

Nonlinear Interactions of Low Frequency Alfven Eigenmodes

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Abstract. To accurately predict confinement properties of energetic particles (EP) in burning plasmas, self-consistent simulations must incorporate kinetic effects of thermal particles, nonlinear interactions of many shear Alfven eigenmodes (AEs), and cross-scale couplings of microturbulence and AE turbulence. Here we report gyrokinetic simulations of nonlinear interactions between beta-induced Alfven eigenmode (BAE) and beta-induced Alfven-acoustic eigenmode (BAAE), low frequency modes that have strong interactions with both thermal and energetic particles.

1. Introduction

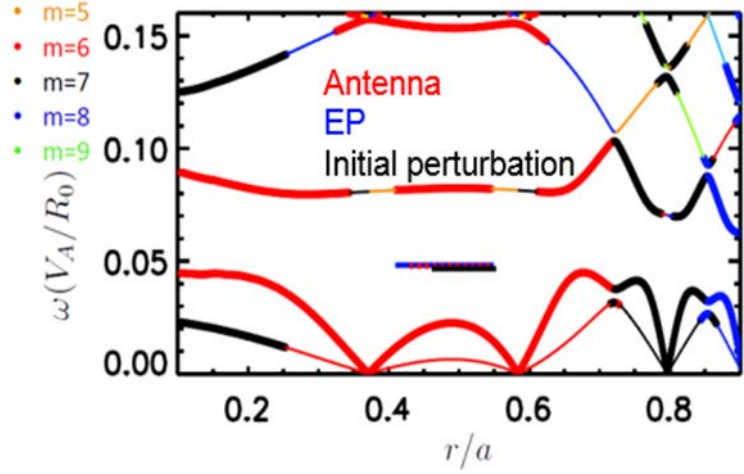
The Alfven eigenmodes, such as the beta-induced Alfven eigenmode (BAE), and beta-induced Alfven-acoustic eigenmode (BAAE) have attracted significant attention in recent tokamak studies. These modes can cause the loss of energetic particles and are deleterious to the plasma performance. The BAAE is formed through the coupling of the shear Alfven continuum and the acoustic continuum in the toroidal geometry. The BAAE has been observed in various tokamaks such as JET, NSTX, and DIII-D. Since the BAAE frequency is dependent on the safety factor in the tokamaks, it can also be used for the diagnosis of the safety factor.

Due to its low frequency, the BAAE can be strongly damped by thermal ions when ion temperature and electron temperature are comparable. Therefore, it has been debated whether the BAAE as predicted by the MHD theory could exist in the collisionless plasmas of fusion interest where BAAE is expected to be heavily damped by ion kinetic effects. A kinetic approach is thus required for the accurate simulation of the BAAE. In this work, the BAAE is verified and studied through global gyrokinetic particle simulations for the first time using the gyrokinetic toroidal code (GTC)^{1,2}.

2. Existence of BAAE

When ion temperature is much lower than electron temperature, the existence of the weakly damped BAAE is verified first in GTC simulations of a reversed shear plasma using initial perturbation, antenna excitation, and energetic particle excitation, respectively (**Fig. 1**). The BAAEs excited by the antenna and energetic particles have almost the same frequency and radial location as that in the initial perturbation simulation, which demonstrating the existence of BAAE. When the ion temperature is comparable to the electron temperature, the unstable BAAE can be excited by realistic energetic particle density gradient, even though the damping rate of the damped BAAE (in the absence of energetic particles) is comparable to the real frequency. In the simulations with reversed magnetic shear, BAAE frequency sweeping is observed and poloidal mode structure has a triangle shape with a poloidal direction similar to that observed in DIII-D tokamak experiments. The triangle shape in the poloidal direction changes and no frequency sweeping is found in the simulations with normal magnetic shear. Furthermore, BAAE frequency sweeping is observed in the reversed shear, consistent with the experimental observations. No frequency sweeping is found in the normal shear simulation³.

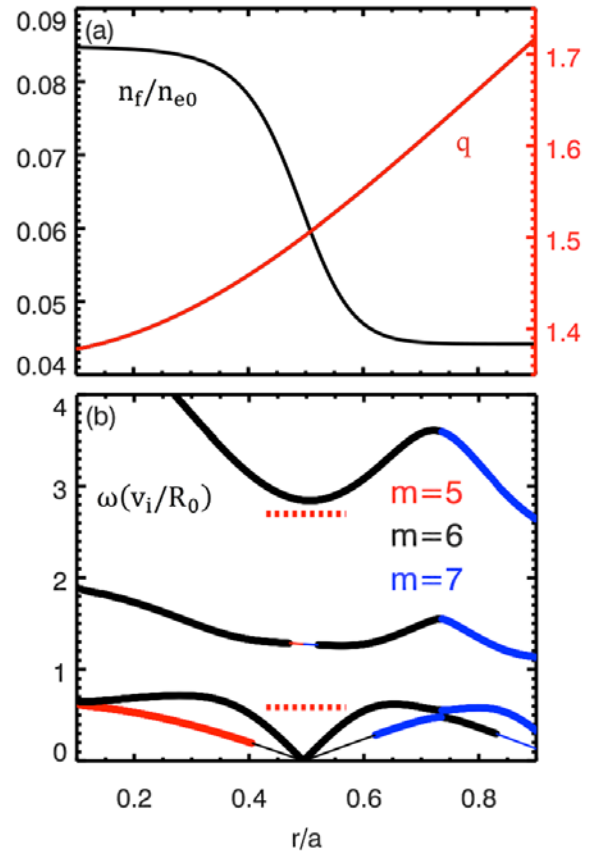
Fig. 1. BAAE frequency and Alfvén continuum for $n=3$ in a reversed shear plasma. The thick lines are Alfvén branches and the thin lines are sound branches. The horizontal lines indicate the BAAE frequencies and radial mode width from initial perturbation (solid black), antenna excitation (dashed red), and energetic particle excitation (solid blue).



3. Excitation of BAAE

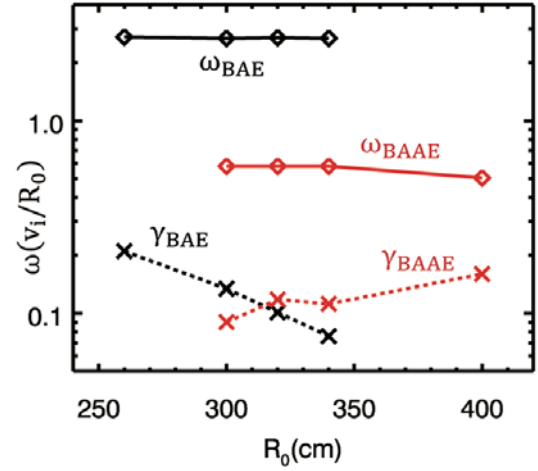
To elucidate the BAAE excitation mechanism, we use GTC to simulate a toroidal mode $n=4$ BAAE in a normal shear tokamak with concentric circular cross-section. The radial profiles of safety factor q and fast ion density n_f are shown in the panel (a) of **Fig. 2**. The Alfvén continua for $n=4$ are plotted in the panel (b), which shows three continua with the accumulation point frequencies of the BAE (upper minimum) and BAAE (lower minimum), and the zero-frequency at the rational surface of $r=0.5a$ with $q=1.5$. The density gradients of fast ions can excite both a BAAE (red dotted line in the lower gap) and a BAE (red dotted line in the upper gap) depending on the tokamak size.

Fig. 2. (a) Radial profiles of safety factor q and fast ion density n_f . (b) Alfvén continuum for $n=4$. The thick lines are Alfvén branches and the thin lines are sound branches. The horizontal dotted red lines indicate BAE (upper line) and BAAE (lower line) frequencies and radial mode width from energetic particle excitation.



By scanning the device size while keeping all other plasma parameters unchanged, we find that the BAAE dominates in the larger tokamak size (similar to ITER). For the smaller tokamak size (similar to current experiments), the dominant mode is BAE⁴⁻⁶. At some machine size, the BAE and BAAE can have the same growth rates (**Fig. 3**), an interesting regime where BAE and BAAE can co-exist and interact nonlinearly as demonstrated in the following nonlinear simulation.

Fig. 3. Frequencies (black) and growth rate (red) of BAAE (square with solid lines) and BAE (triangle with dashed lines) as a function of tokamak major radius.



The mode structure of BAAE is characterized by a triangle shape pointing in the counter-clockwise direction, while the BAE in the clockwise direction (**Fig. 4**). For the intermediate tokamak size, the mode structure has no clear triangle shape due to the coexistence of BAE and BAAE.

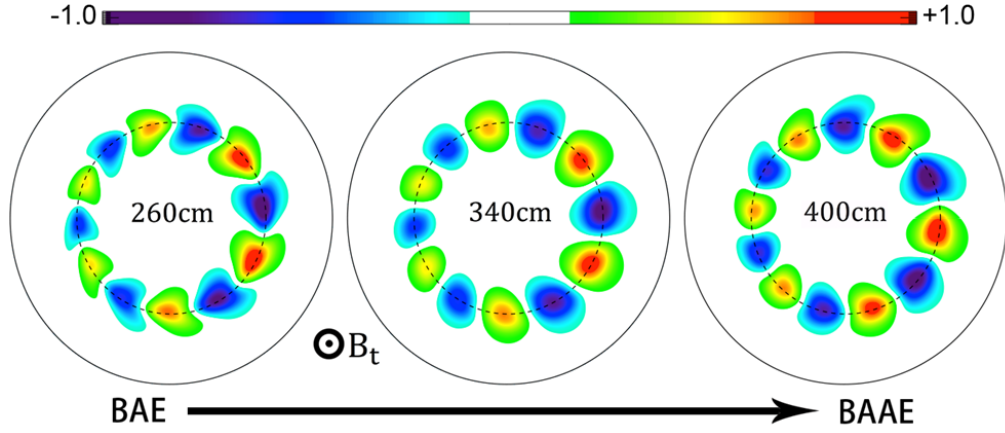


Fig. 4. Poloidal contour plots of electrostatic potential of the BAE and BAAE for various tokamak sizes.

It is remarkable that both BAE and BAAE can be excited simultaneously with the same growth rate, even though the damping rate (normalized by the real frequency) of the BAAE is much larger than that of the BAE. To understand the BAAE excitation mechanism, we examine the mode polarization and wave-particle energy exchanges in **Fig. 5**. The radial profiles of the electrostatic potentials are very similar for excited BAE and BAAE (panels *a* and *e*), and both have a dominant harmonics (6,4) (i.e., poloidal mode $m=6$ and toroidal mode $n=4$) and two sidebands (5, 4) and (7, 4) peaking at their respective rational surfaces with $q=m/n$. The electrostatic and net parallel electric fields structures are also similar between BAE and BAAE (panels *c* and *f*). The net parallel electric fields are much smaller than the electrostatic parallel electric fields, indicating that unstable BAE and BAAE are predominantly Alfvénic. In contrast, the damped BAAE (i.e., without fast ions) have two sidebands that are predominantly electrostatic, i.e., (5,4) and (7,4) are mostly acoustic branches (*c*). On the other hand, the damped BAE mode is still mostly Alfvénic (*g*). This difference in the polarization of the damped modes verifies that BAAE indeed forms due to coupling between Alfvén and acoustic waves. Furthermore, the polarization is quite different between the excited and damped BAAE, indicating the strong non-perturbative effects of the fast ions to the mode structure and polarization of BAAE. Therefore, the perturbative theory of BAAE is not adequate in describing the excitation of BAAE by fast ions.

Furthermore, we calculate the energy exchange between the thermal and fast ions and the waves (d and h). Both BAE and BAAE are excited by perpendicular energy transfer from fast ions, via toroidal precessional resonance with BAAE and transit, drift-bounce, precessional resonances with BAE. The parallel energy transfer from fast ions is quite small for both BAE and BAAE. In contrast, the damping by thermal ions is quite different between excited BAE and BAAE. The BAAE damping is dominated by parallel energy transfer to thermal ions, while BAE damping is dominated by perpendicular energy transfer to thermal ions.

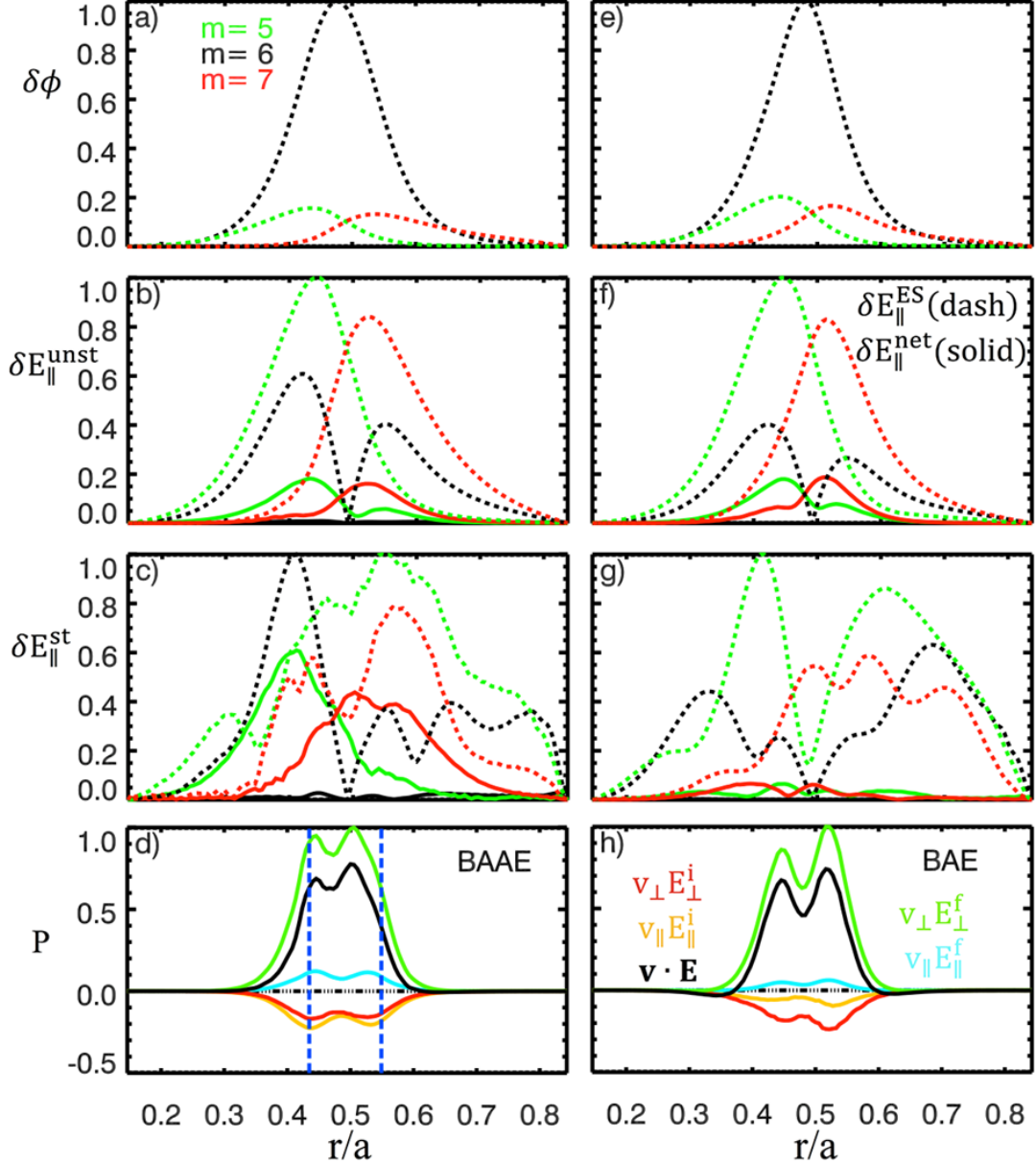


Fig. 5. Radial profiles of BAAE (left column) and BAE (right column) electrostatic potential (a and e), electrostatic and net parallel electric field of excited modes (b and f) and damped modes (c and g), and fast ions-wave energy exchanges of excited modes (d and h).

4. Nonlinear interaction of BAAE and BAE

At some machine size, the BAE and BAAE can have the same growth rates, an interesting regime where BAE and BAAE can co-exist and interact nonlinearly as demonstrated in the following nonlinear simulation. Analysis of fast ion phase space structures shows that the different dependence of toroidal precessional frequency and BAE/BAAE mode frequencies on the machine size induces the transition from BAE to BAAE when tokamak size increases.

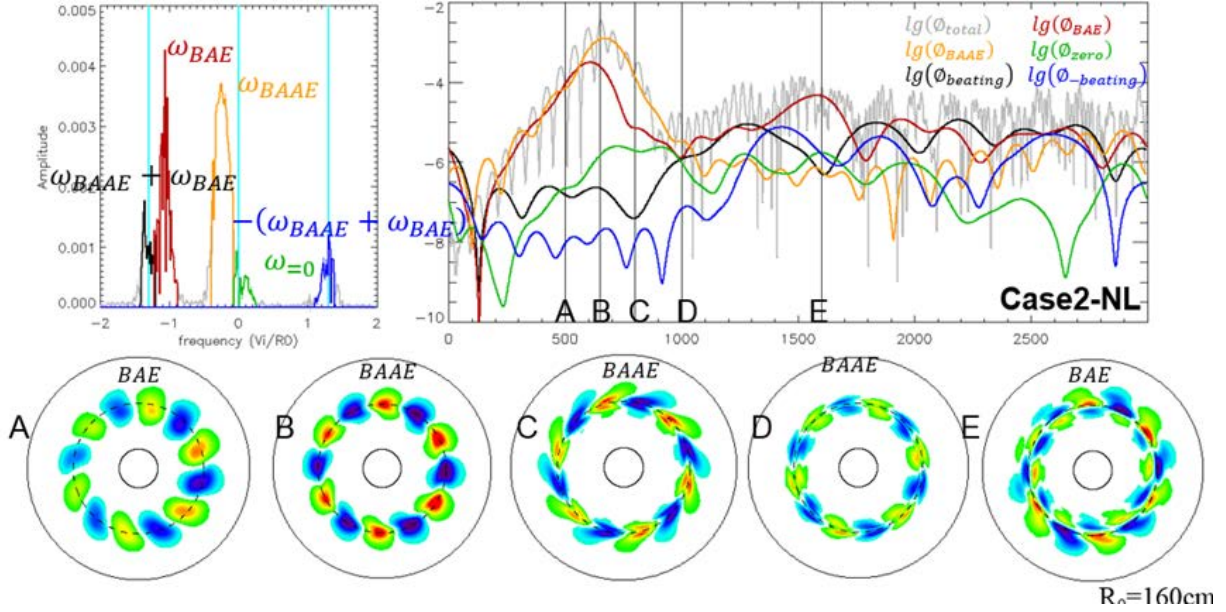


Fig. 6. Upper panels: the frequency (ω) spectrum (left) in the nonlinear simulation of BAE and BAAE, and the time history (right) of total amplitude (grey), ω_{BAE} (red), ω_{BAAE} (yellow), $\omega_{BAE} + \omega_{BAAE}$ (black), $-\omega_{BAE} - \omega_{BAAE}$ (blue), and $\omega \sim 0$ (green) modes. Lower panels: poloidal mode structures of BAE and BAAE at various times indicated in the upper right panel.

The nonlinear simulation results of the tokamak size where BAE and BAAE have the same growth rate are shown in **Fig. 6**. Zonal fields are not included in this simulation. In the linear regime, BAAE and BAE co-exist and have similar amplitudes. The mode structure is the superposition of BAE and BAAE (the time labeled as A). The BAE saturates first, followed by the saturation of the BAAE, which becomes dominant (B). The amplitudes of BAE and BAAE decrease significantly after saturation and various nonlinear modes including a very low frequency ($\omega \sim 0$), a beat wave frequency ($\omega_{BAE} + \omega_{BAAE}$), and its conjugate ($-\omega_{BAE} - \omega_{BAAE}$) (from B to D) are successively excited. The BAE can later become dominant (E). In this long time simulation, the amplitudes of all these modes oscillate with certain phase shift, indicating energy exchanges between these linear and nonlinear modes. Furthermore, we have also performed nonlinear simulation with zonal fields included. Simulation results show that zonal flows reduce the saturation amplitudes of BAE, BAAE, and nonlinear modes.

5. Conclusions

The beta-induced Alfvén-acoustic eigenmode (BAAE) in toroidal plasmas is verified by GTC simulations. The BAAE can be excited by realistic energetic particle density gradient, even though the

stable BAAE (in the absence of energetic particles) is heavily damped by the thermal ions. In the simulations with reversed magnetic shear, BAAE frequency sweeping is observed and poloidal mode structure has a triangle shape with a poloidal direction similar to that observed in tokamak experiments. When we decrease the tokamak size ITER to present-day tokamak, the most unstable modes change from BAAE to BAE (beta-induced Alfvén eigenmode). For a certain tokamak size, BAE and BAAE coexist with similar linear growth rates. At nonlinear stage, BAE modes saturate first, while BAAE modes continue to grow until nonlinear modes with beating wave (sum of BAE and BAAE frequency) and positive frequencies are excited. In the long time simulation, amplitudes of BAE, BAAE, and beat waves oscillate, indicating mode energy nonlinearly transfers between them. Zonal fields suppress the mode coupling and energy transfer between BAE to BAAE, and reduce frequency chirping and saturation amplitudes. The growth rate of the zonal fields is about twice of the linear growth rate of BAE/BAAE.

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