PITCH ANGLE DYNAMICS AND SYNCHROTRON EMISSION OF QUIESCENT RUNAWAY ELECTRONS IN DIII-D

D. DEL-CASTILLO-NEGrete, L. CABAjAL, M. CIANCIOsA
Oak Ridge National Laboratory
Oak Ridge TN 37830, USA
Email: delcastillo@ornl.gov

C. PAZ-SOLDAN
General Atomics
San Diego, CA 92186, USA

E. HOLLMANn, R.A. MOYER
University of California, San Diego
San Diego CA 92186, USA

C.J. LASNIER
Lawrence Livermore National Laboratory
Livermore, California 94550, USA

Abstract

The synchrotron synthetic diagnostic of the Kinetic Orbit Runaway electron Code (KORC) is applied to simulate visible synchrotron radiation of quiescent runaway electrons (QRE) in DIII-D plasmas. In the simulations, the initial RE energy distribution is taken from bremsstrahlung hard-x-rays (HXR) measurements, and the magnetic field corresponds to an EFIT reconstruction of DIII-D plasma #165826. The calculations resolve full-orbit (6-D) effects, in the presence of electric field acceleration and radiation damping. The plasma and impurities profiles are incorporated in the spatial dependence of the collision frequencies of the Monte-Carlo collisions operator. The simulations also incorporate the DIII-D camera geometry (position and tilt), as well as the angular dependence of the synchrotron emission. A numerical study of the dependence of the radiation spatial pattern on the mean radius of the RE beam, $r_0$, and the fall-off decay rate of the pitch angle distribution function tail are presented. It is shown that the synthetic diagnostic is able to reproduce the observed synchrotron radiation pattern of QRE in DIII-D. However, the pitch angle distribution function that provides the best agreement with the experimental data exhibits a faster fall-off (decay of tail) than what is expected from reduced phase-space Fokker-Planck models that neglect the geometry of the magnetic field and RE orbits.

1. INTRODUCTION

In fusion plasmas, high energy relativistic runaway electrons (RE) can be generated by strong electric fields resulting from the rapid cooling of the plasma during magnetic disruptions, see for example Refs.[1,2]. If not avoided or mitigated, RE can cause serious damage to the plasma facing components of fusion reactors, see for example Refs.[3,4]. Among the different processes controlling the dynamics of RE, synchrotron radiation (SR) plays an important role because it provides a limiting mechanism for the maximum energy that RE can reach. Also, the inclusion of SR energy losses is needed for the accurate computation of thresholds for the plasma parameters determining the sustainment and dissipation of RE [5-7]. On the other hand, SR is also important because it is widely used as a diagnostic to infer RE parameters including energy and pitch-angle distributions [8-19]. In particular, SR measurements, complemented with numerical simulations, opens the possibility to validate theoretical models. In this paper we discuss recent progress on this important problem. In particular, using the Kinetic Orbit Runaway electron Code (KORC) [20-22] (see Fig.1) we study the dependence of synchrotron radiation on the RE distribution function and compare the results with synchrotron radiation measurements in DIII-D. Of particular interest is to evaluate the role of spatially dependent effects. This problem has also been

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addressed recently in Ref.[23] following an approach involving different approximations and assumptions as discussed in Ref. [24].

Here, going beyond previous studies, we perform full-orbit simulations that allow the incorporation of detailed orbit effects on SR. As shown in Fig.2, these effects can significantly modify the SR in axisymmetric and stochastic magnetic fields [22]. The calculations also incorporate the geometry of the camera as well as the full angular dependence of the direction of emission which as discussed in [21] can also have an impact on the SR radiation pattern and spectra.

The experiment of interest is the DIII-D plasma #165826, a hot (~keV) low-density (~10^{19} m^{-3}) plasma in which the equilibrium conditions are not significantly affected by a trace-level of quiescent runaway electrons (QRE). The well-controlled, steady-state conditions of this plasma provide a unique setting to perform validation studies of theoretical and numerical models of RE using visible and infrared SR diagnostics. The main conclusion reached is that the KORC synchrotron synthetic diagnostic is able to reproduce the observed synchrotron radiation pattern of QRE in DIII-D. However, the pitch angle distribution function that provides the best agreement with the experimental data exhibits a faster fall-off (decay of tail) than what is expected from reduced phase-space Fokker-Planck models that neglect the geometry of the magnetic field and RE orbits.

2. SIMULATION METHODOLOGY

Our methodology is based on the computation of SR of RE sampled from a given distribution function providing the initial position and momenta of the ensemble. The initial phase space distribution is assumed of the form $f_{0E}(\theta, E, t = 0) = f_0(\theta; E)f_0(E)$. The quantitative modeling of the energy distribution, $f_0(E)$, of QRE in DIII-D is a challenging problem. In particular, as discussed in [19,24] Fokker-Planck models tend to predict distributions that are broader than the experimental observations. Because of this, in the study presented here we use an energy distribution model obtained from a direct fit of the bremsstrahlung hard-x-ray emission measurements [18]. The fitting is based on a distribution of the form

$$f_0(E) = \frac{e^{\alpha - 1}}{\Gamma(\alpha)} \beta^\alpha \exp \left( -\frac{E}{\beta} \right)$$

where $\Gamma(\alpha)$ is the gamma function, $\alpha = 15.38$, $\beta = 0.50$, and $E$ is the energy in MeV. As shown in Fig. 3, these parameters provide the best least-squares fit to the experimental data.
On the other hand, since the pitch angle distribution $f_0(\theta; E)$ is not well resolved in the experiment, we consider the following model

$$f_0(\theta; E) = \frac{A}{2 \sinh A} \exp(A \cos \theta)$$

where $A = \hat{A}C(\gamma^2 - 1)/\gamma$ and $C = 2E/(Z_{\text{eff}} + 1)$. Here $Z_{\text{eff}}$ denotes the effective atomic number of the plasma, $\gamma$ is the relativistic Lorentz factor, and $E$ is the electric field normalized to the Connor-Hastie critical field. When $\hat{A} = 1$ this distribution corresponds to the reduced 2D phase-space Fokker-Planck model that ignores spatial effects and assumes a steady state balance between the pitch angle collision operator and the electric field acceleration $[17, 25]$. As such, this model provides a good starting point to explore the dependence of the SR on the pitch angle degree of freedom. However, to add flexibility to the model, and in particular to explore the possibility of decay rates different to those predicted by the Fokker-Planck model, we treat $\hat{A}$ as free parameter. As shown in Fig.3-b), $\hat{A}$ controls the decay of the pitch angle distribution, in particular $\hat{A} > 1$ increases the fall-off rate.

In addition to the $(\theta, E)$ degrees of freedom, we need to specify the initial spatial distribution, $n_{RE}(r, t = 0)$, of the RE beam. As shown in Fig.3-c)-d), we consider a uniform toroidal distribution centered at the magnetic axis with elliptical cross section approximating the flux surfaces. The mean minor radius $r_0$ of the spatial distribution is used as a second adjustable parameter. That is, the numerical simulations are parametrized by $\hat{A}$ and $r_0$ giving the fall-off of the pitch angle distribution and the size of the runaway beam. In all the simulations presented here, the magnetic field as well as the plasma density, temperature, and impurity concentration ($Z_{\text{eff}}$) spatial profiles shown in Fig.4 were obtained from an EFIT reconstruction of DIII-D QRE plasma #165826.

The total radiation power of an ensemble of RE is given by $P_R = \int d^3r \int_0^\infty dE \int_0^\pi \sin \theta d\theta P_R$ where $P_R = n_{RE}(r, t)f_{RE}(\theta, E, t) p_R(\theta, E, \lambda, B_0)$ is the radiation power distribution function and

$$p_R(\theta, E, \lambda, B_0) = \frac{1}{\sqrt{3} \epsilon_0 \hbar} \frac{N}{\epsilon} (\frac{m_e^2}{e^2})^2 \int_{\lambda/(\epsilon)}^\infty K_{5/3}(w)dw$$

where $N$ is the number density of the impurities.
is the synchrotron radiation per particle located at position \( \mathbf{r} \) with energy \( \mathcal{E} \) and pitch angle \( \theta \), at wavelength \( \lambda \) in a magnetic field \( B_0 \) with \( \lambda_c = 4\pi/(3\kappa\gamma^2) \), \( \kappa \) the curvature of the orbit, and \( K_{5/3} \) the modified Bessel function of order 5/3 [26].

![Graph 1](image1.png)

**Fig. 3** a) Initial energy distribution function, \( f_a(\mathcal{E}) \), in logarithmic scale. Blue triangles show experimental bremsstrahlung hard-x-ray emission measurements [18]. The solid red line shows the least-squares fit of the experimental and the red dashed lines show the 95\%-confidence interval. b) Initial pitch angle distribution, \( \varphi_0(\theta, E) \), for different initial energies and different values of the adjustable parameter \( \Lambda \). c) Spatial initial distribution, \( n_{RE}(r) \), of \( RE \) in a poloidal cross section. d) Radial profile of initial spatial distribution of \( RE \) across a line towards the low field side (LFS) and across a line towards the high field side (HFS).

The main challenge in the accurate evaluation of \( p_\theta(\theta, \mathcal{E}, \lambda, B_0) \) is the computation of the curvature which is determined by the geometry of the orbit. Going beyond previous studies based on approximate expression of \( \kappa \) [27], here we use the exact expression, \( \kappa = \frac{\kappa \times \bar{F}}{r \times \bar{r}} \), where the dots denote derivatives with respect to time and \( \mathbf{r}(t) \) is the \( RE \) orbit according to the full orbit dynamics \( \frac{dp}{dt} = \mathbf{F} \) where \( \mathbf{p} = \gamma m \mathbf{\dot{r}} \) is the relativistic momentum and \( \mathbf{F} \) is the Lorentz force. As discussed in [20-22] inaccuracy in the computation of \( \kappa \) can have important effects on the total radiation power, the radiation spectra, and the spatial distribution of the radiation pattern.

![Graph 2](image2.png)

**Fig. 4** Radial profiles of plasma parameters in the DIII-D plasma #165826 at about 2.5 s after the gas puff see [18,19] for details.
The strong dependence of the synchrotron radiation on the energy and pitch angle of the particle implies that, as shown in Fig. 5, for a given wavelength the main contribution of the observed radiation comes from RE with relatively large energies and pitch angles. Based on this, in the numerical simulations presented here, instead of sampling the whole RE distribution function (filled colored contour in Fig. 5), we limit attention to the subset (yellow box in Fig. 5) of the RE distribution that contributes to 90% of the total radiation. The size and location of this subset depends on the wavelength of the synchrotron emission which in this case we took to be $\lambda = 890 \text{ nm}$ since we are interested in visible radiation. In accordance with DIII-D, the position (in cylindrical coordinates measured from the center of the tokamak) of the SR synthetic camera in KORC is $(R_c, Z_c) = (2.760, 0.076) \text{ m}$, and it is aimed so that its main line of sight is tangent to the magnetic axis. The synthetic camera detector consists of a 2-dimensional array of 75 horizontal X 120 vertical pixels, and it has a band-pass filter in the visible wavelength window $\lambda \in (885, 895) \text{ nm}$.

**Fig. 5** The filled contours show the Initial energy and pitch-angle distribution function $f_\theta(\theta; E)$ in logarithmic scale with $\tilde{A} = 1$. The magenta lines show the isocontours of the radiation power density, $f_{RE}(\theta, E, t = 0) \ p_\theta(\theta, E, \lambda, B_0)$. The yellow box contains 90% of the total synchrotron radiation seen by the DIII-D camera and provides the sampling region for the simulations. Note that the specific size and location of the box depends on $\lambda$, $B_0$, and the distribution function $f_{RE}$, in particular the value of $\tilde{A}$.

**Fig. 6** a) DIII-D camera image of visible synchrotron radiation in plasma #165826 at $t = 5045 \text{ ms}$ [19] b) Synthetic synchrotron radiation image from KORC simulation for optimal model parameter values $(r_0, \tilde{A}) = (0.33)$.  

### 3. Dependence of Synchrotron Radiation on Pitch Angle Distribution Fall-off and RE Beam Radius

Figure 6-a) shows the visible synchrotron radiation pattern in the camera plane measured in the DIII-D plasma #165826. To identify the parameter values that best reproduce the observation we performed a set of simulations...
varying $\hat{A}$ and $r_0$, fixing the rest of the parameters, magnetic field, and initial RE energy distribution as described in the previous section. The figure of merit to assess the agreement between the synthetic and DIII-D cameras is the root mean squared reconstruction error $\Delta = \| I_c - I_s \|$ where $I_c$ and $I_s$ are 75x120 matrices representing the camera measured and simulated radiation intensities respectively, and $\|A\| = \sqrt{\sum_{i,j} A_{ij}^2}$.

![diagram](image)

Fig. 7 Parameter dependence of the mean squared reconstruction error $\Delta = \| I_c - I_s \|$, quantifying the agreement between the KORC simulated and the experimental images in the DIII-D plasma #165826. (a) Computed value of $\Delta$ on a discrete grid of the $(r_0, \hat{A})$ plane. (b) Smooth interpolated version of (a) using Gaussian shape functions. The x at $(r_0, \hat{A}) \approx (0.3, 3)$ marks the location of the minimum of $\Delta$. The magenta shaded area in (a) and the dashed horizontal gray lines in (b) shows the experimentally inferred range of $\hat{A}$ for RE energies in the range $\mathcal{E} \in (6, 9)\text{MeV}$. The solid and dashed vertical magenta lines show experimental estimates of the beam size based on the RE energy distribution and emission [19].

Figure 7-a) shows $\Delta$ for a finite number of simulation plotted on a grid in the $(r_0, \hat{A})$ plane. To guide the interpretation of this set of simulations, Fig.7-b) shows a continuum interpolation of the discrete data using Gaussian cloud functions. It is observed that the minimum of the reconstruction error $\Delta$ is achieved for $(r_0, \hat{A}) \approx (0.3, 3)$. This corresponds to a fall-off of the pitch angle distribution three times larger than the fall off expected from the reduced, $\hat{A} = 1$, model. However, it is important to note that the size of beam, $r_0$, has also an important effect on $\Delta$. Although there are no direct measurements of $r_0$ in DIII-D, indirect measurements based on the RE emission and energy distribution estimate $r_0 \approx (0.25, 0.30)$ (see solid and dashed magenta vertical lines in Fig.7) which is in agreement with the optimal value obtained from the simulations $r_0 = 0.30$.

![diagram](image)

Fig. 8 Comparison of the position and size of simulated and experimental SR images. (a) Difference between the horizontal position of the experimental $R_z$ and the simulated $\hat{R}_z$ SR images. (b) Same as a) but for the vertical position $\hat{Z}$. The X marks the optimal value $(r_0, \hat{A}) \approx (0.3, 3)$.
Further insight on the dependence of the radiation pattern on \((r_0, \hat{A})\) can be gained by using Proper Orthogonal Decomposition (POD) methods. In particular, the singular value decomposition (SVD) of the measured and simulated radiation images in the pixel plane of the camera, \(I_c\) and \(I_s\), can guide a systematic comparison based on optimal modes in the R and Z directions. Based on this, we define the radial and vertical mean position of an image as \(\bar{R} = \sum_j R_i v_i^{(1)}\) and \(\bar{Z} = \sum_j Z_i u_i^{(1)}\) where \(v_i^{(1)}\) and \(u_i^{(1)}\) are the components of the rank-1 SVD modes in the R and Z directions respectively. Here we are assuming that \(v_i^{(1)} > 0\) and \(u_i^{(1)} > 0\) for all \(i\), which is the case for the data under consideration, otherwise similar definitions can be used using the absolute value of the vector elements. Figure 8 shows the difference of the experimental and the synthetic camera mean position as function of \((r_0, \hat{A})\). For \(r_0 \approx (0.25, 0.30)\) a better match in \(\bar{R}\) is observed for \(\hat{A} \geq 3\) while a better match in \(\bar{Z}\) is observed for \(\hat{A} \leq 3\), a result consistent with the previous analysis that show \(\hat{A} \approx 3\) to be an optimal value.

As discussed before, the observed visible synchrotron radiation in the experiment and in the simulations is mostly due to RE with relatively large pitch angles and energies. In particular, the yellow box in Fig.4 contains the RE that contribute to 90% of the total radiation power. This motivates considering the weighted distribution function \(f_{SR} = f_{RE}(\theta, E, t) p_R(\theta, E, A, B_0)/P_E\) which is, up to the normalization factor \(P_E\), the radiation power distribution function. Figure 8(a) shows the initial condition which by construction follows the pattern of the magenta contours in Fig.5. As Fig.9(b) shows, \(f_{SR}\) evolves in time and after \(t=35\) ms shifts downwards and develops a trapped RE tail. It is observed that for low energies the pitch angle dependence of \(f_{SR}\) does not vary significantly. However, for large energies, \(f_{SR}(\theta, t)\) drifts towards smaller values of \(\theta\). However, in interpreting these results it is important to keep in mind the difference between the weighted distribution function, \(f_{SR}\), and the RE distribution function \(f_{RE}\).

![Fig.9 Time evolution of weighted distribution function of RE responsible for the observed SR. Panel (a) shows the initial condition and panels (b) the numerically computed PDFs at \(t=35\) ms logarithmic scale. The high-pitch angle, low energy tail in (b) correspond to trapped electrons.](image)

4. SUMMARY AND CONCLUSIONS

The synchrotron synthetic diagnostic of the Kinetic Orbit Runaway electron Code (KORC) was applied to simulate the visible synchrotron radiation of CRE in DIII-D plasmas. In particular, a numerical study of the dependence of synchrotron emission on the modeling parameters \(r_0\) (mean radius of the RE beam) and \(A\) (fall-off decay rate of the pitch angle distribution function tail) was presented. It was show that the minimum reconstruction error is achieved for a mean radius \(r_0 \approx 0.3\) and a decay rate \(A \approx 3\), bigger than the \(A = 1\) value assumed in simplified 2D phase-space reduced models. In the simulations, the energy distribution function, the EFIT reconstructed magnetic field, and the density, temperature, and impurity radial plasma profiles were taken directly from the experiment.

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