SIMULATION OF TOROIDICITY-INDUCED ALFVEN EIGENMODE EXCITED BY ENERGETIC IONS IN HL-2A TOKAMAK PLASMAS

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Abstract

The toroidicity-induced Alfvén eigenmode (TAE) excited by energetic (fast) ions was first simulated with GTC code, for HL-2A experiments. The simulation results indicate that the fraction of energetic ions (EIs) is about 3%. The critical density of EIs to induce TAE mode is around 0.023 which is in good agreement with theoretical expectation of 0.026. The real frequency of the TAE with toroidal mode number \( n=3 \) is around 211 kHz, being quantitatively in agreement with the experimental observation (Ding et al 2013 Nucl. Fusion 53 043015). The frequency is proportional to the summations of toroidal precession frequency and transit frequency of passing energetic ions, indicating that resonance with passing energetic ions is dominant for the excitation of the modes in the experiment. It is also checked by the phase space structures of \( \partial [f_i] / \partial \xi \) (strength of energetic particle distribution function) that the TAE modes are mainly induced by passing energetic ions instead of by trapped ones in HL-2A. The growth rates of TAE modes increase with increasing of EIs density and its gradient. In addition, lower \( n \) TAE modes, such as \( n=1 \) can also be driven by energetic ions when off-axis heating with higher beam energy is employed in NBI experiment. The half width of radial mode structures for lower \( n \) modes is usually wider than those for higher \( n \) modes. At the same time, the polarization of the mode shows that the perturbed parallel electric field is zero. Thus, the TAE mode is close to an ideal MHD mode.

1. INTRODUCTION

Two Alfvén continuums with same toroidal mode number \( n \) but different poloidal mode numbers, e. g. \( m \) and \( m+1 \), can strongly couple at the position of safety factor \( q=(m+0.5)/n \) and create a frequency gap. The TAE mode lying in the gap was predicted in theory [1]. In general, the mode is stable except for strong enough driving sources, such as NBI heating, alpha particles from the fusion product, existing in the gap [1-4]. When the number and density gradient of energetic particles (Eps) is high enough and the corresponding damping [5] are overcome, the TAE modes can be driven by Eps. In many auxiliary heating experiments, TAE modes are extensively observed, which induce fast particles loss and eject a great amount of Eps [6-11]. Gyrokinetic simulations with GTC code have been applied to investigate the properties of turbulent transport and the discernment of zonal flow for the first time [12]. Since then, many functions of GTC code have been developed to study micro/macro-instabilities [13-16]. This work focuses on interpreting HL-2A experimental phenomenon and acquiring information about Eps from the simulation. The TAE activities, degrading plasma confinement, have also been measured in HL-2A and other devices [17-19]. The main motivations of this simulation are as follows, i) to find out the EIs amount and the corresponding critical density for inducing TAE mode in HL-2A, ii) to simulate the TAE properties in this device, iii) to investigate the excitation requirement of low \( n \) \((m)\) TAE modes. In section 2, the physical model for gyrokinetic simulations are given. The simulated results are presented in section 3 and summarized in section 4.

2. THE PHYSICAL MODEL FOR GYROKINETIC SIMULATION

The gyrokinetic formulations to simulate toroidicity-induced Alfvén eigenmode by GTC code in 5D frame \((\hat{X},\mu,v_\parallel)\) are written as follows [13-15, 20],

\[
[\partial_t + \hat{X} \cdot \nabla + v_\parallel \partial_{v_\parallel} - C_a] f_a = 0
\]

(1)

\[
\hat{X} = v_\parallel B_0 + \hat{v}_x + \hat{v}_y + \hat{v}_z, \quad v_\parallel = -\frac{1}{m_a} \frac{\vec{B} \cdot (\mu \nabla B_0 + Z_a \nabla \phi)}{m_a c} \frac{\partial A_\parallel}{\partial t},
\]

(2)

Here, the subscript \( a \) refers to the species of particles, \( m_a \) and \( Z_a \) are particle mass and electric charge number, respectively. \( \hat{X}, \mu, v_\parallel \) are gyrocenter position, magnetic moment and parallel velocity of particles, respectively. \( C_a \) is collision operator. \( f_a \) is particles distribution function. \( \phi \) is electrostatic potential and \( A_\parallel \) is parallel...
component of vector potential. The drift velocity, the curvature drift velocity and the magnetic gradient drift velocity are respectively written as,

\[ \vec{v}_E = c \vec{B}_0 \times \nabla \phi / B_0, \quad \vec{v}_E = \psi E / B_0, \quad \vec{v}_E = \mu_0 \nabla \times \vec{B} / n_0 \Omega_\alpha. \]  

In this work, we use the fluid-kinetic hybrid electron model [21]. The zero order adiabatic part can be readily obtained by integrating over Eq. (1) and get the following continuity equation,

\[ \frac{\partial \hat{n}_e}{\partial \tau} + \hat{B}_0 \cdot \nabla n_{e0} \hat{\nabla}_e + B_0 \frac{\nabla (n_{e0} - n_{e0}^0)}{B_0} - n_{e0} (\vec{v}_E + \vec{v}_E) \cdot \nabla B_0 / B_0 = 0 \]  

(4)

with \( \vec{v}_E = \hat{B}_0 \times \nabla (\vec{E}_e + \vec{E}_c) / (n_{e0} \Omega_\alpha) \), \( \vec{E}_c = \int e m v B d^3 \nu \), and \( n_{e0} = \int f_{e0} d^3 \nu \). The perturbed parallel velocity of electrons in Eq. (4) can be obtained from Ampere’s law,

\[ \frac{c}{4\pi} \nabla^2 \hat{A}_{e\perp} + \sum_{a=\perp} A_{e\perp} = e n_{e0} \hat{\nabla}_e^2 \hat{A}_{e\perp}. \]  

(5)

Here, the perturbed vector potential can be calculated by making use of the Faraday’s law,

\[ \nabla \times \vec{E}_{e\perp} = \nabla \times (\nabla \phi - \vec{v}_E). \]  

(6)

By expanding Eq. (1) for electrons and keeping the leading order of \( \omega / k^2 v_E \), we can obtain a zero order distribution function \( \phi^0 \) [20-21]. Then, integrating over velocity and assuming the uniform thermal electron density, the effective potential \( \phi_{e\perp} \) can be obtained as follows,

\[ e \phi_{\perp} / T_e = \nabla \phi / n_{e0}, \]  

(7)

Thus, the electron perturbed pressure in Eq. (4) can be written \( \vec{p}_{e\perp} = \vec{p}_{e\perp} = e n_{e0} \phi_{e\perp} \). The closed system can be formed by the gyrokinetic Poisson’s equation,

\[ Z_e^2 n_i (\phi - \tilde{\phi}) / T_i = \sum_{a=\perp, f} Z_a \nabla n_a. \]  

(8)

Where \( \tilde{\phi} \) is the gyrophase-averaged electrostatic potential.

3. GYROKINETIC SIMULATION OF TAE ON THE HL-2A TOKAMAK

For HL-2A, the minor/major radii \( a/R_c=40/165 \) cm, the toroidal magnetic field at magnetic axis \( B_0=1.39 \) T, the electron density \( n_e=2-3 \times 10^{19} \) m\(^{-3}\) during NBI heating, the thermal particles temperature \( T_e=T_i=1 \) keV, the EIs temperature \( T_e=40 T_i \). The Alfvén speed is \( v_A = B_i / \sqrt{\mu_0 n_i m_i} \) and \( n_i, m_i \) are density and mass of thermal ions, respectively, \( \mu_0=4 \times 10^{-7} \) is the permeability. The Alfvén frequency \( \omega_A = k v_A \) and the parallel wave vector \( k_p=(n-m/q)R \). Here, \( n \) is the toroidal mode number, \( m \) is the poloidal mode number and \( q \) is the safety factor.

3.1. Profiles of safety factor and energetic ions on HL-2A

Profiles safety factor and density of EIs are important for inducing TAE modes. The former influences frequency gap of continuous spectrum, whereas the latter is related to main driving source. In order to simulate HL-2A experimental results, the safety factor and the spatial profile of EIs are loaded from the experiment data calculated with EFIT and NUBEAM codes. But the energetic ions are loaded with an isotropic Maxwellian distribution in phase space. Shown in FIG. 1 are the profiles of safety factor and EIs density, calculated based on HL-2A experimental configuration. Here, a polynomial \( q = \sum c_i \psi_i^4 \) was used to fit the experimental profile (red solid line) and given with red diamonds in FIG. 1. The normalized poloidal magnetic flux \( \hat{\psi} = \psi / \psi_w \). \( \psi_w \) is the magnetic flux at plasma boundary, the coefficients \( c_i \) are given as \( c_0=0.99417, c_1=1.11906, c_2=24.007, c_3=104.56, c_4=220.56, c_5=215.19, c_6=79.844. \) Besides, it is difficult to measure the EIs density profile in experiment at present stage. Thus, according to NBI heating condition on HL-2A, the density profile of EIs was calculated with NUBEAM code and shown by dashed line in FIG. 1. A hyperbolic function of magnetic flux (the circles) was employed to fit EIs density profile (dashed line) in FIG. 1 and given by,

\[ \hat{n}_f = \hat{n}_{f0} \left[ 1 + m f \frac{\tanh[|\psi_0 - \hat{\psi}| / \Delta] - 1}{1 + m f \tanh[|\psi_0 / \Delta] - 1]} \right], \]  

(9)

where \( \psi_0 \) is the position of maximum EIs density gradient. The EIs density profiles with NBI on/off-axis heating can be realized by modifying \( \psi_0 \). The index \( \Delta \) is used to describe the density gradient of EIs and \( m_f \)
represents the difference between maximum and minimum in the profile. The EIs density is normalized to $n_{e0}$ (the electron density at magnetic axis), i.e. $\hat{n}_f = n_f / n_{e0}$.

\[\begin{align*}
\mathcal{E} &= n_e/\hat{e}_f(n_f) \\
\hat{e}_f &= n_f / n_{e0}.
\end{align*}\]

**FIG. 1.** The safety factor (red lines) and fast ions density (blue lines) profiles as functions of magnetic flux. The red solid and blue dashed lines are experimental safety factor and fast ion density profiles of HL-2A device calculated with EFIT and NUBEAM codes whereas the red diamonds and blue circles represent polynomial and analytical fit and inputting in GTC.

3.2. Simulations of TAEs observed in HL-2A experiment

TAE modes, essentially being shear Alfven wave, only exists within frequency gap of Alfven continual which can be easily obtained by ALCON code [16]. Shown in FIG. 2 is Alfven continual as function of minor radius plotted based on HL-2A experimental configuration. From the blue and orange lines in FIG. 2, we learn that the frequency gap for $m=3$ is approximately located at the position of 1/3 minor radius. In this simulation, we use 64 grid points in the radial direction, 128 grid points in the poloidal direction and 16 grid points in toroidal direction. The time step size is $\Delta t = 0.005 R_0 / v_{A0}$.

**FIG. 2.** The normalized Alfven continua of $n=3$ in the case of zero-$\beta$ limit based on HL-2A experimental configuration.

It can be seen from the blue curves in FIG. 1, when the parameters in Eq. (9) is set as $n_f=0.49$, $\hat{\psi}_0 = 0.18$ and $\Delta = 0.11$, respectively, the hyperbolic function can approximately fit the experimental profile and describe on-axis NBI heating. The ratio of EIs density to thermal electrons at magnetic axis is $n_f/n_{e0} = 0.05$. The simulated results are presented in FIG. 3, where the lines in (a) are the time evolution of vector potential $A_y$ with $n=3$ and $m=3$ during NBI heating. The black and pink lines are real and imaginary parts of $A_y$, respectively. It shows that, during linear development stage, the perpendicular magnetic field perturbation of TAE mode grows exponentially after the modes are excited by EIs. The line in FIG. 3(b) is the frequency spectrum of vector potential. This spectrum indicates that the real frequency of TAE mode is $0.9 \omega_A$. According to the parameters of HL-2A device, the real frequency of TAE mode is 211 kHz which is well in agreement with experimental observation given in FIG. 1 (g) in Ref. [7].

**FIG. 3.** The time evolution of vector potential $A_y$ (a) and the frequency spectrum of $A_y$ (b) for TAE excited by fast ions in HL-2A with $n=3, m=3/4, T_f=40$ keV, $\hat{\psi}_0 = 0.18$ and $\hat{n}_{f0} = 0.05$. 

3
At present, it is difficult to measure the fraction of energetic particles directly. But, the fractions of EIs and their critical density to excite TAE can be roughly determined by the simulation work during NBI heating. Shown in FIG. 4 are the real frequencies (red solid lines) and growth rates (blue broken lines) of TAE modes as functions of density (a), density gradient (b) and temperature (c) of energetic ions, respectively. Here, the density of EIs and the mode frequencies are normalized to thermal electron density and Alfven frequency on magnetic axis, respectively. Moreover, the data points in FIG. 4 are marked with crosses. The results show that the TAE modes can be excited when the density of EIs is higher than a critical value (0.023). The real frequency of TAE mode almost keeps unchanged with EIs density increasing. The lines shown in FIG. 4 (b) present the dependence of real frequency and growth rate of TAE mode on $\Delta$. In this density profile model, the density gradient of fast ions increases with decreasing $\Delta$. It can be learnt from the FIG. 4(b) that TAE instabilities can be induced when the gradient of EIs is higher than a critical value. In general, for a fixed density profile of EIs, the fraction of EIs can be approximately estimated as the EI density under which TAE mode is excited and the simulation results (e.g. eigen-frequency) are in agreement with experimental observation. The result in FIG. 4 (a) indicates that the normalized critical density of fast ions is around 0.023. Consequently, the fraction of EIs in HL-2A device should be higher than this value. The calculated results show that fine mode structures (not given here) can be obtained when the normalized critical density of energetic ions satisfies $\tilde{n}_f = 0.03$. Therefore, the fraction of EIs in HL-2A experiment is approximately 3%. In order to check the simulation results with reference to the critical density of energetic ions in experiment, we can compare the simulated critical density with the theoretical one and examine the differences between them. The critical density of EIs can also be analytically estimated according to the TAE theory given in Ref. [1], i.e. $\tilde{n}_{f, cr} = \frac{2 T_{nf} \nu_f}{T_f (2 \omega_n / \omega_\varphi - 1) F(x)}$. Here, $\nu_f$ is thermal speed of electrons, $\omega_\varphi$ is diamagnetic frequency of energetic ions and can be readily calculated by using Eq. (9). The function $F(x)=x(1+2x^2+2x^3)\exp(-x^2)$ with $x=\nu_f/\nu_e$. By using the parameters of HL-2A, we can obtain the critical density of energetic ions $n_{f, cr}=0.026$ which is very close to the simulated value 0.023.

It is helpful for understanding the resonant excitation mechanism of TAE modes and determining the kinds of dominant EIs (e.g. trapped/passing ions) in inducing the modes to study the dependence of eigen-frequency on temperature of energetic ions. In general, the resonant conditions satisfy the relation $n_0 \omega_\perp = l \omega_\parallel = \omega_\varphi$. Here, the variables $\omega_\parallel$, $\omega_\perp$ and $\omega_\varphi$ represent real frequency of TAE mode, the toroidal precession frequency, and the bounce/transit frequency of fast trapped/passing particles, respectively. The integer $l$ is taken as $l=0, \pm 1,...$. The toroidal precession and bounce/transit frequencies of EIs are given by [22],

$$\omega_\parallel = \frac{qE}{r \epsilon |B| R_0} \left\{ \begin{array}{ll} 2 \frac{E(k)}{K(k)} - 1 + 4 \omega_\perp \frac{E(k)}{K(k)} \frac{1 + k^2}{1 + 4 \omega_\perp} & \text{if } k > 0 \\ 1 + 2k^2 \omega_\perp \frac{E(k)}{K(k)} & \text{if } k < 0 \end{array} \right\}, \quad \omega_\varphi = \frac{\pi}{qR_0} \sqrt{\frac{m_h}{k}} \left( \frac{1}{2K(k)} \right) \left( \frac{m_s}{K(k)} \right) \left( \frac{1}{2K(k)} \right). \tag{10}$$

Here, $K(k)$ and $E(k)$ are the first and second elliptic functions with the argument $k^2=(1/\alpha-1+\epsilon)/\epsilon$ and $k_+ = 1/k$. $\epsilon$ is the local inverse aspect ratio, $\alpha$ is the pitch angle of energetic ions, $E$ and $m_h$ are the energy and mass of fast ions, and $s$ is the magnetic shear. The neutral beam is injected into the plasma at an angle of 42 degree along the tangential direction in HL-2A. Thus, the pitch angle $\alpha=(\nu_f/\nu_e)=0.4477$ in this device, which means that passing energetic ions are dominant. According to the parameters at the gap position, $\epsilon=0.0848$, $\alpha=0.04569$, the integers $n=3$ and $l=1$, we can estimate the frequency of TAE mode according to the resonant relation given above, $f_{\text{TAE}}=\omega_\parallel/2\pi=203$ kHz which is in agreement with the simulation results. On the other hand, the fractions of trapped particles are proportional to $\sqrt{2\epsilon}$ (about 0.41 at the frequency gap position for HL-2A). As a result, the fraction of trapped fast ions in HL-2A is also large enough. The highest $\omega_\parallel$ and $\omega_\varphi$ can be obtained when we set the elliptical function argument $k=0$ (deeply trapped ions). Meanwhile, the corresponding mode frequency is
$f_{TAE} = \omega_0/2\pi = 125$ kHz for $n=3$, $l=1$, much less than the eigen-frequency of TAE mode. But for higher $l$ harmonic, for example, $n=3$, $l=3$, it is difficult to excite TAE modes by trapped energetic ions due to lower beam injection energy and tangential NBI injection, although the resonant condition seems to be satisfied and the corresponding eigen-frequency $f_{TAE} = \omega_0/2\pi = 213$ kHz. Consequently, the TAE modes are mainly excited by passing fast ions. Shown in FIG. 4 (c) are real frequency (red solid line with crosses and red circles) and growth rate (blue broken line with crosses) as functions of temperature of EIs. The red circles are a nonlinear function of fast ions temperature, $\omega_r = aT_f + b\sqrt{T_f} + c$ with $a=0.0035$, $b=0.00665$ and $c=0.71$, used to fit real frequency (red solid line). It can be learnt from the solid line and the red circles that the real frequency of TAE mode can be perfectly fitted by the analytical function, which analytically demonstrates that the real frequency of TAE mode is just the linear combination of toroidal precession frequency and transit frequency of passing EIs once again, and the toroidal precession and the transit motions of EIs are dominant for resonant excitation of TAE modes.

Besides, the resonant condition can also be identified in phase space of $|\delta f|^2$ as $(E, \alpha)$, and the corresponding contours are shown in FIG. 5. Here, the variable $\delta f_r$ is perturbed distribution function of EIs. The bold black lines are analytical resonant condition ($n\omega_0 + l\omega_f = \omega_{TAE}$) calculated according to Eq. (10). The results show that two resonant zones are in the range of $0.2<\alpha<0.8$, $20<E<100$ (keV) and $0.9<\alpha<1.0$, $40<E<100$ (keV), respectively. Theoretically speaking, both trapped and passing energetic ions can destabilize TAE instabilities via resonant interaction between wave and particles. In fact, it is difficult for trapped energetic ions to excited TAE mode due to the tangential beam injection and lower maximum beam energy (around 40 keV) for HL-2A.

**FIG. 5.** The contour plot of $|\delta f|^2$ in the phase space of energy $E$ and pitch angle $\alpha$ of fast ions. The bold black lines indicate the resonant conditions $n\omega_0 + l\omega_f = \omega_{TAE}$.

Shown in FIG. 6 (a, b) are the poloidal mode structures of electrostatic potential $\phi$ (a) and vector potential $A_r$ (b), and shown in FIG. 6 (c, d) are the radial mode structures of the counterparts with toroidal mode number $n=3$ and poloidal mode numbers $m=3$ (the red lines) and $m=4$ (the blue lines). Note, the radial mode structures of $\phi$ and $A_r$ are normalized to their peak values. The dashed lines in FIG. 6 (d) are vector potential calculated by Faraday’s law. Generally speaking, the ideal mode should satisfy the condition $\delta E_\parallel = 0$. The zero perturbed parallel electric field condition can be checked by polarization of TAE mode. As mentioned above, the vector potential can be analytically calculated by using Faraday’s law, i.e. $\partial_t A_r = -c\partial_t \phi \nabla \phi$ which is the polarization of TAE mode and presented with the dashed line in FIG. 6 (d). The results shown in FIG. 6 (d) indicate that the dashed lines are in good agreement with the solid lines and the perturbed parallel electric field $\delta E_\parallel = 0$ is satisfied. Thus, the mode studied in this work is close to an ideal MHD. Besides, we can also demonstrate that the mode is an Alfvén mode by checking the dependence of eigenmode frequency on electron density. Shown in FIG. 7 is the frequency of TAE mode as function of electron density. The solid line represents the simulation results and the red diamonds, being inversely proportional to square root of electron density, are employed to fit the solid line. The perfect agreement between the simulation result and analytical estimation indicates that the mode frequency satisfies the Alfvén wave frequency scaling.
3.3. Conditions for excitation of low n TAE modes on HL-2A

Although the TAE modes with mode number \( n=3 \) (\( m=3/4 \)) have been observed in experiment and the mode frequency is in agreement with the simulated results, yet the lower \( n \) number modes, such as \( n=2 \) (\( m=2/3 \)) and \( n=1 \) (\( m=1/2 \)) have not been observed in HL-2A experiments. The reasons may include weak driving source, strong damping and the variation of resonant condition coming from changing of toroidal mode number. According to the resonance condition \( n_0\omega_{gy} + \omega_T = \omega_{TAE} \), it can be seen that the original resonance condition may not be satisfied any longer when the mode number \( n \) is changed. Thus, the low \( n \) modes cannot be excited in experiment unless new resonance conditions are satisfied. To be specific, low \( n \) TAE modes cannot be induced unless the beam energy is increased to raise the characteristic frequency of TAE modes (such as the toroidal precession frequency, the bounce/transit frequency of EIs) and satisfy the new resonance conditions. Besides, the positions of maximum density gradient of energetic ions also play a crucial role in destabilizing the TAE mode. Strong enough density gradient of energetic ions within the frequency gap is another important condition for driving TAE instabilities. The frequency gaps for low \( n \) modes, e. g. \( n=2 \), \( m=2/3 \) and \( n=1 \), \( m=1/2 \), can be roughly estimated from TAE theory \([1]\) as located at the positions \( \psi_0 = 0.27 \) and \( \psi_0 = 0.44 \), respectively, for HL-2A parameters. In the case of on-axis NBI heating, the density gradients of EIs at radial position of accumulation point for low \( n \) (especially \( n=1 \)) modes are correspondingly small, which weakens the driving force at these positions. Besides, considering the dependence of toroidal mode number \( n \) on resonant excitation requirement, the toroidal precession frequencies of energetic ions cannot match with the shear Alfvén frequencies any longer at the gap position in low \( n \) cases. The both reasons maybe result in the failures to observe low \( n \) TAE modes in HL-2A experiment. Accordingly, the low \( n \) TAE modes may be excited through modification of the position of maximum density gradient of EIs and raising the injection energy of neutral beam. Theoretically, the resonance relation can be satisfied when the maximum beam energy is raised to \( T_B=44 \) keV and \( T_B=50 \) keV for \( n=2 \) and \( n=1 \), respectively. At the same time, the positions of maximum density gradient need to be shifted to \( \psi_0 = 0.27 \) and \( \psi_0 = 0.44 \) for \( n=2 \) and \( n=1 \), respectively. Then, the low \( n \) modes, \( n=1 \), \( n=2 \) and so on can also be induced by EPs. Note, these parameters about density profile and temperature of energetic ions are only theoretical estimations whereas the real values can be determined by simulation.

According to the above theoretical analysis, the temperature of EIs is first changed to \( T_B=44 \) keV for \( n=2 \) and \( T_B=50 \) keV for \( n=1 \), whereas the corresponding density profile is not changed. The simulated results show that the \( n=1 \) mode cannot be excited by energetic ions at all. The \( n=2 \) (\( m=2/3 \)) mode can be induced, but the mode structure is not well organized, not given here. It is necessary to modify the value of normalized magnetic flux \( \psi_0 \) in Eq. (9), or to change the maximum injection energy of neutral beam so as to satisfy the resonant condition. Considering the little difference of resonance requirements for inducing \( n=2 \) and \( n=3 \) TAE modes, the maximum beam energy is still kept unchanged for \( n=2 \) case, i. e. \( T_B=40 \) keV. The TAE mode for \( n=2 \) can be driven unstable and the results are presented in figure 8 and figure 9, when the normal magnetic flux in Eq. (9) is changed to \( \psi_0 = 0.25 \) (\( n_\| = 0.49 \), \( \Delta = 0.11 \)). In accordance with the above analytical estimation, the mode with lower toroidal mode number, such as \( n=1 \) (\( m=1/2 \)), can be driven unstable when the temperature of fast ions and the normalized magnetic flux are set to \( T_B=50 \) keV and \( \psi_0 = 0.44 \), respectively. Actually, the TAE mode with \( n=1 \) (\( m=1/2 \)) is induced and a perfect mode structure can be obtained when the temperature of EIs \( T_B=80 \) keV and \( \psi_0 = 0.575 \) (\( n_\| = 0.49 \), \( \Delta = 0.11 \)) are employed, respectively. The corresponding simulation results are shown in figures 10 and 11. Meanwhile, the results shown in figures 8 and 9 are evolution of vector potential \( A_y \) and the corresponding frequency spectrums for TAE modes with lower mode numbers \( n=2 \) (\( m=2/3 \)) (e. g. figure 8) and \( n=1 \) (\( m=1/2 \)) (e. g. figure 10). Similar with the results given in figure 3, the black and pink lines are real and imaginary parts of vector potential, respectively. By comparing with those in figure 3, it can be learnt that the mode frequencies decrease with decreasing toroidal mode numbers. Thus, besides the results
presented in figure 4, the contribution of toroidal precession resonance for the excitation of TAE instabilities is checked once again. Especially, for the case of \( n=1 \), the maximum neutral beam energy has been changed to \( T_f=80 \) keV so as to match with the Alfvén frequency and to satisfy the excitation requirement of TAE mode. Presented in figure 9 and figure 11 are the poloidal and radial mode structures for \( n=2 \) \((m=2/3)\) and \( n=1 \) \((m=1/2)\) modes, respectively. From the positions of both poloidal and radial mode structures of TAE, it can be seen that the modes shift outwards with the toroidal mode number \( n \) decreasing and the radial half width of the mode structure becomes larger and larger.

FIG. 8. The time evolution of vector potential \( A_\theta \) (a) and the frequency spectrum of \( A_\phi \) (b) for TAE excited by fast ions in HL-2A with mode number \( n=2 \), \( m=2/3 \), beam energy \( T_f=40 \) keV and the normalized magnetic flux \( \psi_0 = 0.25 \), the density ratio of fast ions to electrons \( \hat{n}_{f0} = 0.05 \).

FIG. 9. The poloidal (a, b) and radial (c, d) mode structures of electrostatic potential \( \phi \) and vector potential \( A_\phi \) for TAE induced by energetic ions based on HL-2A tokamak parameters with toroidal mode number \( n=2 \) and poloidal mode numbers \( m=2/3 \), beam energy \( T_f=40 \) keV and the normalized magnetic flux \( \psi_0 = 0.25 \), the density ratio of fast ions to electrons \( \hat{n}_{f0} = 0.05 \).

FIG. 10. The time evolution of vector potential \( A_\theta \) (a) and the frequency spectrum of \( A_\phi \) (b) for TAE excited by fast ions in HL-2A with mode number \( n=1 \) and \( m=1/2 \), beam energy \( T_f=80 \) keV and the normalized magnetic flux \( \psi_0 = 0.575 \), the density ratio of fast ions to electrons \( \hat{n}_{f0} = 0.05 \).

FIG. 11. The poloidal (a, b) and radial (c, d) mode structures of electrostatic potential \( \phi \) and vector potential \( A_\phi \) for TAE induced by EIs based on HL-2A tokamak parameters with \( n=1 \) and \( m=1/2 \), beam energy \( T_f=80 \) keV and the normalized magnetic flux \( \psi_0 = 0.575 \), the density ratio of fast ions to electrons \( \hat{n}_{f0} = 0.05 \).
4. CONCLUSIONS

The toroidicity-induced Alfvén eigenmode excited by energetic ions was observed in HL-2A neutral beam injection experiment. The numerical simulation has been first performed by making use of GTC code and the results are presented in this work. It is found that TAE modes can be induced when the ratio of EIs densities to thermal electrons is higher than a critical value \((n_0/n_{\text{e0}}=0.023)\) which is very close to the theoretical prediction of 0.026, showing a reasonable agreement with analytic TAE theory. The fraction of energetic ions in HL-2A experiment is approximately 3\%. The growth rates of TAE modes increase with increasing of both the density and its gradient of EIs. The TAE frequency is around 211 kHz and inversely proportional to the square root of electron density, which is quantitatively in agreement with experimental observation. In addition, the frequency of TAE modes is proportional to the combination of toroidal precession and transit frequencies of passing EIs, and increases with temperature of EIs in accordance with the relation \(\omega_\text{e} = aT_f + b\sqrt{T_f} + c\) and keeps almost invariant when the density of energetic ions changes, which indicates that the resonances with toroidal precession and transit of energetic ions play an crucial role in inducing TAE mode. Meantime, the phase space structures of \(|\psi_0|^2\) show that the TAE modes are mainly induced by passing energetic ions instead of by trapped energetic ions based on the parameters of HL-2A. The amplitude of vector potential \(A_\rho\) exponentially increases with time during linear stage of TAE mode. The polarization of TAE mode indicates that the perturbed parallel electric field is zero. Both the conditions indicate that the simulated TAE mode is close to an ideal MHD mode. In current stage of HL-2A NBI heating experiment, the low \(n\) TAE modes were not observed. However, the conditions to excite low \(n\) modes are obtained by simulation. Higher beam energy, for example \(T_f=80\) keV and off-axis NBI heating, for example \(\psi_\theta = 0.575\) are the necessary conditions to drive low \(n\) (e.g. \(n=1\)) modes. The radial structures of low \(n\) TAE modes are usually more global than those of high \(n\) ones.

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