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Kinetics of Relativistic Runaway Electrons (a theory-biased perspective)

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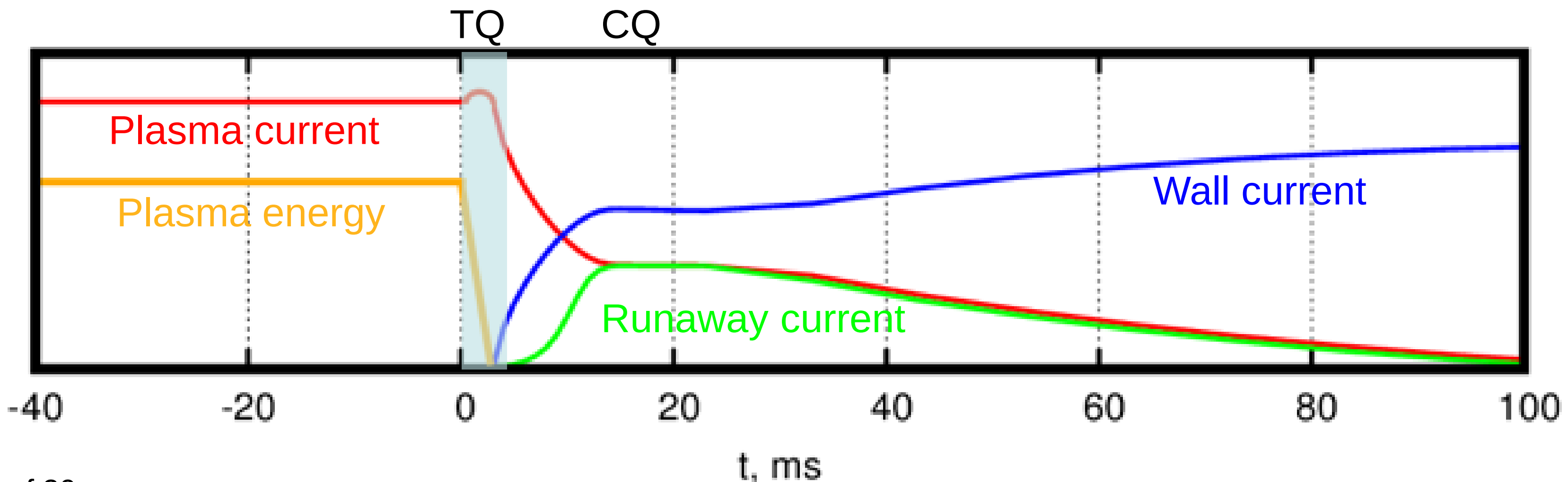
Outline

- Thermal quench and primary runaway production.
- Runaway avalanche and runaway sustainment.
- Damping of runaway current.
- Micro-instabilities.
- Summary.

Chain of events during plasma disruption

- Rapid cooling of plasma electrons (Thermal Quench) makes the plasma more resistive.
- Loop voltage increases to sustain toroidal current.
- Strong toroidal electric field produces seed runaway electrons.
- Runaways multiply via avalanche mechanism and become a large part of the total current (Current Quench).
- Toroidal current decays in line with dissipation of magnetic energy.

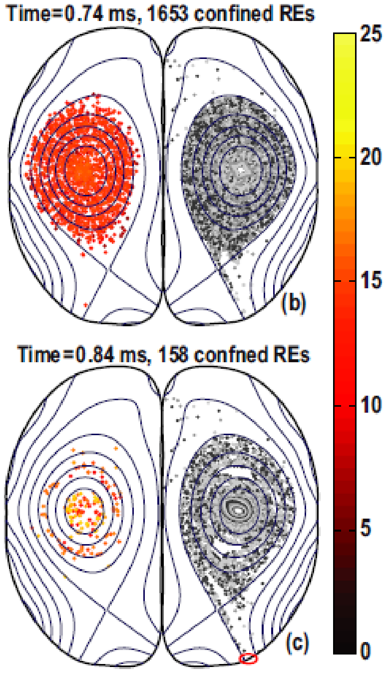


Electron cooling mechanisms 1/2

- Magnetic surface breakup enables electron heat losses along stochastic field lines

  RECHESTER, A. B., and ROSENBLUTH, M. N., PRL **40** (1978) 38.

- Magnetic stochasticity can also degrade confinement of runaway electrons
  - YOSHINO, R. , TOKUDA, S., Nucl. Fusion **40** (2000) 1293.
  - HAUFF, T., and JENKO, F., PoP **16** (2009) 102308.
  - PAPP, G., et al., PPCF **54** (2012) 125008.
  - ABDULLAEV, S.S., Magnetic Stochasticity in Magnetically Confined Fusion Plasmas, Springer-Verlag, Cham-Heidelberg (2014).



V.A. Izzo, et al, PPCF 54, 095002 (2012)

- Difficult items to control:

  electron current to the wall

  the level and spectrum of magnetic fluctuations

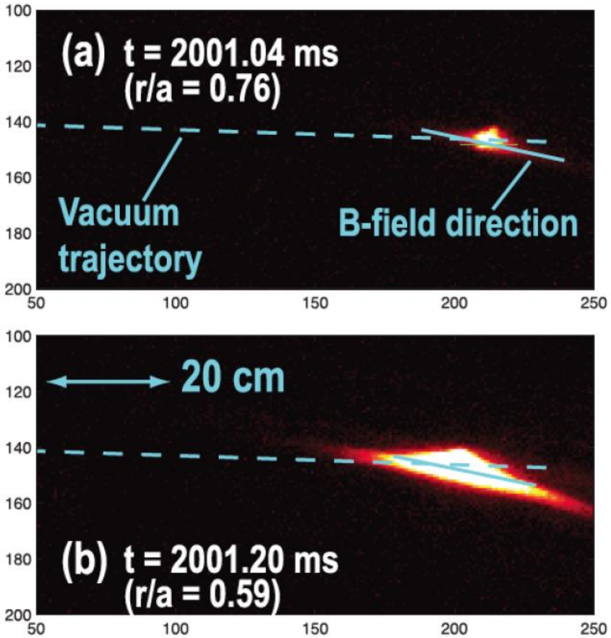
  magnetic islands

Electron cooling mechanisms 2/2

- Impurity influx is controllable experimentally and envisioned as the prevailing disruption mitigation technique.

- Line radiation and bremsstrahlung are expected to govern electron cooling; these two are being analyzed.

- Intrinsic non-uniformity of the impurities during thermal quench provides another powerful cooling mechanism for electrons: ambipolar expansion of the plasma blobs (not yet included in modeling).



HOLLMANN, E., et al. Nucl. Fusion **57** (2017) 016008

- Difficult aspects:

  proper delivery of impurities

  (MGI and pellet injection)

Key ingredients in runaway kinetics

- Driving force from the inductive electric field.
- Electron drag (small-angle collisions with free and bound electrons).
- Elastic scattering (collisions with plasma ions and impurity ions).
- Synchrotron radiation (in partnership with high-Z elastic scattering).
- Knock-on collisions.
- Bremsstrahlung.
- Radial transport.

Runaway kinetics and separation of time scales

- Kinetic equation:

$$\frac{\partial F}{\partial t} + eE\left(\frac{1}{p^2}\frac{\partial}{\partial p}p^2\cos\theta F - \frac{1}{p\sin\theta}\frac{\partial}{\partial\theta}\sin^2\theta F\right) = \hat{C}F + \hat{R}F + \hat{S}F$$

Small-angle collisions:

$$\hat{C}F = \frac{mc}{\tau}\left(\frac{1}{p^2}\frac{\partial}{\partial p}\left(p^2 + m^2c^2\right)F + \frac{(Z_{eff}+1)}{2\sin\theta}\frac{mc\sqrt{p^2+m^2c^2}}{p^3}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta}F\right)$$

Synchrotron radiation reaction:

$$\hat{R}F = \frac{mc}{\tau_{rad}}\left[\frac{1}{m^2c^2p^2}\frac{\partial}{\partial p}p^3\sqrt{m^2c^2+p^2}\sin^2\theta F + \frac{1}{p\sin\theta}\frac{\partial}{\partial\theta}\frac{p\cos\theta\sin^2\theta}{\sqrt{m^2c^2+p^2}}F\right]$$

Knock-on collisions (Møller source): $\hat{S}F$

Time scales:

$$\tau \equiv \frac{m^2c^3}{4\pi n_e e^4 \ln\Lambda}$$

$$\tau_{rad} \equiv \frac{3m^3c^5}{2e^4B^2}$$

- Møller source ($\hat{S}F$) is weaker than electron drag (by Coulomb logarithm).

  MØLLER, C., Ann. Phys. (Leipzig) **14**, 531 (1932)

- Bulk electron density includes ½ of bound electrons.

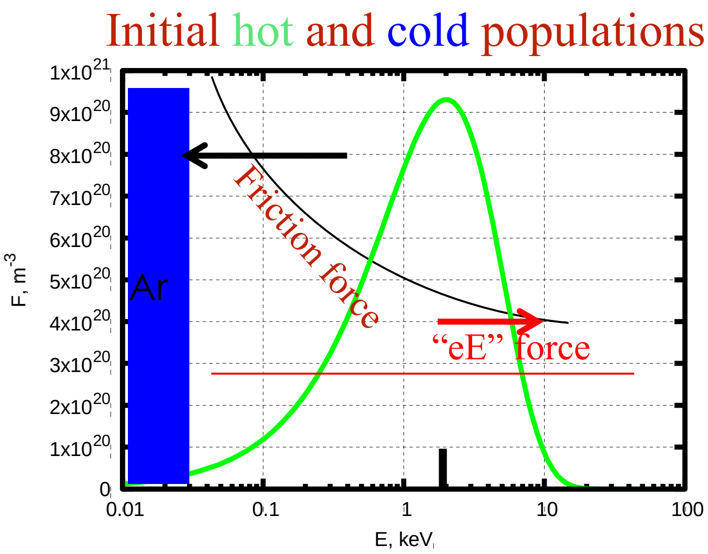
  ROSENBLUTH, M.N., and PUTVINSKI, S.V., Nucl. Fusion **37** (1997) 1355.

- Effective ion charge takes into account bound electron shielding.

  ZHOGOLEV, V.E., KONOVALOV, S.V., VANT series Nucl. Fusion. **37**, (2014) 71.

Components of seed generation model (TH/P4-2)

- Maxwellian pre-quench electrons (n_0, T_0).
- Impurity atoms delivered instantaneously ($n_{cold} > n_0$ after ionization).
- Kinetic description of the hot electrons.
- Spitzer conductivity of the bulk plasma.
- Constant current density during TQ; the electric field evolves accordingly.



- Bulk plasma temperature is determined by:
  - energy release by hot electrons
  - Ohmic heating
  - line radiation SUMMERS, H.P., et al., PPCF **48** (2006) 263.

Seed formation during plasma cooling

Thermal quench scenario:

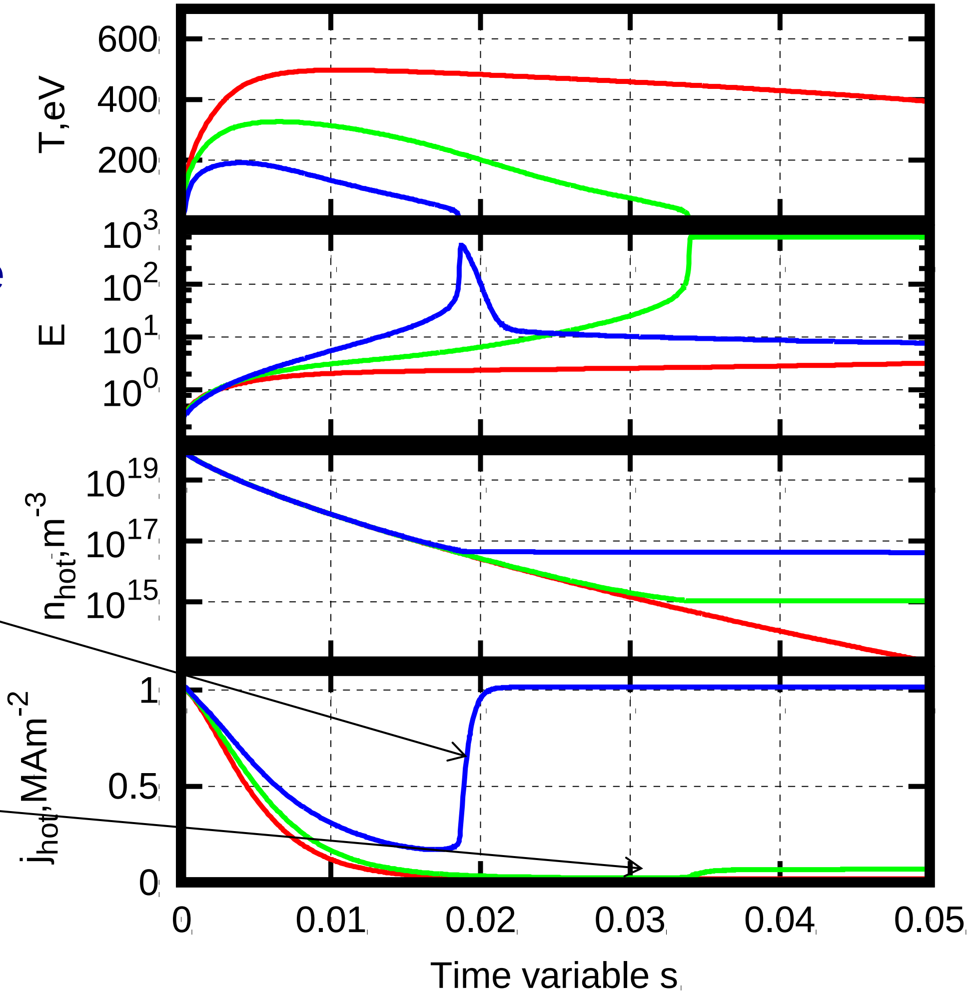
① Hot electrons heat the "cold" bulk
② The bulk overtakes a fraction of the current
③ Bulk conductivity drops due to radiative losses.

There are two possible outcomes:

1. Prompt conversion regime
(low energy REs carry the total current at low electric field).

2. Seed for avalanche regime
(large bulk current at high electric field -> high energy seed + avalanche).

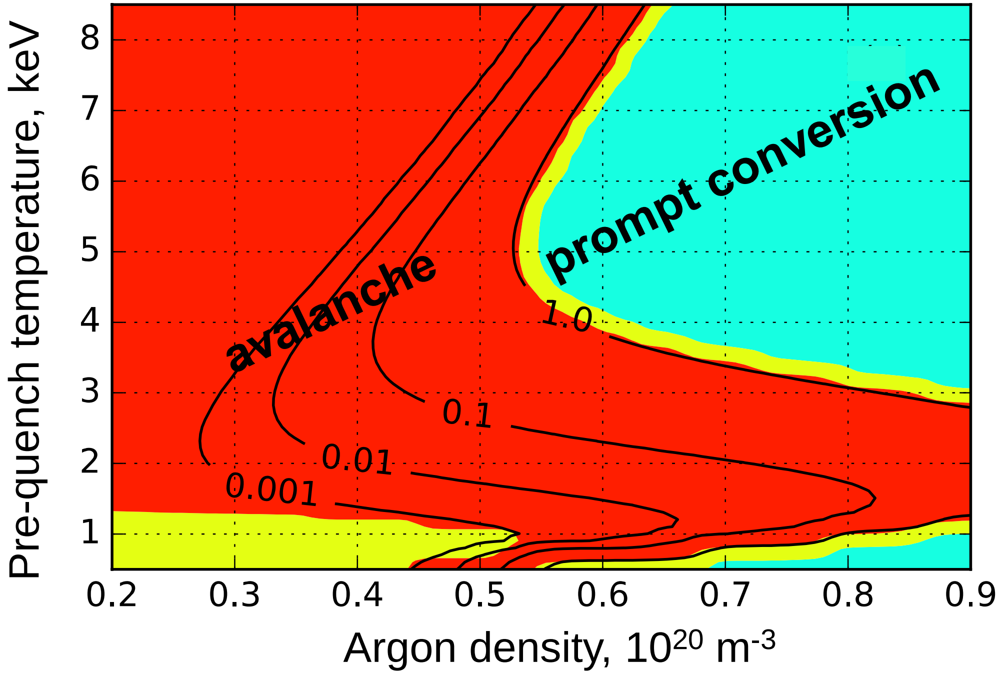
Argon densities: 0.5, 0.3, 0.2



RE seed current is determined by competition between bulk plasma cooling and hot electron cooling

Pre-avalanche RE current

Contour plot of the surviving hot electron population
The **prompt conversion** area represents 100% of sub-MeV RE current



- ❑ The RE seed current grows with impurity density up to prompt conversion (a trend opposite to HARVEY, R.W., et al., PoP **7** (2000) 4590).
- ❑ The RE seed current has a non-monotonic dependence on pre-quench temperature with a maximum at T_0~4keV

Fast avalanche

- Circuit equations for the total current I and runaway current I_{re}:

$$\frac{\partial I}{\partial s} = -(I - I_{re})$$

$$\frac{1}{I_{re}}\frac{\partial I_{re}}{\partial s} \approx \frac{l_i}{\sqrt{Z+5}\,\ln\Lambda}\frac{\left(I - I_{re}\right)}{I_A}$$

$$\ln\frac{I_{re}(\infty)}{I_A} + \frac{l_i}{\sqrt{Z+5}\,\ln\Lambda}\frac{I_{re}(\infty)}{I_A} = \frac{l_i}{\sqrt{Z+5}\,\ln\Lambda}\frac{I(0)}{I_A} + \ln\frac{I_{re}(0)}{I_A}$$

Notations:

l_i - internal inductance (order of unity quantity)

s - time in the units of the Ohmic decay time

I_A - Alfvén current (17kA)

The runaway current grows exponentially and saturates when it takes large part of the total current.

The final runaway current is insensitive to bulk plasma resistivity.

MARTIN-SOLIS, J.R., LOARTE, A., and LEHNEN, M., PoP **22** (2015) 082503

Slow avalanche

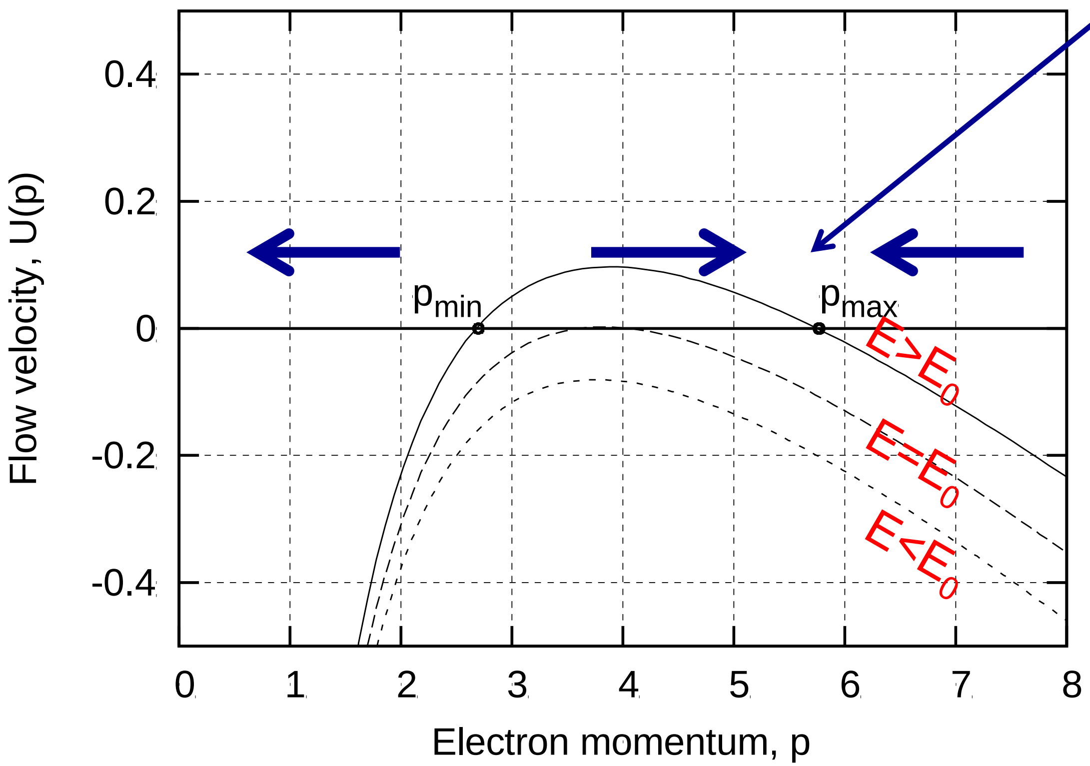
- Seed runaway electrons produce secondary electrons via large-angle (Møller) collisions with the bulk. .
- Møller source is weaker than electron drag (by Coulomb logarithm).

- Two-step description of the runaway avalanche:
  ① Examine sustainment of the runaways in the absence of knock-on collisions
  ② Use the distribution function of the sustained runaways to predict their multiplication or loss due to knock-on collisions.

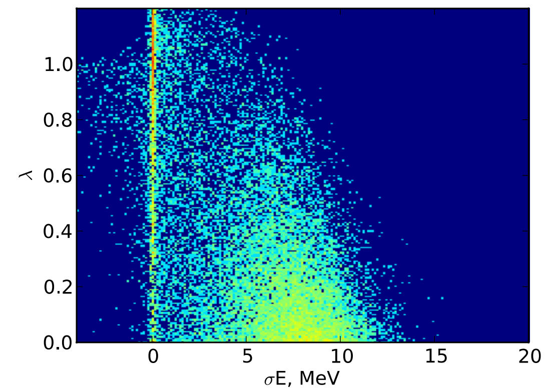
- Items of interest:
  - Critical electric field for avalanche onset
  - Avalanche growth rate
  - Runaway distribution function

Momentum space attractor
(synchrotron losses preclude unlimited acceleration)

Kinetic equation can be reduced to continuity equation in momentum: $\frac{\partial G}{\partial t} + \frac{\partial}{\partial p} U(p;E) G = 0$



Peaking of the distribution function around p_{max}



ALEYNIKOV et al., 25th FEC (2014)
HIRVIJOKI et al., JPP (2015)
DECKER et al., PPCF (2016)

The roots p_{min} and p_{max} merge at a certain electric field $E=E_0$.
This is the minimal electric field required for runaway sustainment.

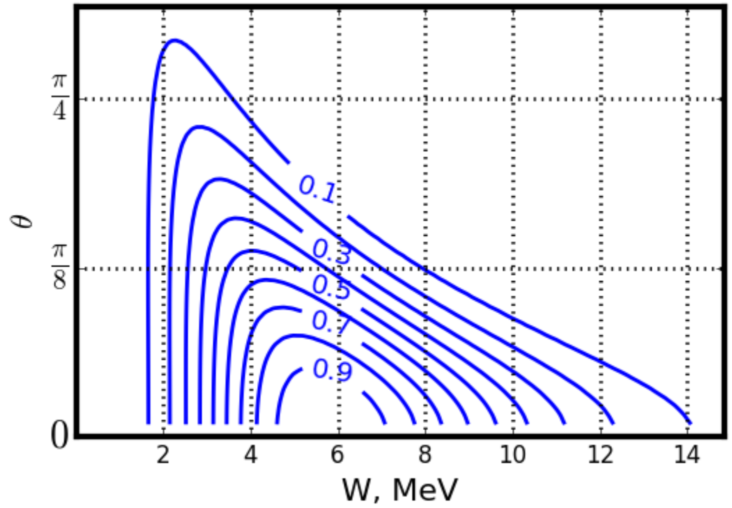
PRL **114** (2015) 155001

Distribution function of the sustained electrons

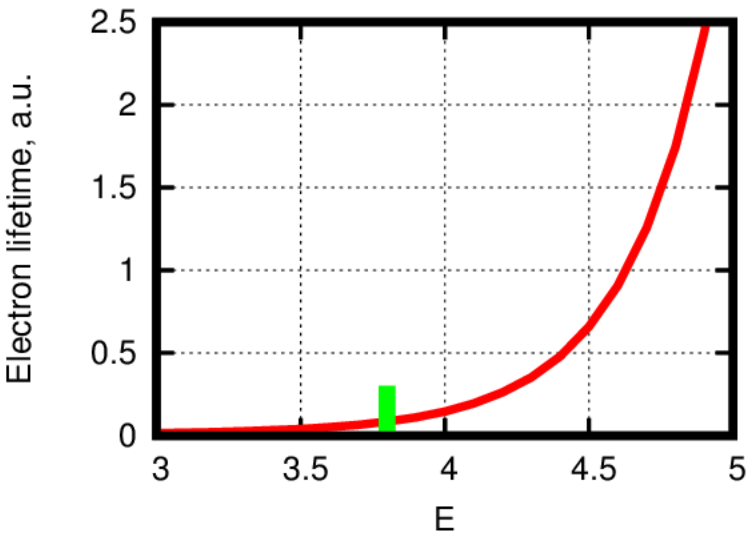
Sustainment condition for fast electrons:

PRL **114** (2015) 155001

$$\frac{E_0}{E_{Connor\_Hastie}} > 1 + \frac{\sqrt{2}\left(Z_{eff}+1\right)/\sqrt{t_{rad}/t}}{\sqrt[6]{1+8\left(Z_{eff}+1\right)^2 t/t_{rad}}}$$



Self-similar electron distribution at the phase space attractor.



Steep growth of the attractor lifetime with the electric field. The threshold field is marked green.

Source of knock-on electrons

The simplified source [ROSENBLUTH and PUTVINSKI (1997)] implies
ultra-relativistic primary electrons without angular spread.

The needs to relax these constraints:

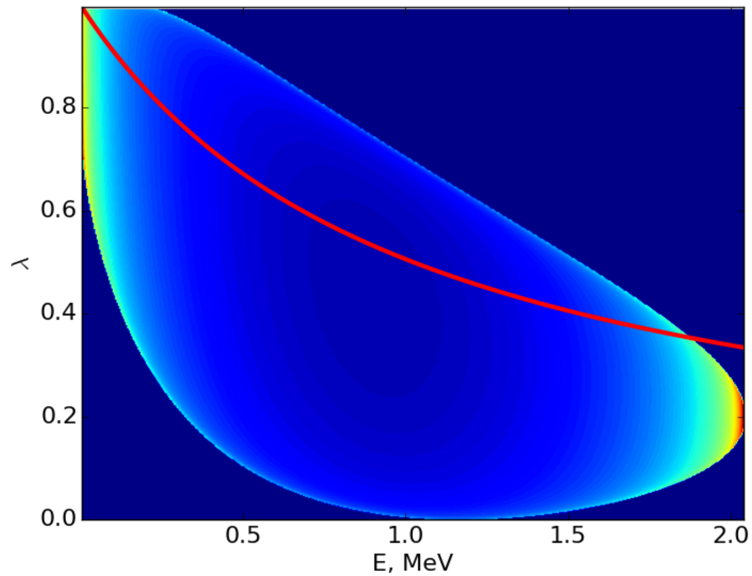
- runaway scattering on high-Z impurities
- energy limitation by synchrotron radiation
- moderate energy of primary electrons at high electric field

Exact source : $$S = n_{cold} c \frac{\sqrt{\gamma_0^2 - 1}}{\gamma_0} \left\langle \delta\left[\cos\theta - \cos\theta_p\right]\right\rangle \frac{d\sigma}{d\Omega}$$

$d\sigma / d\Omega$ = Møller cross-section

The gyro-averaged δ-function is

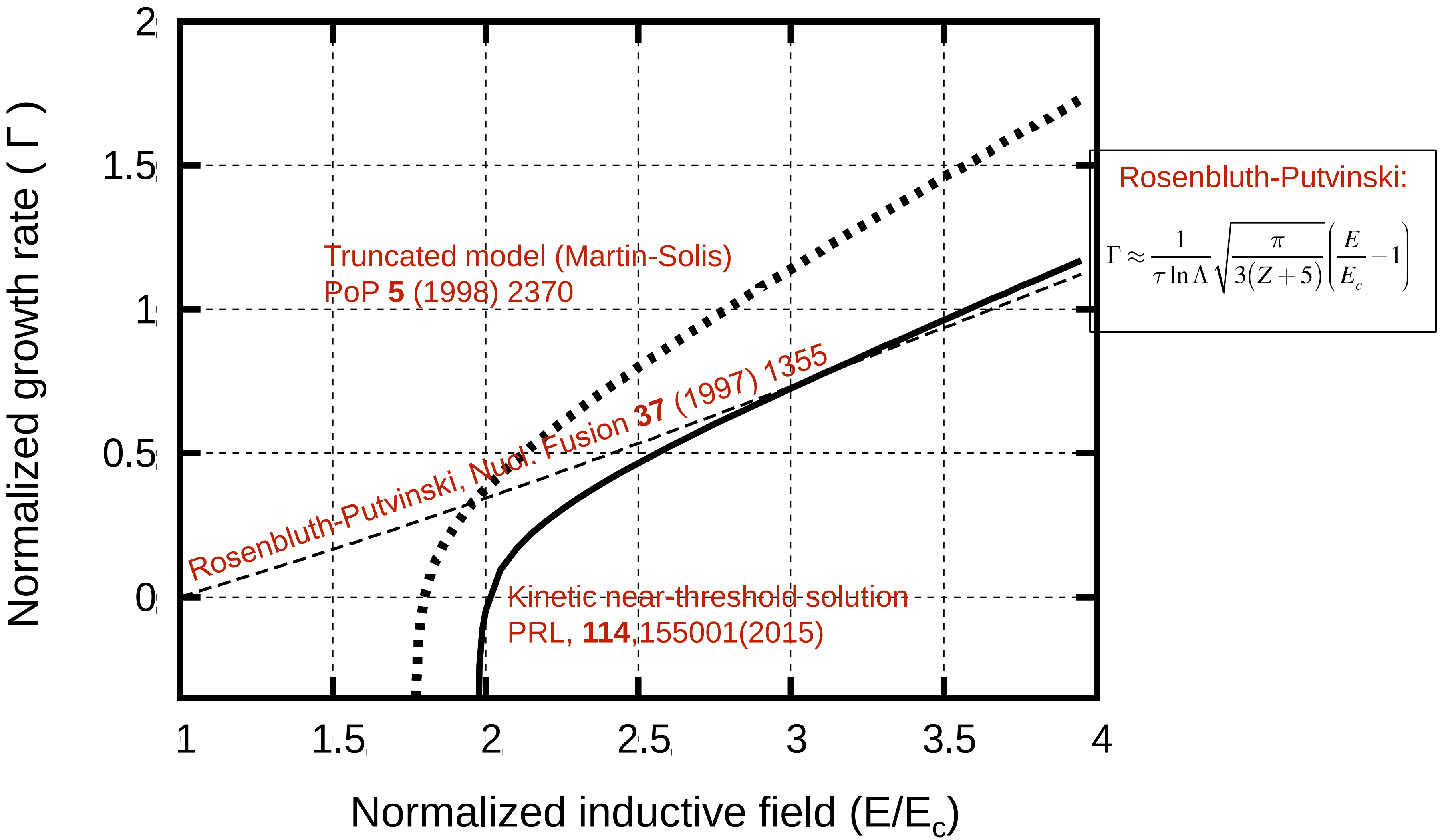
$$\left\langle \delta\left[\cos\theta - \cos\theta_p\right]\right\rangle = \frac{1}{\pi} \frac{1}{\sqrt{\left(\frac{p_\perp p_{0\perp}}{p p_0}\right)^2 - \left(\frac{p_\parallel p_{0\parallel}}{p p_0} - \sqrt{\frac{\gamma-1}{\gamma+1}}\sqrt{\frac{\gamma_0+1}{\gamma_0-1}}\right)^2}}$$



Red curve - shape of the simplified source. Color-coded – exact source for primary electrons with $\gamma_0 \approx 5$ and $\lambda_0 = 0.2$.

ALEYNIKOV, P., et al., Proc. 25th IAEA Fusion Energy Conf, 2014, pp. TH/P3-38.

Avalanche growth rates comparison



Marginal stability model for runaway current decay

- The characteristic time-scale of runaway electron production via avalanche mechanism is much shorter than the current decay time-scale.

- The local inductive electric field must be close to the avalanche threshold $E_0(n; Z_{eff}; B)$ to maintain runaway current at any flux surface.

- Nonlinear "Ohm's law": $j_{||} > 0 \;\Rightarrow\; E_{||} = E_0; \quad j_{||} = 0 \;\Rightarrow\; E_{||} < E_0$

- Faraday law: $\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial E_{\parallel}}{\partial r} = \frac{4\pi}{c^2}\frac{\partial j_{\parallel}}{\partial t}$

- Current decay is roughly linear in time with $t_{decay} = \frac{I_0}{c^2 E_0}$

- Caveat: vertical instability and kink instability may develop on this time-scale (currently under consideration [KONOVALOV, S.V., et al., TH/7-1])

Nucl. Fusion **54** (2014) 072002
PRL **114** (2015) 155001

Microinstabilities

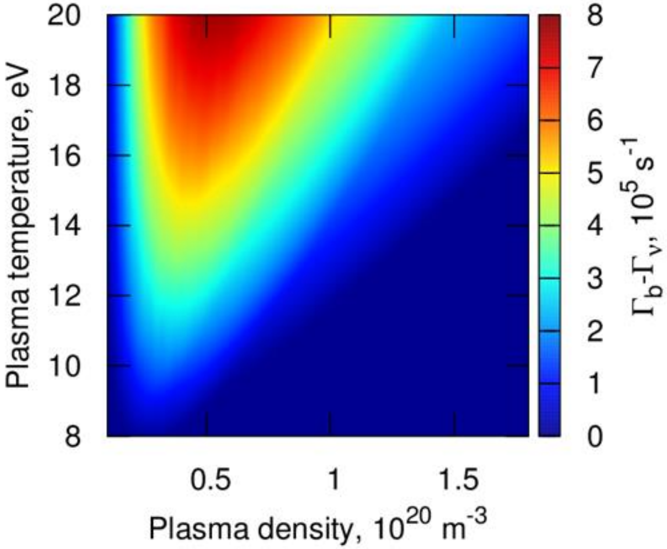
- ❑ Microinstabilities can enhance runaway scattering:
  "Fan" instability observed in early experiments and explained

  ALIKAEV et al., Sov. J. Plasma Phys. (1975)
  PARAIL and POGUTSE, Nucl. Fusion (1978)

Modes of interest:

- ❑ Electron cyclotron waves
- ❑ Magnetized plasma waves
- ❑ Whistlers

  [FÜLÖP, SMITH, and POKOL, PoP **16** (2009) 022502]

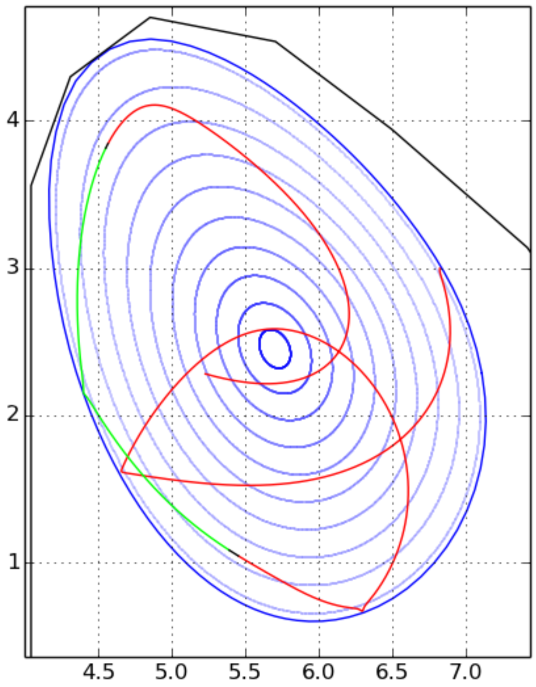


Maximized growth rate shows existence of instability after TQ for ITER-relevant parameters.

Reassessment:

- ❑ Collisional damping, convective damping and instability threshold have been revised substantially [Nucl. Fusion **55** (2015) 043014].
- ❑ Kinetic drive quantified for ray-tracing calculations.

Statistical analysis (COIN code)



Wave transformations:

Red - whistler wave

Green - magnetized plasma wave

Wave trajectories diverge after multiple reflections and mode transformations

This randomness calls for statistical approach:

1. Launch many waves at a reference temperature
2. Find trajectories with maximum amplification factor
3. Scale damping with temperature
4. Find minimal temperature for instability to appear.

Fast analysis of several million wave packets determines instability window

Summary

- Self-consistent kinetic modeling of primary runaway formation during thermal quench (prompt conversion of the plasma current into runaway current is feasible for heavy injection of impurities).

- Kinetic near-threshold theory for runaway sustainment and multiplication in presence of synchrotron losses (enhanced critical electric field found for avalanche onset).

- Marginal stability scenario for runaway-dominated current quench (runaway avalanche threshold determines the current decay time-scale).

- Revised thresholds of runaway-driven micro-instabilities (instability window quantified for ITER-relevant parameters).