Stellarator Nonlinearly Saturated Periodicity-Breaking Ideal Magnetohydrodynamic Equilibrium States

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SYMMETRY BREAKING BY MHD

MHD activity can alter the symmetry properties of magnetic plasmas. The 3D equilibrium solver VMEC [1] has been applied to model the breaking of the axisymmetry in RFPs [2] and tokamaks [3]. In stellarator experiments, very long-lasting oscillations characterized by low n and m, which break the periodic symmetry, have been detected in many devices like the TJ-II Heliac [4] and the Large Helical Device (LHD) [5].

TJ-II NBI-plasmas. Long lasting Llow frequency (4 kHz f 15 kHz) oscillations.
The toroidal mode numbers can be resolved according to the relative phase of the modes detected with three AXUV arrays located at sectors separated π/2 toroidally plus another array located at a fraction of this (see scheme in Figure).
The poloidal mode numbers (for m < 7), can be identified by the poloidal chords.

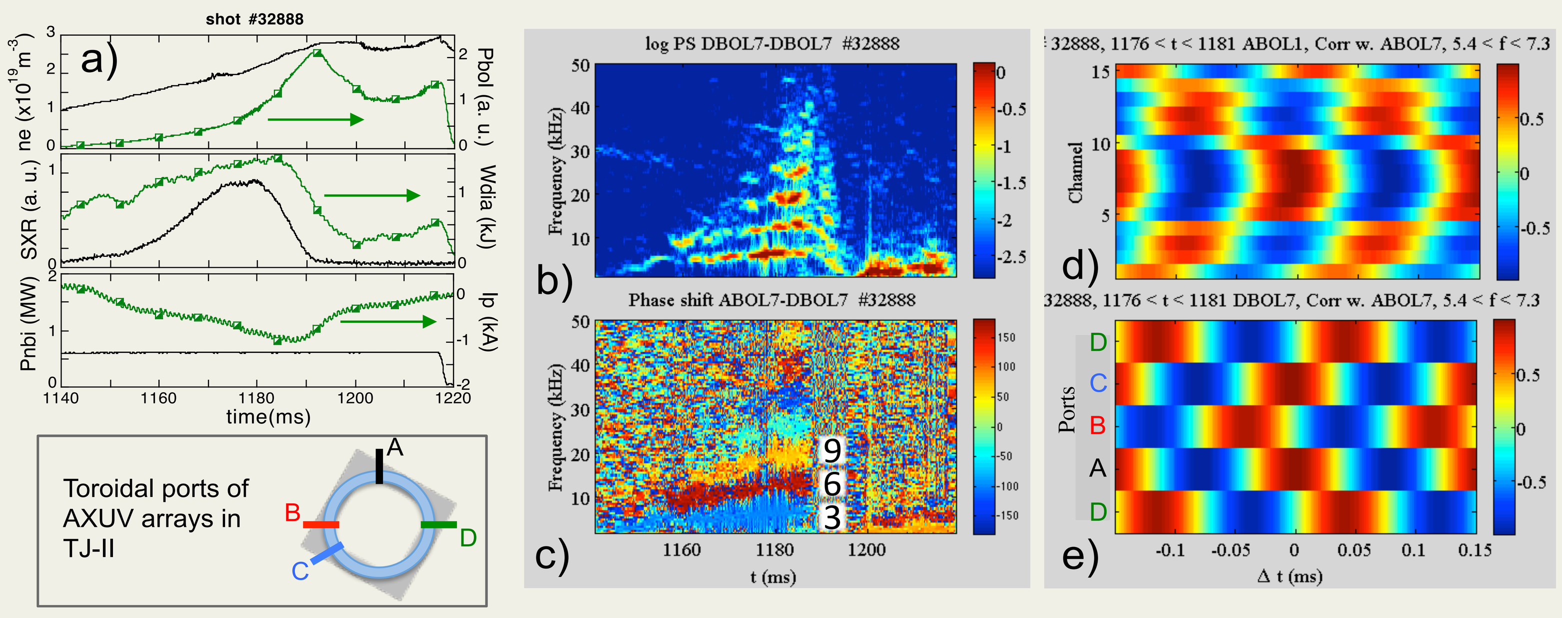


Figure: a) Time traces of TJ-II shot #32888, heated with counter-NBI (Te0 ≈ 200 eV, Ti0 ≈ 100 eV) and top-view scheme of the radiation arrays. b) Spectrogram of a central radiation chord signal (D @ ρ ≈ 0.1). c) Phase of coherence of the different oscillations separated π/2 toroidally (D and A). d) Poloidal correlation of one reference channel (A @ ρ ≈ 0.1) with all poloidal chord signals of bolometer array A for the ≈ 6 kHz mode. e) Toroidal correlation of equivalent chords (@ ρ ≈ 0.1) from each array with the reference channel of A. The labels in c) correspond to the toroidal mode numbers.

In this case, f ≈ 6 kHz and m = 2 and n = 3.
The detected phase shifts of this oscillation correspond to those expected for n=3.
Other oscillations happen for f=12 kHz and n=6, and f=18 kHz and n=9, and m=even.
Modes are rather core localized so they are hardly detected by magnetic coils.
Only in a few cases, mostly (n, m) = (3, 2) and (n, m) = (5, 3), can be detected.
Toroidal mode numbers evidence that the mode structures are non- natural (no multiple of 4),
=> Necessarily break the periodicity imposed by the TJ-II coils set.

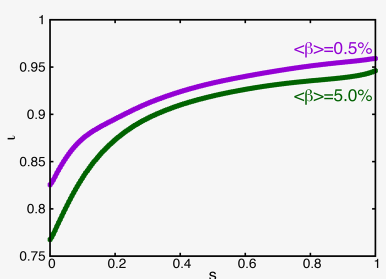
We contend that the deformations observed constitute nonlinear saturated ideal MHD interchange structures with low n driven by the interaction of the pressure gradient with the magnetic field line curvature.
3D free boundary ideal MHD VMEC-equilibrium calculations in a nominally 4-field period Helias reactor including edge distortions that break the periodicity imposed by the coil system.
FINDING: Configurations with broken symmetry are more stable than the symmetric ones.

MHD CALCULATIONS

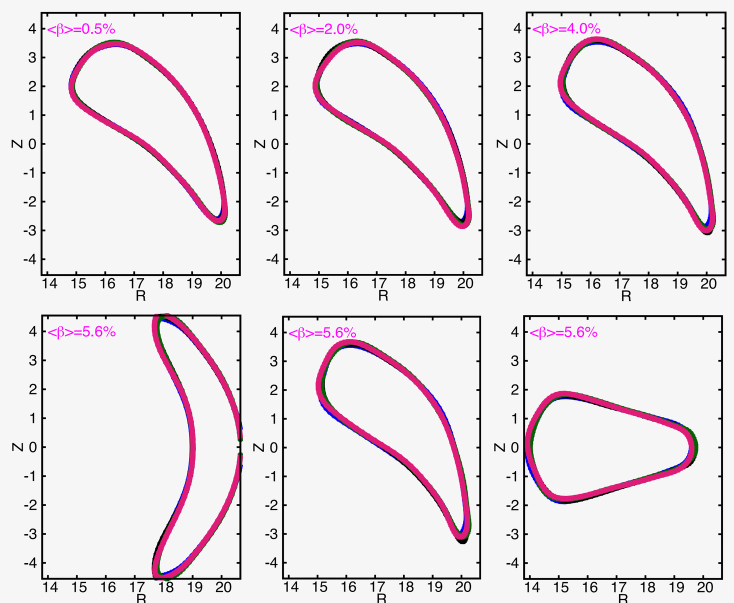
Computation of 3D MHD equilibrium states with the VMEC code imposes nested magnetic flux surfaces [1] without magnetic field lines breaking and reconnecting => ideal MHD.
A 4-field periods Helias reactor configuration has been chosen because the spectrum of modes required is much narrower than that OF TJ-II Heliac. Furthermore, the Helias configuration represents one of the most attractive concepts for stellarator reactor. The plasma energy is given by

$$\mu_0 W = \iiint dx^3 \left[\frac{B^2}{2} + \frac{\mu_0 p(s)}{\Gamma - 1} \right]$$

The equilibrium state is obtained by varying the energy functional with respect to an artificial time variable employing a steepest descent energy minimization scheme. We allow the 4-fold periodicity breaking by including all toroidal mode numbers in the range −20 ≤ n ≤ 20 and poloidal mode numbers 0 ≤ m ≤ 11. The 4-fold periodic Fourier components of R and Z at the LCFS are $R_{0,0}$ = 17.7, $R_{0,4}$ = 0.72, $R_{1,0}$ = 2.15, $R_{1,4}$ = 0.99, $Z_{0,0}$ = 0, $Z_{0,4}$ = 0.33, $R_{1,0}$ = 2.448, $Z_{1,4}$ = 1.33 (all in m).
Edge periodicity-breaking introduced through δ_i, $R_{1,1} = Z_{1,1} = \delta_i$ and $R_{1,-1} = Z_{1,-1} = -\delta_i$.
Null toroidal current J(s) = 0 and pressure profile p(s) = p(0)(1−s)(1−s4).

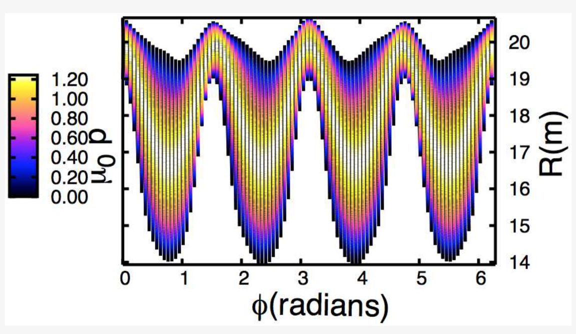


The estimated rotational transform profiles ι(s) for the Helias reactor configuration for ⟨β⟩ = 0.5% and ⟨β⟩ = 5%.
ι = 1 is outside the plasma. the main low order resonances that break the 4-fold periodicity of the coils are n/m = 10/11, 9/10, 7/8, 6/7 and 5/6.



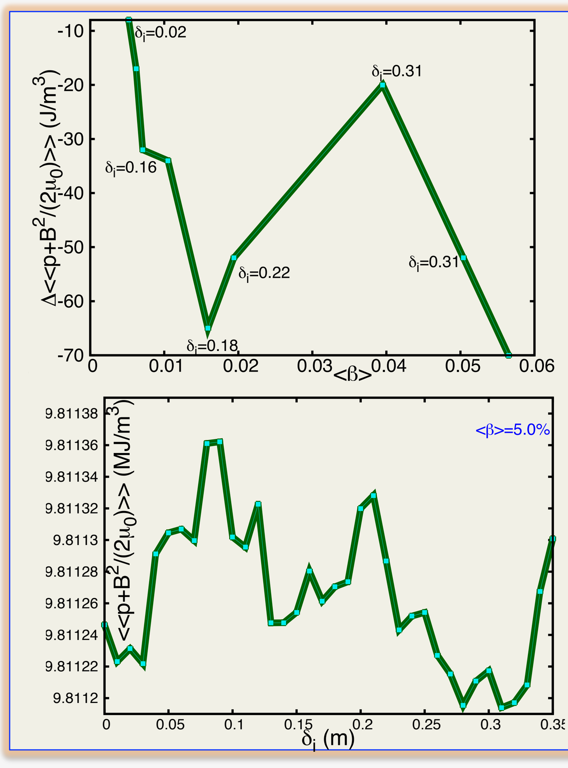
Shapes of the LCFS for the Helias reactor at each of the eggplant cross sections throughout one toroidal transit for ⟨β⟩ = 0.5%, 2% and 4%. There are small but visible deformations.

The LCFS at the bean, eggplant and triangular cross sections over 1 toroidal transit at ⟨β⟩ = 5.6%. A small corrugation of the edge surface appears.



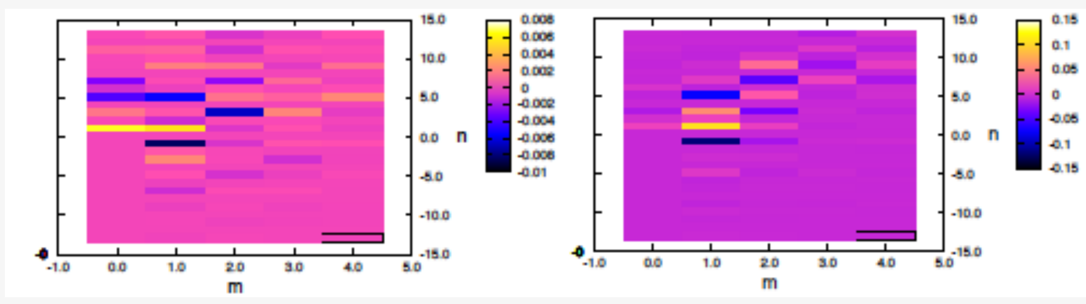
The mid plane pressure distribution for 0 ≤ ϕ ≤ 2π at ⟨β⟩ = 5%. The bulk plasma retains the underlying 4-fold periodicity of the system, while a small n = 1 modulation is visible close to the edge.

MHD STABILITY AND SYMMETRY BREAKING

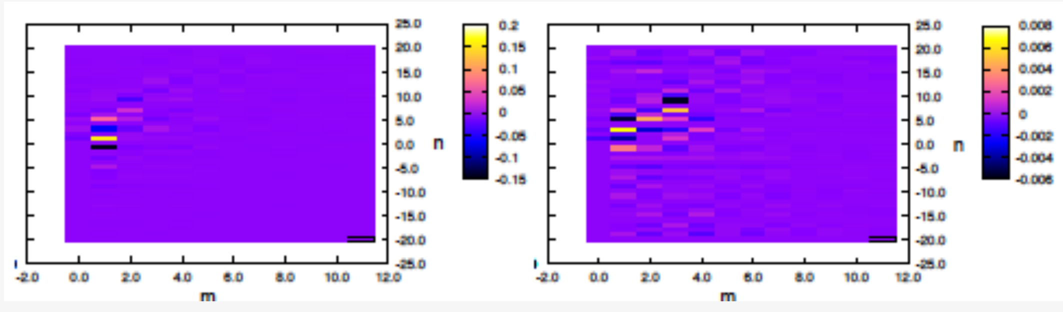


The minimum energy state is obtained in the range ⟨β⟩ = 0.5% – 5.6%
The initial guesses for δ_i that yield the minimum energy states at each ⟨β⟩ are identified in the figure. Δ(p + B2/(2μ0) corresponds to the difference between p+B2/(2μ0) evaluated at finite δ_i and δ_i = 0.
The ordinate values <0 indicating that the volume averaged total plasma energy is lower at finite values of δ_i (in the range of ⟨β⟩ explored).

For ⟨β⟩ = 5%, the most stable values appear for finite values of δ_i ≈ 0.28 and 0.31. Eddy currents in the vacuum chamber produce periodicity-breaking perturbations that can be enhanced by linear MHD instability dynamics (the system investigated is unstable to linear ideal MHD modes when ⟨β⟩ exceeds 0.6%). This justifies increasing δi with ⟨β⟩.



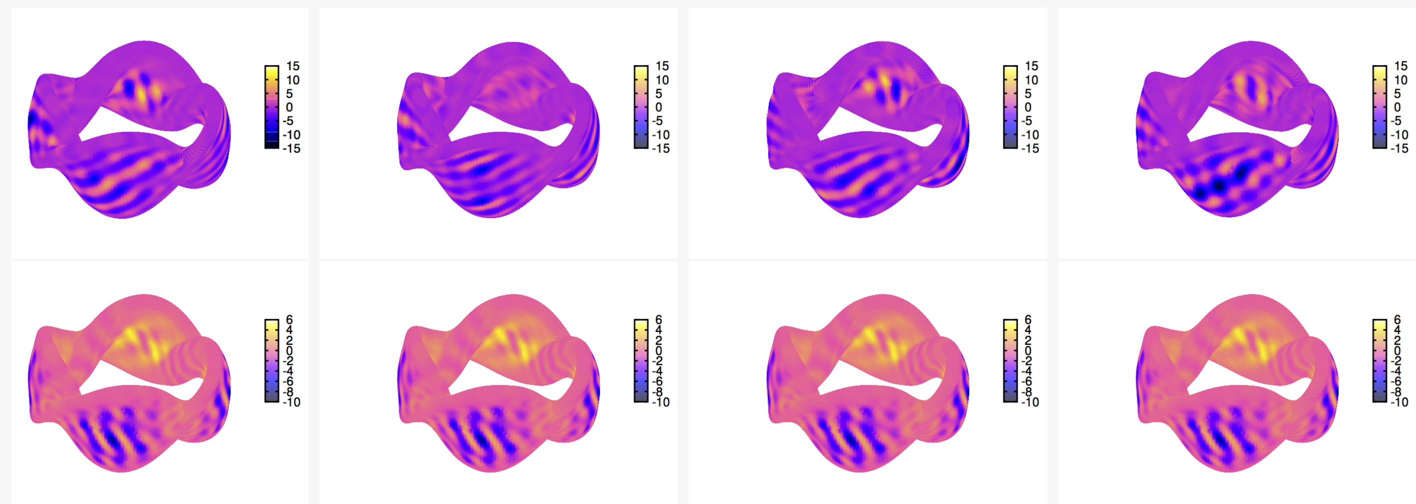
Fourier decomposition of R at the LCFS at ⟨β⟩ = 0.5% (left) and 5.0% (right). The periodicity-breaking terms have been limited to the range 0 ≤ m ≤ 4 and −13 ≤ n ≤ 13 because they are significant only for low m, n.



Left: Fourier decomposition of Z at the LCFS at ⟨β⟩ = 5.0% (right). The periodicity-breaking terms have been limited to the range 0 ≤ m ≤ 11 and −20 ≤ n ≤ 20.
Right: the same for B

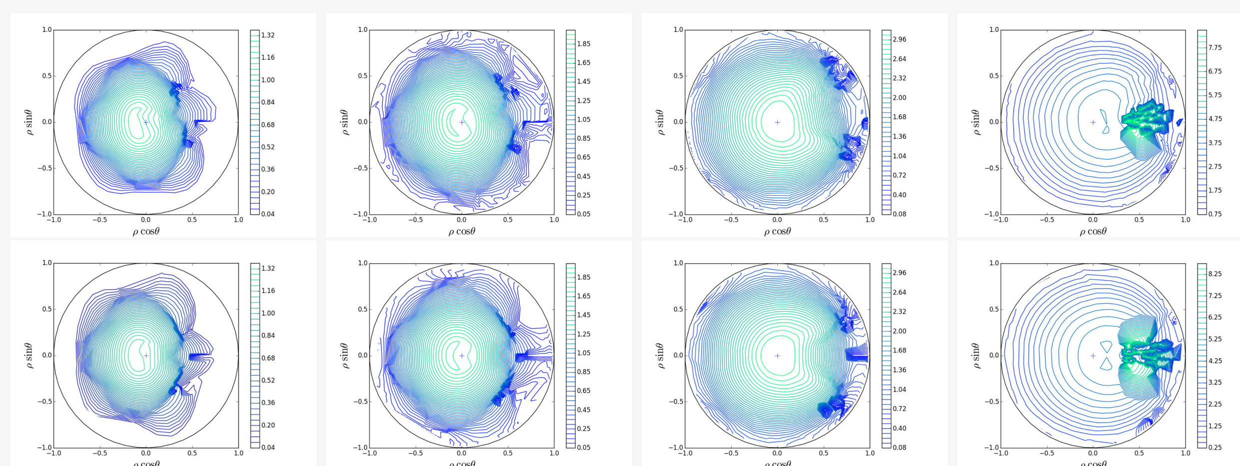
PROPERTIES OF THE NEW CONFIGURATION

In current-free stellarators, MHD instabilities driven by the interaction of the pressure gradient with the magnetic field line curvature. The relevant function is 2√gp′(s)κ · ∇s calculated for ⟨β⟩ = 0.5%
Top row: perturbed configuration (n=1 distortion). Bottom row: periodic configuration.



Structure of 2√gp′(s)κ · ∇s on flux surfaces close to LCFS:
- Strong change from the periodic to the perturbed configuration .
- The most destabilising contribution (most negative) concentrates near the outside edge of one of the bean-shaped cross sections indicating an important n = 1 distortion.
- The structure when the 4-fold periodicity is relaxed is more closely magnetic field aligned.
- The range of values for 2√gp′(s)κ · ∇s is neutral (with respect to positive and negative values) when the periodicity constraint is relaxed indicating marginal stability. However, it tends to be more negative when the 4-fold periodicity is enforced suggesting that this configuration is linearly unstable to ideal MHD, a condition we have confirmed with the TERPSICHORE code [6]. The periodicity-breaking deformations constitute a saturated ideal MHD instability state.

The Helias reactor configuration examined approaches conditions of quasi-isodynamicity [7]. The poloidal closure of the contours of the 2nd adiabatic invariant J in a polar plot for trapped particles imply the good NC confinement of fast and bulk ions.



The relaxation of the periodicity does not modify significantly the contours of the J for different particle pitch angles (⟨β⟩ = 5%). Energetic particle confinement and NC transport should not be seriously impacted when periodicity is broken

CONCLUSIONS

- Long-lived low frequency oscillations with n smaller than the machine periodicity detected in stellarators such as TJ-II.
- We report, for the first time, free boundary VMEC stellarator equilibria with 3D distortions to the LCFS that break the ORIGINAL periodicity.
- The Fourier spectrum of R and Z at the LCFS is dominantly m = 1, n = ±1 at ⟨β⟩ = 5%. The m/n = 1/ ± 1 structures are nonresonant as ι_{max} < 0.95.
- We contend that these geometric deformations correspond to saturated ideal MHD interchanges.
- The 2√gp′(s)κ · ∇s structure confirms a n = 1 modulation around the torus.
- The quasi-isodynamic properties of the Helias configuration are not significantly altered.
- FAVOURABLE conditions for the Helias stellarator as reactor.

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