Max-Planck-Institut für
Plasmaphysik

Advances in stellarator gyrokinetics

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and

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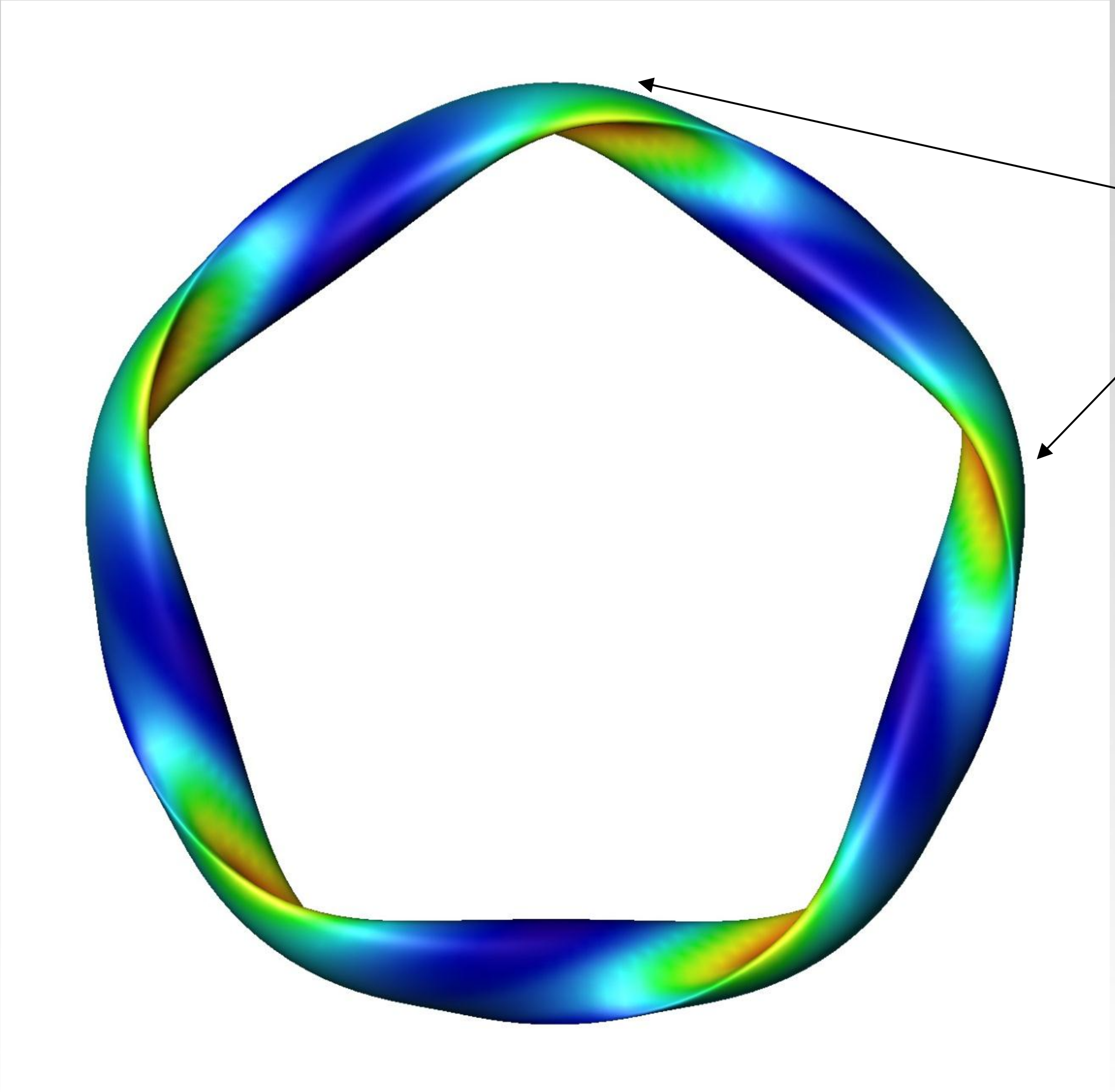
Background

- Wendelstein 7-X will start experiments in 2015
  - optimised for low neoclassical transport
- Turbulence?
- Electrostatic instabilities:
  - ion-temperature-gradient (ITG) driven modes
  - trapped-electron modes

W7-X from above



Bad curvature

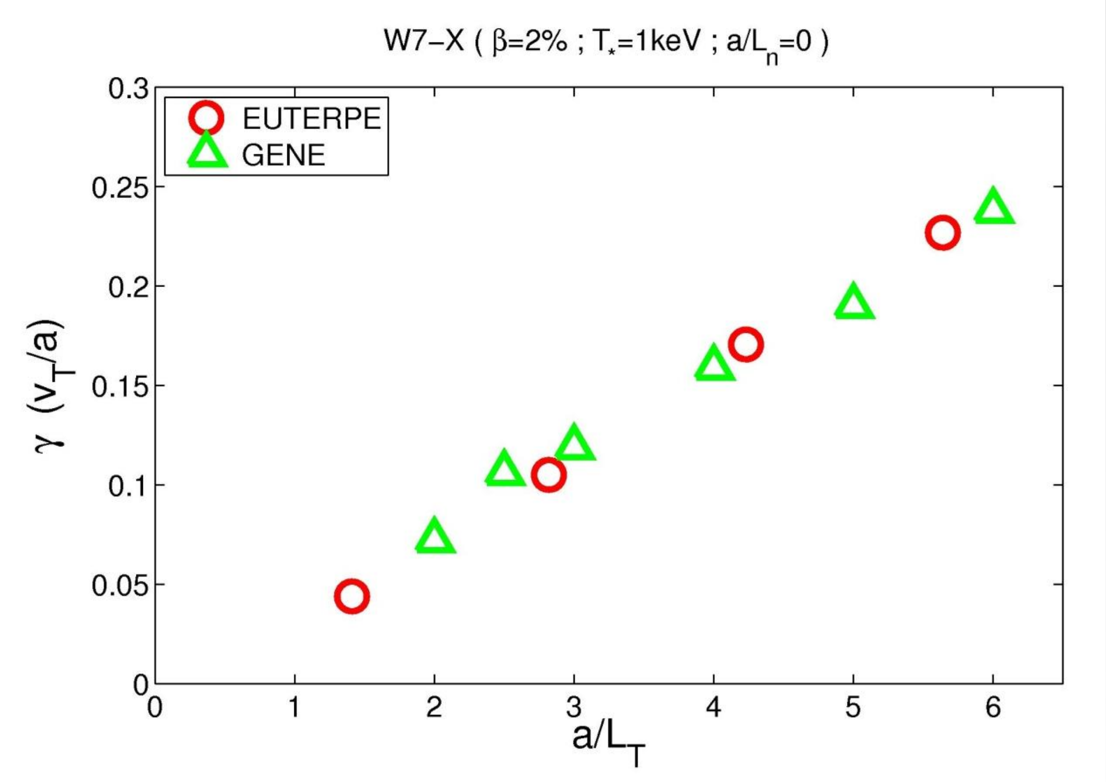
$q'(r) < 0$ everywhere

Gyrokinetic stellarator codes

- EUTERPE
  - global, particle-in-cell, linear in 3D
  - see poster TH/P4-49 by A.Mishchenko
- GENE
  - radially local (flux-tube or full-surface), continuum, nonlinear
- Both codes: electromagnetic, collisions etc.
  - here: collisionless, electrostatic instabilities

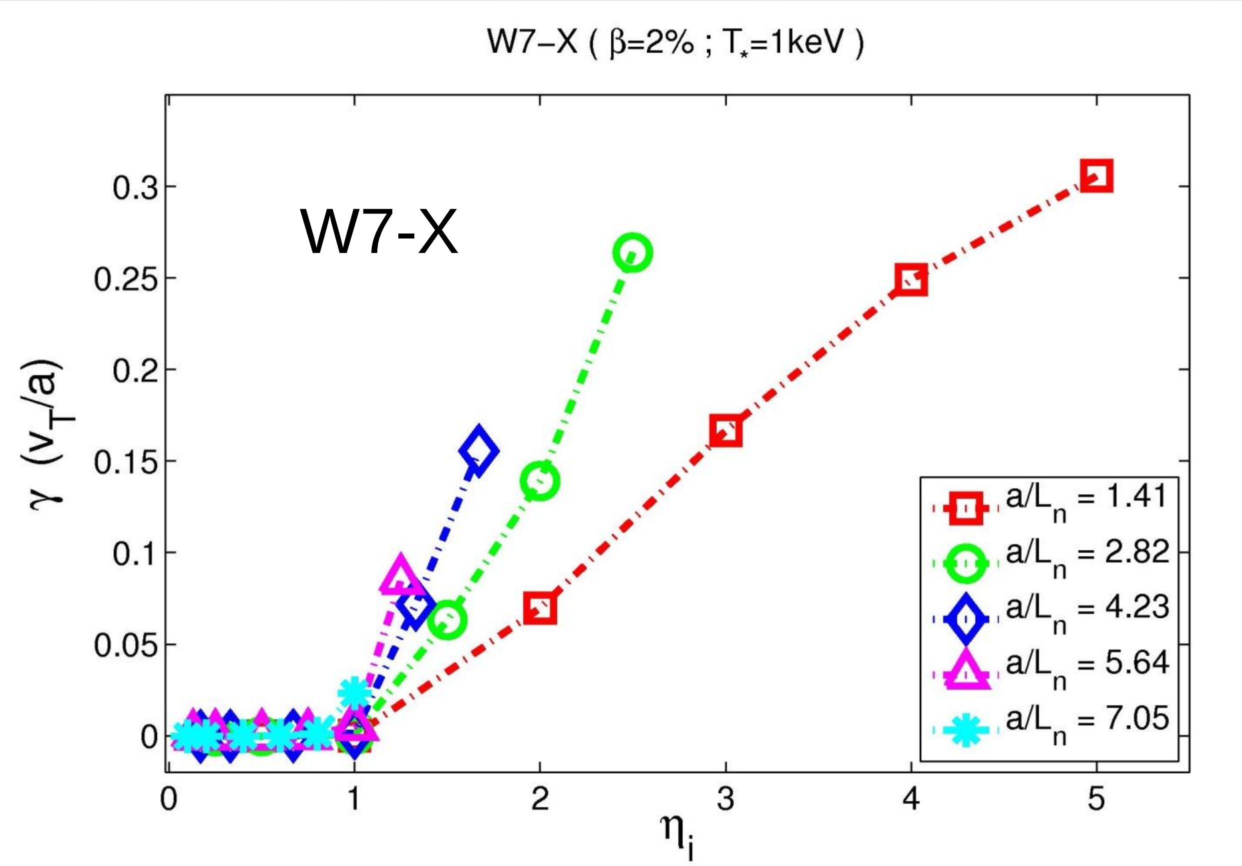
Benchmark

- Linear ITG growth rate with Boltzmann electrons vs ion temperature gradient in W7-X:



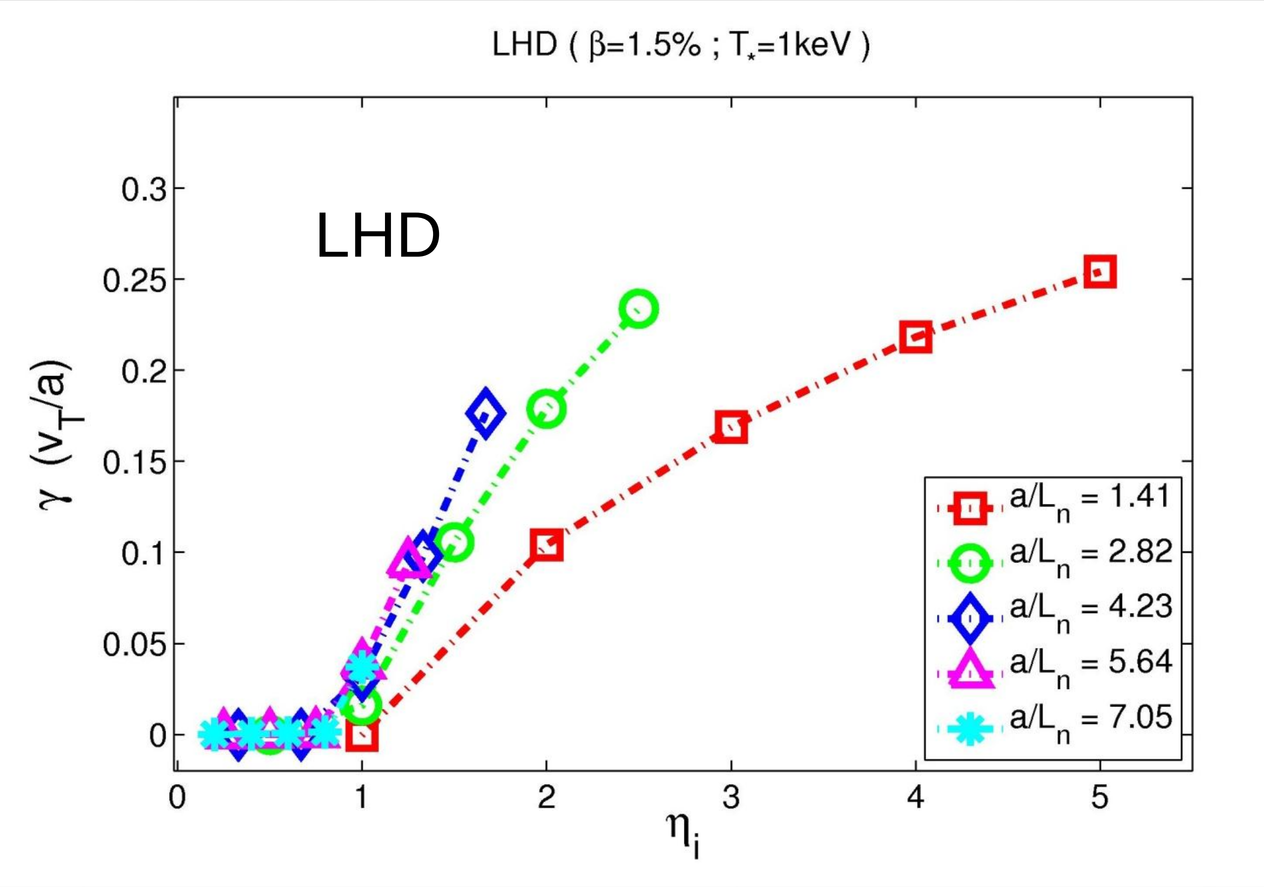
W7-X vs LHD

- Global, linear ITG simulations in W7-X (EUTERPE)



$$\eta_i = |\nabla \ln T_i| / |\nabla \ln n|$$

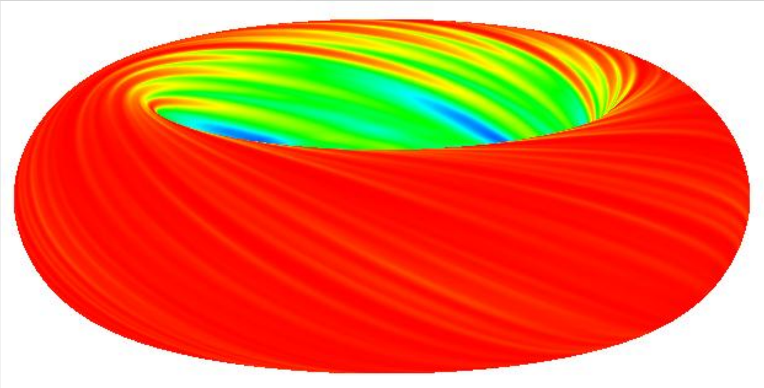
- Global, linear ITG simulations in LHD (EUTERPE)



$$\eta_i = |\nabla \ln T_i| / |\nabla \ln n|$$

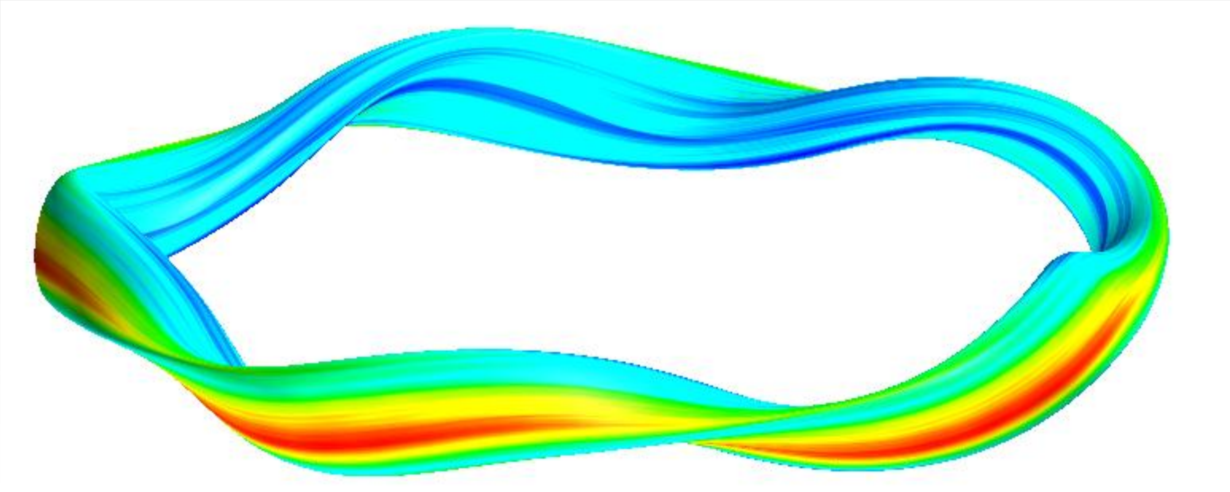
Nonlinear simulations

- ITG turbulence with Boltzmann electrons (GENE): rms potential fluctuations



DIII-D

$$\frac{\overline{\phi}_{\max}}{\overline{\phi}_{\min}} = 1.2$$

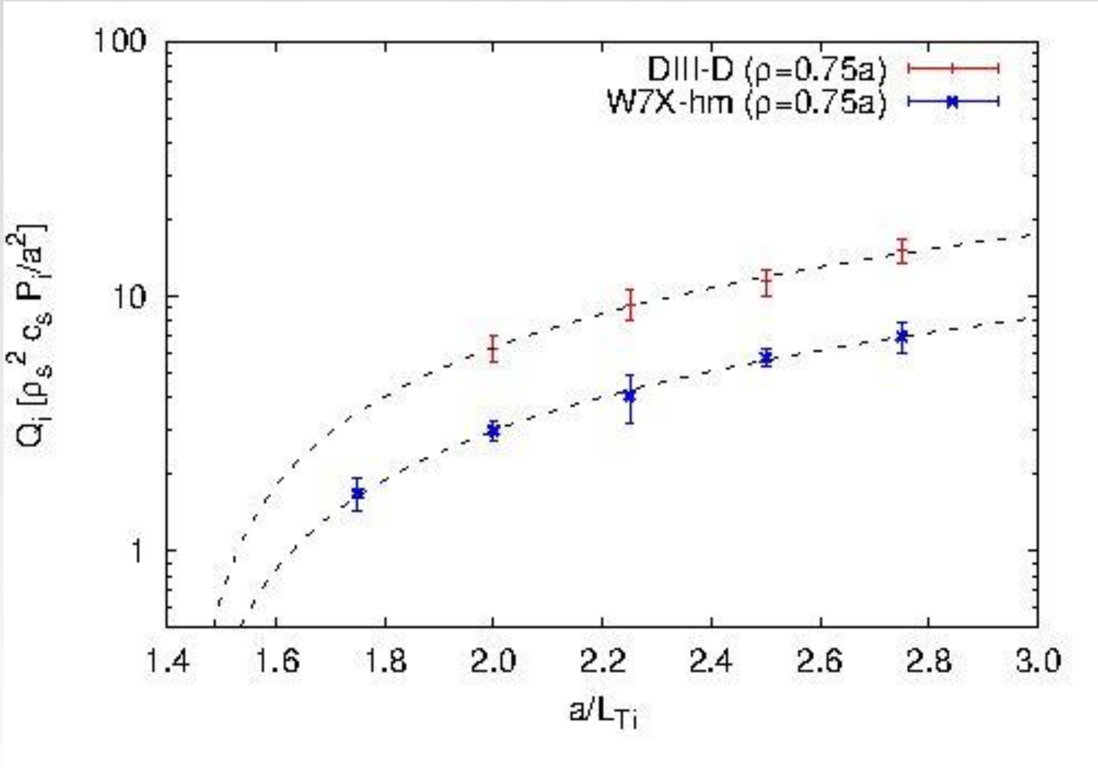


W7-X

$$\frac{\overline{\phi}_{\max}}{\overline{\phi}_{\min}} = 6.5$$

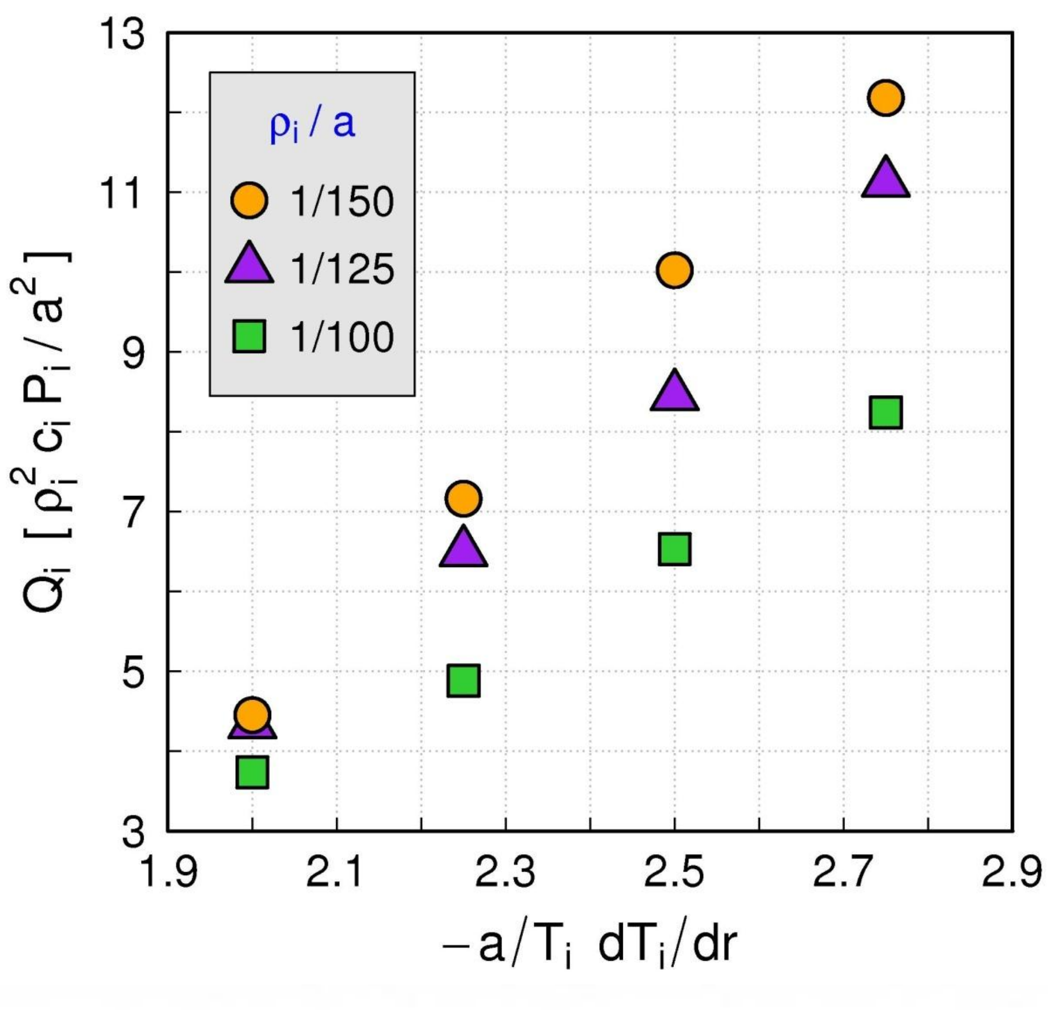
ITGs with Boltzmann electrons

- Nonlinear simulations with Boltzmann electrons (grad T_e=0, ρ*=1/150):
  - heat flux

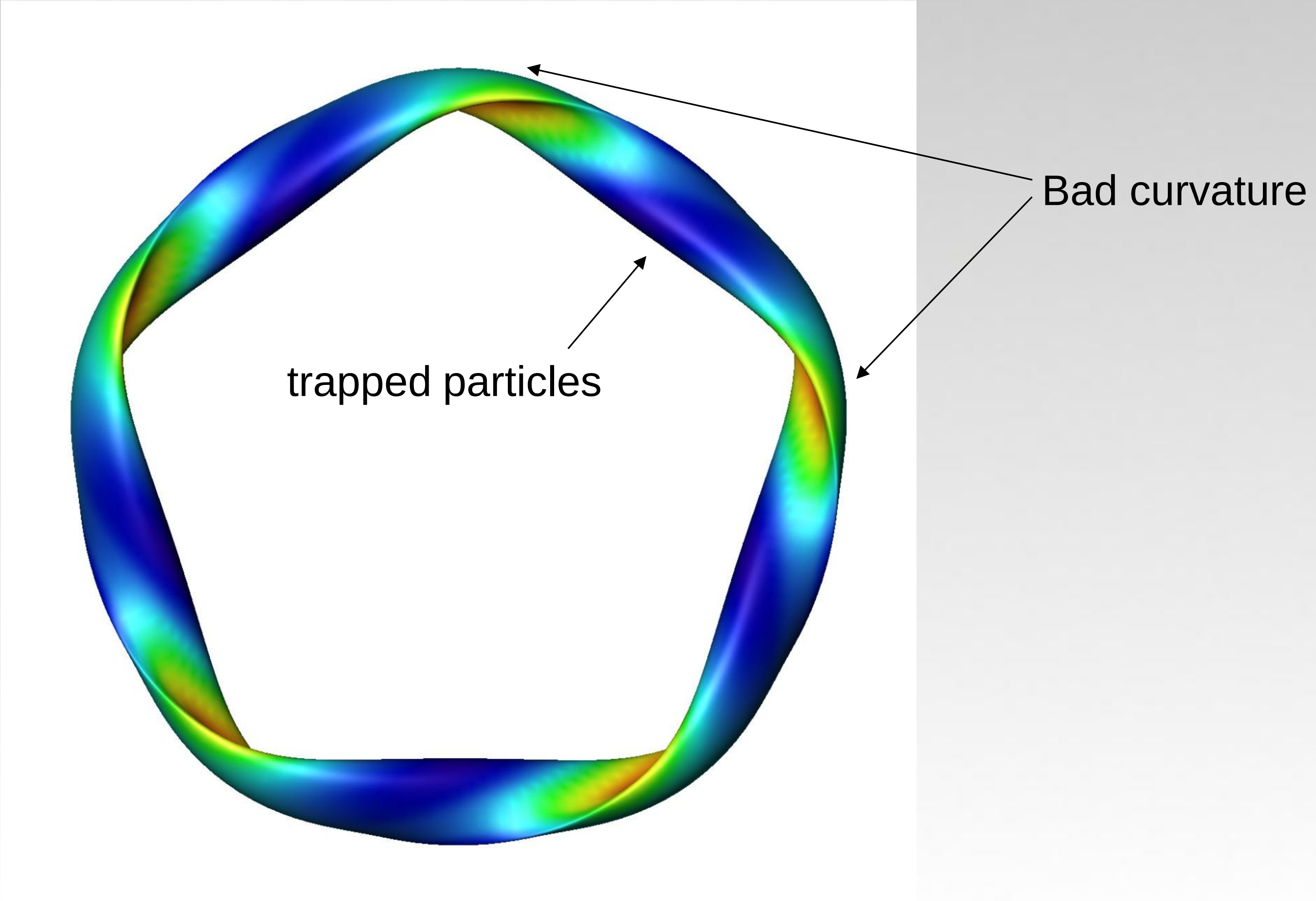


Turbulent transport (ITG ae)

- So far, in W7-X comparable to that in a typical tokamak, but “softer“:
  - depends on ρ*



Trapped-electron modes



Trapped-electron modes

- Instability requires

$$\overline{\omega}_{de}\omega_{*e} > 0$$

where

$$\omega_{*e} = -\frac{T_e k_\alpha}{e}\frac{d\ln n_e}{d\psi}, \qquad \omega_{de} = \mathbf{k}_\perp \cdot \mathbf{v}_{de}$$

$$\mathbf{B} = \nabla\psi \times \nabla\alpha$$

$$\mathbf{k}_\perp = k_\psi \nabla\psi + k_\alpha \nabla\alpha$$

- In an orbit-confining (omnigenous) stellarator

$$\overline{\omega}_{de} = \overline{\mathbf{v}_{de} \cdot (k_\alpha \nabla\alpha + k_\psi \nabla\psi)} = k_\alpha \overline{\mathbf{v}_{de} \cdot \nabla\alpha}$$

Maximum-J configurations

- But the precession frequency can be written

$$\overline{\mathbf{v}_{de} \cdot \nabla\alpha} = \frac{1}{e\tau_b}\frac{\partial J}{\partial \psi}, \qquad J(E,\mu,\psi,\alpha) = \int m v_\parallel dl$$

so

$$\omega_{*e}\overline{\omega}_{de} = -\frac{k_\alpha^2 T_a}{e^2 \tau_b}\frac{d\ln n_a}{d\psi}\frac{\partial J}{\partial \psi}$$

- Stability is thus promoted by “the maximum-J“ condition

$$\frac{\partial J}{\partial \psi} < 0$$

Rosenbluth, Phys. Fluids 1968

Physical picture

- The quantity,

$$J(r, E, \mu) = \int m v_{\|} dl$$

is an adiabatic invariant. E = energy.

- Hence, if a low-frequency instability moves a particle radially, then

$$\Delta J = \frac{\partial J}{\partial r} \Delta r + \frac{\partial J}{\partial E} \Delta E = 0$$

implying that it costs energy to move a particle radially outward

$$\Delta E = -\frac{\partial J / \partial r}{\partial J / \partial E} \Delta r > 0 \qquad \text{if} \quad \frac{\partial J}{\partial r} < 0$$

Trapped-particle modes

- **Theorem:** collisionless trapped-electron and trapped-ion modes are stable if

$$\frac{\partial J}{\partial \psi} < 0 < \frac{d \ \ln T_a}{d \ \ln n_a} < \frac{2}{3}$$

for all species a.

- Favourable bounce-averaged curvature.

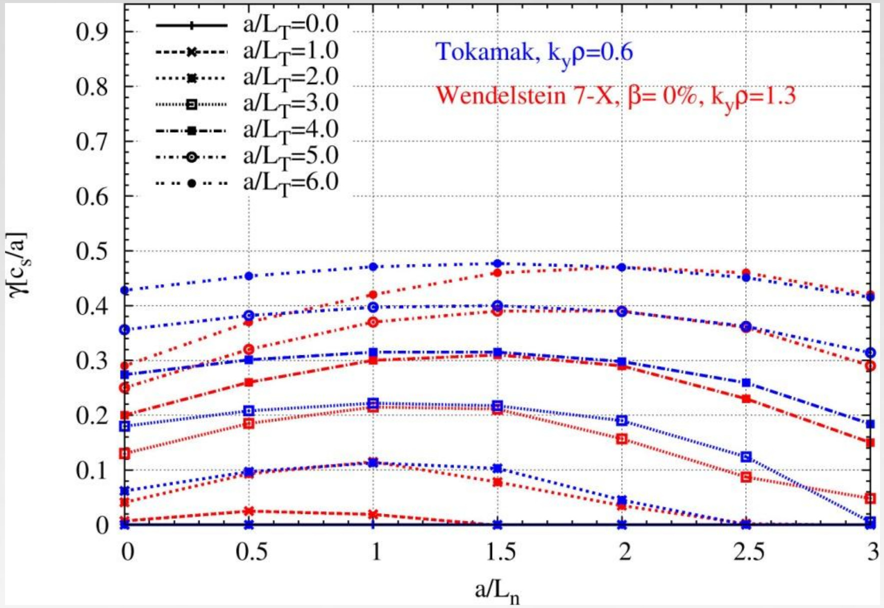
- In a maximum-J device, the precession drift is reversed compared with a tokamak

$$\overline{\mathbf{v}_{de} \cdot \nabla \alpha} = \frac{1}{e\tau_b} \frac{\partial J}{\partial \psi}$$

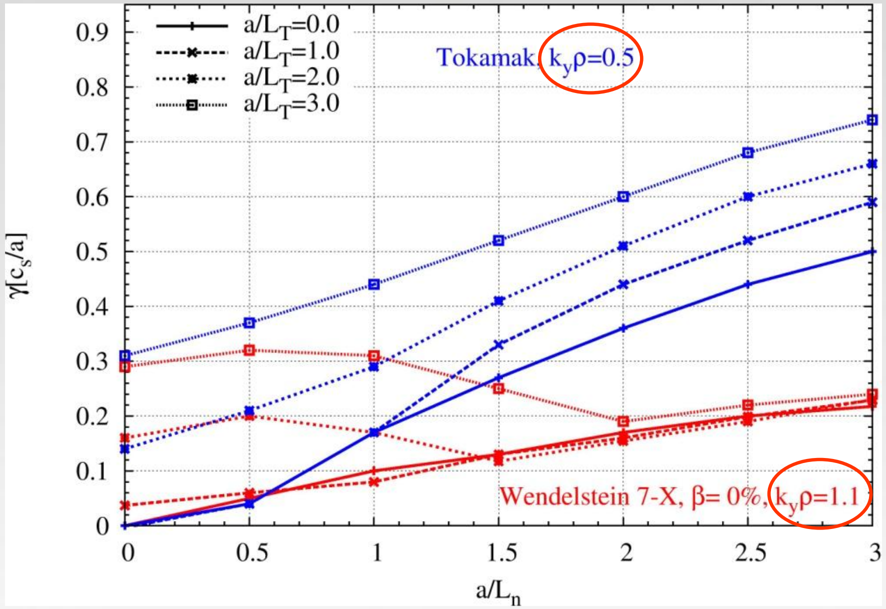
  - no resonance with drift waves.

ITGs and TEMs with kinetic electrons

- Simulations with and without kinetic electrons (grad T_e=grad T_i):
  - growth rate for the most unstable wave number



Boltzmann electrons

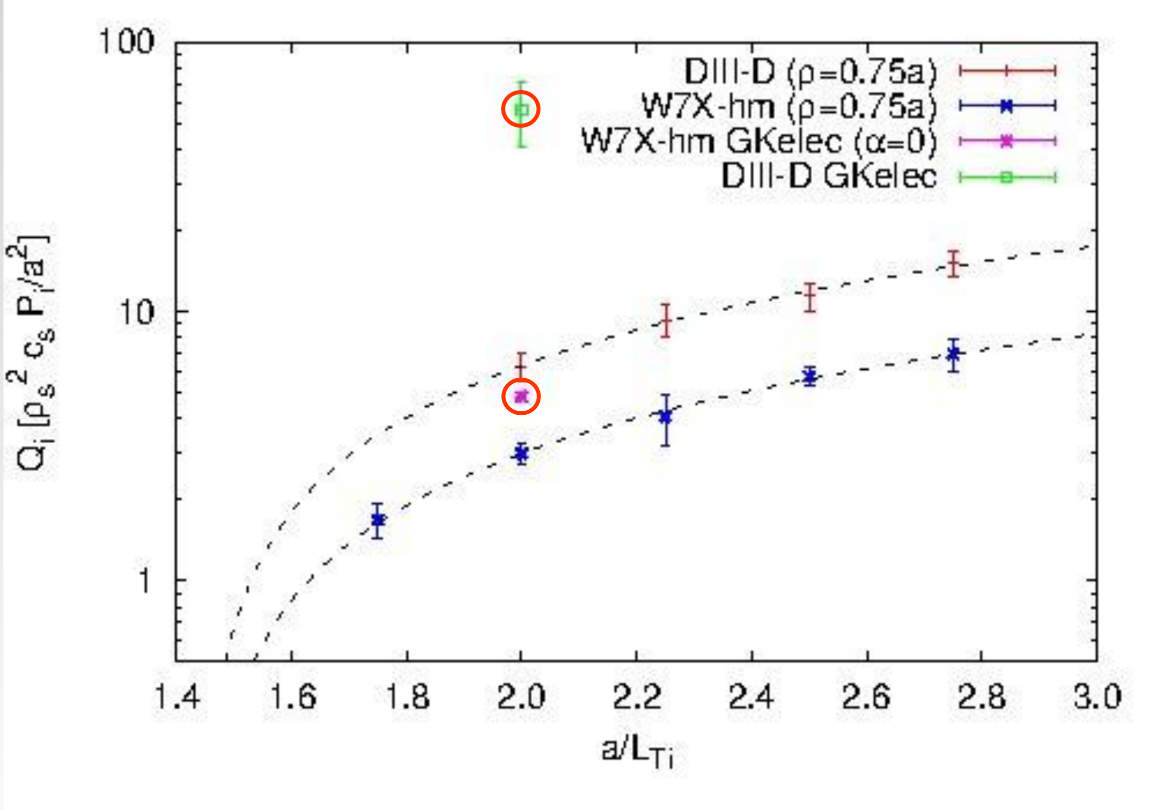


Kinetic electrons

- Kinetic electrons are stabilising.

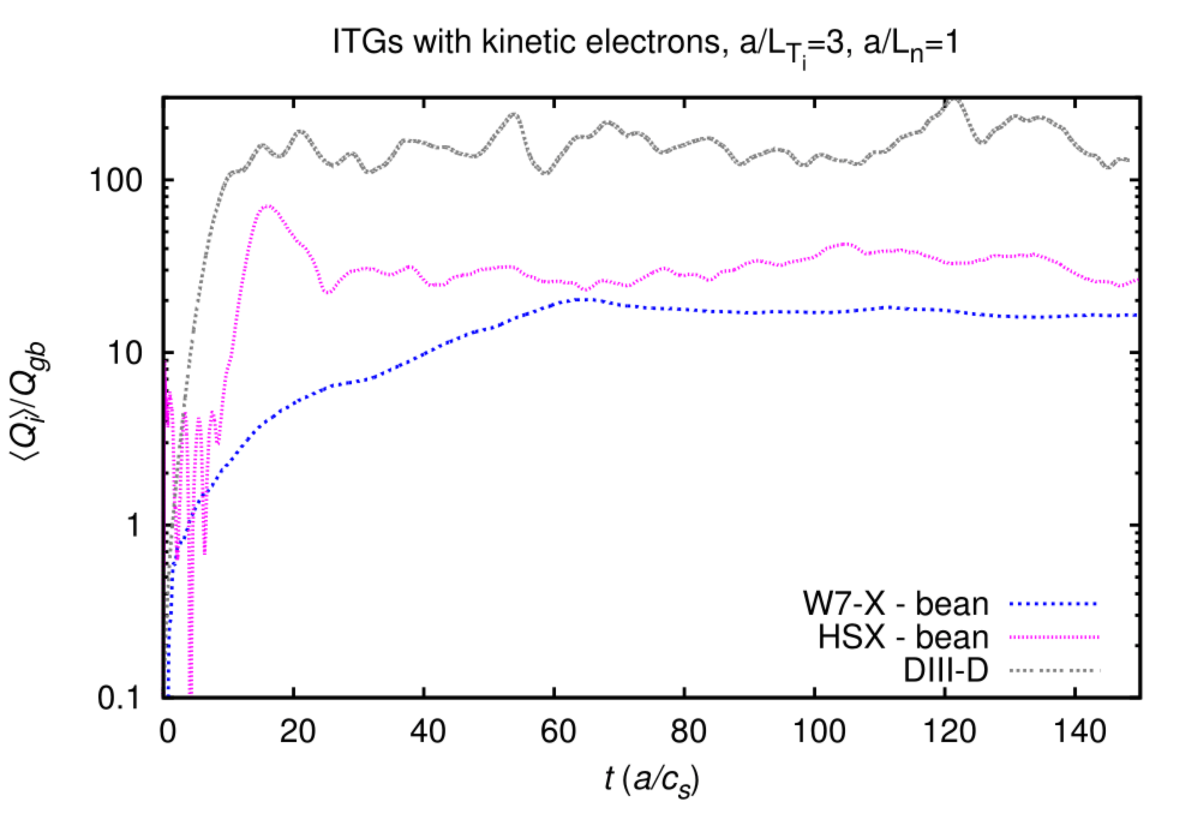
ITGs with kinetic electrons

- Simulations with and without kinetic electrons (grad T_e=0):
  - kinetic electrons in a flux tube



W7-X, HSX and DIII-D

- Another case:



HSX simulations by Benjamin Faber, Madison

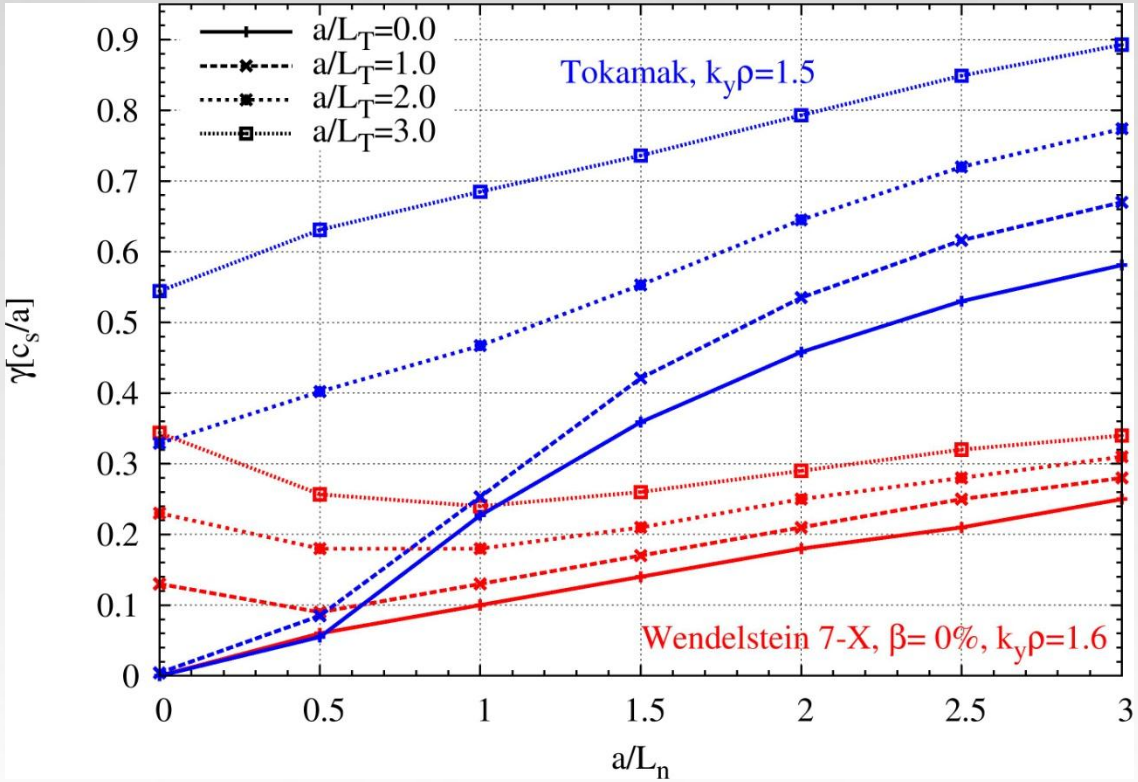
Conclusions

- ITG and TEM modes exist in stellarators, but display qualitative differences.
  - turbulent fluctuations much less evenly distributed.
- Wendelstein 7-X is, to some approximation, a maximum-J device.
  - most orbits have favourable bounce-averaged curvature
- Strongly stabilising for trapped-particle instabilities.
- ITG modes also benefit from stabilising action of the (kinetic) electrons.
- Less turbulent transport than in tokamaks?
  - too early to say

Extra Material

Gyrokinetic calculation of TEMs

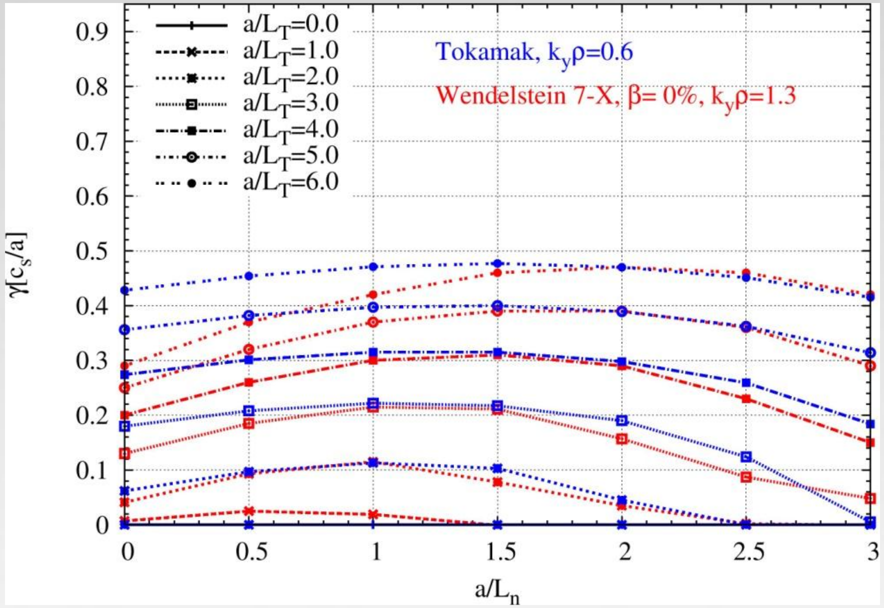
- Linear, flux-tube, electrostatic GENE simulations in DIII-D and W7-X
  - no ion temperature gradient

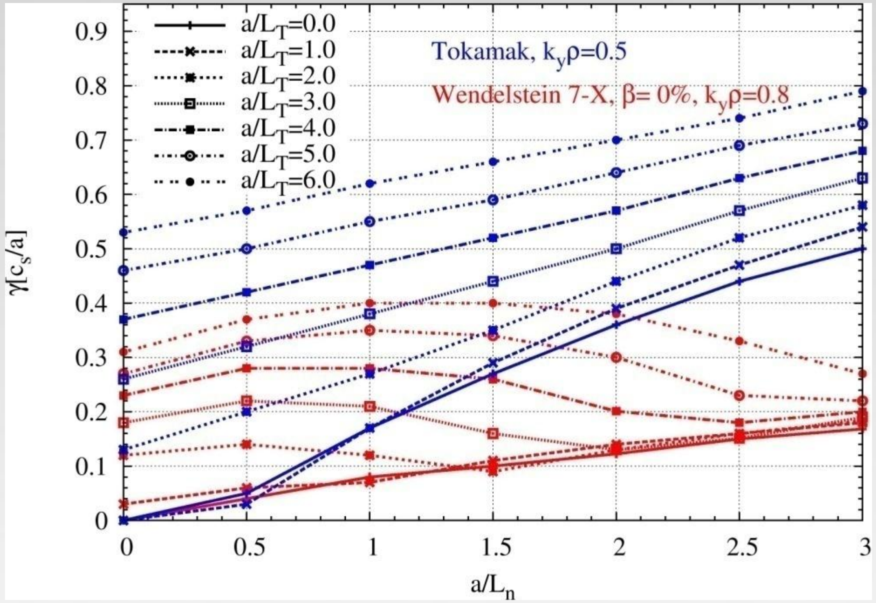


Proll, Xanthopoulos and Helander, submitted to PoP

ITGs with kinetic electrons

- Simulations with and without kinetic electrons (grad T_e=0):
  - growth rate for the most unstable wave number





Boltzmann electrons

Kinetic electrons

- Kinetic electrons are stabilising.

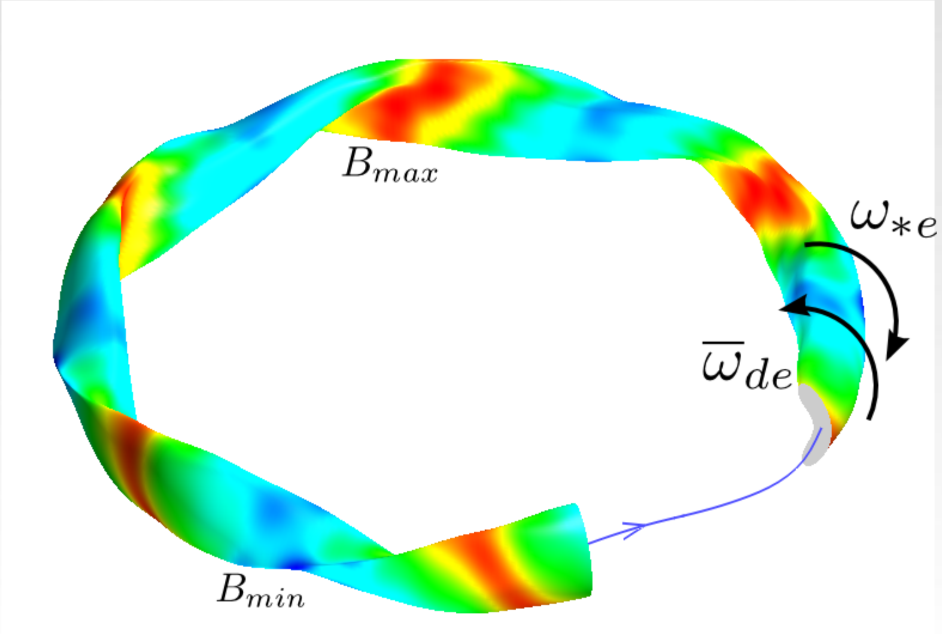
Proll, Xanthopoulos and Helander, submitted to PoP

Another argument for stable TEMs

- In a maximum-J device, the precession drift is reversed compared with a tokamak, since

$$\overline{\mathbf{v}_{de} \cdot \nabla\alpha} = \frac{1}{e\tau_b} \frac{\partial J}{\partial \psi}$$

  – no resonance between precessing electrons and drift waves



Energy balance

- Linear, collisionless, electrostatic gyrokinetics.
  - energy balance:

$$\gamma \sum_a \frac{n_a e_a^2}{T_a} \int (1-\Gamma_0)|\phi|^2 d^3r = \sum_a P_a, \qquad \Gamma_0(b) = e^{-b} I_0(b), \qquad b = (k_\perp \rho_a)^2$$

$$P_a = e_a \int f_{a1} (\mathbf{v}_\| + \mathbf{v}_d) \cdot \nabla \phi \; d^3r d^3v$$

- Substitute the solution of the gyrokinetic equation for fast-moving partices
  - at marginal stability

$$P_a = \frac{\pi e_a^2}{T_a} \int \delta(\omega - \overline{\omega}_{da}) \overline{\omega}_{da} (\omega_{*a}^T - \overline{\omega}_{da}) |\overline{J_0 \phi}|^2 f_{a0} \; d^3r d^3v$$

$$\omega_{*a}^T = \omega_{*a} \left[1 + \eta_a \left(\frac{m_a v^2}{2T_a} - \frac{3}{2} \right) \right], \qquad \eta_a = \frac{d \ln T_a}{d \ln n_a}$$

$$\overline{\omega}_{da} \omega_{*a}^T < 0 \Rightarrow P_a < 0$$

Outline of calculation

- Conventinal drift-wave ordering

$$k_{||} v_{Ti} \ll \omega \ll k_{||} v_{Te}$$

- Expanding in the inverse aspect ratio
  - few trapped particles,

$$\omega \sim \omega_* \gg \omega_d$$

gives electron drift-wave frequency

$$\frac{\omega}{\omega_{*e}} = \frac{\Gamma_0 + \eta_i(\Gamma_1 - \Gamma_0)}{\tau(1 - \Gamma_0) + 1}$$

- In next order, instability from wave-particle resonance only if $\omega \overline{\omega}_{de} > 0$
  - impossible unless $\boxed{\eta_i > 1.64}$

Helander et al, PPCF 2012

Energy balance

- Linear, collisionless, electrostatic gyrokinetics in ballooning space:

$$iv_{\|}\nabla_{\|}g_a + (\omega - \omega_{da})g_a = \frac{e_a\phi}{T_a}J_0(k_\perp v_\perp/\Omega_a)\left(\omega - \omega_{*a}^T\right)f_{a0}$$

$$\sum_a \frac{n_a e_a^2}{T_a}\phi = \sum_a e_a \int g_a J_0 \mathrm{d}^3v$$

$$\omega_{*a}^T = \omega_{*a}\left[1 + \eta_a\left(\frac{m_a v^2}{2T_a} - \frac{3}{2}\right)\right], \qquad \eta_a = \frac{d\ln T_a}{d\ln n_a}$$

- Multiply by $J_0\phi^*$ and integrate over phase space.

- Energy balance:

$$\gamma \sum_a \frac{n_a e_a^2}{T_a}\int \frac{\mathrm{d}l}{B}(1-\Gamma_0)|\phi|^2 = \sum_a P_a, \qquad \Gamma_0(b) = e^{-b}I_0(b), \qquad b = (k_\perp \rho_a)^2$$

$$P_a = e_a \int f_{a1}(\mathbf{v}_\| + \mathbf{v}_d)\cdot\nabla\phi \, d^3r d^3v$$

Energy balance, cont'd

- For fast-moving particles

$$(\omega \ll k_{\|} v_T) \quad \Rightarrow \quad g_a = g_{a0} + g_{a1} + \cdots$$

$$g_{a0} = \frac{e_a \overline{J_0 \phi}}{T_a} \frac{\omega - \omega_{*a}^T}{\omega - \overline{\omega}_{da}} f_{a0}$$

$$i v_{\|} \nabla_{\|} g_{a1} = (\omega - \omega_{*a}^T) \frac{e_a}{T_a} \left(J_0 \phi - \frac{\omega - \omega_{da}}{\omega - \overline{\omega}_{da}} \overline{J_0 \phi} \right) f_{a0}$$

- the energy transfer at marginal stability becomes

$$P_a = \frac{\pi e_a^2}{T_a} \int \delta(\omega - \overline{\omega}_{da}) \overline{\omega}_{da} (\omega_{*a}^T - \overline{\omega}_{da}) |\overline{J_0 \phi}|^2 f_{a0} \, d^3 r d^3 v$$

- Stabilising action if bounce-averaged curvature is favourable:

$$\overline{\omega}_{da} \omega_{*a}^T < 0 \Rightarrow P_a < 0$$

Algebra

- Conventinal drift-wave ordering

$$k_\parallel v_{Ti} \ll \omega \ll k_\parallel v_{Te} \quad \Rightarrow \quad g_i = \frac{\omega - \omega_{*i}^T}{\omega - \omega_{di}} \frac{e J_0 \phi}{T_i} f_{i0}, \qquad g_e^{\rm tr} = - \frac{\omega - \omega_{*e}^T}{\omega - \overline{\omega}_{de}} \frac{e\overline{\phi}}{T_e} f_{e0}$$

- Expanding in the inverse aspect ratio
  - few trapped particles,

$$\omega \sim \omega_* \gg \omega_d$$

gives electron drift-wave frequency

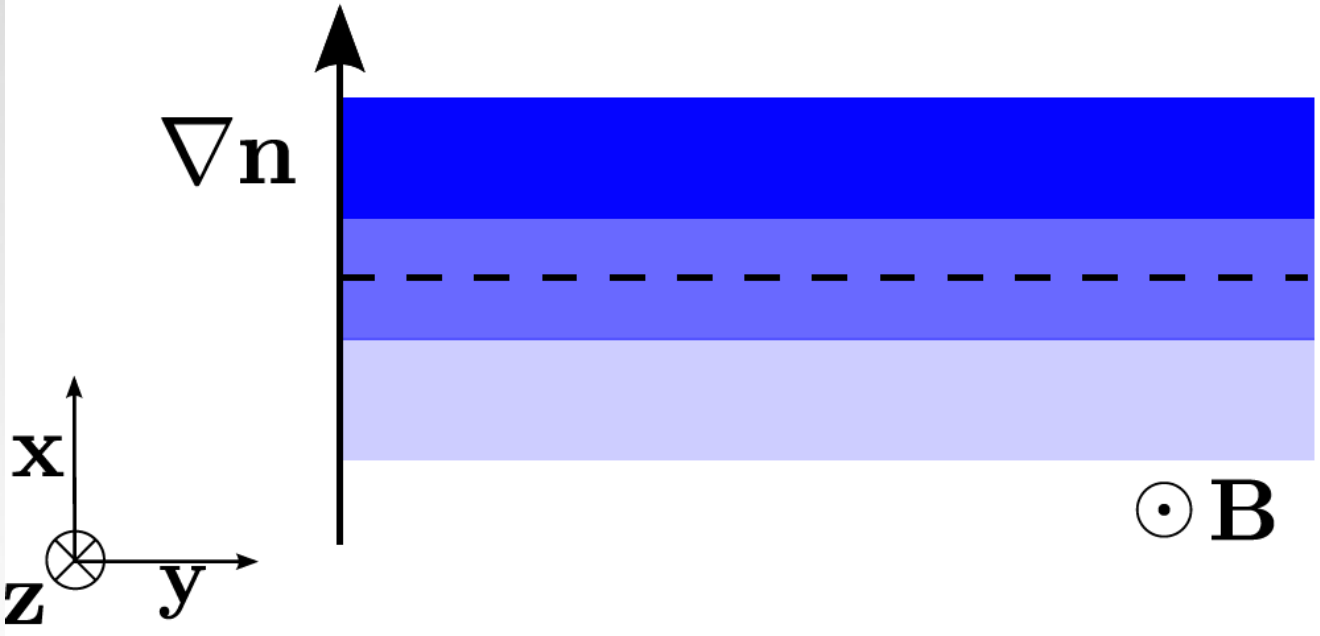
$$\frac{\omega}{\omega_{*e}} = \frac{\Gamma_0 + \eta_i(\Gamma_1 - \Gamma_0)}{\tau(1 - \Gamma_0) + 1}$$

- In next order, instability from resonant denominator only if $\omega \overline{\omega}_{de} > 0$
  - impossible unless $\eta_i > 1.64$

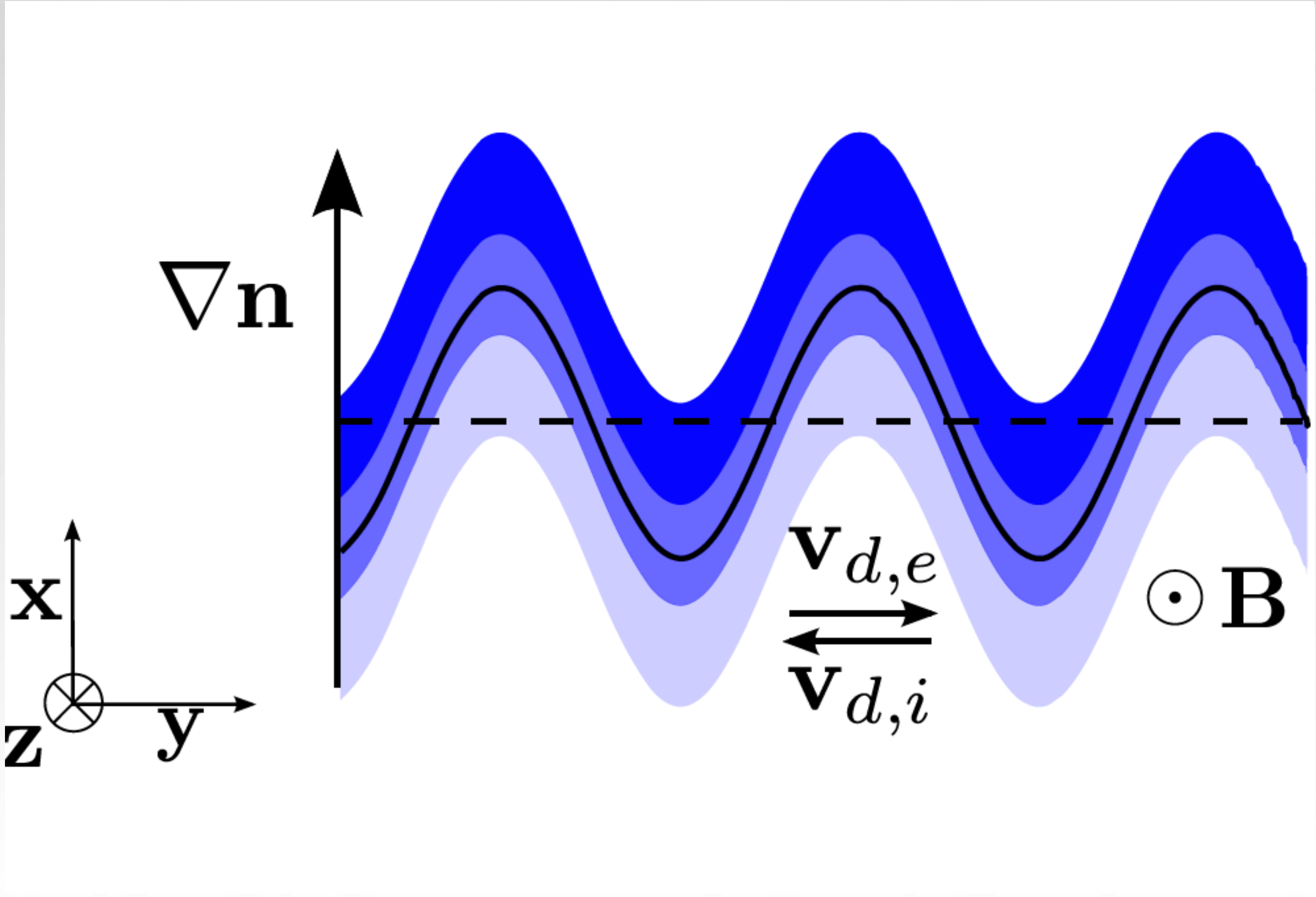
Helander et al, PPCF 2012

Trapped-electron modes

- TEMs result from overlap between
  - bad curvature and
  - trapping regions



Trapped-electron modes



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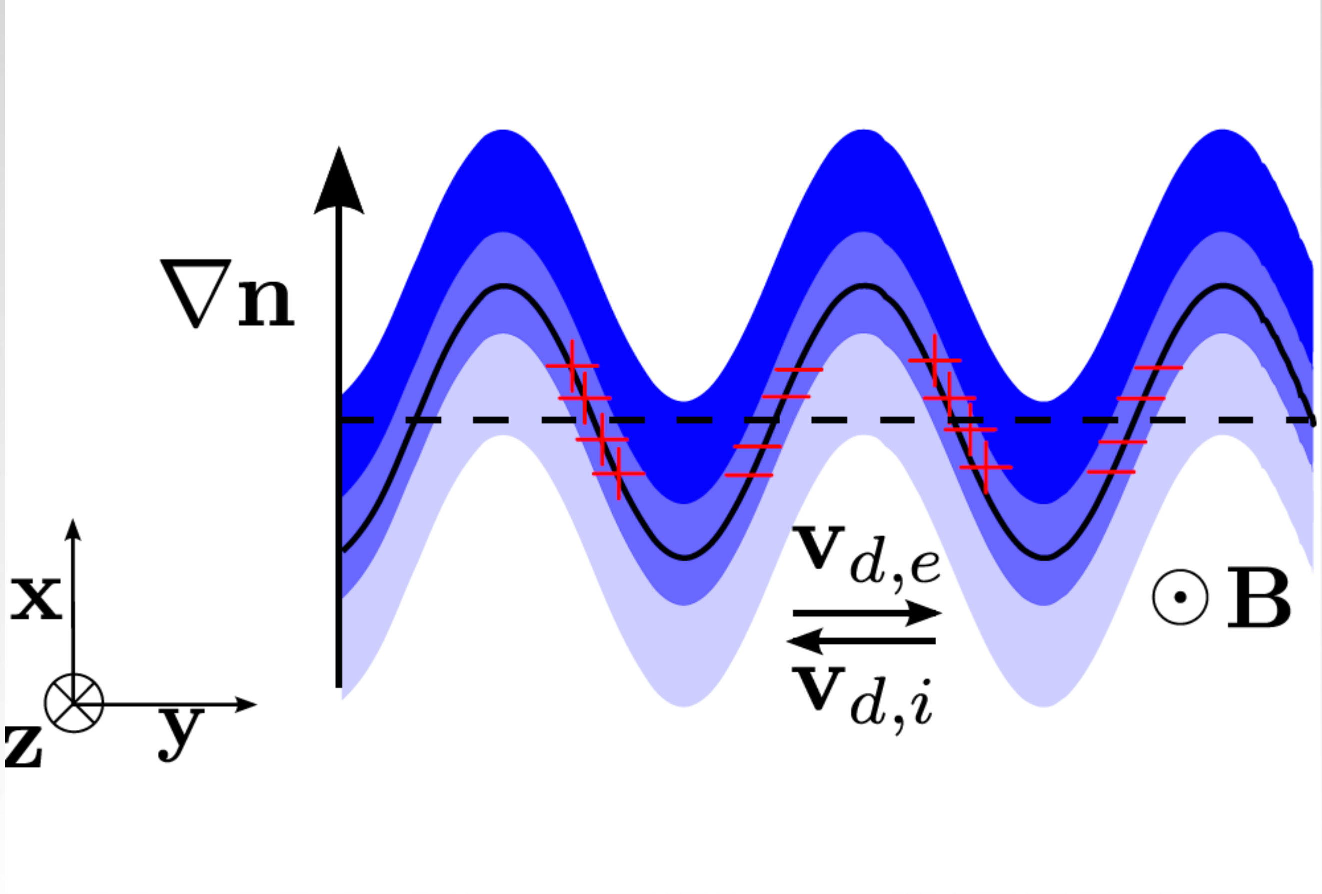
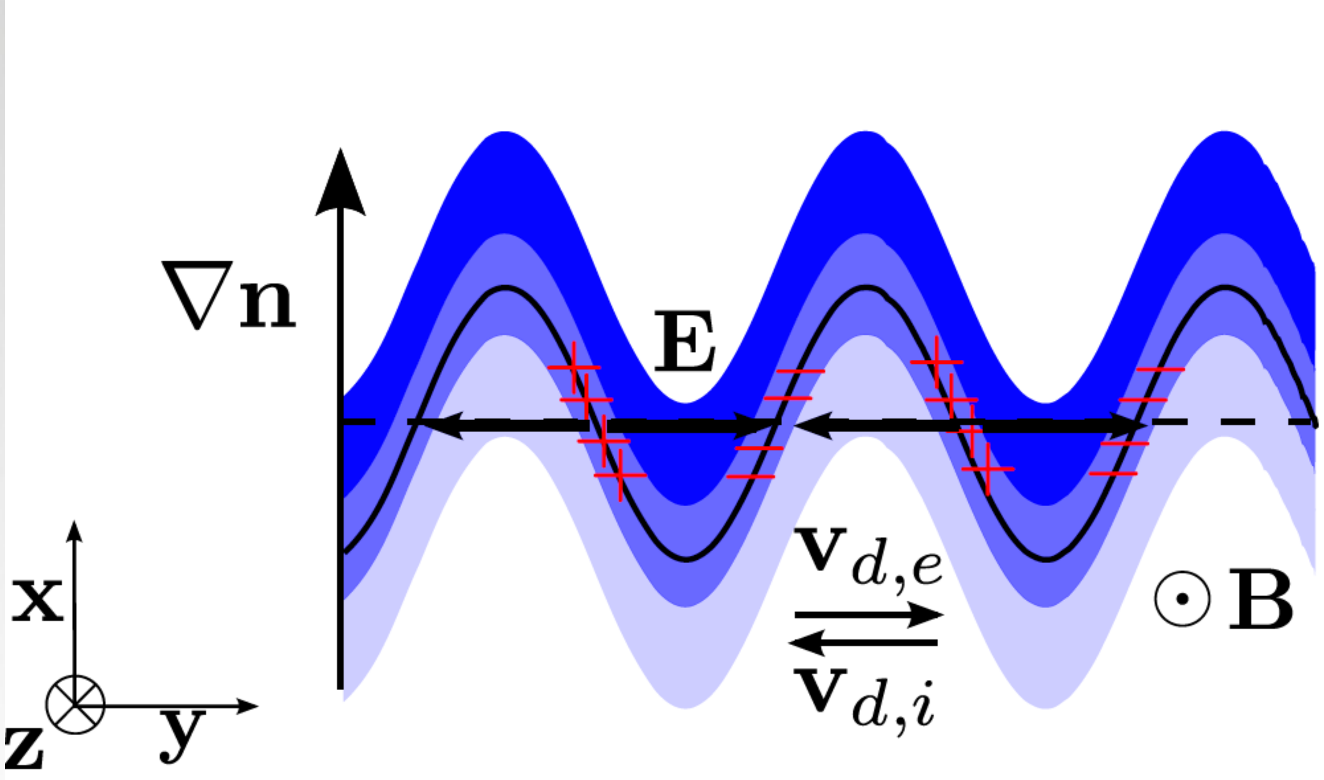


Trapped-electron modes



Trapped-electron modes