Applying the New Principles of Self-organization to Stable Tokamak Plasmas

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Abstract. For *stable* equilibria, parallel current is carried by electrons frozen in flux surfaces that resist deformation. Magnetic perturbations generate a viscous-like force between flux surfaces, similar to forces between the wall and the current with resistive wall modes, leading to self-organization in stable plasmas. The impact of self-organization on the tokamak and the spheromak is discussed from the perspective of electrons being tied to resilient, rotating flux surfaces and agreement with experimental results is presented.

1. Introduction

Matter has the most degrees of freedom in the unmagnetized plasma state, which is the fourth state of matter. Placing the plasma in a magnetic field reduces the freedom of motion of the plasma. In particular, the current has restrictions and forces placed upon it by the magnetic field since, as is well known, the electrons are frozen in the magnetic field (for scale lengths much greater than the electron inertial and collisional skin depths). When the magnetic forces dominate the current, and the plasma currents in turn make significant magnetic fields, the plasma is self-organized. When the magnetic fields form a stable equilibrium, they form a semi-rigid flux surface. Freezing the electrons in this structure limits their freedom to carry current. This simple picture perhaps allows a simple understanding of the Minimum Energy Principle[1], Imposed Dynamo Current Drive[2], and transport barriers[3].

2. Picture of Current in Stable Equilibria

Symmetric flux surfaces allow free differential current flow (non-uniform j_{tor}/B_{tor}) unobstructed by magnetic interference. *(See FIG. 1a)*. Add a magnetic perturbation *(see FIG. 1b)* and differential flow is no longer free. If the perturbation is sufficiently large, the flow locks across flux surfaces (as shown on inner flux surfaces of *FIG. 1c*) giving uniform j_{tor}/B_{tor}.

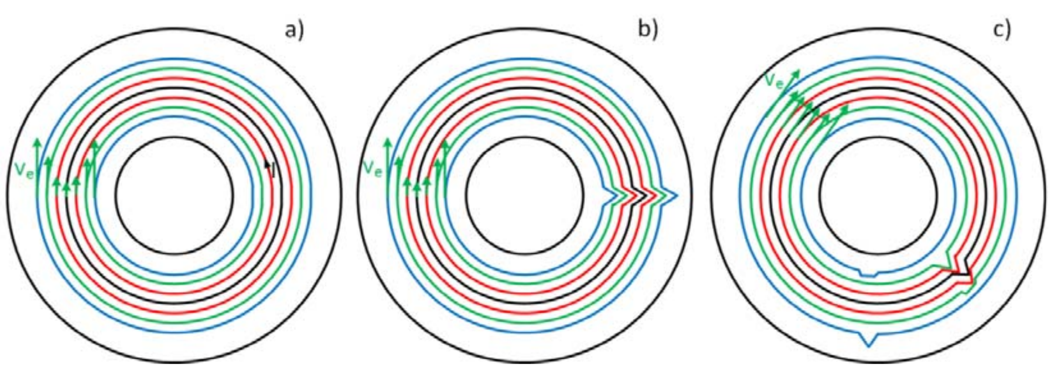

FIG. 1. Toroidal current in a torus with an initially hollow current profile. This is a mid-plane cross section with colors representing flux surfaces. a) Symmetric flow. b) Add perturbations. c) Resultant flow.

If the perturbation is sufficiently small, the differential flow can symmetrize the perturbation and differential flow continues, but the perturbation causes a force. A viscous-like drag force will drive the current inside the symmetrized, and possibly closed flux surface. Both the locking and the symmetrizing forces are current self-organizing (relaxing) across closed flux surfaces towards uniform j_{tor}/B_{tor}. An analysis using the Maxwell stress tensor calibrated by HIT-SI data[2] and tokamak disruption data[4] indicate that the viscous force per unit area needed to slide past the perturbation is approximately $(\delta B_\perp)^2/2\mu_o$ where $\delta B_\perp$ is the component of the perturbation perpendicular to the flux surface. Setting the total viscous force on the flux surface area equal to the force required to drive the current throughout that flux surface volume gives:

$$\int \frac{(\delta B_\perp)^2}{2\mu_o} dA \simeq \int ne(\eta j_{tor} - E_{tor}) dvol \qquad 1$$

This equation can be simplified to:

$$\frac{(\delta B_{\perp rms})^2}{\mu_o} \simeq (\eta j_{tor} - E_{tor}) rne \qquad 2$$

where

$$\frac{\mu_o}{4\pi}\frac{d(l_i I)}{dt} \simeq -E_{tor} \qquad 3$$

where l_i is the dimensionless internal inductance.

3. Application to Experiments

This picture is consistent with several experiments. This mechanism provides an explanation for the level of field error ($\delta B/B = 10^{-4}$) that spoils tokamak performance because it is the level required to drive the current[2]. Since it is enough to drive the current (from Eq. 2 with $E_{tor} = 0$) it is also enough to flatten the current profile. A flat current profile gives a flat q-profile and low β in a conventional aspect ratio tokamak The perturbations are at the level needed for the rate of poloidal flux loss in argon-induced disruptions in DIII-D[4,5]. *FIG. 2* shows data from such a disruption. During the current flattening phase (2.0450 s < t < 2.0462 s) $\Delta(l_i I)/\Delta t$ equals -0.92 GAs^{-1}. Using these data with r = 0.61 m and $n = 4 \times 10^{19}$ m^{-3} in equations 3 and 2, ignoring the resistive term, yields $\delta B_\perp$ of 211 G, which is of similar magnitude to the perturbations observed. During the current quench (2.0462 s < t < 2.0562 s) $\Delta(l_i I)/\Delta t$ equals -0.08 GAs^{-1} yielding a $\delta B_\perp$ of 62 G, similar to that observed, even though ignoring the resistive term may not be as justified.

The change in plasma rotation direction with LHCD in the edge on C-Mod can be understood using this model[5]. In the externally driven regions the drag force brakes electrons so the force is in the direction of the current, driving plasma velocity in that direction. In the dynamo driven regions the force is with the electron flow resulting in plasma flow against the current. Thus, the core plasma rotates with the current in a normal tokamak discharge because the core is externally driven and against the current when LHCD is used in the edge because the core is dynamo driven (seen on C-Mod[6]).

Adding a stellarator field to a tokamak, under some conditions, produces flat I' ($\propto j/B$), as observed in CTH[7], is also consistent with this model. Externally driving the edge, while imposing perturbations, allows the sustainment of stable equilibria[2].

This method is called Imposed Dynamo Current Drive (IDCD). The toroidal current versus time, the injector impedance scaling with *j/n* of the spheromak, and the current profile data of HIT-SI are predicted by this model[2].

Thus, perturbations cause forces that oppose differential rotation and, therefore, forced differential flow produces a symmetrizing force against perturbations. The eigenmodes of non-symmetric instabilities will have difficulty crossing the region that is enforcing symmetry so the region may also inhibit other asymmetric instabilities. There is evidence transport barriers depend on the $\mathbf{E} \times \mathbf{B}$ shearing rate[3]. In ideal, two-fluid MHD $\mathbf{v}_e = \mathbf{E} \times \mathbf{B}/B^2$, and assuming B^2 has a small gradient, the shearing velocity is the electron velocity shear, like *FIG. 1.* The transport barriers might come in "steps" because they only occur where the perturbations are low enough to allow slippage between electron current shells.

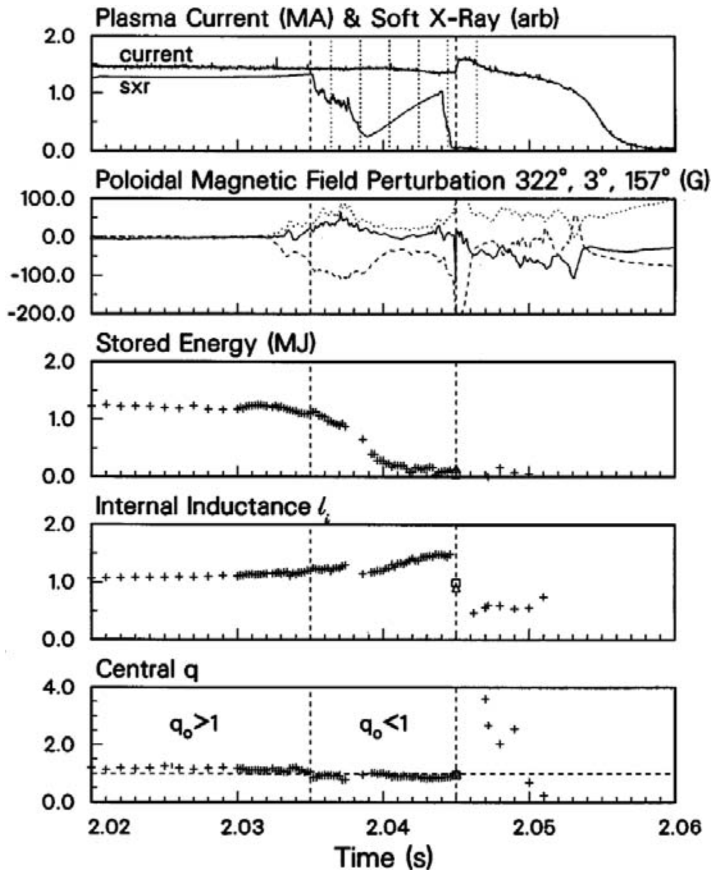

FIG. 2. Data from a disruption on DIII-D produced by argon gas injection[4].

It is well established that magnetized plasma relaxes toward a state of minimum energy while conserving helicity[1] (MECH state). From resistive Hall-MHD, only the resistive term in the generalized Ohm's law gives a non-zero value of helicity dissipation, which is $\propto \mathbf{E} \bullet \mathbf{B}$. Thus, helicity decays on the resistive time scale (the longest characteristic time scale in the system) and the MECH state is stable. With flux boundary conditions (BC) the result is the Taylor state with uniform j/B. In sustained plasmas an arbitrary j/B can be part of the BC and the MECH state does not have uniform j/B. A separatrix can form between the region with j/B defined by the BC and the region where j/B is found by self-organization. This leads to a two-step-like j/B profile observed to be stable in HIT-SI[2].

4. Three Principles of Self-Organization of Stable Equilibria

A summary of this discussion of self-organization is given as three Principles of self-organization of stable equilibria.

1. The first Principle is that magnetized plasma relaxes toward a state of minimum energy that conserves helicity[1] (MECH state) and the MECH state is stable.
2. The second Principle is that imposed perturbations can flatten the current profile of stable equilibria, without instability or reconnection.

3. The third Principle is that differential electron flow produces a local symmetrizing force against perturbations, which may inhibit local asymmetric instabilities as in pedestals and transport barriers.

5. Optimizing the Tokamak

The three Principles make optimizing a normal tokamak difficult. A uniform j/B profile in a normal tokamak has nearly uniform q and has a very low β-limit. The first Principle must be disobeyed by using the inherit stability of the tokamak. The second Principle is defeated by keeping field errors and perturbations low. The third Principle is used to create pedestals and internal transport barriers for high performance. While locally stabilizing, E x B shear can be globally destabilizing → perturbations → uniform j/B → loss of pressure →thermal quench (start of disruption). The solution is to drive the edge current high and impose a perturbation profile that sustains the desired reversed-shear current profile. High edge current prevents the edge from using perturbations to drag down the current in disruptions. This also solves the sustainment problem. (IDCD is over two orders of magnitude more power efficient on a reactor than RF current drive.)

6. An Experiment for Exploiting the Principles

A better solution is to select a high performance equilibrium that has a uniform j/B (low aspect ratio). Rigorously sustain the stable profile by edge current drive and repeated application of non-resonant perturbations of IDCD. The method has been demonstrated on HIT-SI. Again it also solves the sustainment problem.

The spheromak is one of the most self-organized man-made plasmas. It exists in a simply-connected vessel where both the toroidal and poloidal flux are determined by self-organization. Eddy currents in the vessel keep the boundary fields inside the vessel for the short pulse length of present experiments. For DC experiments an equilibrium coil set is needed, and coils to stabilize the tilt and shift resistive wall modes may be necessary. The edge currents are driven by external circuits that supply the power, magnetic flux, and perturbations that form and sustain the spheromak throughout the volume.

HIT-SI data clearly show that helicity injection current drive does not require instability for ramp up and sustainment of the spheromak. An equilibrium is self-organized in the presence of applied perturbations and the spheromak is sustained while remaining stable. If the perturbation does not make the equilibrium unstable and is on for a short time compared to the resistive diffusion time, then it may be possible to apply and remove the perturbations without overly damaging confinement. The perturbation in a reactor would be small enough so that the increased drift losses due to the imposed asymmetry is much less than neoclassical losses. By driving the edge current high with external circuits, while not driving the equilibrium unstable, and imposing perturbations, steady state sustainment is achieved. The perturbation causes a viscous-like force on the electron fluid across the closed flux surfaces, transferring the edge current drive throughout the volume.

The experiment shown in *FIG. 3* through *FIG. 5* is designed, using enhanced technology to overcome some limitations with HIT-SI and to be capable of achieving 100 eV-scale temperatures. HIT-TD (TD is for technology demonstration) will overcome those limitations as follows: a) the uniform-λ, β_{wall}-limit increases from 3% to 10%; b) it is designed for plasma spraying the entire plasma facing surface with a high quality insulating layer; c) passive gas pumping during the discharge will be increased to allow operation well below the Greenwald limit; d) HIT-TD will have a larger size to produce more closed flux (the amount of closed

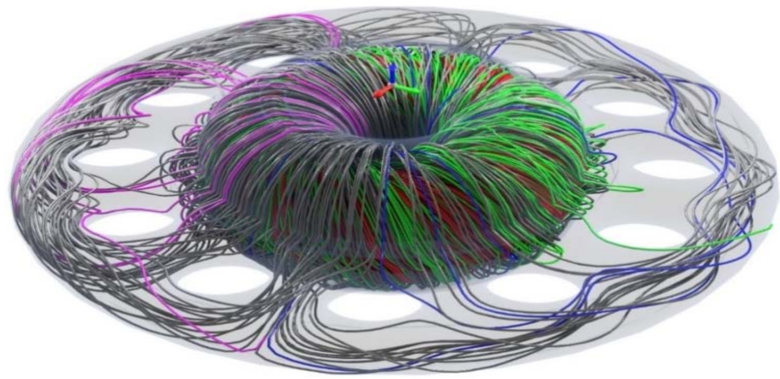

FIG. 3. Geometry of the proposed experiment. HIT-TD has minor radius of 0.37 m (2.4/λ_{sph}) and major radius 0.48 m. Design plasma current is 200 kA with peak temperature of 150 eV. Shown is the geometry with a Taylor state equilibrium with a current gain of 6 and n = 1 perturbation. An injector manifold is used for precise perturbation control.

flux on HIT-SI is not known); e) instead of individual injectors with a limited spectrum of imposed perturbations it will have an injection manifold allowing any combination of *n = 1, 2, 3, 4, 5* and *6* modes.

FIG. 5 shows the flux contours of the experiment. The color contours are the plasma pressure normalized by the average wall magnetic pressure, B_w, which exceeds 16% where B_w is defined as:

$$B_w \equiv \frac{\mu_o I_{tor}}{2\pi a + 2g} \qquad 4$$

where a is the minor radius of the upper and lower half-tori and g is the separation the makes the cross section elongated.

In this case, a Grad-Shafranov equilibrium is found for the region inside the flux conserver, the solid flux contour region is at constant λ (like that produced with IDCD in HIT-SI) with the maximum pressure gradient at the Mercier-limit. The white lines are the flux surfaces. The flux conserver provides the surface fields. For this equilibrium, the pressure gradient is zero near the wall as shown by dashed flux contours.

FIG. 6 shows the resistive power dissipation assuming an edge temperature and uniform density with the temperature gradient corresponding to that allowed by the Mercier pressure gradient. The caption tells how to scale the curve for plasmas of the same shape. Great care must be taken to properly resolve the cooler regions and poor resolution leads to much higher dissipation estimates.

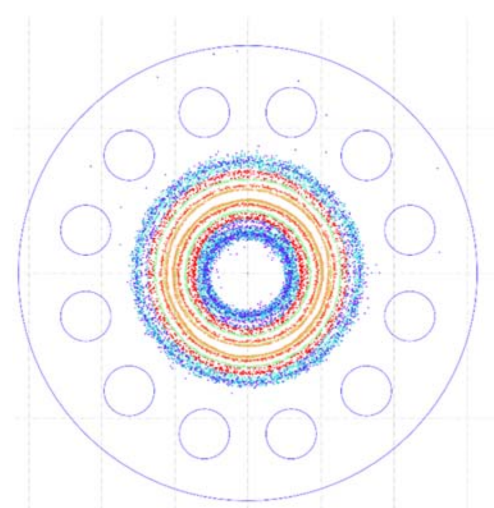

FIG. 4. Puncture plot of the Taylor state shown in FIG.3.

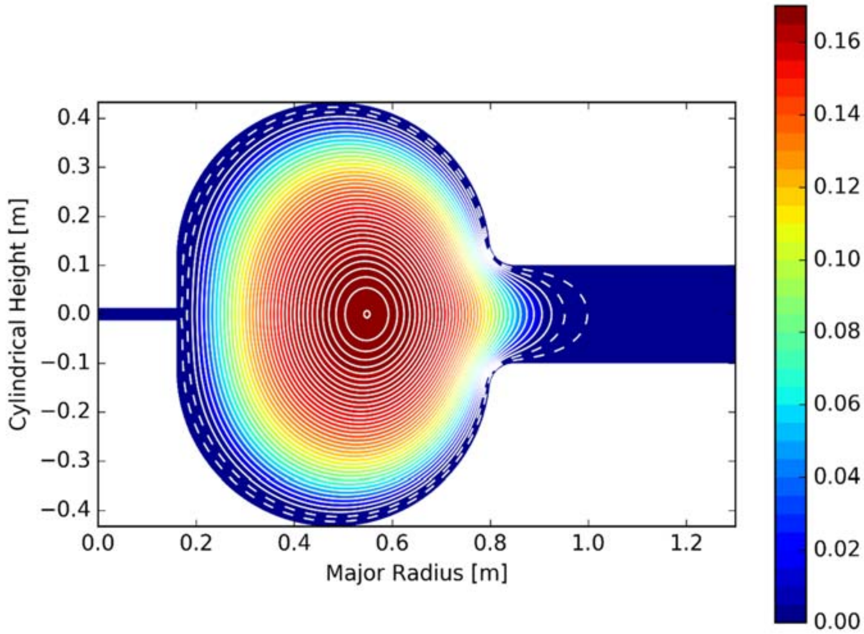

FIG. 5. Flux surfaces (lines) and pressure contours (colors) for the steady-state equilibrium for HIT-PC. The pressure shown in color contours is normalized by the magnetic pressure of B_{wa}.

The high temperature gradients in the edge prevent the low temperature edge from dominating the dissipation. From *FIG. 6* with HIT-TD (n = 2.2 × 10^{19} m^{-3}, a = 0.37 m and T_{peak} = 150 eV) the edge temperature factor is 0.15 and the power factor is 0.55. Thus a plasma with 15 eV edge temperature has a power dissipation of about 550 kW. At 40% efficiency balancing resistive dissipation requires less than 2 MW of injector power.

For this discussion, the edge is the current separatrix where magnetic pressure confinement begins. The resistive power dissipation is estimated assuming an edge temperature and constant density with the temperature gradient corresponding to that allowed by the Mercier pressure gradient. Just outside the edge is the injector-driven, high-λ, warm-plasma region with a nearly uniform temperature and density. The injector currents are expected to exist just outside the edge because such Grad-Shafranov equilibria fit the data the best as in Reference 2, 3D Taylor states give this result as in *FIG. 4*, and the anticipated formation of a stable, symmetric current separatrix because of the third principle of self-organization.

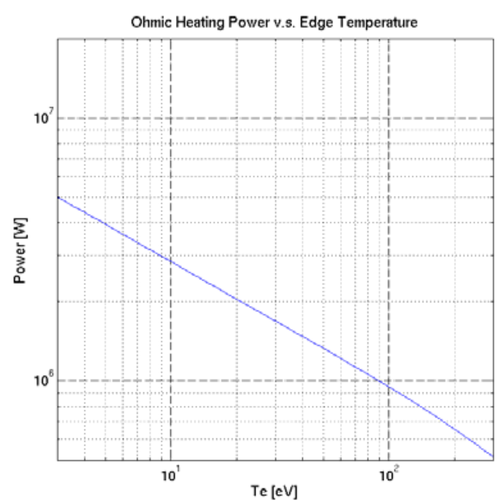

FIG. 6. Ohmic heating power vs edge temperature. T_{peak} = 1 keV and $an/T^{1/2}$ = $1.2x10^{18} m^{-2} eV^{-1/2}$ for the curve. The edge temperature is reduced by the fraction T_{peak} is of 1 keV. The power is reduced by the fraction $an/T_{peak}^{1/2}$ is of $1.2x10^{18} m^{-2} eV^{-1/2}$.

If the current gain is high enough and the equilibrium is kink-stable then nested flux surfaces should form in the confinement region, as shown in *FIG. 5*. The sustained pressure confinement observed in HIT-SI should occur again in HIT-TD. At the edge, the field lines transition from closed flux inside the symmetrizing current separatrix to stochastic outside the separatrix where the injectors impose asymmetry. The large difference in transport will cause a rapid temperature drop. From tokamak and RFP experience this drop should be over 100 eV. We conservatively assume an inside edge temperature of 15 eV requiring a few megawatts to drive the plasma current. Outside the edge, conditions no worse than on HIT-SI (T ~15 eV) should exist requiring a few megawatts to drive the injector current in the edge and in the injector manifold. Thus, less than 10 MW of injector power will be required for sustainment. The conceptual design shown in *FIG. 7* uses a mesh flux conserver. Unlike gettering and wall conditioning, the mesh will provide wall pumping for the entire 15 ms discharge. The first spheromak to reach over 100 eV temperature used a mesh flux conserver with no gettering[8].

7. Summary

Three Principles of self-organization of stable plasma were discussed, namely: The first Principle is that magnetized plasma relaxes toward a state of minimum energy while conserving helicity (MECH state) and the MECH state is stable. The second Principle is that imposed perturbations can flatten the current profile of stable equilibria, without instability or reconnection. The third Principle is that differential electron flow produces a local

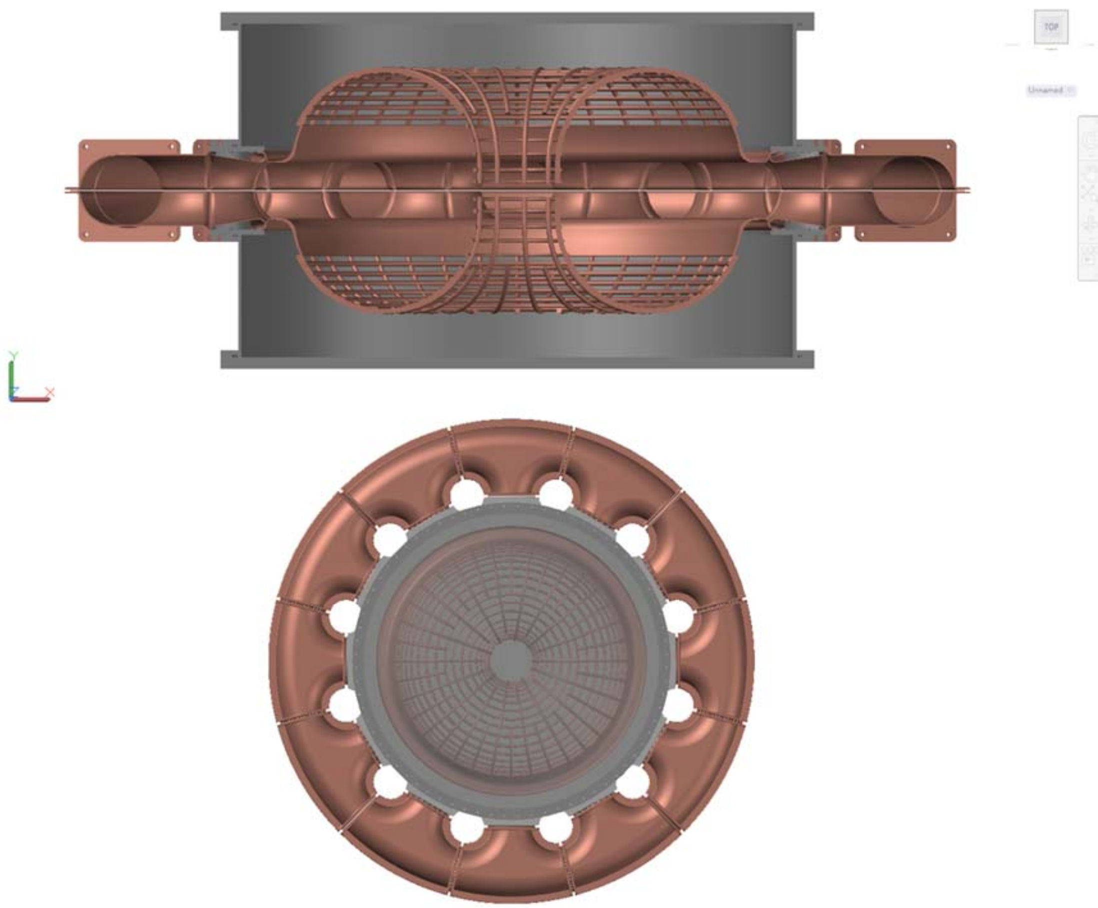

FIG. 7. Conceptual design of the HIT-PC experiment showing the geometry (including the "mesh" flux conserver for passive-pumping density control).

symmetrizing force against perturbations, which may inhibit local asymmetric instabilities as in pedestals and transport barriers. With difficulty the tokamak defeats the first two and takes advantage of only the last Principle. By driving the edge current high and imposing perturbations very power efficient current drive of stable equilibria have been achieved, using the second Principle. The first principle dictates that such a sustained equilibrium has uniform j_{tor}/B_{tor} requiring a low aspect ratio for high shear and high beta.

A new machine is needed to rapidly advance helicity injection current drive to form and sustain tokamaks, spherical tori (ST), spheromaks, and RFPs. A 200 kA experiment with temperature of 150 eV, density of $2 \times 10^{19}\,m^{-3}$, minor radius of 0.32 m, major radius of 0.48 m, elongation 1.3 and a beta relative to the wall B-field of 16% is proposed. The new machine objective is to overcome the limitations of HIT-SI that prevents high temperature operation. Thus, HIT-TD has the pumping required to operate well below the Greenwald density. This density control plus the increased size will increase the closed-flux. The formation of a stable separatrix is expected. HIT-TD is designed for applying a higher quality insulating boundary for satisfactory operation at higher injector voltages. With the perturbation control of the new manifold injectors, density control with passive pumping and the stabilizing effects of IDCD, heating to the beta-limit with Ohmic heating is expected.

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