ELMs onset triggered by mode coupling near rational surfaces in the pedestal

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Abstract

Prior to eruptive events such as edge-localized-modes (ELMs), quasi-coherent fluctuations, referred to as pedestal modes, are observed in the edge of multiple fusion devices. We report on the investigations of nonlinear coupling between these modes leading to the ELM onset in the DIII-D tokamak. Three dominant modes with density and magnetic signatures are identified as the key players. We observe a radial shift of these modes’ radial structure towards the last-closed flux surface as the ELM event approaches. We demonstrate that this shift is due to the generation, via three-wave interactions, of one of these modes. These studies suggest that nonlinear coupling of pedestal modes, associated with radial distortions pushing out of the pedestal, is a possible mechanism for the triggering of an ELM event.

1. Introduction

During an edge localized mode (ELM) event, the plasma suddenly erupts releasing a significant fraction of its stored energy over few microseconds. In some cases, prior to an ELM event, the edge plasma is typically quasi-stationary on multiple transport time scales (e.g., dynamical time of few milliseconds) until the onset of the explosive event. Such quasi-stationary state is reminiscent of metastable state from which ELMs erupt. ELMs can be seen as repetitive eruptions that deposit energy and particles onto the plasma facing components. The ELM phenomenology has been described in many good reviews [1, 2, 3, 4].

The working hypothesis is that ELMs are generated by the destabilization of magnetohydrodynamic instabilities, namely ideal peeling-ballooning modes [2, 5], localized in the pedestal. The pedestal is defined as a narrow boundary layer occurring in high confinement plasma regimes with steep density and temperature gradients. A body of literature [1, 2, 4] supports the paradigm of coupled peeling ballooning (PB) modes as the driver of ELM events. Refs. [6, 7] have alternatively pointed out that ELMs are the result of a basic detonation scenario, where a ballooning instability nonlinearly grows explosively. While the PB theory and the explosive scenario have appealing features that could explain ELMs, the ELM onset mechanism has yet to be demonstrated – specifically when the edge parameters can exist near the stability margin for a substantial part of the period preceding the ELM onset. Here, we show experimental results indicating that three-wave interaction of pedestal modes is observed leading up to the ELM onset. This suggests that ELM events are triggered by nonlinear coupling of pedestal modes leading to radial distortions of the pedestal modes, which push out of the pedestal.

We leverage the many experimental results presented in Refs. [8, 9, 10, 11, 12, 13], in which the fixed pedestal gradients appear to be pinned to the linear marginally stable peeling ballooning profiles prior to the ELM onset – to investigate through experiment nonlinear mode coupling as a possible explanation of the onset mechanism. Specifically, recent experiments have shown that the pedestal density and temperature gradients after an ELM reach a quasi-stationary state [10, 12, 9, 11] during which the pedestal structure (width and height) either evolves slowly or remains quasi-stationary for few milliseconds prior to the ELM onset. During this quasi-stationary phase between ELMs, pedestal localized modes have been observed to grow and saturate (e.g., see Refs [8, 9, 14, 12, 10, 15]). These modes were observed to be correlated with the evolution of the edge profile gradients later in the ELM cycle in multiple devices, namely AUG [10, 11], C-Mod [9], DIII-D [12], and JET [8].

The EPED model predicts the pedestal structure (width and height) as the intersection of local kinetic ballooning and global peeling ballooning criteria near the stability threshold at which the ELM is triggered [16]. Nonetheless, it has been observed in many experiments that the pedestal gradients are nearly stationary for last few milliseconds of ELM cycle. The question that arises is – given that the pedestal can remain locked in this state – which mechanism leads to the onset of the explosive ELM event?
We study the nonlinear interactions of the three dominant modes observed in the pedestal of DIII-D discharges during the inter-ELM phase when the pedestal gradient is pinned to marginally stable peeling-ballooning profiles. We observe that two modes (located inside the pedestal) nonlinearly couple to a mode near the last-closed flux surface. We identify this three-wave interaction mechanism, coupled with a spatial distortion of the pedestal, as the slow (relative to the ELM eruption time scale) mechanism leading to the onset of an ELM.

2. Experiment and Diagnostics

The physical mechanism leading to the onset of an ELM event is studied on the DIII-D tokamak. The discharge is a lower-single null plasma, with plasma current of 1 MA, $\beta_n \sim 1.4$, a stored energy of 0.43 MJ, and line-averaged density of $5 \times 10^{19} \text{ m}^{-3}$ (see Fig. 1). We focus on type I ELMy discharges with low ELM frequency $\sim 20 - 30$ Hz
(higher frequency ELMs have been excluded from this analysis) to capture the evolution of the pedestal parameters as well as the fluctuations leading up to ELMs. Fig. 2 shows an example of PB calculations during the last phase of an ELM cycle, indicating the edge pressure gradient and current are near the stability point 4 ms prior to the ELM onset (see similar observations in refs. AUG [10, 11, 17], C-Mod [9], DIII-D [12], and JET [8], and discussions by Kirk et al. in [13]). The question that arises is why is the pedestal not erupting? The main diagnostics used in this analysis are the fast magnetic probes measuring fluctuations in the poloidal magnetic field (referred to as Bθ) and the spatially resolved beam-emission spectroscopy (BES) diagnostic probing the local density fluctuations [18] (referred to as δnₑ). Figure 3(a) displays the magnetic spectrograms showing quasi-coherent fluctuations between ELMs. This figure shows multiple modes between ELMs. Fig. 3(b) represents a zoomed in version of the spectrogram identifying the three dominant modes (Note each mode’s amplitude and frequency were tracked between ELMs and core modes were excluded – modes whose amplitudes are not affected by the ELMs are identified as core modes.)

Figure 3: Example spectrogram of the magnetic fluctuations for shot 170881. (a) Magnetic spectrogram during multiple ELMs. Here the ELMs are represented by the thick vertical lines. Typical rise time of these ELMs is ≃ 80 µs. (b) Zoomed spectrogram over a shorter time window where the core modes have been filtered out.

Figure 4: Dynamics of the frequency and amplitude of the three dominant modes observed in magnetic fluctuations as a function of ELM cycle [the time relative to an ELM in ms is on the top horizontal axis for reference]. The reference t = 0 is located at the ELM onset. (b) Associated mode amplitude evolution during the ELM cycle, in log-scale. These quantities have been statistically averaged over multiple inter-ELM periods. The shaded area represents the standard deviations.

3. Results

Amplitude and Frequency Dynamics — The inter-ELM dynamics of the most dominant modes are studied in detail using the signals from $\partial_t \tilde{B}_\theta$ with high signal to noise ratio (SNR). We systematically track their amplitude and frequency following local maxima of the spectrogram up to the ELM event. The same color code for the three
modes is used throughout the paper. Figure 4(a) displays these mode frequencies as a function of ELM cycle \( \tilde{t} \), where \( \tilde{t} = t/T_{\text{inter-ELM}} \) and \( T_{\text{inter-ELM}} \) is the normalized duration of each inter-ELM period [\( \tilde{t} = 0 \) corresponds to the ELM onset.] Similarly, Fig. 4(b) shows the associated amplitude evolution. The timeline leading to the ELM onset can be summarized as function of the ELM cycle. During the first half (\( \tilde{t} < -0.5 \)): the mode at \( \sim 69 \) kHz (blue) onset is correlated with the temperature pedestal gradient recovery (such correlation was shown in Refs. [12, 10]) and grows until its amplitude saturates. Similarly, the \( \sim 100 \) kHz (red) mode amplitude fluctuates with a peak at \( \tilde{t} = -0.7 \); and the green mode starts growing from \( \tilde{t} = -0.8 \). During the second half: the blue mode, which started to decay near \( \tilde{t} = -0.6 \), saturates near \( \tilde{t} = -0.3 \); the red mode amplitude increases until saturation, and the \( \sim 39 \) kHz (green) mode continues to increase until saturation at the very end of the ELM cycle \( \tilde{t} \approx -0.9 \).

**Mode radial profile and poloidal wavenumber dynamics** — To further characterize the radial profiles of these three modes and their evolution during the first and second half of the ELM cycles, we analyze the correlation between the magnetic fluctuations \( \dot{B}_\theta \) and the density fluctuations \( \delta n_e \), [measured using the BES system at the positions illustrated in Fig. 5(a)]. The change of magnetic flux is related to magnetic fluctuations (\( \dot{B}_\theta \)), which is associated a parallel current \( (j_\parallel) \) via Ampere’s law. If this parallel current has a finite toroidal mode number then a parallel variation of the parallel current is likely leading to \( \nabla \cdot j_\parallel \approx 0 \). This divergence in parallel current couples density, temperature, and vorticity fluctuations. Subsequently, it is conceivable that the magnetic fluctuations can transfer energy to the density fluctuations. Given this justification, we compute the correlation \( \langle \dot{B}_\theta, \delta n_e \rangle \) to provide the radial profile of each of the three dominant modes.

Figures 5 (b) and (c) display the radial profiles of the modes during the first and second half of the ELM cycle, respectively. The three dominant modes’ contributions to \( \langle \dot{B}_\theta, \delta n_e \rangle \) indicate a transition from a dominant contribution of the blue mode during the first half of the ELM cycle (see Fig. 5(b)) towards a more balanced contribution between the three modes during the second half (see Fig. 5(c)). From the first half to the second one, the blue mode shows a loss in correlation \( \langle \dot{B}_\theta, \delta n_e \rangle \) while the green and red modes display an increase of correlation.

Figure 5(d) represents the combined radial profiles due to the three modes. The profile of the last part of the ELM cycle is clearly shifted towards the separatrix. This shift is due to the contribution of the red mode that peaks near the \( q = 6 \) surface (see Fig. 5(c)), in contrast to the blue and green modes which peak near \( q = 5 \). Given that this correlation provides a proxy for the location of the modes, Fig. 5(d) shows an outwards shift of location of the fluctuations. In addition, Figure 5 (b) suggests that strong coupling between density and magnetic field near \( q = 6 \).
surface. We now investigate the underlying mechanism leading to the increased coupling of the red mode during the last phase of the ELM cycle.

Nonlinear three-wave interaction leading to coupling to the red mode requires the following matching condition:

\[(f, k_\theta)_{\text{green}} + (f, k_\theta)_{\text{blue}} \approx (f, k_\theta)_{\text{red}}.\]  

In the above equation, the frequencies and wavenumbers are measured using BES (see caption of Fig. 6 for more details). The left-hand side of Eq. (1) is represented by the black asterisk in Fig. 6(b) which agrees within error bar with \[(f, k_\theta)_{\text{red}}.\] This analysis suggests that the green and blue modes are coupled to provide energy to the red mode.

**Nonlinear Dynamics** — The three-wave interaction can be described using the Ritz model [19] (sec. IV), where the evolution of a mode’s amplitude is given by:

\[
\frac{\partial P_f}{\partial t} \approx 2 \gamma_f P_f + \sum_{f_1, f_2} T_f(f_1, f_2),
\]

where \(f\) is the frequency of the mode, \(\gamma_f\) its linear growth rate, \(P_f\) its power, and \(f_1\) and \(f_2\) are the frequencies of the other modes composing the triad \(f = f_1 + f_2\) leading to the transfer of energy \(T_f(f_1, f_2)\). Such nonlinear coupling can either lead to the merge of two waves of frequencies \(f_1\) and \(f_2\) into a wave of frequency \(f\); or it can result in the decay of the wave of frequency \(f\) into two waves of frequencies \(f_1\) and \(f_2\).

One useful tool, enabling analyses of the nonlinear coupling between modes and the energy transfer \(T_f\), is the bicoherence \(b^2\) applied to the magnetic signal \(\dot{B}_\theta\) (for simplicity we let \(\dot{B}_\theta = S\)). The bicoherence \(b^2\) is defined by equation (see Ref. [19]):

\[
b^2 = \frac{|\langle S_{f_1} S_{f_2} S_{f_1+f_2}^* \rangle|^2}{|\langle S_{f_1} \rangle|^2 |\langle S_{f_2} \rangle|^2 |\langle S_{f_1+f_2} \rangle|^2},
\]

where \(S_f\) is the signal evaluated at frequency \(f\) (where \(f\) can be \(f_1, f_2, f_1 + f_2\) and \(S_{f}^*\) its complex conjugate.

Figure 7 displays the averaged bicoherence of the magnetic signal (over the last 50% of the ELM cycle), which enables the identification of the triad given by Eq. (1) (labeled in Fig. 7(b) using a red circled cross). A bicoherence of \(\approx 65\%\) clearly indicates that there is a significant nonlinear coupling between the three dominant modes. Therefore, the three-wave interaction is the likely mechanism leading to the energy transfer to the red mode. This is consistent with a decrease of the blue mode’s amplitude between the first and second half of the ELM cycle (see Fig. 4(b)). In addition, the green mode’s amplitude continues to rise, during the last phase of the ELM cycle (see Fig 4(b)), which suggests that this mode’s amplitude growth is dominated by the first term of Ritz model. One can then rule out contributions from the transfer component (second term in Ritz model) for the green mode.

### 4. Conclusions

Nonlinear coupling between the dominant ubiquitous pedestal modes have been studied in type I ELMy H-modes on the DIII-D tokamak. We report that besides possibly regulating the pedestal transport [12, 20], these pedestal
modes play a key role during the onset of an ELM event. Specifically, three dominant pedestal modes, exhibiting both magnetic and density fluctuations, have been identified, and their amplitudes, frequencies, and radial profiles have been tracked during the ELM cycle. We observe a radial shift (toward the last-closed flux surface) of the overall radial structure of the three dominant modes as the ELM event approaches. The mechanism leading to this radial shift is explained by the energy transfer to a mode (the red mode) via three-wave interaction. This red mode is localized near the \( q = 6 \) surface (close to the separatrix), while the other two modes are localized near \( q = 5 \). The three-wave interaction is confirmed using bicoherence analysis, where the triad generating the red mode from the blue and green modes is found. Moreover, the radial shift is consistent with Ref. [21] that suggested that there is a growth in a narrow region (e.g. pedestal) of erupting fingers pushing into the metastable region leading to the process called detonation: ELM event. Such a scenario is consistent with the nonlinear theory suggested by Refs. [7, 22] in which it is shown that explosive onset of events can be attributed to the nonlinearity of MHD ballooning modes. This is consistent with our direct observations which suggest that nonlinear coupling of pedestal modes generating a radially outwards mode as a possible mechanism of the ELM onset. It is worth noting that AUG has shown nonlinear coupling of ELM precursors which was speculated to result in an ELM [23]. Finally, since ITER will be operated with low natural ELM frequency [24] (3 - 4 Hz), it is conceivable the edge parameters might be ultimately clamped in which case nonlinear coupling of pedestal modes might play a key role in the onset of ELMs. As such, controlling the mode coupling between pedestal modes could be envisioned as another possible tool for ELM suppression.

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