Coupling of Current and Flow Relaxation in Reversed-Field Pinches Due to Two-Fluid Effects

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Objective
To understand effects that couple current-profile and flow-profile relaxation (momentum transport) during relaxation events in the Madison Symmetric Torus (MST).

Outline
• Introduction
• Modeling
• Linear results
• Nonlinear island evolution
• Dynamo and momentum transport in multi-helicity relaxation
• Multi-helicity relaxation with background flow
• Discussion and conclusions
Introduction: Measurements on MST point to the importance of two-fluid effects during relaxation events.

- In standard MST operation, magnetic relaxation occurs during discrete sawtooth events.

- Laser polarimetry measurements of $\delta j_\phi$ for the (1,6) mode, and fitted $\delta b$ profiles show a correlation that implies significant Hall dynamo. [Ding, et al., PoP 13, 112306].

- MST parameters have ion-sound-gyroradius ($\rho_s = c_s/\Omega_i$) comparable to the resistive skin depth, so two-fluid linear tearing effects are expected [Mirnov, et al., PoP 11, 4468, for example].
Probe measurements from MST’s edge plasma also indicate the significance of two-fluid effects.

- The existence of Hall dynamo and net parallel Lorentz force density from fluctuations are essentially equivalent.

- Kuritsyn, et al., used an array of magnetic coil triplets to measure Maxwell-stress profiles in the edge of MST [PoP 16, 55903].

- The group also found Reynolds stress contributions from flow fluctuations, measured by Mach probes.

- With respect to momentum transport, the effects largely cancel.
Modeling: We apply two-fluid modeling to investigate these macroscopic RFP dynamics.

\[
\frac{\partial n}{\partial t} + \nabla \cdot (n \mathbf{V}) = 0
\]

-- particle continuity

\[
mn \left( \frac{\partial}{\partial t} + \mathbf{V} \cdot \nabla \right) \mathbf{V} = \mathbf{J} \times \mathbf{B} - \nabla \sum_{\alpha} n T_{\alpha} - \nabla \cdot \Pi
\]

-- flow evolution

\[
\frac{3}{2} n \left( \frac{\partial}{\partial t} + \mathbf{V} \cdot \nabla \right) T_{\alpha} = -n T_{\alpha} \nabla \cdot \mathbf{V}_{\alpha} - \nabla \cdot \mathbf{q}_{\alpha} + Q_{\alpha}
\]

-- temperature evolution

\[
\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \left[ \eta \mathbf{J} - \mathbf{V} \times \mathbf{B} + \frac{1}{ne} \mathbf{J} \times \mathbf{B} - \frac{T_e}{ne} \nabla n + \frac{m_e}{ne^2} \frac{\partial}{\partial t} \mathbf{J} \right]
\]

-- Faraday’s / Ohm’s law

\[
\mu_0 \mathbf{J} = \nabla \times \mathbf{B}
\]

-- low-\(\omega\) Ampere’s law

\[
\nabla \cdot \mathbf{B} = 0
\]

-- divergence constraint

- Initial-value computations with the two-fluid system are solved using the NIMROD code [JCP 229, 5803 (2010)].
The closure for stress ($\Pi$) is a combination of Braginskii ion gyroviscosity and isotropic viscous stress.

$$\Pi_{gv} = \frac{m_i p_i}{4eB} \left[ \hat{b} \times \mathbf{W} \cdot (\mathbf{I} + 3\hat{b}\hat{b}) - (\mathbf{I} + 3\hat{b}\hat{b}) \cdot \mathbf{W} \times \hat{b} \right], \quad \left( \mathbf{W} \equiv \nabla \mathbf{V} + (\nabla \mathbf{V})^T - \frac{2}{3} \mathbf{I} \nabla \cdot \mathbf{V} \right)$$

$$\Pi_\perp \sim -\frac{3p_i m_i^2}{10e^2 B^2 \tau_i} \mathbf{W}, \quad \text{treated as } -nm_i \nu_{iso} \mathbf{W}$$

The computations presented here have finite but uniform background pressure, representing conditions in the RFP core without detailed transport modeling.

$$\mathbf{q} = -n\chi_{iso} \nabla T$$
Linear results: ‘Fast tearing’ was expected, a slower intermediate regime was not.

- Analytical asymptotic results for linear tearing in a slab with uniform equilibrium pressure finds $\gamma > \gamma_{MHD}$ for $\rho_s > \delta$. [Mirnov; also Ahedo and Ramos, PPCF 51, 55018.]

- Our linear slab computations [Sovinec and King, JCP 229, 5803 (2010)] reproduce the analytics.

- Cold-ion results for paramagnetic pinch equilibria also show growth rates that are larger than resistive-MHD.

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<th>MST</th>
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<tr>
<td>$\beta$</td>
<td>0.03</td>
<td>0.1</td>
</tr>
<tr>
<td>$kd_i$</td>
<td>0.7</td>
<td>0.01-6.0</td>
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<tr>
<td>$k\rho_s$</td>
<td>0.05</td>
<td>0.003-1.7</td>
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Warm-ion (gyroviscous, $T_i=T_e$) results for cylindrical geometry find $\gamma < \gamma_{\text{MHD}}$ at intermediate $\rho_s$-values.

Growth rates (solid) and frequencies (open) from warm-ion (red) and cold-ion (black) computations with NIMROD. [log-log scale]

- The effect does not occur for warm-ion computations in slab geometry.
Our analysis of the ion gyroviscous stress indicates drift-tearing effects from $\nabla B$ and poloidal curvature.

- Expanding $\hat{b}_0$ and $k$ about the resonance in tearing ordering and analyzing $2\hat{b}_0 \times \kappa_0 \cdot \nabla \cdot \tilde{\Pi}$ and $-\hat{b}_0 \cdot \nabla \times \nabla \cdot \tilde{\Pi}$ contributions to parallel vorticity evolution finds a drift contribution with frequency [King, et al., PoP 18, 42303]:

$$\omega_{\text{gy}} = \frac{k_\perp}{m_in_0} \frac{p_{i0}}{\Omega_{i0}} \left( \frac{3}{2r} \frac{B_\theta^2}{B_0} - \frac{B'_0}{B_0} \right)$$

Poloidal flux (left) and $B$ (right) for tokamaks (top) and RFPs (bottom). Arrows show projections of $\nabla B$ and curvature drifts.
A simplified model with gyroviscous stress and resistive-MHD Ohm’s law reproduces the stabilizing effect.

- The derivation also neglects compressive responses from pressure, and the dispersion relation is just $\gamma^4 \left( \gamma - i \omega_{*gV} \right) = \gamma_{MHD}^5$.

Two-fluid computations and heuristic dispersion relation (drift & resistive-MHD Ohm’s law) agree until KAW reconnection is significant, $\rho_s \sim \delta$.

- Warm ions in pinch profiles lead to drift-tearing, even with $\nabla p_0 = 0$. 
Nonlinear single-helicity: Nonlinear island saturation with cold ions matches MHD.

- The computations have $R/a = 0.51$ to prohibit multi-helicity dynamics.
- Coupling to $m \neq 1$ components is allowed but is not significant in the results.
- We observe that island evolution reverts to MHD dynamics, i.e. vanishing Hall effect, when the island width exceeds the ion skin depth.

Island-width evolution shows a Rutherford stage for cold and warm ions.

Hall dynamo effect in cold-ion evolution vanishes when $w$ exceeds $d_i (0.17a)$. 
With warm ions, saturation occurs at smaller width.

- Forces from gyroviscous stress supplement Rutherford’s 3rd-order Lorentz force to balance the 1st-order drive.

- The important gv forces result from flows that are out of phase with the standard reconnection flow pattern.

- The out-of-phase flows are influenced by Lorentz forces from perturbed currents resulting from electron-ion decoupling (Hall effect).

- The flows are insensitive to S and viscous dissipation.

- Unlike tokamak diamagnetic drift-tearing, transport does not eliminate large-\( w \) FLR [King, PoP 18, 42303].
Nonlinear multi-helicity: Results at realistic $R/a$ are more directly relevant to standard operation.

- Our multi-helicity two-fluid computations have $R/a=3$ in cylindrical geometry, Hall effect, and gyroviscous stress from warm ions.

- Plasma parameters have $S=8 \times 10^4$, $P_m=1$, $\beta=0.1$, and $\rho_s =0.05a$.

- Overall evolution at $\Theta=1.6$ shows familiar fluctuation dynamics and field reversal

- Dynamics remain nearly ‘force-free.’

- Magnetic energy in $m=0$ fluctuations is relatively weak after the first event.

Evolution of fluctuation energies in the two-fluid computation.
MHD computations of the same conditions have larger core fluctuation amplitudes and more nonlinear coupling.

MHD computations started at $t=0$ with $0$-$\beta$ (below left) and at later time with finite-$\beta$ (below right) develop larger $m=0$ fluctuations than the two-fluid model (top).

Smaller fluctuation levels in the two-fluid computation leads to less reversal than in the MHD computations for the same conditions.
Two-fluid multi-helicity computations with \((\rho_s/a)\)-values relevant to MST exhibit Hall dynamo during relaxation.

Our computations at ion sound gyroradius \((c_s/\Omega_i)\) \(\rho_s=0.05a\) show significant Hall and MHD dynamo effects that are comparable in magnitude during a relaxation event.

As in single-fluid computation, the combined dynamo emfs act to reduce parallel current in the core and drive it near the edge.

Evaluating for MST, \(5\times10^{-5}v_A B_0\) is 40 V/m, so the predicted magnitude is comparable to the Hall emf measured by Ding et al. [PRL 93, 2004].
There is also significant momentum transport during two-fluid relaxation with m=0 mode activity.

- Simulations show transport of parallel momentum driven by fluctuation-induced forces from Maxwell, Reynolds, and gyroviscous stresses.
- The fluctuation-induced Maxwell stress is linked to the Hall dynamo.
- Similar to measurements by Kuritsyn et al. [Phys. Plasmas 16, 2009] in the edge of MST, the Maxwell and Reynolds stresses tend to cancel.
- The warm-ion computations also show significant gyroviscous forces.
Similar to MST, the large relaxation event increases parallel flow in the core and reduces it near the edge.

- The net change in flow profile is only significant with two-fluid effects.
- These simulations do not include other transport effects that maintain flow profiles.
- The relevance of analytical two-fluid relaxation theory [papers by Steinhauer, Hegna, for example] is intriguing but needs further study.

Profiles of parallel (black) and perpendicular (blue) flow generated by the first simulated relaxation event.

Temporal evolution of parallel flow at three radii in MST. [Kuritsyn, PoP 16, 55903]
Multi-helicity with background flow: MST develops a parallel-flow profile between events.

- We use a background parallel-flow profile that matches the experimental measurements between events at the probe locations shown above.
- Computing with $S=5000$ facilitates scanning different configurations.

This background parallel-flow profile is used in all of the following results.
Background flow breaks the symmetry of relaxation relative to positive and negative current.

- Results with negative current, $\mathbf{J}_0 \cdot \mathbf{B}_0 < 0$, exhibit approximately twice as much field-reversal during the first relaxation event.
- Somewhat more nonlinear coupling occurs, as evident from the larger spectral width, $N_s = \left(\sum_n W_{1,n}\right)^2 / \sum_n W_{1,n}^2$, where $W_{1,n}$ is the magnetic fluctuation energy in the $(m=1,n)$ component.

Evolution of reversal parameter ($F$) and spectral width ($N_s$) for positive current.

Evolution of $F$ and $N_s$ with negative current.
Background flow alters Hall and MHD dynamo effects.

- At reversal the sum is similar for the two current orientations, but the contributions oppose each other with positive current.

Dynamo contributions at initial reversal with (a) $J_0 \cdot B_0 > 0$ and (b) $J_0 \cdot B_0 < 0$ and at greatest reversal with (c) $J_0 \cdot B_0 > 0$ and (d) $J_0 \cdot B_0 < 0$. 
Background flow also affects quasilinear dynamo.

- This set of quasilinear profiles is computed from linear tearing-mode ($\Delta' = 48$) eigenfunctions for $S=80,000$, $Pm=0.1$.

Dynamo effects from tearing-mode results with $J_0 \cdot B_0 > 0$ (a) Hall with warm ions, (b) MHD with warm ions, (c) Hall with cold ions, (d) MHD with cold ions.
Flow does not affect dynamo from ideal-unstable modes.

Dynamo effects from ideal-mode results with $J_0 \cdot B_0 > 0$ (a) Hall with warm ions, (b) MHD with warm ions, (c) Hall with cold ions, (d) MHD with cold ions.

- Flow can change the sign of Hall & MHD dynamo from warm-ion tearing.
At reversal in the 3D cases, Maxwell-stress forces are stronger than Reynolds-stress forces for both $J_0$ orientations.

Fluctuation-induced parallel force-densities at initial reversal with (a) $J_0 \cdot B_0 > 0$ and (b) $J_0 \cdot B_0 < 0$ and at greatest reversal with (c) $J_0 \cdot B_0 > 0$ and (d) $J_0 \cdot B_0 < 0$. 
Although Reynolds-stress contributions increase by the time of peak reversal, sequencing is important for the parallel flow profile evolution.

- For both orientations, parallel flow evolves in the direction of $\mathbf{J}_0$, consistent with Hall dynamo relaxation of the current profile.

- In MST, $\mathbf{J}_0 \cdot \mathbf{B}_0 < 0$, and this two-fluid computation evolves flow in the opposite direction of the observations in Ref. [Kuritsyn, PoP 16, 55903.]
Conclusions

• First-order FLR modeling of warm ions produces $\nabla B$ and poloidal-curvature drift-tearing effects in pinch profiles.
  • At large-$\rho_i$, tearing decouples from ions, so ion kinetics may not be critical.

• Nonlinear island evolution in pinch profiles is influenced by ion FLR.
  • With cold ions, two-fluid saturation is equivalent to MHD.
  • When modeling experimentally relevant parameters with first-order FLR, warm-ion effects reduce island widths.

• Two-fluid multi-helicity simulations show non-MHD effects similar to those observed in MST: Hall dynamo and competition between Maxwell and Reynolds stresses.

• Background flows affect the profile evolution.
  • Both the Hall and MHD dynamo profiles differ from results without background flow.
  • Background flow breaks the symmetry between conditions with parallel and anti-parallel current.
Future Work

• Investigate profiles with nonuniform background pressure.
  • Incorporates pressure-driven effects.
  • Changes the $\rho_s$-profile to decrease in $r$.
• Include parallel viscosity in the model.
  • Fluctuations in $\hat{b}$ and $\nabla_\parallel V$ can affect momentum transport.
• Investigate possible effects from the toroidal-field circuit.
• Apply the same modeling in toroidal geometry.