large plasma current and strong magnetic field, the generation of runaway electrons in a typical disruption event can be much more significant. The major generation mechanism is the hot-tail generation, which means that the hot electrons in the Maxwellian tail are accelerated by the strong inductive electric field in disruptions and become runaway electrons. Then these seed runaway electrons can further grow exponentially in population, called “runaway electron avalanche” [3], which is caused by large-angle collisions between a runaway electron and a cold electron. If left unmitigated and uncontrolled, these runaway electrons can be accelerated to very high energy, and finally cause severe damage to the tokamak device. Several strategies of runaway electron mitigation have been proposed now for ITER, including massive gas injection and shattered pellet injection. Given the risks of testing these methods in ITER, it is very important to have a good understanding of runaway electron physics and a quantitatively correct model to predict the runaway electron phenomena in ITER, in order to test these methods using simulations.

One important discrepancy between the current modelling of runaway electrons and the experimental results is the critical electric field for runaway electron growth, that the experiments give as about 5-10 $E_{\text{CR}}$ (Connor-
Hastie critical electric field \([4]\) whereas theory predicts about \(2 \, E_{\text{crit}}\). The discrepancy indicates the existence of unknown physics mechanisms determining the rate of runaway electron population loss. One candidate is the kinetic instabilities driven by the anisotropic distribution of runaway electrons, which can excite whistler waves in plasma and scatter the resonant electrons to isotropize the distribution. The kinetic instabilities of runaway electrons and the plasma waves excited have been studied by scientists in both theory and experiment, but a self-consistent numerical model to study the interaction between plasma waves and runaway electrons is still missing. In terms of that, we develop a simulation model to study the wave-particle interaction using quasilinear approximation.

2. SIMULATION FRAMEWORK

The framework includes both the modelling of runaway electrons and the plasma waves. The runaway electron distribution function is modelled using a continuum representation. In the current model only the distribution in momentum space is studied, assuming the distribution is homogeneous in space. The coordinates for momentum space are \((p, \zeta)\), where \(p\) is the momentum and \(\zeta\) is the cosine of pitch angle. The plasma waves are modelled using a spectrum method, where the amplitude of every modes is stored in a grid of \((k, \theta)\), where \(k\) is the wavenumber and \(\theta\) is the wave vector direction with respect to the local magnetic field. For each \((k, \theta)\), the wave frequencies for different wave branches can be calculated following the cold plasma dispersion relation, including the slow wave branch (shear Alfvén waves), fast wave branch (compressional Alfvén waves and whistler waves), and extraordinary electron waves. The wave amplitudes of different branches are stored separately. For the current work the whistler waves will be the focus.

The evolution of electron distribution function in momentum space is advanced following the kinetic equation. The kinetic equation we solve is

\[
\frac{\partial f}{\partial t} + \frac{eE_{\parallel}}{mc} \left( \zeta \frac{\partial f}{\partial p} + \frac{1 - \zeta^2}{p} \frac{\partial f}{\partial \zeta} \right) + C[f] + \frac{\partial}{\partial p} \left( F_{\text{rad}} f \right) + D[f] = S_A[f]
\]

Where \(E_{\parallel}\) is the parallel electric field and \(C\) is the collision operator. \(F_{\text{rad}}\) is the synchrotron radiation reaction force. \(D\) is the diffusion operator from the excited waves. \(S\) is the source term for the avalanche.

The wave energy evolves according to

\[
\frac{dE(k, \theta)}{dt} = 2\Gamma(k, \theta)E(k, \theta) + K(k, \theta)
\]

Where \(\Gamma\) is the growth rate of wave amplitude, and \(K\) represents the fluctuation electromagnetic field energy from radiation. This provides the initial amplitudes of the mode.

The wave particle interaction is represented by the term \(\Gamma, K\) and \(D\). The growth (or damping) rate of the mode can be calculated using

\[
\Gamma(k, \theta) = \frac{\omega_p^2}{D} \int d^3 p \sum_{n=\infty} Q_n \pi \delta (\omega - k_v \zeta - n\omega_{ce} / \gamma)(p^2 / \gamma) \hat{L} f
\]

where

\[
\hat{L} = \frac{1}{p} \frac{\partial}{\partial p} - \frac{1}{p^2} \frac{n\omega_{ce} / \gamma - \omega(1 - \zeta^2)}{\omega \zeta} \frac{\partial}{\partial \zeta}
\]

\[
Q_n = \left[ \frac{n\omega_{ce}}{\gamma k_v} J_n(k_v \rho) + E_z \xi J_n(k_v \rho) + iE_z \sqrt{1 - \zeta^2} J'_n(k_v \rho) \right]^2
\]
Here $\omega_p$ and $\omega_{ce}$ are the plasma frequency and electron cyclotron frequency, $J_n$ is the nth order Bessel function, $v$ is the particle velocity, $\gamma$ is the relativistic factor, and $\rho$ is the Larmor radius. $E_y$ and $E_z$ are wave polarization normalized to $E_x$.

In addition to the damping caused by gradient of distribution function (like Landau damping), the collisional damping of plasma waves is also included in the model. The damping rate is calculated by adding the electron-ion collision frequency into the dielectric tensor. The collisional damping is weak compared to Landau damping in flattop runaway electron experiments but becomes dominant in disruption scenarios.

The value of $K$ can be calculated similarly,

$$K(k, \theta) = \frac{\omega_p^2}{D} \int d^3p \sum_{n=-\infty}^{\infty} Q_n \pi \delta(\omega - k_n v \xi - n\omega_{ce} / \gamma) mv^2 f$$

Which, different from $I$, depends on the value of $f$ instead of gradient of $f$.

Finally, the diffusion of resonant electrons can be calculated using a quasilinear diffusion model in magnetized plasma,

$$D[f] = \frac{e^2}{2D} \int d^3k \hat{L} [p_\perp \delta(\omega - k_n v \xi - n\omega_{ce} / \gamma) \mathcal{E}(k, \theta) Q_n p_\perp \hat{L} f]$$

3. FAN INSTABILITIES OF RUNAWAY ELECTRONS

In this study we focus on the results of simulation based on DIII-D runaway electron experiments in flattop scenarios. In these experiments, the density of plasma is kept low and the MHD instabilities are suppressed using RMP coils to cancel the error fields. The runaway electrons can be generated given the large value of $E/E_{CH}$, first through Dreicer generation and later through avalanche. In this first stage of experiment, the value of $E/E_{CH}$ is around 9. Then after some time as the signal of runaway electrons (like hard X-ray or ECE) reaches a certain level, gas puffing is launched which increases the density of plasma. This results in a drop of $E/E_{CH}$ and a suppression of runaway electron population growth. The numbers we used in the simulation is closed to the diagnostic result of plasma core.

In our simulation, we find that in the first stage (before gas puffing), the runaway electron population can have an exponential growth, just like the avalanche theory predicts. The growth rate is also consistent with the theory initially. Then as the runaway electron density reaches a certain level, the whistler mode with frequency around 1-5GHz first become unstable, and the mode amplitude grows to significant level in a short time. The runaway electrons in resonance with these waves are in the energy regime around 7-10 MeV. The resonance is the first harmonic anomalous Doppler resonance ($\omega - k_n v = n\omega_{ce}$, $n<0$), which is driven by the anisotropic distribution of runaway electron tail. Then as the mode amplitude grows, these resonant electrons will experience quasilinear diffusion caused by these modes, and get strongly scattered in pitch angle. This effect is called “fan instability”. The distribution function thus looks different from the simulation result without plasma waves involved (Fig. 1). Then in the later time, as the RE population keeps growing, more modes in the whistler wave branch become unstable, include the high frequency whistler waves with frequency 5-20 GHz (Fig. 2). According to the whistler wave dispersion relation and the anomalous Doppler resonance condition, the higher frequency waves can interact with electrons in lower energy regime, thus in this case the lower energy runaway electrons (2-5MeV) are also scattered. The simulation shows that these electrons can be scattered to larger pitch angle than high energy REs, and some can even reach $p_\parallel = 0$.

In the simulation of the second stage (after gas puffing), as the value of $E/E_{CH}$ decreases to around 4, the total runaway electron population shift from growth to decay. Note that in this case of value of $E/E_{CH}$ is still larger than 1, and according to the avalanche theory, the RE population should still grows. To find the reason for this discrepancy, another simulation is conducted with the same initial condition but with plasma waves turned off. In this case, the RE population grows which is consistent with the avalanche theory. On the other hand, the simulation result agrees with the experimental observations. Further analysis of the simulation shows that the decay is caused by the loss of runaway electrons in lower energy regime, which is also in agreement with the DIII-D experiment gamma-ray imaging diagnostic [5].
According to the simulation result, the decaying of runaway electron population is caused by the diffusion effects from whistler waves. This is because whistler waves can scatter REs to large pitch angles in the low-energy regime through anomalous Doppler resonance. These electrons become less susceptible to the electric force acceleration and can more easily lose energy. This mechanism provides a new channel for RE loss, and is further illustrated in Fig. 1, where we show the directions of flux in momentum space calculated from the kinetic equation. Compared to the case without wave diffusion, the whistler wave can form a vortex structure in runaway electron momentum space. This vortex can hinder REs from going into the higher energy, resulting in a bump-on-tail distribution. On the other hand, in the low-energy regime the electron flux is stochastic since the dynamics are dominated by diffusion rather than advection. The strong diffusion in this region comes from all three types of resonances of whistler waves (Cherenkov resonance, anomalous Doppler resonance, and normal Doppler resonance). Electrons entering this region can be diffused from low pitch angle to high pitch angle, losing energy to the waves, and finally returning to the thermal population.

Note that this mechanism is different from the first stage. In the first stage with large value of $E/E_{CH}$, the excited whistler waves can cause Cherenkov resonance, and the resonance region overlaps with the runaway-loss separatrix. Then the resonance can help thermal electron to cross the separatrix and become runaway electrons. Thus, in this case, the whistler waves enhance the avalanche growth rate. In the second stage the runaway-loss separatrix shifts to higher momentum due to the reduction of $E/E_{CH}$, which does not overlap with the Cherenkov resonance region anymore. Therefore, the diffusion across the separatrix is less significant.

By varying the density of the second stage, we find a new trend of runaway electron population growth or decaying rate as a function of $E/E_{CH}$ (Fig. 2). For small values of $E/E_{CH}$, the avalanche is suppressed by the fan instabilities, but or large values of $E/E_{CH}$, the avalanche is enhanced by the Cherenkov resonance. Compared with the simulation result without plasma waves, we find the new model now predicted a new value of threshold electric field around $6 \ E_{CH}$. This threshold value is much larger than previous models, and agrees well with the DIII-D experiments [6].

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**FIG. 1.** (a) Distribution function of runaway electron tail without kinetic instabilities. (b) Momentum space trajectories of electrons without wave-particle interaction. (c) Distribution function with kinetic instabilities excited. (d) Momentum space trajectories with wave effects.

**FIG. 2.** (a) Spectrum of whistler waves excited in flattop runaway electron experiments. (b) Growth (decay) rate of RE population as a function of $E/E_{CH}$ in stage II, with and without whistler wave diffusion.
4. EFFECTS OF FAN INSTABILITIES ON ECE

The electron cyclotron emission (ECE) has been widely used in tokamak experiments to diagnose the electron temperature. The diagnostic is based on the radiation of electron cyclotron waves (ECWs), because of the gyromotion of electrons around magnetic field lines. ECWs can be emitted and absorbed by the electrons when the resonance condition is satisfied. The high-energy runaway electrons can also produce ECE during gyro-motions. In many RE experiments in tokamaks including both the flattop and disruptions scenarios, ECE signals have been observed that are much more intense than the ECE from thermal electrons.

In the DIII-D flattop runaway electron experiments, ECE plays an important role in diagnosing the runaway electrons. It is found that during the RE population growth in the first stage, the ECE can experience an abrupt growth in the later time, with growth rate much larger than other signals from RE like hard X-rays or gamma rays. The level of ECE signals observed is much higher than the normal ECE signals from thermal electrons. In addition, the high frequency ECE signals grows faster than lower frequency ones and can surpass the lower frequency signal if represented by the electron temperature [7]. These phenomena indicate ECE signals reflects not only a population increase of REs, but also a change of distribution function shape. However, most of the current ECE diagnostic model are based on a Maxwellian distribution, which cannot be applied to study the runaway electron tail.

To study the ECE from runaway electrons, an ECE synthetic diagnostic model for arbitrary electron distributions is developed. Unlike the forward method used in previous codes, in our model we use a backward method to calculate the ECE power based on the reciprocal theorem, which separates the wave emission and absorption by introducing an artificial wave propagating against the radiation along the same ray path. The RE distribution is obtained from the quasilinear simulation framework including the fan instabilities.

The calculation of wave emission and absorption is like the calculation of $I$' and $K$ terms in section 2, which depends on the gradients and the values of distribution function $f$ respectively. As shown in the equations, for the interaction to happen, the resonance condition $(\omega - k_{\parallel} V_F = n \omega_{ce})$ must be satisfied. Given that the magnetic field in tokamak varies as $1/R$ in space, for thermal electrons, this condition can only be met in a thin layer where $\omega_{ce}$ satisfied the equation. Thus, for thermal electrons, the interaction with electron cyclotron waves is local, and the emitted wave amplitude only depends on the property in this resonance layer. If the plasma is optically thick with respect to this wave, it can be further proved that the wave amplitude only depends on the electron temperature rather than density. However, for runaway electrons, the relativistic factor $\gamma$ appearing in $\omega_{ce}$ can vary significantly. Therefore, the wave resonance becomes non-local, and at different radial location, the wave interacts with REs with different energy. In addition, at one location the resonance can happen with several different $n$ number, where higher energy REs correspond to larger $n$.

However, the emission and absorption of electron cyclotron waves by runaway electrons can be limited by their small pitch angles. As shown in the equations of emission and absorption, the interaction coefficients depend on Bessel function $J_\nu$. For runaway electron tail generated through Dreicer and avalanche, the pitch angle is usually very small due to the weak collisional scattering in high-energy regime. Although the value of $p$ can be large, the value of $p_\perp$ is relatively small. Furthermore, for higher energy runaway electrons the interaction with electron cyclotron waves is even weaker, since the resonance condition requires large $n$, and Bessel function with large $n$ requires larger argument to have a significant value.

This can be confirmed with our simulation results. Fig. 3 shows the ECE signals obtained from the synthetic diagnostic tool, using the RE distribution from quasilinear simulation for the first stage. In the early time (before 2s), the signals stay silent, and the value is close to the thermal electron radiation. This is because in this case the RE population is still much smaller than the thermal electrons, and their small pitch angles weaken the interaction with electron cyclotron waves. In the later time (after 3s), the signals start to grow significantly after fan instabilities are triggered, which is consistent with experiments [7]. This result shows that the pitch angle scattering caused by fan instabilities can enhance the ECE from runaway electrons. To further illustrate this effect, we use the ECE diagnostic tool to analyse the distribution with no plasma wave diffusion, and found that the ECE signals barely grows. In other words, the abrupt growth of ECE signals observed is not just a result of runaway electron avalanche, but also be regarded as a indicator of fan instability.
FIG. 3. (a) ECE signals from synthetic diagnostic using RE distribution from kinetic simulation in the first stage. The dashed lines are results without wave diffusion. (b) Spectrum of ECE signals at different time from kinetic simulation.

The simulation result also shows that the higher frequency ECE signals (correspond to third harmonics of $\omega_{ce}$ at core) grows faster than the lower frequency ECE signal (second harmonic), which also agrees with experiments. This effect inspires us to study the evolution of whole ECE spectrum during RE growth. We find that at the beginning when the distribution is close to a Maxwellian, the spectrum is like a step function. At later time as the runaway electron tail grows and fan instabilities are excited, the ECE signals grow at all frequencies and the spectrum becomes flatter, meaning that REs give larger contributions to ECE at higher frequency than lower frequencies. This flattening behaviour is consistent with the ECE spectrum observed in experiments.

To further illustrate the connection of ECE power and fan instabilities, we can study the weight function of the ECE signal power in runaway electron momentum space. Fig. 4 shows the values of the weight function $W(p)$ and the product $W(p)f(p)$, where $f(p)$ is the runaway electron distribution function obtained from the quasilinear simulation including fan instabilities. It is shown that only runaway electrons with large pitch angle and relatively small energy can give significant contribution to the ECE power. Compared with the RE distribution without fan instabilities, these electrons are mainly generated by quasilinear diffusion of whistler waves. This explains why the ECE signals only have prompt growth after the fan instabilities are excited.

5. SUMMARY

With the help of a newly-developed simulation model for interactions between runaway electrons and plasma waves, we can advance our understanding of runaway electron avalanche. We find that in the flattop case of discharge with runaway electrons generated, the RE population can excite fan instabilities due to their anisotropic distribution. The instability can alter the distribution of runaway electrons in momentum space, resulting in significant pitch angle scattering in both low and high energy regimes. In addition, the pitch angle scattering can create a new channel for runaway electron loss. Using this model, we can explain the decrease of runaway electron population in the lower energy regime observed in DIII-D experiments, even if the electric field is still larger than the Connor-Hastie critical electric field. The new threshold electric field after considering the fan instability agrees with the experimental observation.

FIG. 4. (a) Weight function of ECE signal in RE momentum space. (b) Value of weight function multiplied by the RE distribution function in the first stage including fan instability.
Based on the results of fan instabilities, we also study the ECE radiation from runaway electrons. We develop a new synthetic diagnostic tool for ECE in tokamaks, which can be applied to any electron distribution function. The results show that the fan instability plays a critical role in the abrupt growth of ECE signal power from runaway electrons. The result is consistent with the experimental observations. Analysis of the weight function of the ECE power shows that most of the radiation comes from electrons in the lower energy regime with large pitch angles. These electrons are absent in the runaway tail unless fan instabilities happen. This simulation work proves that the ECE can be used as a method to diagnose fan instabilities of runaway electrons.

These results suggest possibilities of mitigating runaway electrons generated in ITER disruptions using fan instabilities and whistler waves. The whistler waves have recently been observed directly in DIII-D experiments [8], which are found to have a correlation with the ECE signals from REs. In addition, similar phenomena have been observed in disruption experiments in several tokamaks, which indicates that the kinetic instabilities also play an important role in the disruption scenario. However, to have these waves excited, the driving force from runaway electrons must overcome the collisional damping, which is much stronger in disruptions due to extremely low plasma temperature. A new simulation model targeting kinetic instabilities in disruption scenarios is now under development, the results will be presented in the future.

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