PREDICTING SCRAPE-OFF LAYER PROFILES AND FILAMENTARY TRANSPORT FOR REACTOR RELEVANT DEVICES

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Abstract

The paper describes a novel statistical framework that was developed to derive radial profiles of thermodynamic quantities in the Scrape-Off Layer (SOL) of tokamak devices starting from basic properties of filamentary fluctuations that generate them. The framework changes the emphasis from interpreting and predicting diffusive/advective coefficients to describe SOL transport to understanding the statistics and dynamics of the filamentary structures. Experimental and numerical tools were developed to provide this input. In particular, it was developed a novel fast camera analysis technique based on wide angle pseudo-inversion of the light emitted by the filaments coupled with convolutional neural networks. Probability density functions for filament widths, amplitudes, waiting times and toroidal separation were obtained, finding, for example, that filaments do not have a clear modal structure. A Bayesian analysis of Langmuir probe data at the midplane shows that filaments are well matched by individual independent events and that are not generated in the SOL. 3D numerical simulations confirm that filaments that are sufficiently far apart (~5 widths) do not interact. Electromagnetic effects, important for inter-ELM filaments, show that the electrical connection to the target can be lost at sufficiently high $\beta$ or long enough connection lengths, leading to faster filaments and increased cross field transport. Finally, MAST and JET data were successfully matched with profiles calculated with the statistical framework.

1. INTRODUCTION

In magnetic confinement devices, boundary turbulence is responsible for transporting across the magnetic field plasma and energy from the well-confined region towards the material surfaces [1]. It is well known that the plasma exhaust associated with this transport, if uncontrolled, can severely harm reactor relevant machines, challenging the viability of magnetic confinement fusion as an energy source [2]. In particular, the first wall will be less protected than the divertor and equipped with delicate plasma facing structures such as resonant heating antennas. It is therefore essential to develop a solid first principles understanding of the mechanisms behind the transport in the edge of the plasma.

As universally observed, the midplane boundary plasma is characterised by large fluctuations, often called filaments or blobs [1], which dominate the particle transport and determine the loads, fatigue and erosion of the plasma facing components in steady state conditions. Importantly, filamentary transport is present in every operation regime [3], including H-mode, where it will determine the dominant wall flux in ELM mitigated regimes, and L-mode, a necessary transient phase at the beginning and the end of pulsed reactor operations.

The paper will review a novel approach to predict and interpret particle and power exhaust physics in the Scrape-Off Layer (SOL) as mediated by L-mode and inter-ELM filaments [4-6]. We will present a comparison between the theoretical and numerical work recently carried out at CCFE and experimental measurements obtained with innovative techniques on MAST and JET. In the final part of the paper, the emphasis will be on extrapolating the knowledge we gained to future machines like ITER and to advanced divertor solutions relevant to MAST-U and potentially DEMO. This will be done by exploiting the so called “universality” of the SOL turbulence and robust results from present-day machines.

2. THEORETICAL FRAMEWORK

The theoretical framework [4-6] was developed to clarify the relation between radial SOL profiles and the fluctuations that generate them. Our work was originally inspired by the seminal work of Garcia and his group [7-8], which approached the theoretical interpretation of single point measurements of SOL turbulence using a
rigorous statistical treatment. The prediction based on this theoretical formulation of the problem found excellent confirmation in experimental measurements in several machines [9-12]. Our theoretical framework extended this successful approach in order to go from single point interpretations (i.e. at a specific radial location in the SOL) to a full radial and toroidal description by introducing information on the cross field and parallel dynamics of isolated filaments. As a consequence, the calculation presented in this paper inherits all the successful features of the Garcia model and allows for a radially dependent description.

Our theoretical framework can therefore predict and interpret the experimental features of the profiles and of the turbulence statistics simultaneously on the basis of simple properties of the filaments, such as their radial motion and their draining towards the divertor. Importantly, the fundamental equations correlating the filament dynamics and statistics with the radial profiles and turbulence properties do not require specifying the former. This means that the framework is flexible and can be customized by employing a filament model of the user’s choice, which can be extracted from either theoretical considerations or experimental observations. In this respect, the framework can be used as a consistency test for interpretations of how the filaments determine the profiles, especially when it is combined with data from experiments.

At the base of our statistical calculation is the assumption that L-mode and inter-ELM filaments can be described as a Poisson process in which each event is independent, i.e. the filaments do not interact with each other. In a toroidal plane, a single event has the form:

\[ n_i(x,y,t) = n_{0,i} D(t) \lambda \left( x - \int_0^t V_{x,i}(t')dt', y - y_0 - \int_0^t V_{y,i}(t')dt', w_{x,i} w_{y,i} \right), \]  

(1)

where \( n_i \) is the density associated with a single filament that moves in the radial (x) and toroidal (y) direction. While density is used in Eq. (1), any thermodynamic quantity associated with the filament could be used as well. Here, \( n_{0,i} \) measures the peak filament amplitude and \( \lambda \) is the filament spatial shape function with \( w_{x,i} \) and \( w_{y,i} \) the widths in the two directions and \( y_0 \) the toroidal position where the filament crosses the separatrix (\( x=0 \)). The dynamics of the filament is determined by \( D \), which is a function representing its draining in the parallel direction (i.e. outside the toroidal plane), and by \( V_{x,i} \) and \( V_{y,i} \) its radial and toroidal velocity. Experimentally, many filaments are observed in the SOL at the same time and they are ejected continuously from the core. This can be captured by summing individual (independent) events:

\[ \eta(x,y,t) = \sum_{i=1}^k n_i(x,y,t - t_{0,i}). \]  

(2)

where \( \eta \) is now the 2D instantaneous distribution of the density and \( t_{0,i} \) is the time at which the \( i \)th filament is ejected. It is easy to see that a radial profile at a given toroidal position \( y=y_0 \) is simply the time integral of Eq. (2). In many SOL cases, it is such an integral that is actually measured experimentally since diagnostics can have a finite and rather coarse exposure time (~millisecond). This generates the misconception that midplane SOL profiles are relatively smooth, while they are actually quite bumpy.

In a deterministic approach, in order to calculate \( \eta \) and hence the profiles, we would need to provide all the quantities with subscript \( i \) in Eqs. (1) and (2). Indeed, even in the same discharge, filaments have characteristics (e.g. \( n_{0,i}, w_{x,i}, t_{0,i}, \ldots \)) that are statistically distributed [13]. However, Eqs. (1) and (2) represent an ergodic process, for which time averages can be replaced with ensemble averages, which are much easier to calculate once the probability distribution functions of the filament’s characteristics are known. We will skip the technical details of calculations, which can be found in [4-6], and give only the final result:

\[ N(x) = \frac{1}{w_i} \int_{-\infty}^{\infty} dy_0 \int_{-\infty}^{\infty} dt \int_0^{\infty} d\lambda \int_0^{\infty} dw \int_0^{\infty} d\eta \int_0^{\infty} dw \int_0^{\infty} dw \eta(x,y,t) P_{y_0}(y_0) P_{v_0} (v_0) P_{w} (w), \]  

(3)

where \( \tau_{w} \) is the waiting time of the Poisson process (i.e. the typical time separation between a filament and the next) and the symbol \( P \) represents the probability distribution function of the quantity in the subscript. For the sake of simplicity, we took \( w_{x,i}=w_{y,i} \) in Eq. (3), but this is not a necessary assumption and extension to the general case is straightforward. Note that all the symbols in Eq. (3) have lost their subscript \( i \) when passing from the time to the ensemble average. We have now correlated the filament dynamics \( |D(t)|, V_{x,i}(t) \) and \( V_{y,i}(t) \) and statistics \( (P_{y_0}, P_{v_0} \) and \( P_{w} \) to the radial profile, \( N(x) \). In order to determine the turbulence statistics (variance, skewness, PDF of the fluctuations,…) the same procedure can be repeated with different integrands in the integrals. The variance, in particular, is given by:
\[
\sigma_n(x)^2 = \frac{1}{\tau_{\text{wp}}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dt \int_{0}^{\infty} dn_{0} \int_{0}^{\infty} dw \left[ n(x,y,t)^2 \right. \\
\left. \times \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dy_{0} \int_{-\infty}^{\infty} dt_{0} P_{y0}(y_{0}) P_{n0}(n_{0}) P_{w}(w) \right].
\]  

(4)

As mentioned earlier, Eqs. (3) and (4) are entirely consistent even without specifying the features of the filament population. In the rest of the paper we discuss experimental and numerical results that provide useful information with respect to the validity of the framework (i.e. when the assumptions of the model hold), possible models for the dynamics and statistics of the filaments and finally its applicability to reactor relevant exhaust problems.

3. EXPERIMENTAL RESULTS AND TOOLS IN SUPPORT OF THE FRAMEWORK

Innovative experimental analysis techniques, making use of deep machine learning and Bayesian approaches, were developed and tested on MAST data in order to provide a statistical basis for the input of the framework, but also to thoroughly characterise the filaments. Due to its open configuration, MAST was ideally suited for SOL transport studies: measurements of filaments over a large database of discharges were performed with fast unfiltered wide-angle visual cameras (framerate: 100kHz; exposure time ~3\(\mu\)sec) [14] and with a midplane reciprocating probe.

The number of filaments captured by the cameras in each discharge is of the order of one million, and hence a non-human based procedure for their detection and tracking is essential. The newly developed ELZAR code [14] automates data analysis by using a pseudo-inversion of the 2D fast camera images, which relies on the alignment of the filaments with the magnetic field. Each field line, projected on the camera view, is treated as a homogeneous light emitter and the actual camera image is treated as a linear combination of such emitters. Mathematically, \([I] = [W][B]\), where \([I]\) is the matrix representing the intensity of the pixels in the camera picture, \([B]\) is the matrix of basis functions associated with each emitter and \([W]\) is the matrix that weights the contribution of each field line to the camera image. The pseudo-inversion is then performed by calculating the relative weight of each field line given the camera image, \([W] = [I][B]^{-1}\), and, knowing the position of the field lines in the midplane, provides a mapping of the light intensity as a function of radial and toroidal position. Filaments in this mapping appear as 2D blobs of high light intensity.

Once the mapping is available, the position, width and (relative) intensity of the filaments can be assessed. The identification process can be performed with ellipse fitting algorithms or, more recently, with Region proposal Convolutional Neural Networks (R-CNN). In particular, a faster R-CNN was used to identify the filaments in the mapped image. In order to train and test the network, a set of 5,000 synthetic images with a total of ~40000 filaments was produced. These images were generated by randomly choosing filaments extracted from a statistical population based on experimental PDFs. The training set was composed of 4500 images and the testing set of 500. Even with this limited dataset, the recognition network was very successful with a precision of 93% (percentage of correctly detected items among all the detected ones, including false positive) and with a recall of 73% (fraction of items that are correctly detected, i.e. ratio between true positive an actual number of filaments in the image). These encouraging results can be probably improved by using a much larger dataset, which led us to generate half a million synthetic images, currently being processed by a new neural network. Once this network is trained, it will be deployed on actual experimental data in order to create a large database of filament characteristics to be used in the theoretical framework.

The ELZAR code has already produced a large number of results using standard detection techniques as the ones described in [15]. In particular, preliminary measurements of PDFs of amplitude, widths at the separatrix and toroidal ejection position of the filaments were measured in a typical MAST Ohmic L-mode discharge [6]. It was found that the widths \([P_{\text{wa}}]\) and \([P_{\text{wo}}]\) of Eqs. (3) and (4) were well described by a log-normal distribution with average radial value around 1-2 centimeters (the toroidal value is larger, but it depends on the pitch angle at the midplane). The PDF of the amplitude \([P_{\text{a}}]\) had an exponential behavior over almost 4 orders of magnitude, while the toroidal separation \([P_{\text{t}}]\) was exponential for separations above ~12cm and rolling over at smaller values. This is consistent with the theoretical result that filament separations above ~5 typical widths lead to no interaction [16] and hence an exponential decay as in a spatial Poisson process, but closer filaments do affect each other (see next Section). Filaments are therefore emitted uniformly and randomly, thus lacking a well-defined toroidal mode number. This result, together with the many observations of exponential decay of the waiting times in the far SOL with reciprocating probes, justifies the use of the independence assumption in our theoretical framework. However, the rollover of the PDF at small separations and the possible filament interactions in the near SOL suggest that care must be taken in using our calculations in the near SOL and that some residual diffusive turbulence might be present.

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Reciprocating probes were used to assess the background plasma level in the SOL, which could be generated by such small amplitude turbulence. In particular, a Bayesian fitting of ion saturation current signals at different radial positions was carried out. Time traces collected at different radial positions during the plunge of the probe were fitted by using the following functional form:

$$S(t) = B + \sum_{i=1}^{K} n_{0,i} \left( e^{\frac{t-t_i}{\tau_{1,i}}} H(t-t_i) + e^{\frac{t-t_i}{\tau_{2,i}}} H(-t+t_i) \right),$$  \hspace{1cm} (5)$$

where $B$ is a stationary background level, $K$ is the number of filaments fitted, $n_{0,i}$ is the amplitude of the $i$th filament, $\tau_{1,i}$ and $\tau_{2,i}$ are the raise and decay times of the event, $H$ is the Heaviside function and $t_i$ is the time at which the event reaches its peak value. It is evident that Eq. (5) is a reformulation of Eq. (2) for a point-wise measurement with a specific definition of $\Theta$. Note that the plasma background can be interpreted as originated from the collisional leakage from the core and a low level turbulent environment. The Bayesian fitting based on Eq. (5) shows an excellent match with the experimental data, see Fig. 1. This approach removes the arbitrariness in selecting the filaments with a specific threshold condition, i.e. by taking events in the signal that are above a certain number of standard deviations. The statistical analysis of the fitting parameters shows that throughout the SOL, the waiting times between filaments of similar relative amplitude (i.e. filament amplitude divided by maximum amplitude of the signal in the interval analysed) remain constant and has a double exponential dependence. This self-similar behaviour, which is not present in the core data, suggests that the filaments are generated by a mechanism that sets such a trend before the filaments reach the SOL. In addition, the value of the stationary background in the SOL appears to be extremely small (~0.5%) compared to the typical filament amplitude. A possible interpretation of this result is that the background is not stationary, but rather fluctuating in time and formed by low amplitude filaments.

**FIG. 1** (left panel) Ion saturation current from midplane reciprocating probe (blue) fitted by a synthetic signal calculated with Eq.(5). (right panel) Waiting time between filaments of different amplitudes as a function of the relative amplitude.

### 4. NUMERICAL RESULTS AND TOOLS IN SUPPORT OF THE FRAMEWORK

High performance numerical simulations were used to test the framework assumptions as well as to establish reliable models for its inputs (e.g. filament parallel draining and perpendicular velocities). The STORM code, a 3D drift fluid solver for boundary turbulence which is used in our investigations, was used to compare numerical and experimental dynamics of isolated filaments, finding agreement within the experimental errorbars [15]. This validation gave us confidence in the results of STORM that was later employed to assess the level of interaction of filaments seeded at different distances from each other [16]. It was found that the electric field generated by the filament affects neighboring filaments only if they are closer than ~5 typical filament widths. This weak interaction is limited to a short range by the fact that the electric field is dipolar in nature and therefore decaying in space rather quickly. The typical separation between filaments can be estimated by using the intermittency parameter introduced in [7] and extended in [4-6], in particular filaments are expected to interact when their dwelling time (persistence of the filament in a pointwise measurement) is comparable with their waiting time (time separation between events). This roughly corresponds to the transition between the near and the far SOL, where the turbulence statistics goes from Gaussian to skewed and flattened. This suggests that although the theoretical framework can reproduce Gaussian behavior in the limit of small intermittency...
parameter, its validity could be anyway questionable in the near SOL due to the fact that the independence hypothesis does not hold there.

One of the essential inputs of the statistical framework is a model for the dynamics of the filaments. Many papers have already addressed this issue, both from a theoretical and experimental perspective (see [1] for a review). Early theoretical considerations were based on dominant balance estimates of the main mechanisms driving the filaments and were followed by simple 2D simulations of the cross field motion of the filaments (see e.g. [18]). More recently, full 3D simulations have confirmed the correctness of the previous simplified treatments and elucidated the basic mechanisms at play [19-23]. In particular, the inertial and sheath regime were recovered with their non-monotonic trend (the radial filament velocity scales like the square root of the width at small widths, and like the inverse of the square for large widths). Importantly, the boundary between the two asymptotic regimes depends on plasma and equilibrium parameters, such as collisionality (or resistivity) and magnetic shear, as theoretically predicted by Ref.[24] but also the plasma $\beta$ [22] and the background configuration [23]. With our 3D simulations we found that an increase in the plasma resistivity [20] can favor the inertial regime (which extends to larger widths), but three orders of magnitude changes lead only to marginal increases $(x2)$ in the radial Mach number of filaments around the critical threshold width [25], which is where experimental filaments seem to cluster [17]. Importantly, a reduction of the filament velocity is actually observed if the resistivity is self-consistently increased by decreasing the plasma temperature [23], i.e. by moving from a sheath to a conduction limited divertor regime.

A similar effect occurs when electromagnetic corrections are taken into account at finite $\beta$. For inter-ELM filaments in reactor relevant machines, these corrections might be required to properly capture the dynamics. In this case, the Alfvén waves, which communicate the potential perturbation between the filament and the target, have a finite velocity, which can be too slow to transmit information about the existence of the target to a radially moving filament. This electrically disconnects the filament from the sheath, resulting in faster radial filaments at the midplane, and slower at the target, see left panel of Fig. 2. Importantly, this transition depends on the parallel connection length, a longer value making the electromagnetic effects more significant. This has repercussions on large major radius machines, where the connection length scales with $R$, but also on advanced divertor configurations [26,27], that require longer connection length.

While isolated filament simulations can provide insight into their dynamics, fully turbulent simulations can also clarify the generation mechanisms of the perturbations (and therefore their statistics) and the strength of their interactions. Full 3D simulations in full MAST geometry and realistic plasma parameters have been performed with STORM for the first time and will be used to thoroughly test the assumptions and predictions of the theoretical framework, see central panel of Fig. 2. An example of the code output is given in the right panel of
Fig. 2, where we show the pressure perturbations in on a flux surface in the outer low field side SOL. In this region, filaments tend to closely follow the structure of the magnetic field lines as shown by the dashed line in the plot, which represents the trajectory of a single field line. Note that the n=4 toroidal regularity of the perturbations is only apparent as the image was constructed by patching together a simulation that described only a toroidal quarter of the machine.

5. COMPARISON WITH EXPERIMENTS

The final test of any model and theory should be the comparison with experimental data. The theoretical framework was used to interpret JET [29] and MAST [6] data. In the first case, simultaneous match of profiles and turbulence data at the midplane wall was obtained in different regimes of fueling, see Fig. 3. While low density, almost exponential profiles could be matched with filaments following simple dynamics (constant radial velocity and exponential in time draining), high density discharges showing a broadening of the profiles required a simultaneous increase in the outward velocity and an increase of the typical draining time. Such a change in the filament dynamics could be correlated with stronger plasma/neutral interactions, as suggested in [3-6] and observed with divertor diagnostics in the modelled discharges [29]. In particular, charge exchange could ‘clog’ the divertor, thus slowing down the motion of the plasma towards the target while neutral wind [30] at the midplane could increase the filaments’ drive and therefore induce an acceleration.

**FIG. 3 Upper panel: comparison between radial profile of the experimental light intenity and theoretical match of the framework. Bottom panels: density profile from JET Li-beam and theoretical framework prediction (left);Comparison of experimental and theoretical probability distribution functions of the fluctuations at the midplane wall.**

Similarly, MAST low density radial profiles of the mean and variance, obtained with visual cameras and processed with ELZAR, were properly matched by the theoretical ones by using again constant velocity of the filaments and exponential draining. It is useful to remark that, for both MAST and JET, the theoretical velocity and draining time used were compatible with those typically observed in experiments (i.e. radial velocity 1-5% of the sound speed and draining time comparable with the parallel transit time).

6. SUMMARY AND CONCLUSIONS

We have developed a theoretical framework that rigorously relates the fluctuations in the SOL with the profiles that they generate. Novel experimental and numerical tools have been developed and applied to inform its input parameters (statistics and dynamics of the filaments). Experimental results obtained with fast visual camera data show that one of the fundamental hypotheses of the framework - the independence of the filaments - is valid as the filaments emerge from the separatix as uncorrelated events without a precise toroidal wave number. Also, the camera data were used to estimate probability distributions of perpendicular widths, waiting times and toroidal distribution. Analysis of waiting times between filaments of different amplitude, measured with midplane reciprocating probes, showed that the filaments evolve in a self-similar way at different radial positions, which indicates that the filaments do not originate in the SOL.

Numerical simulations provided a theoretical basis for the filament independence hypothesis, showing that the dipolar field generated by the filaments, which drives their outward motion, decays quickly and leads to
negligible interactions when the separation between filaments is larger than ~5 widths. Fundamental considerations related to the electromagnetic effects associated with high $\beta$ conditions, such as those associated with the inter-ELM phase, lead to the conclusion that filaments with large perpendicular width have higher radial velocities than in the $\beta$=0 case. These filaments would be slowed down by the sheath impedance in the electrostatic case, but for finite $\beta$, the parallel propagation of the Alfvén waves cannot catch up with the moving filaments and the electrical connection with the target is lost. In addition, we acquired the capability to carry out 3D simulations of SOL turbulence in realistic MAST geometry and will be used to determine the statistics of the filament generation. Finally, the framework was applied to JET and MAST data, finding a good match in several channels simultaneously.

After gaining some confidence in the ability of the frame to reproduce experimental results, we can now turn our attention to the predictions for future reactor relevant machines. Experimental measurements show that the turbulent fluctuations follow a rather universal behavior, i.e their PDFs collapse if properly normalized to the mean and standard deviation of the data [17]. As a consequence, these are probably the only two quantities that need to be predicted in order to have the full characterisation of the turbulence. This can easily be done with our framework once the filament dynamics and statistics can be extrapolated to ITER relevant conditions. This should motivate a strong experimental and theoretical effort aimed at understanding the filament motion in the SOL. What the framework suggests is that the density profile should tend to become broader if the filaments have a radial acceleration or if the parallel transport becomes less efficient. Analysis of JET data shows that both effects are probably needed simultaneously. Interactions with neutral particles might provide an explanation for both effects as the plasma undergoes strong charge exchange in the divertor and is accelerated by neutral wind caused by wall recycling (which might also increase the local ionization source). Finally, our electromagnetic results show that a longer connection length, resulting from larger size of the machine or the use of alternative divertor configurations, might lead to stronger radial transport, and therefore broader profiles, as electrical connection with the sheath is lost. As a consequence, the radial SOL profiles seem to be affected by the divertor design and conditions, an essential piece of information for reactor relevant machines.

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