Temperature Anisotropy in Magnetized Fusion

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Abstract. In the present work, the electronic distribution function for magnetized plasma, taking into account the electron-ion collisions is explicitly calculated.The basic equation in this investigation is the Fokker-Planck equation where some justified approximations for fusion and astrophysical magnetized plasmas are used. By computing the second moment of the distribution function, we have expressed the electrons temperatures in the parallel direction and in the perpendicular plane to the magnetic field. It has been shown that this temperature is anisotropic and this anisotropy is due to competition between magnetic field effect and the collisions effect.

1. Introduction

A magnetized plasma is one in which an ambient magnetic fieldis strong enough to significantly alter the particle trajectories. This kind of plasma isa good environment for various physical phenomenon's which are intensively studied in literature, Alfvèn wave [1], Cyclotron instabilities [2], magnetic field reconnection [3]. The magnetized plasma presents an anisotropy in temperature which can be interpreted in the microscopic way by an anisotropic distribution function.Usually, in the literature, this distribution function is supposed a to be bi-maxwellienne: $f_{BM} = \left(\frac{m_e}{2\pi}\right)^{3/2} \frac{n_e}{T_\perp {T_\parallel}^{1/2}} \exp\left(-\frac{m_e v_\parallel^2}{2T_\parallel}\right) \exp\left(-\frac{m_e v_\perp^2}{2T_\perp}\right)$.

In this paper, we aim to calculate analytically the electronstemperature anisotropy for magnetized plasma, in the frame of the kinetic theory. This investigation can found applications in several research axis, such in magnetic fusion experiments.

In microscopically level of magnetized plasma, there are charged particles of deferent species in thermal motion with different velocities, each particle has a fast gyration motion around the magnetic field with a perpendicular velocity, $v_\perp$, and a parallel motion non affected by the magnetic field. The time dependent electron velocity can be written as: $\vec{V}(t) = \vec{V}_\parallel + \vec{V}_\perp(\mathrm{t})\exp(i\omega_{ce}t)$, where $\omega_{ce} = \frac{eB}{m_e}$ is the electron cyclotron frequency which is proportional to the magnetic field and it is the same for all electrons in the plasma.

In order to compute the electronic distribution function, we consider the one particle kinetic theory with a 6D phases space: $(\vec{r}, \vec{v})$. Then, the Fokker Planck equation is the suitable equation for describe these kind of plasmas.

In the present investigation, we have considered two time scales for the evolution of the electrons distribution function: a fast time scale relative to the cyclotron motion of electrons around the magnetic field lines, $\sim\frac{1}{\omega_{ce}}$ (typically $\omega_{ce}\sim 10^{11}s^{-1}$ for magnetic thermonuclear fusion experiments) and an hydrodynamic slow time scale.

This paper is organized as follow: in section 2., we present the basic equationused for this investigation. In section 3.,we compute the distribution function under some justified approximations. In section 4.we compute the high frequency distribution function. In section 5., we compute the static distribution function. In section 6.,we compute the electrons temperature in the parallel and the perpendicular direction of the magnetic field and we represent the anisotropy in temperature. Finally, in sec 7., a conclusion is given for obtained results.

2. Basic equation

The basic equation in this investigation is the Fokker-Planck (F-P) equation. The F-P equation can be presented for homogeneous plasma, in the presence of the Lorenz force due to a statistic magnetic field, $\vec{F}_L = -e\vec{V}(t) \times \vec{B}$, taking into account the e-i collisions, following the Braginskii notation [4]as follows:

$$\frac{\partial f}{\partial t} + \frac{\vec{F}_L}{m_e}.\frac{\partial f}{\partial \vec{v}} = C_{ei}(f). \quad (1)$$

We suppose that the magnetic field is oriented in the x direction, $\vec{B} = B\hat{x}$, and the electronsoscillate in the (y, z) plane, where: $\overrightarrow{v_\perp}(t) = \frac{v_\perp}{\sqrt{2}}(\hat{z} - i\hat{y})\exp(i\omega_{ce}t)$. In this geometry, the Lorentz force is presented as:

$$\vec{F}_L = \frac{-m_e\omega_{ce}v_\perp}{\sqrt{2}}(\hat{y} + i\hat{z})exp\,(i\omega_{ce}t). \quad (2)$$

This force is alike to the force due to the presence of acircularly polarized laser wave in the plasma [5,6,7,8]. Taking into account to the equation (2), the F-P equation (1) is written as:

$$\frac{\partial f}{\partial t} - \frac{\omega_{ce}v_\perp}{\sqrt{2}}(\frac{\partial f}{\partial v_y} + i\frac{\partial f}{\partial v_z})exp\,(i\omega_{ce}t) = C_{ei}(f), \quad (3)$$

where$f = f(\vec{v}, \vec{r}, t)$ is the electrons distribution function. $C_{ei}(f)$ represents thee-i operator and$\omega_{ce} = \frac{eB}{m_e}$ is the electron cyclotron frequency. We point out that equation (3) is similar to that characterizes a homogenous plasma in interaction with a circularly polarized laser wave [6,7]. Then we expect anisotropy in temperature due to the presence of magnetic field.

3. Distribution function

The motion of individual particle in plasma in presence of a static magnetic field,can be decomposed in a parallel motion which is not affected by the magnetic field and a perpendicular gyration motion. Typically the gyration period time is very small compared to hydrodynamic evolution time of plasma. Then it is judicious to separate the time scales in the F-P equation,(3),by supposing that the distribution function is the sum of oscillating distribution function and a static one, so:

$$f(\vec{v}, t) = f^s(\vec{v}, t) + Real\{f^h(\vec{v}, t)\}, \quad (4)$$

where $f^h(\vec{v}, t) = f^h(\vec{v})\exp\,(i\omega_{ce}t)$. (5)

The separation of time scales in F-P equation (3), using equation (4), allows to a system of two coupled equations: a high frequency equation representing the spatio-temporal evolution of f^h and a slow time variation F-P equation representing the spatio-temporal evolution of f^s , $\frac{\partial f^h}{\partial t} - \frac{\omega_{ce} v_\perp}{\sqrt{2}} (\frac{\partial f^s}{\partial v_y} + i \frac{\partial f^s}{\partial v_z}) exp\,(i\omega_{ce} t) = C_{ei}(f^h)$, (6)

$$\frac{\partial f^s}{\partial t} - \frac{\omega_{ce} v_\perp}{\sqrt{2}} \langle Real(exp\,(i\omega_{ce} t) Real(\frac{\partial f^h}{\partial v_y} + i \frac{\partial f^h}{\partial v_z})\rangle = C_{ei}(f^s). \quad (7)$$

The symbol$\langle \quad \rangle$ means the average value on the cyclotron period time. Note that the average of quantities proportional toexp $(i\omega_{ce} t)$ vanishes.

4. High frequency distribution function

Using equation (5), f^h can be calculated from equation (6), where $\frac{\partial f^h}{\partial t} = i\omega f^h$, as function of f^s, so:

$$i\omega_{ce} f^h - C_{ei}(f^h) = \frac{\omega_{ce} v_\perp}{\sqrt{2}} (\frac{\partial f^s}{\partial v_y} + i \frac{\partial f^s}{\partial v_z}) exp\,(i\omega_{ce} t). \quad (8)$$

The collisions operator,$C_{ei}(f)$, is expressed under Landau form [9,10] in the limit of immobile ions by:

$$C_{ei}(f^s) = \frac{A}{v^3} \frac{\partial}{\partial v_i} \left(v_i v_j - v^2 \delta_{ij}\right) \frac{\partial f^s}{\partial v_j} \quad . \quad (9)$$

$A = \frac{v_t^4}{2\lambda_{ei}}$, $\lambda_{ei} = \frac{4\pi\varepsilon_0 T_e{}^2}{n_e e^4 Z ln\Lambda}$ is the mean free path, $\nu_{ei} = \frac{1}{2} \frac{v_t}{\lambda_{ei}}$ and $v_t = \sqrt{T_e/m_e}$ is the thermal velocity. Note that in equation (9) the Einstein's notation is used.

The e-i collisions operator (9) has spherical harmonics [11,12] like proper functions. Then it is judicious to use the spherical system $(v, \mu = \frac{v_x}{v}, \varphi = arctg \frac{v_y}{v_z})$. The right hand side of equation (8) is written as:

$$\frac{\omega_{ce}}{\sqrt{2}} exp(i\omega_{ce} t + i\phi) \times \left((1-\mu^2)^{3/2} \left(v \frac{\partial f^s}{\partial v} + \mu \frac{\partial f^s}{\partial \mu} \right) \right). \quad (10)$$

This shows that $\boldsymbol{f^h}$ is proportional to $\boldsymbol{exp(i\phi)}$ and $\boldsymbol{f^s}$ is independent on $\boldsymbol{\phi}$. Then it is practical to develop $\boldsymbol{f^s(\vec{v}) = f^s(\mu, v)}$, on the Legender polynomials, $\boldsymbol{P_l(\mu)}$: $\boldsymbol{f^s(\mu, v) = \sum P_l(\mu) f_l^s(v)}$, and to develop $\boldsymbol{f^h(\vec{v}, t) = f^h(\mu, v) exp\ i(\omega_{ce} t + \varphi)}$, on the on the spherical harmonics, $\boldsymbol{Y_l^1(\mu, \varphi)}$, of order $(\boldsymbol{l, m = 1})$: $\boldsymbol{f^h(\mu, v, \varphi) = \sum Y_l^1(\mu, \varphi) f_l^h(v) = exp\,(i\varphi) \sum P_l^1(\mu) f_l^h(v)}$, where $\boldsymbol{P_l^1(\mu)}$ is the associated Legender polynomial of order $(\boldsymbol{l, m = 1})$.

Using these developments the high frequency equation, (8), can be presented as:

$$\left(i\omega_{ce} + l(l+1)\frac{\alpha}{v^3}\right) \sum P_l^1 f_l^h(v) = \frac{\omega_{ce}}{\sqrt{2}} \left\{ (1-\mu^2)^{3/2} \left(v \sum P_l \frac{\partial f_l^s}{\partial v} + \mu \sum \frac{\partial P_l}{\partial \mu} f_l^s \right) \right\} . \quad (11)$$

After some algebra using recurrence relations between Legender polynomials and associated Legender polynomials [11,12], we have demonstrated that:

$$\sum_l v \frac{\partial f_l^s}{\partial v} . (1-\mu^2)^{3/2} P_l = P_l^1 \sum - \left\{ \frac{(2l+1)(2l+3)(2l+5) - (l+1)^2(2l+5) - (l+2)^2(2l+1)}{(2l+1)(2l+3)^2(2l+5)} + \frac{l(l+1)}{(2l+1)(2l-1)(2l+3)} \right\} v \frac{\partial f_{l+1}^s}{\partial v} + \left\{ \frac{(2l-3)(2l-1)(2l+1) - (l-1)^2(2l+1) - (l)^2(2l-3)}{(2l-3)(2l-1)^2(2l+1)} + \frac{l(l+1)}{(2l-1)(2l+1)(2l+3)} \right\} v \frac{\partial f_{l-1}^s}{\partial v} + \frac{(l+2)(l+3)}{(2l+3)(2l+5)(2l+7)} v \frac{\partial f_{l+3}^s}{\partial v} - \frac{(l-2)(l-1)}{(2l-5)(2l-3)(2l-1)} v \frac{\partial f_{l-3}^s}{\partial v} \quad . \quad (12)$$

and

$$(1-\mu^2)^{3/2}\,\mu\frac{\partial P_l}{\partial \mu}f_l^s = P_l^1(\mu)\sum_l\{-\frac{(l-3)(l-2)(l-1)}{(2l-5)(2l-3)(2l-1)}f_{l-3}^s + \frac{l(l-1)^2(2l+1)(2l+3)-(2l-3)(l-1)l^2(2l+3)+(l-1)l(l+1)(2l-1)(2l-3)}{(2l-3)(2l-1)^2(2l+1)(2l+3)}f_{l-1}^s + \frac{l(l+1)(l+2)\quad(2l+3)(2l+5)-(l+2)(l+1)^2\,(2l+5)(2l-1)+(2l+1)(l+1)(l+2)^2(2l-1)}{(2l-1)(2l+1)(2l+3)^2(2l+5)}f_{l+1}^s - \frac{(l+2)(l+3)(l+4)}{(2l+3)(2l+5)(2l+7)}f_{l+3}^s\}. \quad (13)$$

By using the above equations, (12) and (13), the equation (11) is presented as follows:

$$\sum_l\left(i\omega_{ce}+\frac{\alpha}{v^3}l(l+1)\right)P_l^1(\mu)f_l^h(v) = -\frac{\omega_{ce}}{\sqrt{2}}\sum P_l^1(\mu)\left(-\left\{\frac{(2l+1)(2l+3)(2l+5)-(l+1)^2(2l+5)-(l+2)^2(2l+1)}{(2l+1)(2l+3)^2(2l+5)}+\frac{l(l+1)}{(2l+1)(2l-1)(2l+3)}\right\}v\frac{\partial f_{l+1}^s}{\partial v} + \left\{\frac{(2l-3)(2l-1)(2l+1)-(l-1)^2(2l+1)-(l)^2(2l-3)}{(2l-3)(2l-1)^2(2l+1)}+\frac{l(l+1)}{(2l-1)(2l+1)(2l+3)}\right\}v\frac{\partial f_{l-1}^s}{\partial v} + \frac{(l+2)(l+3)}{(2l+3)(2l+5)(2l+7)}v\frac{\partial f_{l+3}^s}{\partial v} - \frac{(l-2)(l-1)}{(2l-5)(2l-3)(2l-1)}v\frac{\partial f_{l-3}^s}{\partial v} + \frac{(l+1)(l+2)\quad(2l+3)(2l+5)-(l+2)(l+1)^2\,(2l+5)(2l-1)+(2l+1)(l+1)(l+2)^2(2l-1)}{(2l-1)(2l+1)(2l+3)^2(2l+5)}f_{l+1}^s + \frac{l(l-1)^2(2l+1)(2l+3)-(2l-3)(l-1)l^2(2l+3)+(l-1)l(l+1)(2l-1)(2l-3)}{(2l-3)(2l-1)^2(2l+1)(2l+3)}f_{l-1}^s - \frac{(l+2)(l+3)(l+4)}{(2l+3)(2l+5)(2l+7)}f_{l+3}^s - \frac{(l-3)(l-2)(l-1)}{(2l-5)(2l-3)(2l-1)}f_{l-3}^s\right). \quad (14)$$

The projection of this equation on the associated Legender polynomial, $P_l^1(\mu)$, allows to compute the f_l^h as function of $f_{l-3}^s, f_{l-1}^s,\ f_l^s, f_{l+1}^s$ *and* f_{l+3}^s, so:

$$f_l^h(v) = \frac{i}{\sqrt{2}}\left(\left\{\frac{(2l+1)(2l+3)(2l+5)(2l-1)-(l+1)^2(2l+5)(2l-1)-(l+2)^2(2l+1)(2l-1)+l(l+1)(2l+3)(2l+5)}{(2l+1)(2l+3)^2(2l+5)(2l-1)}\right\}v\frac{\partial f_{l+1}^s}{\partial v} - \left\{\frac{(2l-3)(2l-1)(2l+1)(2l+3)-(l-1)^2(2l+1)(2l+3)-(l)^2(2l-3)(2l+3)+l(l+1)(2l-3)(2l-1)}{(2l-3)(2l-1)^2(2l+1)(2l+3)}\right\}v\frac{\partial f_{l-1}^s}{\partial v} - \frac{(l+2)(l+3)}{(2l+3)(2l+5)(2l+7)}v\frac{\partial f_{l+3}^s}{\partial v} + \frac{(l-2)(l-1)}{(2l-5)(2l-3)(2l-1)}v\frac{\partial f_{l-3}^s}{\partial v} - \frac{l(l+1)(l+2)\quad(2l+3)(2l+5)-(l+2)(l+1)^2\,(2l+5)(2l-1)+(2l+1)(l+1)(l+2)^2(2l-1)}{(2l-1)(2l+1)(2l+3)^2(2l+5)}f_{l+1}^s - \frac{l(l-1)^2(2l+1)(2l+3)-(2l-3)(l-1)l^2(2l+3)+(l-1)l(l+1)(2l-1)(2l-3)}{(2l-3)(2l-1)^2(2l+1)(2l+3)}f_{l-1}^s + \frac{(l+2)(l+3)(l+4)}{(2l+3)(2l+5)(2l+7)}f_{l+3}^s + \frac{(l-3)(l-2)(l-1)}{(2l-5)(2l-3)(2l-1)}f_{l-3}^s\right). \quad (15)$$

This equation, the high frequency approximation, $\omega_{ce} > \nu_{ei}$ is used.

The first three components of $f^h(v)$ are presented as follows:

$$f_1^h(v) = \frac{i}{\sqrt{2}}\left(\frac{120}{525}v\frac{\partial f_2^s}{\partial v}+\frac{12}{15}v\frac{\partial f_0^s}{\partial v}-\frac{12}{315}v\frac{\partial f_4^s}{\partial v}-\frac{180}{525}f_2^s+\frac{60}{315}f_4^s\right), \quad (16)$$

$$f_2^h(v) = \frac{i}{\sqrt{2}}\left(\frac{840}{6615}v\frac{\partial f_3^s}{\partial v}+\frac{60}{315}v\frac{\partial f_1^s}{\partial v}-\frac{20}{693}v\frac{\partial f_5^s}{\partial v}-\frac{1260}{6615}f_3^s-\frac{60}{315}f_1^s+\frac{120}{693}f_5^s\right), \quad (17)$$

$$f_3^h(v) = \frac{i}{\sqrt{2}}\left(\frac{2898}{31185}v\frac{\partial f_4^s}{\partial v}+\frac{630}{4725}v\frac{\partial f_2^s}{\partial v}-\frac{30}{1287}v\frac{\partial f_6^s}{\partial v}+\frac{2}{15}v\frac{\partial f_0^s}{\partial v}-\frac{5040}{31185}f_4^s-\frac{630}{4725}f_2^s+\frac{210}{1287}f_6^s\right). \quad (18)$$

5. Static distribution function

The second term in the left hand side of the static distribution function equation, (7), can be presented using the spherical coordinates as:

$$-\frac{\omega_{ce}v_{\perp}}{\sqrt{2}}\langle Real(exp\,(i\omega_{ce}t)Real(\frac{\partial f^{h}(v,\mu,\varphi,t)}{\partial v_y}+i\frac{\partial f^{h}(v,\mu,\varphi,t)}{\partial v_z}))\rangle =$$

$$-\frac{\omega_{ce}}{2\sqrt{2}}(1-\mu^2)^{3/2}\left\{v\frac{\partial f^{h}(v,\mu)}{\partial v}-\ \mu\frac{\partial f^{h}(v,\mu)}{\partial \mu}+\frac{f^{h}(v,\mu)}{(1-\mu^2)}\right\}. \qquad (19)$$

The static distribution function is then presented in the spherical coordinates as:

$$\frac{\omega_{ce}}{2\sqrt{2}}(1-\mu^2)^{3/2}\left\{v\frac{\partial f^{h}(v,\mu)}{\partial v}-\ \mu\frac{\partial f^{h}(v,\mu)}{\partial \mu}+\frac{f^{h}(v,\mu)}{(1-\mu^2)}\right\}=\frac{\alpha}{v^3}\left(\frac{\partial}{\partial\mu}(1-\mu^2)\frac{\partial f^{s}(v,\mu)}{\partial\mu}\right)\ . \qquad (20)$$

We develop, as in the section 4, $f^{s}(v,\mu)$on the $P_l(\mu)$ and $f^{h}(v,\mu)$ on the $P_l^1(\mu)$, so:

$$\frac{\omega_{ce}}{2\sqrt{2}}\left\{\sum v\frac{\partial f_l^h}{\partial v}(1-\mu^2)^{3/2}P_l^1(\mu)-\sum(1-\mu^2)^{3/2}\mu\frac{\partial P_l^1}{\partial\mu}\frac{f_l^h}{v}+\sum(1-\mu^2)^{3/2}\frac{P_l^1 f_l^h}{(1-\mu^2)}\right\}$$

$$=\frac{\alpha}{v^3}\sum l(l+1)P_l f_l^s\ . \qquad (21)$$

After some algebra using recurrence relations betweenLegender polynomials, $P_l(\mu)$, and associated Legender polynomials, $P_l^1(\mu)$, Equation (17) is presented as follows:

$$\frac{\omega_{ce}}{2\sqrt{2}}\left(\sum_l P_l\left\{-\frac{(l-3)(l-2)(l-1)l}{(2l-5)(2l-3)(2l-1)}v\frac{\partial f_{l-3}^h}{\partial v}+\right.\right.$$

$$\left(\frac{(2l-3)(2l-1)(2l+1)(2l+3)(l-1)l-(l-2)(l-1)l^2(2l+1)(2l+3)+l(l+1)(2l-3)(2l+3)(l-1)^2-(2l-3)(2l-1)(l-1)l(l+1)(l+2)}{(2l-3)(2l-1)^2(2l+1)(2l+3)}\right)v\frac{\partial f_{l-1}^h}{\partial v}$$

$$\left(\frac{(l+1)(l+2)(2l+1)(2l+3)(2l+5)(2l-1)-(2l-1)l(l+1)(l+2)^2(2l+5)+(2l-1)(l+1)^2(l+2)(l+3)(2l+1)-(2l+3)(2l+5)(l-1)l(l+1)(l+2)}{(2l+3)^2(2l-1)\quad(2l+1)(2l+5)}\right)$$

$$\frac{(l+1)(l+2)(l+3)(l+4)}{(2l+3)(2l+5)(2l+7)}v\frac{\partial f_{l+3}^h}{\partial v}-\frac{(l+4)^2(l+2)(l+3)(l+1)}{(2l+3)(2l+5)(2l+7)}f_{l+3}^h-$$

$$\left(\frac{(l)^3(l-1)(l-2)(2l+3)(2l+1)-(l)^2(l-1)^2(l+1)(2l+3)(2l-3)-(2l-3)(2l-1)(l-1)^2(l)(l+1)^2-(2l-3)(2l+1)(2l+3)l(l-1)(2l-1)}{(2l-3)(2l-1)^2(2l+1)(2l+3)}\right)$$

$$\frac{(l-3)^2(l)(l-2)(l-1)}{(2l-1)(2l-3)(2l-5)}f_{l-3}^h-$$

$$\left(\frac{(2l+3)(2l+5)(l)^2(l+1)(l+2)^2+\ (l+1)^2(l+2)^2l(2l-1)(2l+5)\ -(l+1)^3(l+3)(l+2)(2l-1)(2l+1)-(2l-1)(2l+1)(2l+5)(2l+3)(l+1)(l+2}{(2l-1)(2l+1)(2l+5)\ (2l+3)^2}\right.$$

$$\frac{\alpha}{v^3}\sum l(l+1)\,P_l\ \ f_l^s(v)\ . \qquad (22)$$

This equation coupled with the $f_l^h(v)$ formula, (eqs. 15-18), allows to determinate the different components, $f_l^s(v)$, of the static distribution function, so:

For the zero order ($l=0$):

$$\frac{\omega_{ce}}{2\sqrt{2}}\left(\frac{36}{45}v\frac{\partial f_1^h}{\partial v}-\frac{24}{105}v\frac{\partial f_3^h}{\partial v}-\frac{96}{105}f_3^h+\frac{36}{45}f_1^h\right)=0. \qquad (23)$$

For the first order ($l=1$):

$$f_1^s(v) = \frac{1}{4\sqrt{2}}\frac{\omega_{ce}}{\vartheta_{ei}(v)}\left(\frac{648}{525}v\frac{\partial f_2^h}{\partial v} - \frac{120}{315}v\frac{\partial f_4^h}{\partial v} - \frac{600}{315}f_4^h + \frac{36}{525}f_2^h\right). \quad (24)$$

For the second order ($l = 2$):

$$f_2^s(v) = \frac{1}{12\sqrt{2}}\frac{\omega_{ce}}{\vartheta_{ei}(v)}\left(\frac{180}{315}v\frac{\partial f_1^h}{\partial v} + \frac{9936}{6615}v\frac{\partial f_3^h}{\partial v} - \frac{360}{693}v\frac{\partial f_5^h}{\partial v} - \frac{2160}{693}f_5^h + \frac{348}{315}f_1^h - \frac{432}{6615}f_3^h\right) . \quad (25)$$

By neglecting the higher order components behand the f_0^s component by considering that $f_{l+2}^s \ll f_l^s$, this last equation can be presented as:

$$f_2^s(v) = \frac{\omega_{ce}}{\vartheta_{ei}(v)}\left(+0.037v\frac{\partial f_0^s}{\partial v} + 0.028v\frac{\partial}{\partial v}\left(v\frac{\partial f_0^s}{\partial v}\right)\right). \quad (26)$$

Note that the equation (23) presents a recurrence relation between different components of f^s. This allows to determinate the distribution function by knowledge of f_0^s as a boundary condition.

Physically, the zero order static distribution function corresponds to the electrons non perturbed distribution function by the magnetic field. It can be supposed by considering the thermodynamic equilibrium as a Maxwell function. At this order (zero) the high frequency function vanishes.

6. The temperature anisotropy

The high frequency distribution function does not contribute to the temperature because its average on the cyclotron period time vanishes. By limiting our development on the Legender polynomials at the second order, the parallel temperature, $T_\parallel = \overline{m_e v_\parallel^2}$, where the symbol $\overline{}$ means the average value on the velocities distribution, is given by:

$$n_e T_\parallel = m_e \int v_\parallel^2 f d^3\vec{v} = \pi m_e \int \mu^2 v^4 \{f_0(v) + P_1(\mu) f_1(v) + P_2(\mu) f_2(v)\} dv d\mu$$

$$= \frac{4}{3}\pi m_e \int v^4\{f_0(v)\}dv - \frac{8}{15}\pi m_e \int v^4\{f_2(v)\}dv. \quad (27)$$

By supposing that the zero order distribution function is maxwellian :$f_0(v) = \frac{n_e}{v_t^3(2\pi)^{3/2}}\exp\left(-\frac{v^2}{2v_t^2}\right)$, the second anisotropic distribution function, eq. (26), can be written as follow

$$f_2^s(v) = -n_e\frac{\omega_{ce}}{\alpha}\left(0.0047\frac{v^5}{v_t^5} - 0.0012\frac{v^7}{v_t^7}\right) \; exp\left(-\frac{v^2}{2v_t^2}\right). \quad (28)$$

By computing the integral, the explicitexpression of $T_\parallel$is found as:

$$T_\parallel = T\left(1 + a\frac{\omega_{ce}}{\nu_{ei}}\right), \quad (29)$$

Where ν_{ei} is the collisions frequency and $a \approx 4.7$ is a constant.

The perpendicular temperature, $T_\perp = \frac{1}{2}\overline{m_e v_\perp^2}$, is calculated using the same equations as:

$$T_\perp = \frac{1}{2}m_e \int v_\perp^2 f d^3\vec{v} = \pi m_e \int (1-\mu^2) v^4 \left\{f_0(v) + \mu f_1(v) + \frac{1}{2}(3\mu^2 - 1) f_2(v)\right\} dv d\mu =$$

$$\frac{4}{3}\pi m_e \int v^4\{f_0(v)\}dv - \frac{4}{15}\pi m_e \int v^4\{f_2(v)\}dv. \quad (30)$$

In the case of the maxwellian isotropic distribution function, the $T_\perp$ is calculated as:

$$T_\perp = T\left(1 + \frac{a}{2}\frac{\omega_{ce}}{\nu_{ei}}\right). \quad (31)$$

The temperature anisotropy is then given by:

$$\frac{T_{\parallel}}{T_{\perp}} = \frac{1+a\frac{\omega_{ce}}{\nu_{ei}}}{1+\frac{a\omega_{ce}}{2\,\nu_{ei}}}. \quad (32)$$

It is well clear that this anisotropy depends on the ratio of the cyclotron frequency to the collisions frequency.

7. Conclusion

In conclusion, in this paper we have analytically calculated the distribution function for highly magnetized plasma. Using this distribution function we have calculated the temperature in the parallel direction and in the perpendicular direction. It has been shown that the temperature is anisotropic and it is depend on the magnetic field and the collisions frequency. In this study, we have limited our development to the second order. The plasma is hotter in the parallel direction.

This study can found applications for several phenomenon's in magnetized plasmas namely, transport, alfvèn wave, instability. As extension to this work, we'll calculate the anisotropic in temperature for relativistic plasma.

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