Developing and validating predictive models for fast ion relaxation in burning plasmas

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- Future burning plasma (BP) experiments motivate developing predictive models for Energetic Particle (EP) confinement.
- EPs, i.e. fusion products, auxiliary heating particles, in BPs are superalfvenic $\Rightarrow$ capable of driving deleterious Alfvénic instabilities.
- EP physics challenge: can we predict EP profiles in BP conditions?

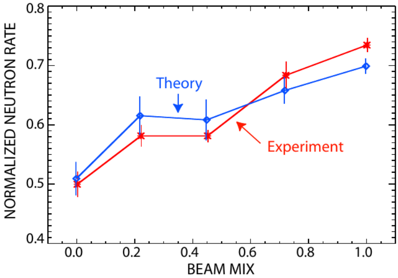
  models exist: need to validate systematically!

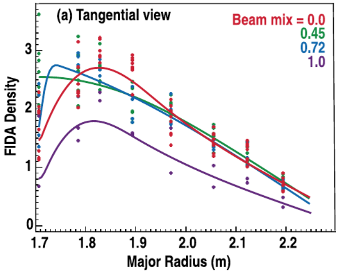
Outline

1. **Critical Gradient Model (CGM)** or 1.5D reduced quasi-linear (QL) model for EP pressure profiles relaxation & losses
   - model outline
   - applications to DIII-D
   - application go NSTX
2. **Hybrid model:** perturbative effect of Alfvén Eigenmodes (AE) on EP population *(N.N. Gorelenkov et al., NF'99,Y. Chen, Ph.D. PPPL 98)*

- formulation; model provides detailed EP phase space dynamics
- employ dynamic growth rate (for multiple AEs) calculations to evolve test case amplitudes, *i*.e. ideal MHD (NOVA) & guiding center (ORBIT) codes.

Promising initial applications of CGM were done on DIII-D





On-axis ↑ On+Off-axis ↑ *(W. Heidbrink et al., NF'13)*

- Critical Gradient Model allows fast evaluation of EP profile relaxation
- Time averaging was required; reduced error bars
- Linear instability theory should to be robust
- 1.5D model motivates more accurate 2D quasi-linear (QL) theory development

Should 1.5D model work robustly? - need to systematically validate CGM!

1.5D QL/crit. gradient model OUTLINE (K.Ghantous et.al.PoP'12)

(red color is unique for 2D QL; blue is unique for 1.5D CGM; black is common)

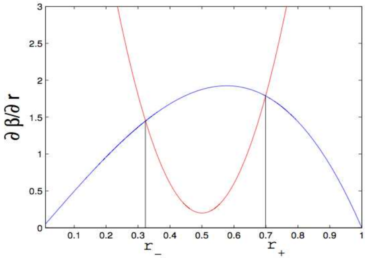
- large number of unstable localized modes
- fast EP diffusion within velocity/phase island
- fixed background dampings, plasma profiles
- critical gradient $\partial\beta_{EP}/\partial r$ due to AE instabilities
  - "improve" linear calculations with accurate evaluation of the growth/damping rates (use NOVA-K): ion Landau, trapped electron are key in used conditions - should include rotation, realistic continuum;
  - 1.5D produces analyt. expressions to keep the parametric dependences outside of comput. domains;
- integrate critical EP beta to compute (i) relaxed profiles; (ii) losses;
- account for distrib. in a simple form *[Kolesnichenko NF'80]*, i.e. simple resonance $v_{\parallel} \sim v_A$ ($\rightarrow$ 0.5D) (too optimistic approximation?)

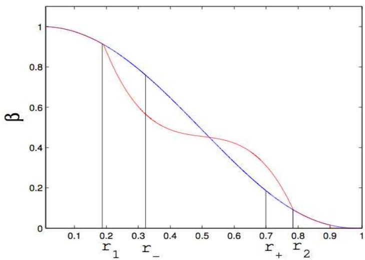
Illustration of EP CGM application for *AE instabilities

Key equation:

$$\frac{\partial \beta_{EPcr}}{\partial r} = -\frac{\gamma_{iL} + \gamma_{ecoll} + \gamma_{rad}}{\gamma'_{EP}}, \; \gamma'_{EP} = \gamma_{EP}/(\partial \beta_{EP}/\partial r)$$

Three damping mechanisms are dominant in DIII-D, ITER: ion Landau, electron collisional, radiative (high-n's) $\rightarrow$ essentially nonlocal!! $\Rightarrow$ 1.5D should rely on global(?) stability analysis.



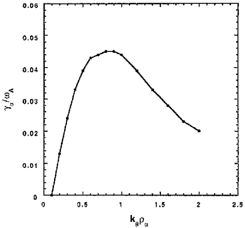


Use particle conservation law $\int_0^a r(\beta_{EP} - \beta_{EPrelax})\,dr = 0$ to compute **profile broadening** and EP losses.

Condition $|\beta'_{EP}| \leq |\beta'_{EPcrit}|$ is used to compute the relaxed EP profile. It is broadened from initially unstable: $r_{\pm} \rightarrow r_{1,2}$

CGM/1.5D model is ready for applications in two ways

1. **Analytic:**
   - employ TRANSP or analytic profiles
   - analytic distribution for EP αs or beams, slowing down
   - n is taken at the maximum growth rate $k_\perp \rho_{EP} \sim 1$



G.-Y. Fu, C.Z. Cheng, PoP'92

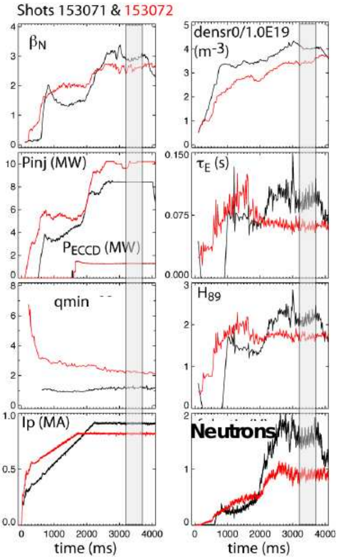
B.N. Breizman, S.E. Sharapov, PPCF'95

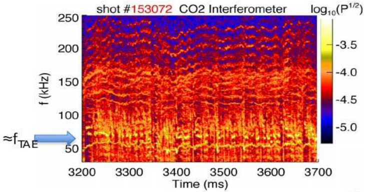
IAEA 2012 by EP ITPA group

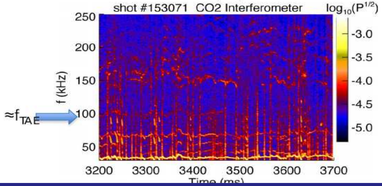
2. **Numerical** (via NOVA-K - this poster):
   - nonlocal *AE growth rates for normalization of the analyt. growth rates
   - scan n/z_{EP} (around $k_\perp \rho_{EP} \sim 1$) in growth rate in search for plateau
   - use n from the above procedure for growth rate normalization;

     stabilizing finite orbit width effects are seen numerically
     (Gorelenkov et al., PoP'99; ITPA, IAEA'10,'12)

Further validate CGM against elevated q_{min} discharges on DIII-D



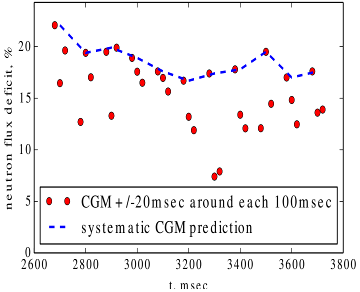




Apply to multiple unstable AEs on #153071 plasmas

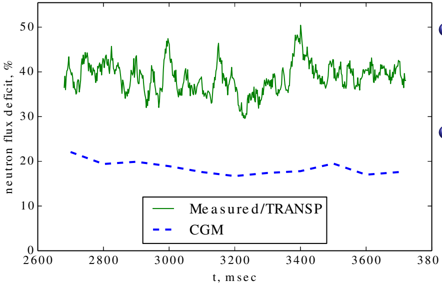
- Compare deficit of neutron loss vs time
- rely on TRANSP for neutron loss predictions
- use NOVA-K normalization foe CGM
- Mirnov spectrum indicates virulent AEs, frequencies are not steady

For systematic study of #153072 time window is subdivided by $\Delta t = 100msec$ and each time t_0 is subdivided again by $\delta t = 20msec$, i.e. $t_{0-} = t_0 - \delta t$ and $t_{0+} = t_0 + \delta t$.



1. Slowing down time of D beam ions is large $\tau_{se} \simeq 250msec \gg \Delta t, \delta t$.
2. Growth rate scales as $\gamma_{beam}/\omega \sim q^2$ - damping has different ingradients.
3. CGM assumed fast diffusion implies the choice of most unstable case

Compare with TRANSP classic predictions

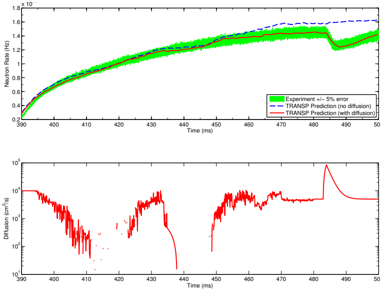


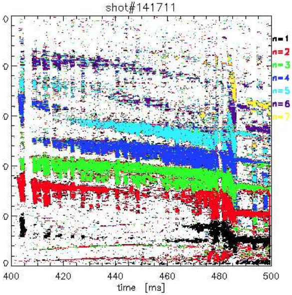
- Significant discrepancy: CGM underpredicts the neutron loss by a factor of 2.
- Linear stability theory suggests: either drive is too weak or the damping is too strong. Why?

- Our conjecture: nonlinear regime drives nonperturbative modes, EPMs, rTAEs: more unstable & localized *(Z. Wang, PRL'13 and Yang Cheng et al., PoP'10).*

NSTX provides another validation case

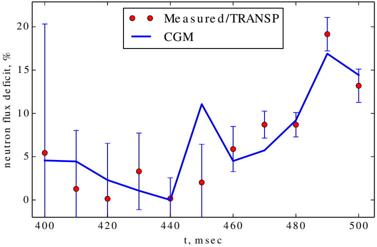
- Special exercise within TRANSP/Nubeam codes infers diffusion coefficient up to $< 1m^2/sec$.
- Use CGM method as NOVA/TRANSP postprocessor to computed neutron losses.
- Relatively "non-virulent" instability case is chosen, but still chirping modes.





CGM & NOVA stability analysis show good agreement for one case

- NOVA improved stability calculations account for plasma rotation, thermal ion drift orbit effects
- Had strong/important effects on growth/damping rate predictions.
- Near threshold (as indicated by NOVA) conditions are appropriate for linear theory applications.



CGM (linear theory) works better in considered NSTX plasma!! Why???
low aspect ratio, more coupling for harmonics? avalanches are present even near threshold!

Hybrid model formulation (ORBIT + NOVA)

Straight field line Boozer coordinates ψ_p, θ, ζ, with $d\zeta/d\theta = q(\psi)$
Normalized parallel velocity $\rho_\parallel = v_\parallel/B$
Covariant representation $\vec{B} = g(\psi_p)\nabla\zeta + I(\psi_p)\nabla\theta + \delta(\psi_p,\theta)\nabla\psi_p$
Canonical momentum, toroidal flux ψ, $d\psi/d\psi_p = q$

$$P_\zeta = g\rho_\parallel - \psi_p, \qquad P_\theta = \psi + \rho_\parallel I,$$

Hamiltonian $H = \rho_\parallel^2 B^2/2 + \mu B + \Phi$,

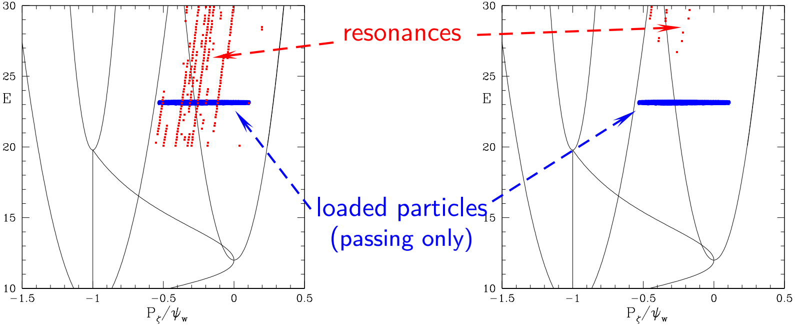
$$\dot{\theta} = \frac{\partial H}{\partial P_\theta} \qquad \dot{P}_\theta = -\frac{\partial H}{\partial \theta}$$
$$\dot{\zeta} = \frac{\partial H}{\partial P_\zeta} \qquad \dot{P}_\zeta = -\frac{\partial H}{\partial \zeta}.$$

Perturbation $\delta\vec{B} = \nabla \times \alpha\vec{B}$ and $\alpha = \sum_{m,n} \alpha_{m,n}(\psi_p) sin(n\zeta - m\theta - \omega_n t)$, Φ
Mode frequency much less than cyclotron frequency, so μ is constant

Determination of domains of broken quasi-periodic (KAM) surfaces in Hybrid model

- Closely spaced pair of orbits, defining a phase vector in P_ζ, θ plane.
- Resonance identification: phase vector angle χ rotates without bound in the island.

Resonances shown in plane of E, P_ζ for two modes:
Low frequency $f = 30$ kHz mode; high $f = 120$ kHz - less resonances.

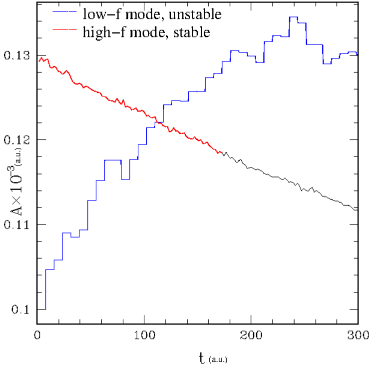


Main equations within Hybrid model are for mode amplitude and phase

$$\frac{dA_n}{dt} = \frac{-v_A^2}{N\omega_n}\sum_{k,m}\left\langle\left[\rho_\parallel B^2\alpha_{mn} - \Phi_{mn}(\psi_p)\right] cos(\Omega_{mn})\right\rangle - \gamma_d A_n$$

$$\frac{d\phi_n}{dt} = \frac{-v_A^2}{N\omega_n A_n}\sum_{k,m}\left\langle\left[\rho_\parallel B^2\alpha_{mn} - \Phi_{mn}(\psi_p)\right] \times \times sin(\Omega_{mn})\right\rangle$$

- Example of time evolution of $m/n = 7/2$, 30 kHz, $m/n = 5/2$ and 120 kHz mode amplitudes $\Rightarrow$ infer growth rates γ/ω.
- Modes initiated with small amplitude. The low frequency mode grows to saturation and the high frequency mode is damped.



- 1.5D/Critical Gradient Model validations against DIII-D and NSTX plasmas have challenges for the linear perturbative theory
  - DIII-D comparison shows less neutron losses by a factor of 2 throughout the discharge
    - nonperturbative solutions could be responsible and should be sought in experiments
  - Applications to NSTX high-q discharge shows good agreement using NOVA normalization for the growth rates
- Some applications do not require details of distribution function and can rely on CGM:
  - EP density profile knowledge is sufficient (normal TRANSP analysis)
  - calculations are fast
- Hybrid model (ORBIT + NOVA) is presented for predictive accurate modeling. Systematic validation is required.