Nonlinear excitation of subcritical energetic particle-driven mode by a supercritical chirping mode

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Abstract:
In collisionless plasmas, it is known that linearly stable modes can be destabilized (subcritically) by the presence of structures in phase-space. The growth of such structures is a nonlinear, kinetic mechanism, which provides a channel for free-energy extraction, different from conventional inverse Landau damping. However, such nonlinear growth requires the presence of a seed structure with a relatively large threshold in amplitude. Recently, we demonstrated that, in the presence of another, linearly unstable (supercritical) mode, wave-wave coupling can provide a seed, which can lead to subcritical instability. The mechanism hinges on a collaboration between fluid nonlinearity and kinetic nonlinearity. The subcritical instability can be triggered, even when the frequency of the supercritical mode is rapidly sweeping (chirping). The model recovers key features of the bursty onset of geodesic acoustic modes (GAM) in a LHD experiment. These previous studies suggest that the strongest GAM bursts in this experiment are subcritical instabilities, with sustained collaboration between fluid and kinetic nonlinearities. These results were obtained by a hybrid model, where the subcritical mode is modeled kinetically, but the impact of the supercritical mode is modeled by simple wave-wave coupling equations. In this paper, we generalize the study to the interactions between chirping energetic particle-driven modes, based on a fully kinetic model (1D Vlasov-Poisson). Preliminary results suggest that a supercritical chirping mode can destabilize a subcritical mode. This occurs when a phase-space vortex associated with the chirping of the supercritical mode approaches the resonant velocity of the subcritical mode.

1 Introduction

A major concern in burning plasmas is that Energetic Particles (EPs) can excite plasma instabilities in the frequency range of Alfvén Eigenmodes (AEs) or Geodesic Acoustic Modes

(GAMs). These EP-driven modes significantly enhance the transport of EPs themselves, threatening both confinement quality and first-wall integrity. We are concerned with the impacts of the trapping of particles in a well of electrostatic potential (nonlinear electrostatic trapping). This is different from the well-known trapping due to the geometry of the magnetic field in toroidal plasmas, which yields banana orbits.

In many toroidal plasma experiments with strong beam injection systems, EP-driven modes are frequency observed to split into two branches that sweep upwardly and/or downwardly, by 10 to 100% of the linear mode frequency, on a timescale much faster than the equilibrium evolution. This phenomenon is called as chirping. Similar chirping modes are spontaneously generated within the Berk-Breizman (BB) model [1, 2], which is a generalization of the bump-on-tail instability [3]. This model includes a collision operator, and an external wave damping, which accounts for background dissipative mechanisms at a constant rate γ_d. In the context of the BB model, chirping has been shown to correspond to the evolution of phase-space structures [2]. This has later been confirmed in 3D simulations [4]. These structures are BGK-like vortices in phase-space, with a depletion or surplus of density, which are formed by electrostatic self-trapping.

The BB model was successfully applied to recover quantitatively the nonlinear evolution of EP-driven modes in toroidal fusion plasmas. Firstly, it was applied to Toroidal Alfvén Eigenmodes in MAST [5] and JT-60U [6]. More recently, we developed an extension of the BB model, which combines the kinetic description of a linearly stable (subcritical) mode with the nonlinear fluid coupling with a prescribed linearly unstable (supercritical) mode [7, 8, 9]. In this paper, we refer to this model as the hybrid two-waves BB model. This model was successfully applied to reproduce qualitatively an experimental observation in the helical plasma of the LHD, which was described in Refs. [10, 11]. The hybrid two-waves BB model interprets this experiment as follows: a subcritical GAM is dormant in the plasma until a supercritical Energetic particle-driven GAM (EGAM) excites it.

Let us now take a more academic point-of-view. Phase-space structure formation is a kinetic nonlinearity, in the sense that it cannot be described by fluid models, unlike other nonlinearities such as higher harmonic generation, mode coupling, fluid vortex, etc. Theory predicts that these structures can tap free energy where wave excitation cannot, and lead to subcritical instabilities, where the kinetic nonlinearity enables the growth of a mode that is linearly damped [12, 13]. However, such subcritical growth requires a large-amplitude seed perturbation. Several scenarios could provide the seed for kinetic nonlinear growth of a linearly stable mode:

1. the presence of large thermal noise or an external source of wave excitation, or
2. a hysteresic path from supercritical to subcritical regime, or
3. a transfer of energy from another, linearly unstable mode, or
4. a large perturbation of the distribution function at the resonance, due to a phase-space structure originating from an other, supercritical mode.

Previous works on kinetic subcritical instabilities assumed some initial, relatively large amplitude (at least, compared to thermal noise) perturbation [14, 15, 16] for the subcritical mode, corresponding to scenario 1. The hysteretic behavior, corresponding to scenario 2, was obtained in a COBBLES simulation, and will be the subject of a future paper. This situation is linked to the existence of purely nonlinear steady-state regimes, which are consistent with tokamak conditions [20]. Another work explored an artificial scenario, where a seed phase-space hole is imposed at $t = 0$ [17].

In Refs. [7, 9], we explored the third scenario with the hybrid two-waves BB model. We showed that the supercritical mode can provide a seed by transfer of energy, for the nonlinear growth of the subcritical mode. In Section 2, we summarize these findings.

In the present paper, we aim at exploring the fourth scenario, although a combination between the third and fourth scenario will not be ruled out. To this aim, we adopt the standard BB model (fully-kinetic), and set up an initial distribution function such that one mode is unstable and chirping, an other one is barely stable, and we filter out any but those two modes. We expect that when a phase-space structure corresponding to the chirping of the supercritical mode approaches the resonant velocity of the subcritical mode, the subcritical mode may be destabilized. Our preliminary results, which are shown in Section 3, suggest that this is indeed the case.

2 The hybrid two-waves BB model

2.1 Model

In the hybrid two-waves BB model, as described in Ref. [9], the electric field E is split between two waves, $E = E_1 + E_2$. The subcritical (daughter) mode (E_1) is treated by the kinetic 1D model, and the supercritical (mother) mode (E_2) is treated as a simple medium for nonlinear energy transfer. For E_2, we prescribe the initial amplitude $Z_{2,0}$ and time-evolution of frequency $\omega_2(t)$. We assume that the impact of the mother on the particles near the resonant location of the daughter is negligible. The interaction between the two waves is modeled by the equations for period doubling.

We adopt a perturbative approach, and cast the equations for wave-particle interactions in a reduced form, which describes the time evolution of the beam particles only [18]. In this model, the linear frequency of the wave E_1 is fixed. Even when chirping occurs, ω_1 does not change. Chirping, when it occurs, is due to the nonlinear evolution of the amplitude and phase of E_1, rather than the evolution of ω_1.

The evolution of the energetic particle distribution, $f(x, v, t)$, in the neighbourhood of the resonance of the daughter mode E_1, is given by a kinetic equation [18, 6],

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} + \frac{qE_1}{m}\frac{\partial f}{\partial v} = \frac{\nu_f^2}{k_1}\frac{\partial \delta f}{\partial v} + \frac{\nu_d^3}{k_1^2}\frac{\partial^2 \delta f}{\partial v^2}, \tag{1}$$

where $\delta f \equiv f - f_0$, and $f_0(v)$ is the initial velocity distribution. The r.h.s. is a collision operator, where ν_f and ν_d are input parameters characterizing dynamical friction and velocity-space diffusion, respectively.

The evolution of the two parts of electric field is given by

$$\begin{aligned}\frac{\mathrm{d}Z_1}{\mathrm{d}t} = & -\frac{m\omega_1^3}{4\pi q^2 n_0}\int f(x, v, t)\, e^{-i(k_1 x - \omega_1 t)}\,\mathrm{d}x\,\mathrm{d}v \\ & - \gamma_d\, Z_1 - \imath\frac{V}{\omega_1} Z_2 Z_1^* e^{-\imath\theta t},\end{aligned} \tag{2}$$

$$\frac{\mathrm{d}Z_2}{\mathrm{d}t} = -\imath\frac{V}{\omega_2}Z_1^2 e^{\imath\theta t}, \tag{3}$$

where $E_j \equiv Z_j \exp\left[\imath(k_j x - \omega_j t)\right] + c.c.$, and n_0 is the total density.

The term proportional to γ_d is an external wave damping, which is a model for all linear dissipative mechanisms of the wave energy to the background plasma [18]. We note that in this model, we split the electric field into two parts, and assume that there is one class of particles (distribution f) which does not interact with one of the two parts of the electric field. We consider a system composed of the two waves and the latter class of particles. In this sense, this model system is an open system.

2.2 Phenomenology

Equation (2) contains two nonlinear terms, which we refer to as kinetic nonlinearity (the term proportional to $\int f e^{-\imath(k_1 x - \omega_1 t)}$), and fluid nonlinearity (the term proportional to $V Z_2 Z_1^* e^{-\imath\theta t}$). In the references, we showed that the fluid nonlinearity and the kinetic nonlinearity can work in collaboration to drive a subcritical instability to relatively large amplitude.

2.3 Comparison with experiment

In a toroidal device, the linear structure, linear frequency and linear growth rate of an energetic particle-driven mode is determined by 3D calculations. These linear properties evolve on a slow timescale of mean field evolution ($\sim$ 100 ms). However, the kinetic nonlinear effects, which induce chirping and subcritical instability, are linked with the evolution on a fast timescale ($\sim$ 1 ms). They can be treated perturbatively in the BB model [2], by taking advantage of the timescale separation. This reduced 1D model is linked to the 3D mode by a perturbative expansion of a gyrokinetic Hamiltonian around a resonant surface in phase-space [5].

In the LHD, chirping bursts of Energetic particle-driven GAM (EGAM), with dynamical evolution of frequency (chirping) are routinely observed, with a 10 ms duration [21]. These primary EGAM bursts are sometimes accompanied by a secondary, stronger burst [10, 11]. The secondary burst has a 1 ms duration, and a peak amplitude that significantly exceeds that of the primary burst. The hybrid two-waves BB model interprets this experiment as follows: the secondary (daughter) mode is a subcritical instability, which is dormant until the primary (mother) mode excites it. A detailed comparison is given in Refs. [7, 9].

3 The fully kinetic two-waves BB model

In this section, we still consider two waves in the BB model. However, we model both waves and the distribution function in a self-consistent, fully kinetic way.

3.1 Model

We consider a 1D plasma with a distribution function $f(x, v, t)$. In the initial condition, the velocity distribution $f_0(v)$ comprises a Maxwellian bulk of density n_M, thermal velocity v_{th}, and a beam of high-energy particles, of density n_B ($n_M + n_B = n_0$), thermal velocity v_{TB}, and drift velocity v_B. To ensure charge neutrality, we assume a fixed background population of the opposite charge with a distribution function $f_0(v)$.

The evolution of the distribution is given by the kinetic equation,

$$\frac{\partial f}{\partial t} + v\,\frac{\partial f}{\partial x} + \frac{qE}{m}\frac{\partial f}{\partial v} = \frac{\nu_f^2}{k_1}\frac{\partial \delta f}{\partial v} + \frac{\nu_d^3}{k_1^2}\frac{\partial^2 \delta f}{\partial v^2} \tag{4}$$

where E is the total electric field, and k_1 is the smallest wave number of the system.

In the expression of the electric field, we filter out all but two wave numbers k_1 (corresponding to the system size) and $k_2 = 2k_1$. The displacement current equation,

$$\frac{\partial E}{\partial t} = \frac{q}{\epsilon_0}\int v\,(f - f_0)\,\mathrm{d}v \; - \; 2\,\gamma_d\,E, \tag{5}$$

yields the time evolution of the wave. In the initial condition we apply a small perturbation, $f(x, v, t = 0) = f_0(v)(1 + \epsilon \cos k_1 x)$, and the initial electric field is given by solving Poisson's equation. In Eq. (5), an external wave damping has been added to model all linear dissipation mechanisms of the wave energy to the background plasma that are not included in the previous equations [22].

3.2 Setup and preliminary results

In this paper, we choose $n_B = 0.2\,n_0$, $v_{TB} = 4.0\,v_{th}$, $v_B = 10.0\,v_{th}$, and a system size $L = 2\pi/k_1$ with $k_1 = 0.05\,\lambda_D^{-1}$ (hence, $k_2 = 0.1\,\lambda_D^{-1}$). Then we adjust γ_d, ν_f and ν_d such that mode 1 (of wavenumber k_1) is linearly unstable and chirping, and that mode 2 (of wavenumber k_2) is linearly stable, but close to marginal stability (barely stable). Here, we choose $\gamma_d = 0.32\,\omega_p$, $\nu_f = 0.002\,\omega_p$, and $\nu_d = 0.01\,\omega_p$.

Fig. 1 shows the time evolution of components $k = k_1$ and $k = k_2$ of the electric field (for short, E_1 and E_2). Fig. 1 shows the corresponding spectrogram, with k_1 and k_2 components split up as well. The supercritical (mother) mode E_1 grows as predicted by linear theory, until it saturates with significant chirping, consistently with earlier nonlinear theories. It is also similar to the way it behaves in isolation, as we have tested by running the same simulation but with the component k_2 filtered out.

Let us now focus on the evolution of mode E_2. This is a subcritical mode, which, if isolated, would quickly fade out to zero amplitude, as we have tested by running the same simulation but with the component k_1 filtered out. However, when the two components k_1 and k_2 are allowed (unfiltered), the mode grows to an amplitude comparable to that of the mother mode E_1. Restricting ourselves to the early ($t < 650$) evolution of this daughter mode E_2, we can distinguish three phases.

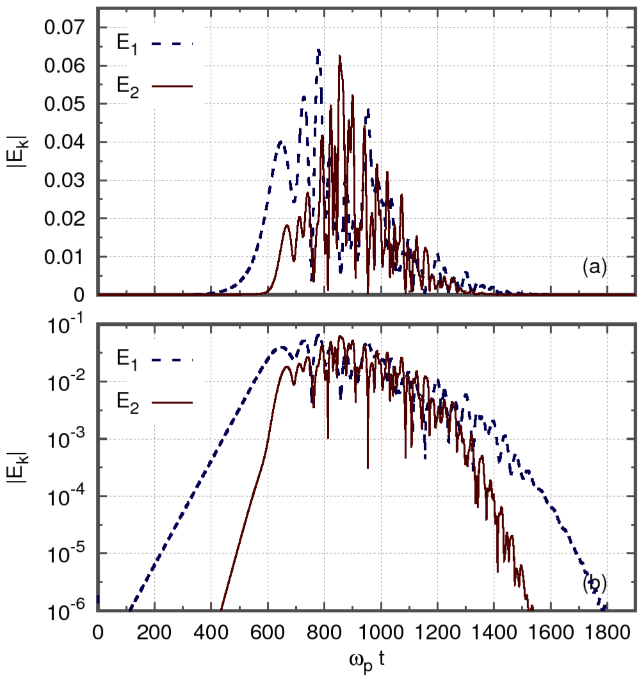

FIG. 1: Time evolution of the amplitude of electric field, split into components E_1 and E_2, which correspond to the supercritical mode and the subcritical mode, respectively. Here, the electric field is normalized by $mv_{th}\omega_p/q$. (a) linear scale, (b) semi-logarithmic scale.

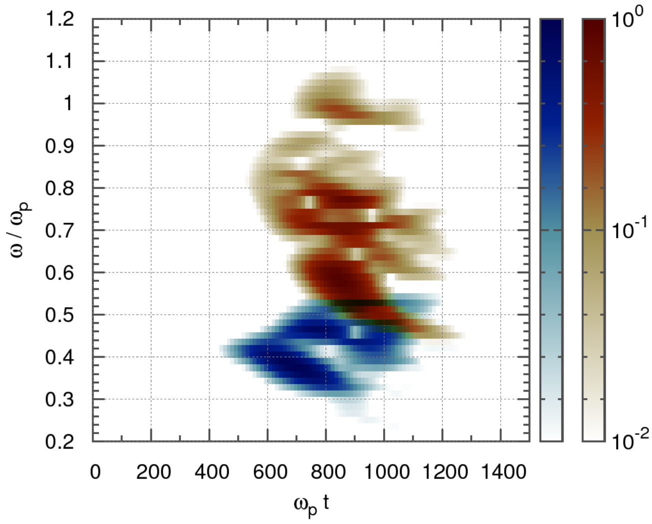

FIG. 2: Spectrogram of electric field. The supercritical mode k_1 is colored in blue, and the subcritical mode k_2 is colored in red. Colorbars on the right span show the values of the power spectrum (normalized by a maximum value), spanning two orders of magnitude.

In a first phase, $t < 500$, the amplitude grows exponentially, at twice the rate of the mother. In a second phase, $500 < t < 600$, the growth of both the mother and the daughter slows down, which can be interpreted as the mother saturating and the daughter responding to the mother. However, at $t \approx 600$, the growth of the daughter re-accelerates (even though the growth of the mother continues to slow down). In the spectrogram, we observe that this reacceleration phase, $600 < t < 650$, is associated with a frequency $\omega_1 \approx 0.71$, which is the linear frequency of the daughter. Furthermore, we observe that this occurs when a chirping branch of the mother approaches a frequency $\omega_2 \approx 0.36$, which is about half the above daughter frequency.

Since $k_2 = 2k_1$, we conclude that, here, the reacceleration of the daughter corresponds to a time where the velocity of a phase-space structure corresponding to a chirping branch of the mother approaches the resonant velocity of the daughter. Indeed, the linear resonant velocity of the daughter is $v_1 = \omega_1/k_1 \approx 7.1$, and the velocity of the phase space structure corresponding to the down-chirping branch of the mother at $t \approx 600$ is $v_2 = \omega_2/k_2 \approx 7.2$.

In similar simulations as well, we found this correspondence when a subcritical daughter emerges. However, we have only performed a handful of such simulations with varying parameters. Further work is required to present the evidence that this is not just fortuitous, confirm the implied mechanism, and investigate the underlying mechanism.

4 Conclusions

We have reviewed the developement of reduced models for energetic particle-driven, nonlinear excitation of subcritical instabilities. The first model, hybrid two-waves BB model combines a 1D kinetic equation with equations for period doubling, which models wave-particle interactions between two modes. This model was shown to reproduce key aspects of the experimental observation of Refs. [10, 11]. It interprets an abrupt GAM burst as a manifestation of the collaborative fluid-kinetic subcritical instability. In contrast with previously-known kinetic subcritical instabilities, the amplitude stays below the kinetic threshold, and chirping seems to be limited by a quasi-phase-matching condition with the mother mode. These results imply a new channel of mode excitation, which modifies the flow of energy in the system.

The novelty of this paper concerns the second model, the fully-kinetic two-waves BB model. We investigated whether two energetic particle-driven modes can interact via phase structures in the distribution function. This can be seen as a generalization of the previous study. Preliminary results suggest that a supercritical chirping mode can destabilize a subcritical mode. This occurs when a phase-space vortex associated with the chirping of the supercritical mode approaches the resonant velocity of the subcritical mode. Further analysis is underway in order to assess whether this is the dominant underlying physics, or whether other mechanisms (such as fluid-like energy exchange between waves) play important roles as well.

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