ENDOGENOUS AND ASYMMETRIC MAGNETIC RECONNECTION WITH ASSOCIATED PROCESSES OF RELEVANCE TO FUSION BURNING PLASMAS

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Abstract

An endogenous magnetic reconnection process is characterized by a driving factor that lays within the layer where a drastic change of field topology occurs. This kind of process is shown to take place in the presence of an electron temperature gradient in a well-confined plasma referring to quasi-collisionless regimes where the resulting electron temperature fluctuations can be anisotropic. Then a class of (radially) localized reconnecting modes is identified. These involve a transverse field $B_x$ of odd parity (as a function of the radial variable), and have finite (phase) velocities of propagation contrary to commonly considered reconnecting modes. The width of the relevant reconnection layers remain significant even when large macroscopic distances are considered. Given that there are plasmas in the Universe with considerable electron thermal energy contents, these features can be relied upon in order to produce magnetic field generation, or conversion of magnetic energy into particle energy when the coupling of the localized odd modes to extended even modes can be significant. In any case, the resulting magnetic islands are not symmetric. With their excitation these modes can extract momentum from the main body of the plasma column which should recoil in the opposite direction. The excitation of antisymmetric endogenous modes is shown to be relevant to the electron temperature heating due to the reaction products in a fusion burning plasma as, in this case, the longitudinal thermal conductivity on selected rational magnetic surfaces is decreased, relative to its collisional value, by the effects of reconnection. The best agreement between theory and experiments concerning the onset of magnetic reconnection is (probably) represented by the theory of the internal kink mode. The observed accelerated reconnection rate, following the onset is suggested as being explained by the formation of a relatively large magnetic island with a local steepening of the electron temperature gradient.

1. INTRODUCTION

The main characteristic of an endogenous magnetic reconnection process is that its driving factor lays within the layer where a drastic change of magnetic field topology occurs. This kind of process is shown to take place when a significant electron temperature gradient is present in a magnetically confined plasma and in regimes where the evolving electron temperature fluctuations can be anisotropic [1]. Then [2] two classes of reconnecting modes are identified. The radially localized class of mode involve a reconnected field $\tilde{B}_x$ of odd parity and have finite (phase) velocities of propagation contrary to commonly considered reconnecting modes. The width of the relevant reconnection layers remain significant even when large macroscopic distances are considered. The other class of modes is characterized by $\tilde{B}_x$ amplitudes that are even functions of the radial variable within the reconnection region and extend well beyond it. In view of the fact that there are plasmas in the Universe with considerable electron thermal energy contents, these features can be relied upon in order to produce generation of magnetic fields and conversion of magnetic energy into particle energy when the coupling of the localized odd modes to extended even modes can be significant. The resulting magnetic islands are not symmetric.
The simplest confinement configuration that we refer to is represented by \( \mathbf{B} = B_0 \mathbf{e}_z + B_y(x) \mathbf{e}_y \), where \( B_0 = \text{const} \) and \( B_y' \gg B_y'' \). The electron and ion temperatures, \( T_e \) and \( T_i \), are considered to be nearly isotropic in the unperturbed state, but the evolution of \( T_e \) in the perturbed state is considered to be anisotropic, i.e., \( \hat{T}_{e\perp} \neq \hat{T}_{e\parallel} \), on account of the large anisotropy of the electron thermal conductivity.

The (normal) modes that we consider involve magnetic field perturbations represented by \( \mathbf{B} = \mathbf{B}(x) \exp(-i\omega t + ik_yy + ik_zz) \). The gradients of the particle density and temperatures are assumed to be significant for \( |k| = |x_0| > 0 \), as we choose to analyze modes such that \( \mathbf{k} \cdot \mathbf{B}(x = x_0) = k_x B_0 + k_y B_y(x_0) = 0 \) and \( \mathbf{k} \cdot \mathbf{B}((k_y B_y')(x - x_0)) \) around \( x = x_0 \).

We suggest that a sequence of mode-particle resonances [3] starting from the excitation of the oscillatory modes considered here may be proposed [4] for the generation of experimentally observed high energy electron populations following magnetic reconnection events. An attractive feature of processes of this kind is that the considered modes involve relatively large amounts of particle thermal energies and can channel the energy gained, with their excitation, into that of very few particles.

The modes that we have found have significant phase velocities [1,4] with characteristic magnitudes and directions. Therefore, these modes acquire momentum, along the main magnetic field component \( B_0 \). Then the plasma column is expected to recoil in the opposite direction and the momentum extracted from it to be transported away from it by the same modes. When these considerations are transferred to toroidal confinement configurations we may argue that “reconnecting” modes excited within the main body of the plasma can induce a “spontaneous rotation” of it [5]. Relevant experimental observations [6] are consistent with this theoretical indication.

2. ELECTRON THERMAL ENERGY BALANCE EQUATIONS: SIMPLEST LIMITS

We refer to weakly collisional regimes where \( v_{ce} \), the electron-electron collision frequency, is considerably smaller than \( |\omega| \). Since, \( |\omega|^2 \ll \Omega_{ce}^2 \), where \( \Omega_{ce} \) is the electron cyclotron frequency, we are justified in using the “equation of state”

\[
\frac{d}{dt} \left( \frac{p_{e\perp}}{nB} \right) = 0,
\]

where \( d/dt \) is the total time derivative. The perturbed form of Eq. (1) yields

\[
-i\omega \hat{T}_{e\perp} + \hat{u}_{e\perp} T_e = 0,
\]

for \( |\hat{B}_y / B| \ll |\hat{\mathbf{R}}_{e\perp} / T_e| \). That is,

\[
\hat{T}_{e\perp} = -\frac{\mathbf{e}_e}{\omega_{ce}} T_e',
\]

for \( -i\omega \mathbf{e}_e = \hat{u}_{e\perp} \).

On the other hand we may estimate \( \hat{T}_{e\parallel} \) by considering the effective large longitudinal thermal conductivity limit \( |\omega|^2 < k_\perp^2 v_{de}^2 \), where

\[
\left( B \cdot \nabla T_{e\parallel} \right) = 0.
\]

In this case

\[
\hat{T}_{e\parallel} = -\frac{1}{ik_x} \frac{dT_e}{dx} \frac{\hat{B}_x}{B}
\]
Then we note that if \( \tilde{B}_i(x-x_0) \neq 0 \), \( \tilde{\tau}_{\alpha}(x-x_0) \) has a singularity at \( x = x_0 \) that can be removed by including the transverse electron thermal conductivity term in the relevant analysis. If instead \( \tilde{B}_i(x-x_0) \) is odd \( \tilde{\tau}_{\alpha}(x-x_0) \) can be a regular even function of \( x - x_0 \).

3. MOMENTUM CONSERVATION EQUATIONS

The perturbed total momentum conservation equation can be written as

\[
-i\omega m \nabla \hat{u}_i = -\nabla \left( \hat{p}_i + \frac{\tilde{B}_i \cdot \mathbf{B}}{4\pi} - \frac{n}{B^2} B \left[ i (\mathbf{k} \cdot \mathbf{B})(\tilde{\tau}_{\alpha} - \tilde{\tau}_{\alpha}) \right] \right) + \frac{1}{4\pi} \left[ i (\mathbf{k} \cdot \mathbf{B}) \hat{B}_i + \frac{d}{dx} \right] \mathbf{B}. \tag{6}
\]

Applying the operator \( \mathbf{B} \cdot \nabla \times \) to Eq. (6) and considering modes for which \( \nabla \cdot \hat{u}_i = 0 \), we find that the equation of interest within \( |x - x_0| \sim \delta_m \), where \( \delta_m \) represents the width of the reconnection layer, is

\[
-\omega(\omega - \omega_{\alpha})(\frac{d^2 \tilde{\tau}^2}{dx^2} - k_{\perp}^2 \tilde{\tau}) + \frac{1}{m \eta} \frac{k_B}{B^2} \frac{d}{dx} \left[ n \mathbf{k} \cdot \mathbf{B} \right] (\tilde{\tau}_{\alpha} - \tilde{\tau}_{\alpha}) = \frac{i}{4\pi m \eta} \left[ \mathbf{k} \cdot \mathbf{B} \left( \frac{d^2 \tilde{B}_i}{dx^2} - k_{\perp}^2 \tilde{B}_i \right) - k_{\perp} \frac{d^2 B}{dx^2} \tilde{B}_i \right].
\]

Here we have taken \( \hat{u}_i = -i(\omega - \omega_{\alpha}) \tilde{\tau}_{\alpha} \), with \( \omega_{\alpha} = k_{\parallel} \left[ c/(enB) \right] \left( \frac{d\rho_i}{dx} \right) \). We note that

\[
d^2 B_i/dx^2 = (4\pi/c)(dJ_z/dx), \quad J_z \text{ being the equilibrium current density.}
\]

The adopted longitudinal electron momentum conservation equation that has a key role in the onset of magnetic reconnection is

\[
-i\omega m \hat{n}_i = -\nabla \left( \hat{p}_i + \frac{\tilde{B}_i \cdot \mathbf{B}}{4\pi} \right) \frac{\hat{B}_i}{B} - \frac{n}{en} \left[ \hat{E}_i - \left( \eta_{\alpha} - i\omega L_i \right) \hat{J}_i \right], \tag{8}
\]

where \( \hat{p}_i = n \tilde{J}_i + n \tilde{\tau}_{\alpha} \), \( \eta_{\alpha} \) is a small but finite electrical resistivity, and \( L_i \) is the inductivity that was introduced in Ref. [7]. Then, taking \( \hat{u}_i = -\hat{J}_i/(en) \), Eq. (8) becomes

\[
(\omega - \omega_{\alpha} - \omega_{\perp}) \tilde{B}_i = i \mathbf{k} \cdot \mathbf{B} \left[ (\omega - \omega_{\alpha}) \tilde{\tau}_{\alpha} + \omega_{\perp} \tilde{\tau}_{\alpha} \right] + \omega \left[ d^2_{\perp} + d^2_{\parallel} \right] \left( \frac{d^2}{dx^2} \right) \tilde{B}_i,
\]

where \( T'_{\parallel} = dT_{\parallel}/dx \), \( d^2_{\parallel} = c^2/\omega_{\perp}^2 \), \( d_{\parallel} \) is the “inductive skin depth” defined by \( L_i = (4\pi/c^2) d_{\parallel}^2 \), and \( D_{\parallel} = c^2 \eta_{\alpha} / (4\pi) \) is the magnetic diffusion coefficient that in the collisional limit is \( \alpha_{\eta} \eta_{\alpha} d_{\parallel}^2 \) where \( \alpha_{\eta} \) is an appropriate numerical coefficient. The two characteristic frequencies appearing in Eq. (9) are \( \omega_{\alpha} = -k_{\parallel} \left[ cT_{\parallel}/(enB) \right] (\n' \n') \) and \( \omega_{\perp} = -k_{\parallel} \left[ cT_{\parallel}/(enB) \right] \), where \( \n' \equiv dn/dx \). The term \( d^2_{\perp} \) arises from the electron inertial term and \( d^2_{\parallel} \ll d^2_{\parallel} \) is assumed. We note that, for ultra-relativistic electrons found in astrophysical plasmas the \( d^2_{\perp} \) term can be significant.

With appropriate dimensionless variables Eqs. (9) and (7) become

\[
\tilde{Z}_s - i(x-x_0) \tilde{Y}_s = \frac{\alpha d^2_{\perp} + D_{\parallel}}{\omega - \omega_{\alpha}} \nabla^4 \tilde{Z}_s, \tag{10}
\]

for \( \nabla^4 \equiv d^4/dx^4 - k_{\perp}^2 \), \( \tilde{Z}_s = \tilde{B}_i / B'_{\parallel} \) and \( \tilde{Y}_s = \tilde{\tau}_{\alpha} / k_{\perp} \), and
\[ -\omega (\omega - \omega_a) \frac{1}{k_y^2} \nabla_y^2 \tilde{Y}_y - i \omega \sigma^2 \frac{d}{dx} \left[ \tilde{Z}_x - i(x - x_0) \tilde{Y}_x \right] = i \omega \sigma^2 \left[ (x - x_0) (\nabla_y^2 \tilde{Z}_x) - \left( \frac{1}{B_y^*} B_n^* \right) \tilde{Z}_x \right], \tag{11} \]

where

\[ \omega \sigma^2 = \frac{B^*}{4 \pi n m_i} \quad \text{and} \quad \omega \sigma^2 = - \frac{dT_x}{dx} \frac{1}{2m_i} \frac{d}{dx} \left( \frac{B}{B} \right)^2. \tag{12} \]

In particular, \( \omega \sigma^2 = (T_x / m_i) (B / B)^2 \left( \frac{r_i}{r_B} \right) \) where \( \omega \sigma^2 = - \left( dT_x / dx / T_x \right) \) and \( \left( 1 / r_B \right) \equiv \left( dB / dx / B \right) \), and the ratio

\[ \epsilon_A = \frac{\omega \sigma^2}{\omega_n \epsilon} = - \frac{dT_x}{dx} \frac{4 \pi n m_i}{B} \frac{B_0}{B} = \frac{4 \pi n m_i}{B_0^*} \frac{r_B^*}{r_B} \ll 1 \quad \text{in well confined plasmas.} \tag{13} \]

4. **LOCALIZED MODES INVOLVING DIAMAGNETIC AND “DRIFT” FREQUENCIES**

Now we refer to the case where the frequencies \( \omega \sigma \) and \( \omega_n \) cannot be neglected in comparison with \( \omega \) while, for the sake of simplicity, we take \( D_m / |\omega| \ll d_x^2 \). Then considering the limit where \( \epsilon_A \ll 1 \) we can verify that \( \tilde{Z}_x \) can be neglected relative to \( (x - x_0) \tilde{Y}_x \) in Eqs. (10) and (11). We define \( \tilde{\Omega} \equiv \omega / (\omega_n \epsilon) \) and rewrite

\[ -\tilde{\Omega} (\tilde{\Omega} - \tilde{\Omega}_a) \frac{d^2 \tilde{Y}_y}{dx^2} - \frac{d}{dx} \left( \frac{\tilde{\Omega} - \tilde{\Omega}_a}{\tilde{\Omega}} \right) \tilde{Y}_y = 0, \tag{14} \]

if we take \( \xi = (x - x_0) / \tilde{\Omega}_a \), \( (\tilde{\Omega}_a)^2 = \epsilon_A d_x^2 \), and \( \tilde{\Omega}_a \ll B_0^* / B_n^* \). In this case the solution

\[ \tilde{Y}_y = \tilde{Y}_y (1 - \lambda \xi^2) \exp \left( - \frac{\sigma}{2} \xi^2 \right) \tag{15} \]

requires that \( \tilde{\Omega} (\tilde{\Omega} - \tilde{\Omega}_a) (\sigma + 2 \lambda) = 1 \), i.e., \( \tilde{\Omega} (\tilde{\Omega}_a / 2) = \left[ \tilde{\Omega}_a / 4 + 1 / (\sigma + 2 \lambda) \right]^{1/2} \). In addition

\[ \tilde{\Omega} (\tilde{\Omega} - \tilde{\Omega}_a) \sigma^2 - \sigma + \left( 1 - \frac{\tilde{\Omega}_a}{\tilde{\Omega}} \right) = 0 \tag{16} \]

and

\[ \tilde{\Omega} (\tilde{\Omega} - \tilde{\Omega}_a) (\sigma^2 + 5 \sigma \lambda) - (\sigma + 3 \lambda) + \left( 1 - \frac{\tilde{\Omega}_a}{\tilde{\Omega}} \right) = 0 \tag{17} \]

that implies

\[ \sigma = \frac{3}{5} \tilde{\Omega} (\tilde{\Omega} - \tilde{\Omega}_a) \quad \text{and} \quad \lambda = \frac{1}{5} \sigma. \tag{18} \]

As pointed out previously [1] this solution corresponds to

\[ \tilde{B}_y = \tilde{B}_y^* \exp \left( - \sigma \frac{\xi^2}{2} \right). \tag{19} \]

The resulting dispersion relation is

\[ (\tilde{\Omega} - \tilde{\Omega}_a) (\tilde{\Omega} - \tilde{\Omega}_a) = \frac{6}{25}, \tag{20} \]

That is,

\[ \left( \frac{\omega}{k_y} - v \right) \left( \frac{\omega}{k_y} - v \right) = \frac{6}{25} \frac{\omega_n^2}{\omega_n} d_x^2. \tag{21} \]

In the limit when \( \tilde{\Omega}_a \sim \tilde{\Omega}_n \gg 1 \) we have \( \tilde{\Omega} = \tilde{\Omega}_a + \delta \tilde{\Omega}_a \) where

\[ \delta \tilde{\Omega}_a = \frac{6}{25} \left( \tilde{\Omega}_a - \tilde{\Omega}_a \right) \tag{22} \]
and
\[ \sigma = \frac{5}{2} \left( 1 - \frac{\Omega_v}{\Omega_d} \right) \sim 1 \]  
(23)
corresponding to
\[ \delta^2 - \left( \delta^2_n \right)^2 - \sigma - \left( \delta^2_n \right)^2 = 0. \]  
(24)
Alternatively, \( \Omega = \Omega_v + \Delta \Omega_v \) where \( \Delta \Omega_v = (6/25)/(\Omega_v - \Omega_d) \) and
\[ \sigma = \frac{3 \Delta \Omega_v}{5 \Omega_v(\Omega_v - \Omega_d)} \ll 1, \]  
(25)
meaning that
\[ \delta^2 - \frac{5}{3} \left( \delta^2_n \right)^2 \Omega_v (\Omega_v - \Omega_d) = \frac{5 V_{pe}(V_{pe} - V_{id})}{\omega_{sd}^2} = \frac{5 \left( cm \right)^2}{3 (eB_f)^2} \frac{r_x}{r_{x^*}} \left( 1 + \frac{T_e}{T_i} \frac{r_e}{r_{i*}} \right) \frac{r_y}{r_{y*}}. \]  
(26)
We can verify that as long as \( \epsilon_d \ll 1 \) the adopted “quasi-electrostatic” approximation is valid.

5. “RESISTIVE” MODES

The simplest case to analyze is that for which \( \omega > \omega_{sd} > \omega_{pe} \), \( D_n \) is considerably larger than that given by the collisional theory and we can take \( D_n/\omega d_j \gg 1 \). Then, if we define \( \bar{x} = (x - x_0)/\delta_m \) and \( \Gamma^2 = -\left[ \omega/k D_n \delta_m \omega_{sd} \right]^2 \), Eq. (11) becomes
\[ \Gamma^2 \frac{d^2 \bar{Y}_r}{dx^2} - \Gamma^2 \bar{Y}_r = \frac{d}{dx} \left( \bar{x} \bar{Y}_r \right) \]  
(27)
when we take
\[ \delta_m = \left| \frac{D_n \omega_{sd}}{\omega_{sd}, k} \right| = \left| \frac{D_n \omega_{sd}}{k \omega_{sd}} \epsilon_d \right|. \]  
(28)
Now, it is easy to see that \( \Gamma^2 \) cannot be real. In particular, the solution (15) with \( \Re \sigma > 0 \), leads to find
\[ \Gamma^2 (\sigma + 2 \lambda) = -1, \]  
\[ \Gamma^2 \sigma (\sigma + 5 \lambda) + (\sigma + 3 \lambda) = \Gamma, \]  
\[ \Gamma^2 \sigma^2 + \lambda = \Gamma. \]  
Therefore we obtain, \( \Gamma^2 = -6/25 \), \( \sigma = -3/(5 \Gamma^2) \) and \( \lambda = \sigma/3 \). Consequently, the mode growth rate and frequencies are given by \( \Gamma = (6/25)^{1/3} \left( 1 \pm i \sqrt{3} \right)/2 \), and
\[ \omega = i \Gamma \left| \frac{D_n k_v \omega_{sd}}{\omega_{sd}} \right|^\frac{1}{3}. \]  
(29)
Clearly, these are overstable modes with larger frequencies than their growth rates as \( \Re \omega = \sqrt{3} \Im \omega \). An important feature of the expression for the growth rate is that it shows that the electron temperature gradient is the driving factor of the instability and is proportional to \( d T_e/dx \)^\frac{1}{3}. The necessary condition to neglect \( \omega_{pe} \) and \( \omega_{sd} \) is, considering that \( \left| \omega_{pe} \right| \sim \left| \omega_{sd} \right| \),
\[ \left| \frac{\omega_{pe}^4}{\omega_{sd}^4} D_n k_v^2 \right| \geq k_y^2 \frac{V_{pe}^4 \omega_{pe}^2}{\omega_{sd}^4} \]  
(30)
and this means that
\[ D_n \geq k_y \frac{V_{pe}^4 \omega_{pe}^2}{\omega_{sd}^4} = \left| k_y \frac{V_{pe}^4 \omega_{pe}^2}{\omega_{sd}^4} \right| \left( \frac{c^2}{\alpha_{pe}^2} \frac{T_e}{T_i} \frac{r_e}{r_{i*}} \right)^2 \]  
(31)
We shall refer to the magnetic configuration characterized by a transverse field of the kind given by Eq. (19) as antisymmetric reconnection.
6. COMPOSITE SOLUTION AND ASSOCIATED ASYMMETRIC RECONNECTION

The presence of the current density gradient term \( \frac{dJ_e}{dx} \) in Eq. (7) introduces an even parity component for \( \tilde{B}_e(x-x_0) \) to be added to the lowest order “endogenous” solution given, for instance, by Eq. (19). We represent this component as \( \tilde{B}_e^{\text{od}} = \tilde{B}_0^{\text{od}} \hat{W}_0(\tilde{x}^2) \), and add the even component \( \tilde{B}_e = \tilde{B}_0^{\text{ev}} \left[ 1 + \varepsilon_e \phi_e(\tilde{x}^2) \right] \), where \( \varepsilon_e \ll 1 \).

Then we have to analyze the odd component of Eq. (7) that is

\[ -\omega (\omega - \omega_e) \left( \frac{d^2}{dx^2} - k_y^2 \right) \tilde{\xi}_x + i \frac{T_e^y k_y B_y}{m} \frac{d}{dx} \left( \tilde{B}_e^{\text{ev}} - i (\mathbf{k} \cdot \mathbf{B}) \tilde{\xi}_x^{\text{od}} \right) \]

\[ \equiv \frac{i}{4\pi \mu_n} \left[ (\mathbf{k} \cdot \mathbf{B}) \left( \frac{d^2 \tilde{B}_e^{\text{ev}}}{dx^2} - k_y^2 \tilde{\xi}_x^{\text{ev}} \right) - k_y^2 \frac{d^2}{dx^2} \tilde{\xi}_x^{\text{od}} \right] , \]

obtained under the conditions indicated in Section 2 but observing that in Eq. (32) \( \tilde{B}_e^{\text{ev}} \) should be more appropriately replaced by \( \tilde{B}_e^{\text{od}}(x-x_0) / \left[ (x-x_0)^2 + (\delta^e)^2 \right] \) in order to take care of the singularity pointed out at the end of Section 2. Here \( (\delta^e)^2 \) depends on the ratio \( D_e^c/D_e^t \) of the relevant thermal conductivity (see Section 7). The r.h.s. of Eq. (32), where the \( \frac{dJ_e}{dx} = J_e^y \) term that couples \( \tilde{B}_e^{\text{ev}} \) to \( \tilde{B}_e^{\text{od}} \) enters, is proportional to

\[ \tilde{G} = \delta_m \tilde{\xi}_x \left[ \frac{1}{\delta_m^2} \frac{d^2 \Phi_0}{dx^2} - k_y^2 \right] \tilde{B}_0^{\text{od}} = \frac{1}{\varepsilon_e} \tilde{B}_0^{\text{od}} , \]

and we may take

\[ \varepsilon_e \sim \frac{\delta_m}{\varepsilon_e} \tilde{B}_0^{\text{od}} \] for \( \frac{1}{\varepsilon_e} = \frac{J_e^y}{J_e^y} \) and \( k_y^2 \lesssim \frac{1}{\varepsilon_e} \tilde{B}_0^{\text{od}} \).

Moreover, using the expression indicated previously for \( \tilde{B}_e^{\text{od}} \) within \( \delta_m \) and using the longitudinal electron momentum conservation equation given by

\[ \tilde{B}_e^{\text{ev}} = \frac{i}{\varepsilon_e} \tilde{\xi}_x^{\text{ev}} + \frac{d^2}{dx^2} \left( \frac{\omega}{\omega - \omega_e} \left( \frac{d^2}{dx^2} - k_y^2 \right) \tilde{B}_e^{\text{ev}} \right) \]

we obtain

\[ \frac{d^2}{dx^2} \left( \frac{\omega}{\omega - \omega_e} \left( \frac{d^2}{dx^2} - k_y^2 \right) \tilde{B}_e^{\text{ev}} \right) = \left( \frac{\omega}{\omega - \omega_e} \right)^2 \tilde{B}_e^{\text{ev}} \]

or

\[ \varepsilon_e^2 = \frac{k_y^2 \delta_m^2}{\varepsilon_e} \]

\[ \text{and } \tilde{\xi}_x^{\text{ev}} \sim \frac{1}{\varepsilon_e} \tilde{B}_0^{\text{od}} d^2_e / \left( \varepsilon_e \delta_m \right) \].

This implies \( \varepsilon_e \sim \left( \frac{\delta_m}{\varepsilon_e} \right) \). Then we have

\[ \Gamma_0 \left( \frac{d^2}{dx^2} - \varepsilon_e^2 \right) \tilde{\xi}_x^{\text{ev}} - \alpha_e^d \frac{d}{dx} \tilde{\xi}_x^{\text{ev}} = \tilde{\xi}_x^{\text{ev}} - 1 \]

where \( \alpha_e^d \equiv -T_e^y B_y \left( \Gamma_0 B_x^y \right) \), \( \delta_m^e = \varepsilon_e d_e^2 / \alpha_e^d \) and \( \Gamma_0 \equiv \omega (\omega - \omega) \alpha_e^d / \left( k_y^2 \delta_m^e \omega \varepsilon_e \right) \).

The condition for the asymptotic matching of the solution for \( \tilde{B}_e^{\text{ev}} \) inside the \( \delta_m \)-region with that (ideal MHD) outside the \( \delta_m \)-region involves the integral \( \int_{-\infty}^{\infty} dx (1 - \tilde{\xi}_x^{\text{od}}) = \mathcal{O} \). According to Eq. (38)

\[ \tilde{\xi}_x^{\text{od}} - 1 = C_0 \tilde{W}_0 (\tilde{x}^2) + \frac{\Gamma_0}{\tilde{x}} \left( \frac{d^2}{dx^2} - \varepsilon_e^2 \right) \tilde{\xi}_x^{\text{od}} - \frac{\alpha_e^d}{\tilde{x}} \frac{d}{dx} \tilde{\xi}_x^{\text{od}} , \]

where
Then,

\[ -\mathcal{Z} = C_0^o + \int_{-\infty}^{\infty} d\xi \left[ \frac{\Gamma_0}{\lambda} \left( \frac{d^2}{d\xi^2} - \epsilon_{\xi}^2 \right) \gamma_{\xi} - \alpha_{\xi}^2 \frac{d}{d\xi} \left( \gamma_{\xi} \right) \right]. \]  

(41)

where

\[ C_0^o = C_1 \int_{-\infty}^{\infty} d\xi W_0 \left( \xi^2 \right). \]  

(42)

Since the well known asymptotic matching condition becomes

\[ \mathcal{Z} = \frac{d^2 \Delta}{\delta_m} \frac{\hat{B}_0^o}{B_0^o}, \]

where \( \Delta = 1/a \) represents the jump of the logarithmic derivative of \( \hat{B}_0^o \) for \( x-x_0 \rightarrow \pm0 \) in the outer region, we may regard \( \mathcal{Z} \ll 1 \) as a significant limit to consider. Then, \( C_0 \) is given by

\[ C_0 = -\int_{-\infty}^{\infty} d\xi \left[ \frac{\Gamma_0}{\lambda} \left( \frac{d^2}{d\xi^2} - \epsilon_{\xi}^2 \right) \gamma_{\xi} - \alpha_{\xi}^2 \frac{d}{d\xi} \left( \gamma_{\xi} \right) \right] \int_{-\infty}^{\infty} d\xi W_0 \left( \xi^2 \right). \]  

(43)

The process resulting in the formation of magnetic islands due to the combination of symmetric and antisymmetric \( \hat{B}_0 \) reconnection may be referred to as asymmetric reconnection and we note that the theory of the “internal (reconnecting) kink mode” \([8]\) involves a composite solution mode of the sum of \( \delta_{\xi} \) even and \( \delta_{\xi} \) odd.

7. LONGITUDINAL ELECTRON THERMAL CONDUCTIVITY IMPACTED BY ENDOGENOUS RECONNECTION

A physically relevant case illustrating the significant influence of endogenous reconnection on the longitudinal electron thermal conductivity is ignition in magnetically confined plasmas that can develop from small layers around rational surfaces \([9]\). To analyse this case we refer to a large aspect ratio axisymmetric toroidal configuration that can be simulated by the plane geometry model introduced in Section 1 assuming that the surface \( x = x_0 \) corresponds to a rational toroidal magnetic surface.

Then we consider the simplest form of the perturbed electron thermal energy balance equation, that is

\[ \gamma \tilde{T}_e = \gamma_e \tilde{T}_e + D_{\parallel} \frac{\partial^2 \tilde{T}_e}{\partial x^2} + D_{\perp} \frac{\partial \tilde{T}_e}{\partial y} \left[ ik \tilde{T}_e + \frac{\tilde{B}}{B} \frac{dT_e}{dx} \right] \]  

(44)

where \( \gamma_e \) represents the rate of net thermonuclear heating of the electron population, \( \tilde{T}_e = \tilde{T}_e \) \( x-x_0 \) \[ \exp \left( \gamma t + ik y + ik_z \right) \] \( \tilde{T}_e \) \( x-x_0 \) is an even function of \( x-x_0 \), \( k = k_{\parallel} \) \( (x-x_0) \) and \( D_{\parallel} \), \( D_{\perp} \) represent the relevant electron thermal conductivities with \( D_{\parallel} \approx D_{\perp} \). We consider \( \tilde{B}_0 \) to be representative of an axisymmetric reconnection process and define \(-ik \tilde{B}_{\parallel} \equiv \hat{B} \) \( (dT_e/dx) \). Then in the large \( D_{\parallel} \) limit we have \( \tilde{T}_e \) \( x-x_0 \) \( T_\parallel \) \( x-x_0 \) and if we define \( \Delta \tilde{T} = \tilde{T}_e - \tilde{T}_n \), Eq. (44) reduces to

\[ \left( \gamma_e - \gamma \right) \tilde{T}_n + D_{\parallel} \frac{d^2 \tilde{T}_n}{dx^2} - D_{\perp} \left( k_{\parallel} \right)^2 \frac{d}{dx} \left( \frac{\tilde{T}_n}{x-x_0} \right)^2 \Delta \tilde{T} = 0. \]  

(45)

This can have a simple solution for \( \Delta \tilde{T} \propto \tilde{T}_n \) \[ \tilde{T}_n \exp \left[ -\left( \sigma/2 \right) \left( x-x_0 \right)^2 \right] \] and \( \left| \Delta \tilde{T} \right| \ll \left| \tilde{T}_n \right| \), that yields

\[ \left( \gamma_e - \gamma \right) = D_{\perp} \sigma, \]  

(46)

and

\[ D_{\parallel} \sigma^2 = D_{\parallel} \left( k_{\parallel} \right)^2 \frac{\Delta \tilde{T}}{\tilde{T}_n}. \]  

(47)
Then, for $\sigma \equiv 1/\delta_F$ and $\delta_F^2 \equiv \left(D_F/\gamma_F\right)^{1/2}/k_0^2$, Eq. (49) becomes

$$\frac{\delta_T^4}{\delta_F^4} = \frac{\Delta T}{T_m},$$

while

$$\delta_T = \left( \frac{D_T}{\gamma - \gamma} \right)^{1/2} < a.$$  \hspace{1cm} (49)

Here $\delta_F$ represents the radial width of the heated helical structure and $a$ is the radius of the plasma column. Clearly $\delta_F > \delta_T$ and $\gamma_F > \gamma + D_F/a^2$ is required. This analysis illustrates the fact that the thermonuclear burning in a magnetically confined plasma has to be studied as a 3-dimensional process and, since antisymmetric endogenous reconnection can have an important influence on it, the relationship between the evolution of the thermonuclear instability and reconnection calls for further attention.

8. FINAL REMARKS

The best agreement between theory and experiments concerning the onset of magnetic reconnection is (probably) represented by the theory for the onset of the internal kink mode [8]. The proposed interpretation for increase of the reconnection rate, about 200 $\mu$s after the onset [10], is that the relevant process involves the formation of a relatively large magnetic island with a local steepening of the electron temperature gradient. Thus, the effects of a magneto-thermal reconnecting mode [4], associated with the electron temperature gradient, are compounded with those (e.g. total plasma pressure gradient) that lead to initial excitation of the $(m^o = 1)$ reconnection mode.

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