

Objectives

- Conventional Fault Tree technique is the main tool currently used for safety analysis of complex engineering systems such as Nuclear Power Plants (NPPs).
- It utilizes failure rates of every individual basic event constructing the tree to calculate the occurrence probability of the top undesired event.
- However, it is very difficult to get the exact estimate for the failure probabilities of occurrence when fault events are imprecise such as human error.
- That is due to due to insufficient data, environment changing or new components.
- To overcome the over and under estimation of the failure probability of basic events as well as the probability of the undesired event of Fault Tree Analysis (FTA) Fuzzy numbers can be applied to estimate failure rates by handling linguistic terms.
- An approach for the integration of fuzzy theory and FTA has been presented to solve the conventional FT problem.
- In this approach fuzzy fault rate (FFR) are derived from the conversion of fuzzy possibility scores (FPS) using a transformed function. The procedures are presented.

Fuzzy FTA Approach

It is natural for the safety engineers to employ linguistic expression to estimate imprecise or ill-defined event. The fuzzy numbers, the linguistic variables, the converting fuzzy number into FPS and the transforming FPS into FFR have been presented.

- Fuzzy probability is used to determine the fuzzy failure rate of the basic events based on the statistical data, influencing factors and expert judgments.

Fuzzy numbers

- To represent the linguistic values, fuzzy number is used.
- Fuzzy number is used to handle imprecise information such as 'High', 'Low' reliability,
- A fuzzy set in probability space represents a fuzzy number between 0 and 1 which can be assigned to the probability of an event.
- There are many forms of fuzzy numbers used to represent the linguistic values. e.g. Triangular fuzzy number and Trapezoidal fuzzy number. Among the various shapes of fuzzy number, Triangular fuzzy number is the most popular one.
- The triangular fuzzy number is defined by a triplet $A = [a, b, c]$ with a membership function $f_A(x)$ and $\mu_A(x)$ as shown in the Figure below (1):
- Fuzzy number also called fuzzy probability which is expressed by a fuzzy set and characterized by its membership function as shown in the following Equation.

Let $x, a, b, c \in (\text{Real line } -R)$. A triangular fuzzy number is a fuzzy number A in R , if its membership function $f_A: R \rightarrow [0, 1]$ is;

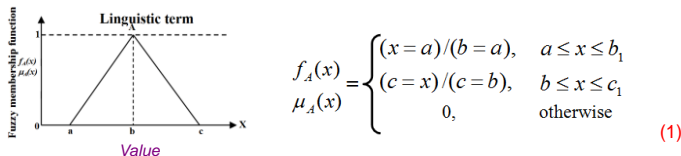


Fig.1 Triangular fuzzy number

Where $f_A(x)$ or $\mu_A(x)$ is the membership value of the element x in the subset A

- The parameter 'b' is the mean of the fuzzy number and parameters 'a' and 'c' are the lower and upper bounds.
- The left and right expansions of triangular fuzzy number and the confidence level of probability of uncertain events can be obtained from statistical data and expert judgment of the system.

Linguistic Variable

- It is natural for the safety engineers to employ linguistic expression to estimate imprecise or ill-defined event.
- Linguistic variables are variables whose values are words or sentences.
- A fuzzy set is useful for transforming these linguistic terms into graphical or mathematical representations.
- The triangular and trapezoidal typed membership functions for seven linguistic variables, Very Low (VL), Low (L), Fairly Low (FL), Medium (M), High (H), Fairly High (FH), and Very High (VH) are utilized to represent the likelihoods of occurrence of fault events as shown in the following Figure (2).
- These terms are defined as a fuzzy set whose members are probabilities of failures.

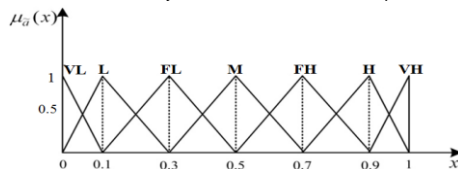


Fig.2 Fuzzy numbers represent linguistic value

Fuzzy Possibility Score Evaluation

- Since the final ratings obtained by using Eq. (1) are fuzzy numbers, it is necessary to convert the fuzzy number to a crisp score, referred as (FPS). FPS is a crisp score that represents experts' belief of the most possible value that an event may occur.
- For group evaluation, it is required to aggregate different expert opinions into one.
- The aggregation is a result of the union of two or more membership functions. Fuzzy max-min aggregation operator is employed because of its simplicity and efficiency. and given by the following expression

$$\mu_A(x) = \max(\min[\mu_1(x), \mu_2(x), \dots, \mu_n(x)]) \quad (2)$$

where $\mu_A(x)$ is the membership value of the element x in the subset A ; $\mu_1(x), \mu_2(x), \dots, \mu_n(x)$, are membership values representing the 1st, 2nd, ..., and the nth expert's assessment, respectively.

The Union in Fuzzy set theory is the equivalent of the OR operation in Boolean algebra.

$$\mu_{A \cup B}(x) = \max(\mu_A(x), \mu_B(x)) \quad \text{for all } x \text{ (fuzzy union)}$$

The Center of Area (COA) defuzzification method is applied because of its computational simplicity and efficiency. Defuzzification is the process of converting the degrees of membership of output linguistic variables into numerical values.

where COA is the center of area, x is the value of the linguistic variable, x_{\min} and x_{\max} represent the range of the linguistic variable.

Fuzzy Fault Rate Estimation

Most hardware failure rate data can be obtained from reliability data handbook. In order to ensure compatibility between the non-fuzzy failure rate of hardware (real number) and the fuzzy possibility score of experts' evaluation data, the fuzzy failure rate (FFR) can be obtained from the FPS and is defined as:

Fuzzy logic operator

$$COA = \frac{\int_{x_{\min}}^{x_{\max}} f(x) \cdot x \, dx}{\int_{x_{\min}}^{x_{\max}} f(x) \, dx} \quad (3)$$

$$FFR = \begin{cases} \frac{1}{10^6} & FPS \neq 0 \\ 0 & FPS = 0 \end{cases}$$

where

$$K = \left[\frac{1-FPS}{FPS} \right]^{1/5} * 2.301 \quad (4)$$

Supposing that the fuzzy triangular number P_1 and P_2 can be separately expressed by (a_1, b_1, c_1) and (a_2, b_2, c_2) , where P_1 and P_2 are the probability values of basic event. The algebraic algorithms about the fuzzy number P_1 and P_2 for the logic operations of AND gate and OR gate are shown as follows:

1. "OR gate" fuzzy operators. In conventional FTA, "OR gate" operator is: $POR = \Pi (1 - P_i)$ in which P_i represents the precise probability of event i is P_i , then "OR gate" fuzzy operator can be denoted by:

$$P_{OR} = 1 - \prod_{i=1}^n (1 - P_i) = 1 - \left[\prod_{i=1}^n (1 - a_i), \prod_{i=1}^n (1 - b_i), \prod_{i=1}^n (1 - c_i) \right] \quad (5)$$

If the events are dependent, then the algorithm is $POR = \max(P_1, P_2, \dots, P_n)$

Where Π denotes the fuzzy multiplication

2. "AND gate" fuzzy operators. In conventional FTA, "AND gate" operator is: $PAND = \Pi P_i$ in which $P_i (i = 1, 2, \dots, n)$ represents the precise probability of the event i is P_i . Fuzzy operator is:

$$P_{AND} = \prod_{i=1}^n P_i = \left[\prod_{i=1}^n a_i, \prod_{i=1}^n b_i, \prod_{i=1}^n c_i \right] \quad (6)$$

If the events are dependent, then the algebraic algorithm is: $P_{AND} = \min(P_1, P_2, \dots, P_n)$.

Illustrative Example

- Assume that 2 grinding machines are working next to each other. What is the possibility that people coming into the vicinity of the machines are injured by getting a chip into the eyes?

The fault tree for the main event that somebody will be injured can be constructed as shown in Figure 3. Since any of these events is capable of producing the top undesirable event, an OR gate is utilized.

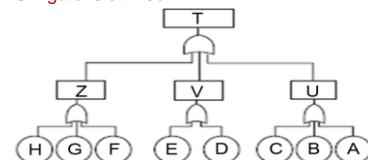


Fig.3 Fault tree for the accident

- The basic events contributing to the accident are summarized.
- The results of expert judgments to assess fault likelihood of each event are given in Table 1.

Table 1: Opinion of the expert

Event	1 st	2 nd	3 rd	4 th	5 th	6 th
A	H	VH	VH	H	H	M
B	VH	H	VH	VH	VH	H
C	H	M	M	H	H	H
D	H	VH	VH	H	H	H
E	H	VH	H	VH	H	VH
F	M	VH	M	M	VH	H
G	M	M	VH	M	H	M
H	H	M	H	H	M	H

- By the use of the foregoing procedures and the whole expert's evaluation, the FPS and the corresponding FFR for each event can be derived.
- By using the estimated FFR and Eq.(5), the failure rate for event A is determined.
- Likewise the failure possibilities for events B and C are calculated and then the failure rate of the top event (T) can be calculated.

Conclusion

- Fuzzy fault tree approach to overcome the drawback of FTA was proposed.
- The combination of FTA with fuzzy logic will help in artificially generating the unavailable data
- The details of fuzz number design are described and an application example of the method also provided
- Fuzzy numbers were applied to evaluate failure rates by handling linguistic terms.
- In the suggested approach, opinions of experts are taken into consideration while considering the probability of occurrence of each basic event of the fault tree.
- The procedures for determining FPS and the method for converting FPS into FFR using a transformed function were presented.