Burning plasmas in future fusion reactors are an example of complex systems where multiple scales are intrinsically linked. In this work, the self-consistent interaction between Alfvén modes (AM), turbulence, and zonal structures (ZS), like zonal flows and geodesic acoustic modes, is investigated. Numerical simulations are performed with the global gyrokinetic particle-in-cell code ORB5. A magnetic equilibrium with a reversed shear is considered, taken from Ref. [1]. The value of $\rho^*$ is chosen to fall in the regime of typical turbulence simulations of the cyclone base case. Micro-turbulence modes are driven unstable by means of temperature gradients of the thermal species. When an EP population is added to the electromagnetic turbulence, the perturbed saturated field is observed to be enhanced by the presence of AMs. In the considered regime, AMs grow to higher levels than the turbulence. The heat transport is also observed to reach higher levels, due to the contribution of the nonzonal field of the AM. At the same time, strong ZSs are excited by the nonlinear interaction with the AMs. The effect of the ZSs is found to be the modulation of the heat flux with low-frequency oscillations. The radial structure of the perturbed field is also described. Finally, the indirect contribution of the EP to the heat transport, by means of the combined interplay of AMs and ZSs, is discussed.

1. INTRODUCTION

As is widely appreciated in the fusion community, to accelerate the development of fusion energy, it is highly desirable to develop a truly predictive simulation capability which is able to capture various types of multi-physics and multi-scale couplings self-consistently. In this context, it is getting increasingly clear that many phenomena which have traditionally been investigated separately are actually intrinsically linked. This may lead to unexpected emergent plasma
behavior violating a simplistic superposition principle, especially when ultimately dealing with burning plasmas. One outstanding example along these lines - which will be investigated in the present contribution - is the interaction between Alfvén modes (AM) [2, 3], turbulence, and zonal structures (ZS) [4, 5, 6].

Burning plasmas are characterized by the presence of energetic particles (EP) which are the product of fusion reactions as well as of auxiliary heating and current drive systems. AMs, driven unstable by resonances with EPs, are recognized as potentially problematic due to their role in redistributing the energetic ion population [2, 3]. However, in the presence of turbulence, the dynamics of AMs can change fundamentally, especially in the outer part of the tokamak core, where AM and turbulence co-exist. Evidence of strong interaction of AM and turbulence has been documented even in present day tokamaks (see, for example, Ref. [7]). ZS can be generated in a turbulent environment in the presence of Alfvénic fluctuations. In particular, EP can excite zonal and nonzonal modes [8], and can also directly interact with the turbulence [9]. These zonal sheared flows - toroidally and poloidally symmetric and with finite radial wavenumbers - are known to play an important role in the turbulence self-regulation [10, 11]. The aim of this work is to analyze the nonlinear interplay of Alfvénic fluctuations, turbulence, and ZS by means of self-consistent gyrokinetic simulations, thus contributing to the development of a predictive simulation capability.

Recently, a strong interest was raised in the fusion community by the possibility of generating ZS via nonlinear interaction with global modes like Alfvén instabilities. Theoretical investigations have been carried out numerically with local simulations or hybrid codes [12, 13, 14] and analytically with the gyrokinetic theory [15, 16]. Although these two mechanisms of ZS generation - by turbulence or by AMs - have been studied separately with different models and in different regimes, no comprehensive study exists, treating on the same footing all nonlinearities and kinetic effects. Due to the complex dynamics of such a problem, it is crucial to adopt a self-consistent global gyrokinetic model, where all modes can interact with each other in a fully nonlinear way. This is the main novelty of the results presented here. In fact, by using hybrid models which do not treat the electrons fully (drift-)kinetically, the electromagnetic dynamics of the zonal structure would not be properly understood. Similarly, by using local simulations, the profiles would not be able to evolve in time, and this would neglect one of the main means of saturation of AMs, namely the radial EP redistribution. Here, a nonlinear global gyrokinetic model is adopted.

In this work, the interaction of AMs, turbulence and ZS is studied with the code ORB5. This model treats ions and electrons respectively as gyrokinetic (GK) and driftkinetic. ORB5 is a nonlinear global particle-in-cell code, developed for turbulence studies [17] and extended to its electromagnetic multi-species version [18]. The discretized Lagrangian formulation used to derive the model equations of ORB5 [19], makes this model ideal for rigorous nonlinear electromagnetic simulations of global instabilities in the presence of EP and turbulence, where all nonlinearities are treated on the same footing in a self-consistent way. ORB5 has been successfully verified/benchmarked for the linear dynamics of ZS against analytical theory and other GK codes [20], and benchmarked for the linear dynamics of AMs against other hybrid MHD-GK and GK codes [1, 21, 22]. Recently, the importance of the kinetic electron effects in the ZS dynamics has also been emphasized with ORB5 [23]. ORB5 has also been used to investigate the generation of ZS by AM in the absence of turbulence [24], confirming that a force-driven excitation [16] is the main mechanism of generation, in the considered low-shear regime. In this work, we extend the previous studies, to the case of turbulent plasmas.
2. MODEL

ORB5 is a global nonlinear GK particle-in-cell code, which was originally written for studying electrostatic turbulence in tokamak plasmas [17], and extended to treat multiple kinetic species (i.e. thermal ions, electrons, EP, impurities, etc) and electromagnetic perturbations [18, 19]. The equations for the marker trajectories for the thermal ions and fast ions (in the electrostatic version of the code) are [18, 19]:

\[
\begin{align*}
\dot{\mathbf{R}} &= \frac{1}{m_s} \left( p_\parallel - \frac{q_s}{c} \tilde{A}_\parallel \right) \frac{\mathbf{B}^*}{B_\parallel} + \frac{c}{q_s B_\parallel^2} \mathbf{b} \times \left[ \mu \nabla B + q_s \nabla (\tilde{\phi} - \frac{p_\parallel}{m_s c} \tilde{A}_\parallel) \right] \\
\dot{p}_\parallel &= -\frac{\mathbf{B}^*}{B_\parallel} : \left[ \mu \nabla B + q_s \nabla (\tilde{\phi} - \frac{p_\parallel}{m_s c} \tilde{A}_\parallel) \right] \\
\dot{\mu} &= 0
\end{align*}
\]

(1)  

The set of coordinates used for the phase space is \( (\mathbf{R}, p_\parallel, \mu) \), i.e. respectively the gyrocenter position, canonical parallel momentum \( p_\parallel = m_s v_\parallel + (q_s/c) \tilde{A}_\parallel \) and magnetic momentum \( \mu = m_s v_\parallel^2 / (2B) \) (with \( m_s \) and \( q_s \) being the mass and charge of the species). \( v_\parallel \) and \( v_\perp \) are respectively the parallel and perpendicular component of the particle velocity. The gyroaverage operator is labeled here by the tilde symbol (used for the ions, whereas the electrons are treated with a drift-kinetic model). \( \mathbf{B}^* = \mathbf{B} + (c/q_s) p_\parallel \nabla \times \mathbf{b} \), where \( \mathbf{B} \) and \( \mathbf{b} \) are the equilibrium magnetic field and magnetic unitary vector. Dotted variables are meant to be subjected to time derivative. The time-dependent fields are the scalar potential \( \phi \) and the parallel component of the vector potential \( \tilde{A}_\parallel \).

The gyrokinetic Poisson equation is [18, 19]:

\[
-\sum_{s \neq e} \nabla \cdot \frac{n_{0s} m_s e^2}{B^2} \nabla_\perp \phi = \sum_s \int dW_s q_s \delta f_s
\]

(4)

with \( n_{0s} = \int dW_s f_{0s} \). The summation over the species on the left-hand-side is performed for the bulk ions and for the EP. Here \( \delta f_s = f_s - f_{0s} \) is the gyrocenter perturbed distribution function, with \( f_s \) and \( f_{0s} \) being the total and equilibrium (i.e. independent of time, assumed here to be a Maxwellian) gyrocenter distribution functions. The integrals are over the phase space volume, with \( dW_s = (2\pi/m_s^2) B_s \delta p_\parallel d\mu \) being the velocity-space infinitesimal volume element.

In this paper, finite-larmor-radius effects are considered, for both thermal and fast ions.

The Ampère equation is [18]:

\[
\sum_s \frac{\beta_s}{\rho_{Ls}} \tilde{A}_\parallel = -\nabla_\perp^2 A_\parallel = \mu_0 \sum_s \delta j_||
\]

(5)

where \( \rho_{Ls} = \sqrt{2m_s T_s/q_s B} \) is the Larmor radius of the species \( s \), and \( \beta_s = 2\mu_0 n_s T_s/B_0^2 \) is the thermal to magnetic pressure ratio of the species \( s \). The currents are defined by \( \delta j_|| = q_s \int dW(p_\parallel/m_s) \delta f (\mathbf{R} + \rho - \mathbf{x}) \), with \( \rho \) being the vectorial gyroradius.

For a description of the discretisation of ORB5, the reader is referred to Ref. [18, 19].

3. EQUILIBRIUM AND LINEAR DYNAMICS

The tokamak geometry and magnetic field is taken consistently with Ref. [1], for the case referred to as “energetic particle driven modes” (EPM). The major radius is \( R_0 = 1.0 \) m, the minor radius is \( a = 0.1 \) m, and the toroidal magnetic field at the axis is \( B_0 = 3.0 \) T. Circular
concentric flux surfaces are considered. The safety factor has a value of 1.85 at the axis, it decreases from $\rho=0$ to $\rho=0.5$, where the minimum value is located ($q(\rho=0.5)=1.78$), and then it raises to the edge, where it reaches the maximum value ($q(\rho=1)=2.6$). Here $\rho = r/a$ is the normalized radial coordinate. The value of the shear is always higher than -0.5.

We choose a reference radial position of $\rho_r = 0.5$. The ion and electron temperatures are taken equal everywhere, $T(i) = T(e)$. Here, differently from Ref. [1], a value of $T(e)(\rho = \rho_r)$ corresponding to $L_x = 2/\rho^* = 350$, i.e. $\rho^* = \rho_s/a = 0.00571$, is chosen (with $\rho_s = \sqrt{T_e/m_i}/\Omega_i$ being the sound Larmor radius). This is the turbulence regime of the cyclone base case. This corresponds to a value of temperature of $T(\rho_r) = 140.56$ eV. The electron thermal to magnetic pressure ratio of $\beta_e = 0.5e-3$. The ion cyclotron frequency on axis is $\Omega_i = 1.44e8$ rad/s. At the reference radial position, the Alfvén velocity is $v_A = 5.19e6$ m/s, and the sound velocity is $c_s = 8.21e4$ m/s.

An analytical function is used for the profiles of the equilibrium density and temperature, for the three species of interest (thermal deuterium, labelled here as “d”, thermal electrons, labelled here as “e”, and hot deuterium, labelled here as “EP”). For the EP density, for example, the function is written as:

$$n_{EP}(\rho)/n_{EP}(\rho_r) = \exp[-\Delta \kappa_n \tanh((\rho - \rho_r)/\Delta)]$$

(6)

The value of $\Delta$ is the same for all species, for both density and temperature: $\Delta = 0.208$. Deuterium and electrons have $\kappa_n = a/L_n = 0.3$ and $\kappa_T = a/L_T = 2.0$, and the EP have $\kappa_n = a/L_n = 10.0$ and $\kappa_T = a/L_T = 0.0$. The EP temperature is given by $T_{EP}/T_e = 100$.

The distribution function of the EP population is Maxwellian in velocity space. The EP averaged concentration is $<n_{EP}>/n_e = 0.005$. Once the EP population is loaded into ORB5, we...
choose to satisfy the quasineutrality for the considered simulations. This means that ORB5 automatically re-calculate the electron density profile, in order to have \( n_{EP}(\rho) + n_i = n_e(\rho) \) for all values of \( \rho \).

An annular region \( 0.2 < \rho < 0.8 \) is considered. A filter allows poloidal and toroidal mode numbers with \(-128 < m < 128\) and \(0 < n < 48\) to develop. Unicity boundary conditions are imposed at \( \rho = 0.2 \) and Dirichlet at \( \rho = 0.8 \). A white noise initial perturbation is set at \( t=0 \).

The electron mass is chosen as \( m_e/m_i = 0.005 \). A Krook operator is applied to the thermal deuterium and electrons.

The linear dynamics is investigated by running electrostatic and electromagnetic simulations with unperturbed trajectories for the markers. A linear ITG spectrum is observed with the instability peak around the toroidal mode numbers \( n = 25 \). Note that, for burning plasmas, even higher peaks (\( n \sim 100 \)) of the turbulence spectra are expected, and AM peaked around \( n = 20 \). These regimes will be investigated as an extension of the present study. No sensible effect of the EP on the linear ITG instability is found in the considered regime. The effect of EP on the linear dynamics of ZS is also studied, by running electrostatic Rosenbluth-Hinton tests with ORB5 like in Ref. [20], but with the additional third species of EP with Maxwellian distribution function as described above. The effect of kinetic electrons is neglected in these tests, to focus on the interaction with the EP. In the considered regime, the effect of EP on the zero-frequency zonal flow (i.e. the residual level) is found negligible (although some analytical derivation predict a non-negligible effect in other regimes [25]). Preliminary results show that EP with a Maxwellian distribution function enhance the Landau damping of the GAM, due to the higher effective finite-orbit-width.

4. NONLINEAR DYNAMICS OF AM AND TURBULENCE

4.1. Identification of the phases in the evolution in time

In this section, we investigate the effect of the EP on the nonlinear dynamics of the nonzonal and zonal components of the perturbation. By comparing different simulations with and without EP, and with and without gradients of the thermal species (which constitute the drive of the turbulence), we can identify different phases in the evolution of the perturbed potential in time.
When a nonlinear simulation without EP is performed, a first “pseudo-linear” ITG phase is observed, where the ITG modes grow nearly linearly, from $t = 0$ to $t = 5000 \, \Omega_i^{-1}$. When the nonlinear interactions among modes become non-negligible, we have a transition phase, from $t = 5000 \, \Omega_i^{-1}$ and $t = 10000 \, \Omega_i^{-1}$. Finally, a saturated phase is observed, with level of the perturbed potential remaining constant in time, after $t = 10000 \, \Omega_i^{-1}$. When EP are added, the first “pseudo-linear” ITG phase is found to be not modified, and a second “pseudo-linear” AM phase is observed, where AMs grow nearly linearly, up to $t = 15000 \, \Omega_i^{-1}$, and the saturation occurs only at $t > 15000 \, \Omega_i^{-1}$, and at higher levels with respect to the case without EP. An analysis of the zonal and nonzonal components, shows that the ZS grows not only in the “pseudo-linear” ITG phase, but also during the “pseudo-linear” AM phase, due to the nonlinear interaction with the AM.

### 4.2. Heat transport

In this section, we compare the heat transport for the two nonlinear simulations without and with EP, shown in Fig. 3.-left. The corresponding heat fluxes are shown in Fig. 5. As it can be seen, the heat flux of the simulation with EP grows linearly in the “pseudo-linear” ITG phase, then continues growing during the “pseudo-linear” AM phase, consistently with the weaker AM growth rate, and then enters a phase with big oscillations, where the value of the heat flux oscillates above the value without EP. The fact that the average is higher than the case without EP, reflects the presence of the nonzonal potential of the AM, which gives a contribution to the heat transport. The oscillations in time are caused by the ZS.

### 4.3. Radial structure

In this section, we investigate the radial structure of the perturbed field, in a simulation with turbulence and AMs. In the “pseudo-linear” ITG phase, the characteristic ballooning structure of the ITG is observed, with stronger scalar potential at the low field side, and weaker signal at the high field side, and a large value of the poloidal mode number $m$ (see Fig. 6.-left). Similar structure of the vector potential as previously investigated with ORB5 [26] has also been recovered. In the “pseudo-linear” AM phase, the AM grows on top of the turbulent plasma, driven by the EP free energy source (see Fig. 6.-right). Finally, the AM saturates at a higher level with respect to the turbulence perturbed field, due to mode-mode coupling (see also Ref. [27]) and nonlinear...
interaction with the EP population.

5. CONCLUSIONS AND DISCUSSION

Burning plasmas are characterized by an intrinsic nonlinear interplay of different scales - from microscopic turbulence modes, to mesoscopic zonal structures (ZS) to macroscopic global instabilities, like Alfvén modes (AM). AMs, excited by energetic particles (EP, product of fusion reactions), are particularly important for their role in redistributing the EP population, and therefore modifying the heating localization. In this work, the nonlinear self-consistent interaction of AM, ZS and turbulence is investigated with the gyrokinetic code ORB5. In previous studies, ORB5 had been extensively verified and benchmarked on the different physics of the turbulent modes, of the AMs excited by EPs, and on the ZSs. The detailed verification/benchmark process has given trustability to the code, and has increased the knowledge of the different pieces of physics. This has paved the way to the more complex, nonlinear study, where all the modes are let evolve and interact selfconsistently.

An equilibrium with a reverse shear has been chosen, and a regime with $\rho^*$ of the order of the cyclone base case has been adopted. An EP population with Maxwellian distribution function in velocity space has been considered. A linear investigation has shed light on the spectra of the micro-turbulence modes, and on the properties of the EP-driven AMs. Nonlinear electromagnetic simulations with and without EPs have been performed. Without EPs, microturbulence is formed and ZS are driven due to modulational instability. When EPs are present, AMs are found to develop on top of the turbulent plasma, saturating with higher amplitude fields with respect to the saturated levels of the turbulence. This higher amplitude nonzonal fields are found to be associated to higher averaged levels of radial heat fluxes. At the same time, AMs also couple nonlinearly to generate ZSs, by force-driven instability (see also Ref. [24], for a more dedicated study with ORB5). The combined effect of the high-amplitude saturated fields of the AMs and ZSs excited by turbulence and AMs, is shown to be an enhanced averaged level of the radial heat flux, modulated with low frequency oscillations. The quantitative estimation of the saturated levels of the AMs, and the corresponding effect on the EPs redistribution and heat flux, and the dynamics of the ZSs, in experimental magnetic configurations and with experimental profiles,
is the next step of our investigation. Comparisons with the GK Eulerian code GENE [28, 29] are also in progress.

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