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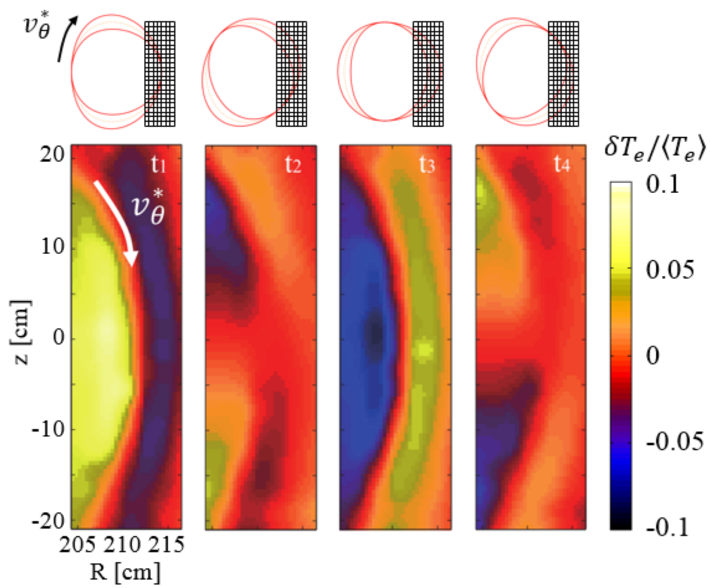
EX/P8-15, Green 8 & 9, Friday 14:00—18:45

Accurate Estimation of Tearing Mode Stability Parameters in the KSTAR using High-resolution 2D ECEI diagnostic



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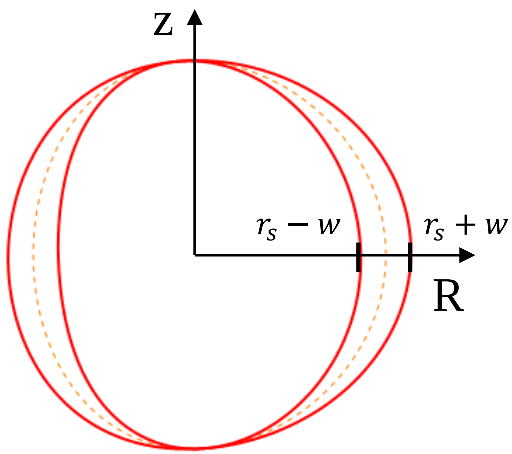
Introduction



Magnetic flux surface of
$m = 2$ magnetic island

- Evolution of the tearing mode island size (w) is described with

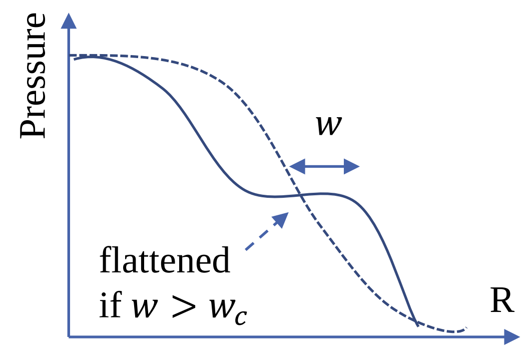
$$a_1 \frac{\tau_r}{r_s}\frac{\mathrm{d}w}{\mathrm{d}t} = r_s\Delta' + a_2\sqrt{\epsilon}\frac{\beta_\theta}{w}\frac{L_q}{L_p}\frac{w}{w^2 + (w_c)^2} + \cdots$$

$\tau_r = \mu_0 r_s^2/\eta$: the current diffusion time
η : the plasma resistivity
$\epsilon = r_s/R$: inverse aspect ratio
β_θ : the plasma poloidal beta
$L_q = q/(\mathrm{d}q/\mathrm{d}r)$ and $L_p = p/(\mathrm{d}p/\mathrm{d}r)$ where q is the safety factor and p is the plasma pressure

a_1 and a_2 : coefficients related to flux geometry of the magnetic island

$\Delta'(w) \sim \tilde{B}_\theta(r_s + w) - \tilde{B}_\theta(r_s - w)$ classical stability index

$w_c = \sqrt{\frac{RqL_q}{m}\left(\frac{\kappa_\perp}{\kappa_\parallel}\right)^{1/4}}$ critical width for pressure flattening



- Parameters such as $\boldsymbol{a_1}$, $\boldsymbol{a_2}$, $\boldsymbol{\Delta'}$, and $\boldsymbol{w_c}$ can not be "measured"
  → Method to estimate those parameters is required!

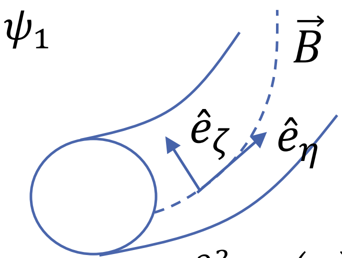
Method to estimate Δ' and w_c

- Electron temperature profile near the magnetic island (away from heat sink or source) by heat flow equation $\nabla \cdot (-\kappa_\parallel \nabla_\parallel T - \kappa_\perp \nabla_\perp T) = 0$

$$\left[\kappa_\parallel \nabla_\parallel^2 + \kappa_\perp \nabla_\perp^2\right] T_e \approx \left[\frac{\kappa_\parallel}{\kappa_\perp} \nabla_\parallel^2 + \nabla^2\right] T_e = 0 \qquad w_c = \sqrt{\frac{RqL_q}{m}} \left(\frac{\kappa_\perp}{\kappa_\parallel}\right)^{1/4}$$

where magnetic field is represented with the helical flux function $\psi = \psi_0 + \psi_1$

$$B = \nabla\psi \times \hat{e}_\eta + B_\eta \hat{e}_\eta$$



then,

$$\nabla_\parallel = \hat{b} \cdot \nabla \approx \frac{1}{|B|} \begin{pmatrix} -\dfrac{m}{r_s} \psi_1 \sin\zeta \\ -\dfrac{\partial \psi_0}{\partial r} - \dfrac{\partial \psi_1}{\partial r} \cos\zeta \end{pmatrix} \cdot \begin{pmatrix} \dfrac{\partial}{\partial r} \\ \dfrac{m}{r_s} \dfrac{\partial}{\partial \zeta} \end{pmatrix} = \frac{m}{r_s |B|} \left(\psi_1 \sin\zeta \frac{\partial}{\partial r} + \psi_0' \frac{\partial}{\partial \zeta} + \psi_1' \cos\zeta \frac{\partial}{\partial \zeta} \right) \qquad \nabla^2 \approx \frac{\partial^2}{\partial r^2} + \left(\frac{m}{r_s}\right)^2 \frac{\partial^2}{\partial \zeta^2}$$

with helical angle $\zeta = m\theta - n\phi$ and

J.P. Meskat et al., PPCF (2001)

$$\psi_0(r) = \frac{\mu_0 I_0}{8\pi} \left(\left(\frac{r}{a}\right)^2 - \left(\frac{r_s}{a}\right)^2 \right)^2$$

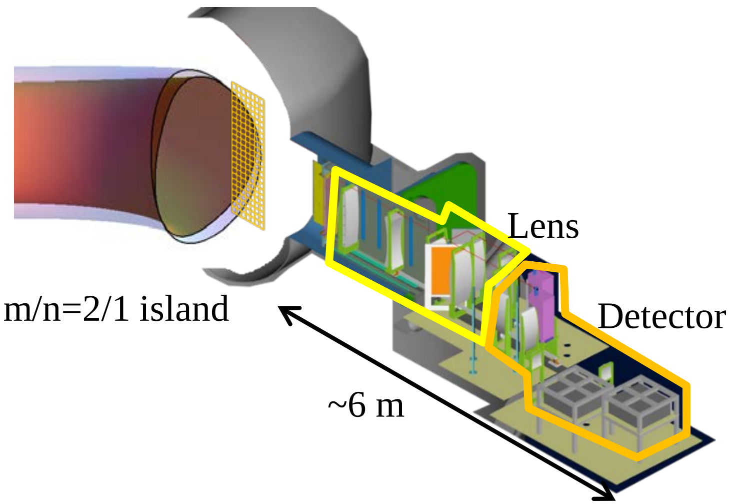
$$\psi_1(r) = \frac{\mu_0 I_0}{8\pi} \alpha \left(\frac{r}{r_s}\right)^m \left(1 - \beta \frac{r}{r_s}\right) \quad \text{for } r \le r_s$$

$$= \frac{\mu_0 I_0}{8\pi} \frac{\alpha(1-\beta) - \gamma + \gamma r / r_s}{(r/r_s)^{m+1}} \quad \text{for } r > r_s$$

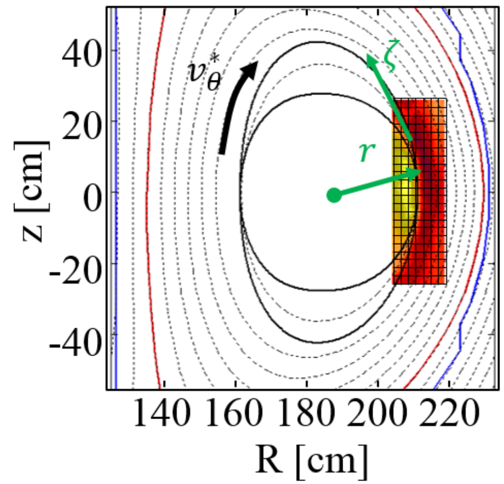
$$\Delta' \equiv \left[\left(\frac{d\psi_1}{dr}\right)_+ - \left(\frac{d\psi_1}{dr}\right)_- \right] / \psi_1$$

- Electron temperature profile solution $T_e(r, \zeta)$ becomes a function of parameters α, β, γ, and $\frac{\kappa_\parallel}{\kappa_\perp}$ → a function of $\Delta'(\alpha, \beta, \gamma)$ and $w_c = \sqrt{\frac{RqL_q}{m}} \left(\frac{\kappa_\perp}{\kappa_\parallel}\right)^{1/4}$

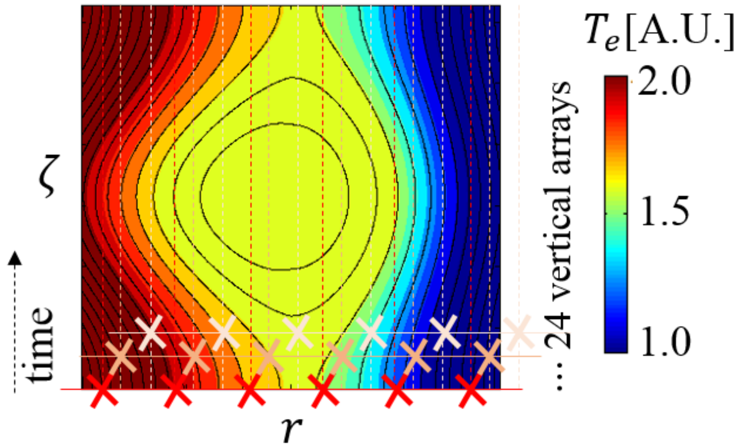
- Fine 2D electron temperature fluctuation ($\delta T_e / \langle T_e \rangle_t$) measurement near the island by the KSTAR ECEI diagnostic



Local 24 (vertical) X 8 (radial) = 192 measurement points



ECEI channels on (R, z) space



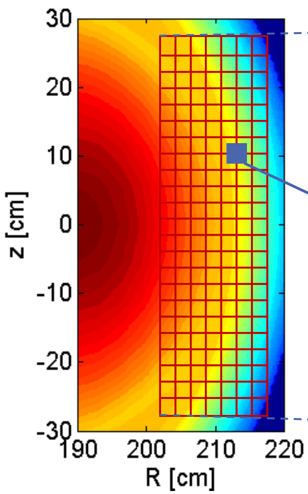
ECEI channels on (r, ζ) space

Spatial resolution in r direction < 1cm

- ECEI measurement reveals detail $\boldsymbol{T_e}$ structure of tearing mode on $(\boldsymbol{r}, \boldsymbol{\zeta})$ space
→ can be compared with the $\boldsymbol{T_e}$ model to estimate $\boldsymbol{\Delta'}$ and $\boldsymbol{w_c}$

- The $T_e(r,\zeta)$ model → synthetic $\delta T_e/\langle T_e\rangle_t$ for the direct comparison with the measured $\delta T_e/\langle T_e\rangle_t$ images by the ECEI diagnostic

2D T_e model



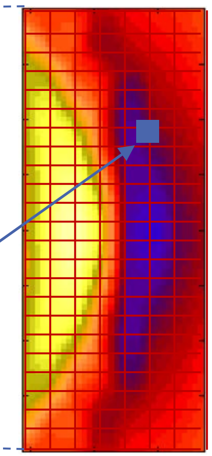
F_{inst} is instrumental 2D response function of each channel

$$T_{e,\text{syn}} = \frac{\int T_e F_{\text{inst}}(R,z)\,drdz}{\int F_{\text{inst}}(R,z)\,drdz}$$

$$\frac{\delta T_{e,\text{syn}}}{\langle T_{e,\text{syn}}\rangle_t} = \frac{T_{e,\text{syn}} - \langle T_{e,\text{syn}}\rangle_t}{\langle T_{e,\text{syn}}\rangle_t}$$

Normalization with the time average

Synthetic ECE image



- Find best matching $(\alpha, \beta, \gamma, \kappa_\perp/\kappa_\parallel)$ between $\delta T_{e,\text{syn}}/\langle T_{e,\text{syn}}\rangle_t$ and $\delta T_{e,\text{ECEI}}/\langle T_{e,\text{ECEI}}\rangle_t$

Monte-Carlo Method for initial values

$\boldsymbol{p}_{\textbf{int}} = [\alpha, \beta, \gamma, \kappa_\perp/\kappa_\parallel]$ → $\delta T_{e,\text{syn}}/\langle T_{e,\text{syn}}\rangle_t$ ↔ $\delta T_{e,\text{ECEI}}/\langle T_{e,\text{ECEI}}\rangle_t$

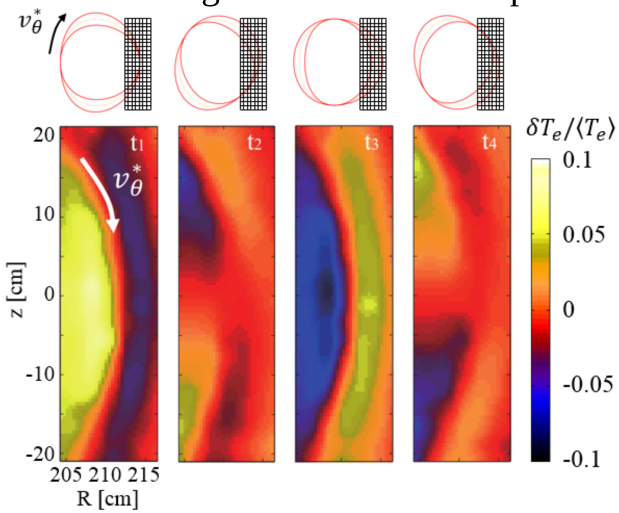
Is difference smaller enough?

Yes → Matching parameters $\boldsymbol{p}_{\textbf{fin}} = [\alpha, \beta, \gamma, \kappa_\perp/\kappa_\parallel]$

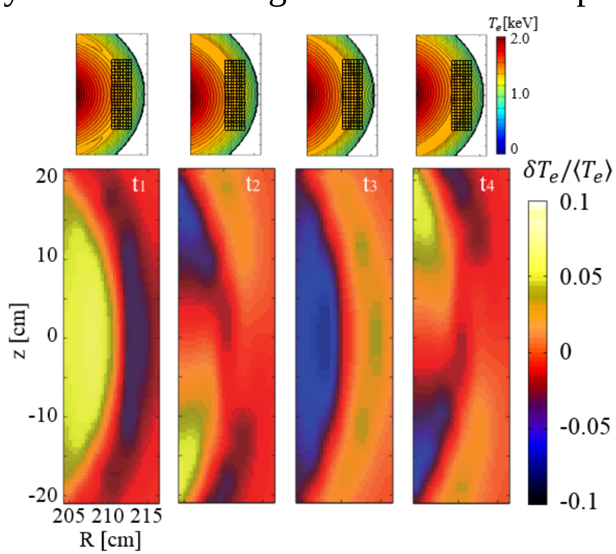
Estimate Δ' and w_c

No → Update the parameters with Levenberg-Marquardt Alogrithm

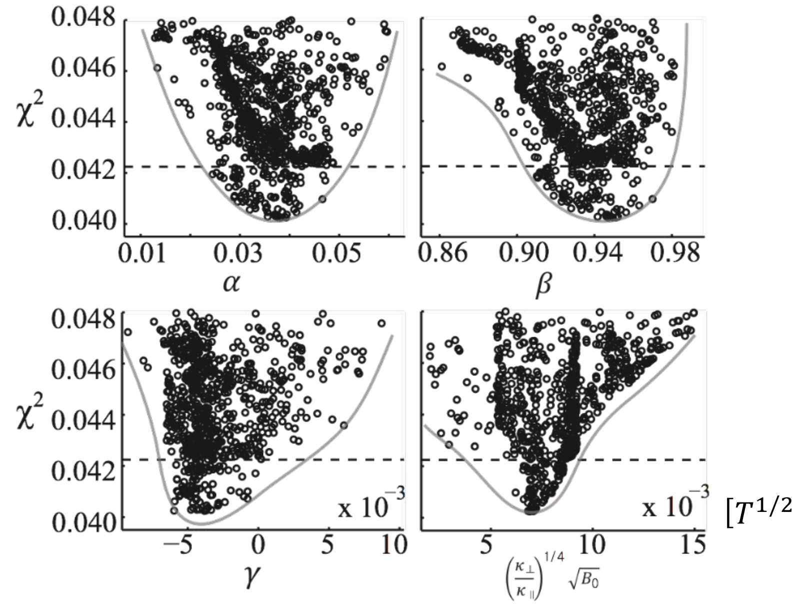
The illustration for the m=2 island and the measured ECE images at different time points



The T_e model solution at given $\boldsymbol{p} = [\alpha, \beta, \gamma, \kappa_\perp/\kappa_\parallel]$ and the synthetic ECE images at different time points



$\chi^2(\boldsymbol{p}) = \frac{1}{2}\sum_{i=1}^{N}\left[y_{\mathrm{syn}}(i) - y_{\mathrm{ECEI}}(i)\right]^2$ difference is calculated with different $\boldsymbol{p}$s (y represents all data points of four ECE images). The global minimum χ^2 point is found over $\boldsymbol{p}$ space.



The parameter sets whose $\chi^2(\boldsymbol{p}) < 0.0422$ (under the dashed line) are selected to estimate Δ' and w_c

M.J. Choi et al., NF (2014)

$\rightarrow r_s\Delta' = -1.633 \pm 1.265$

$\rightarrow w_c = 0.612 \pm 0.0726$ cm

$r_s\Delta'$ is found to be negative (classically stable)

$w_c < w$ implies that the pressure profile inside the island is flat and the lost bootstrap current is destabilizing

Method to estimate a_1 and a_2

- Unknown parameters in modified Rutherford equation (MRE) for the KSTAR

$$a_1 \frac{\tau_r}{r_s}\frac{dw}{dt} = r_s\Delta' + a_2\sqrt{\epsilon}\frac{\beta_\theta}{w}\frac{L_q}{L_p}\frac{w}{w^2+(w_c)^2} + \cdots$$

can be estimated by
the ECE images

H.R. Wilson, FST (2004)

a_1 and a_2 are integration coefficients which depend on magnetic geometry of the island, and they can be determined by fitting the measured island size evolution with the MRE

- Stepwise approach to estimate a_1 and a_2 for more accuracy

First, consider the plasma such that $a_2\sqrt{\epsilon}\frac{\beta_\theta}{w}\frac{L_q}{L_p}\frac{w}{w^2+(w_c)^2} \ll r_s\Delta'$, then the equation returns to the original Rutherford equation

$\mu_0 r_s^2/\eta$ where η is Spitzer resistivity

estimated for the KSTAR plasma geometry

$$a_1 \frac{\tau_r}{r_s}\frac{dw}{dt} = r_s\Delta'$$

estimated by the ECE images

estimated by the magnetic fluctuation measurement (Mirnov coil)

Estimation of a_1 for the KSTAR plasma

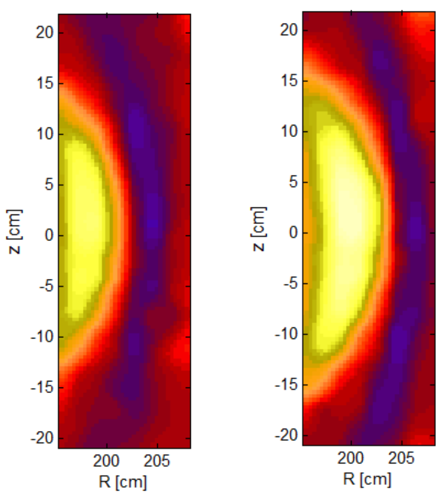
- The low β_θ plasma with the constant $m/n = 2/1$ island growth rate

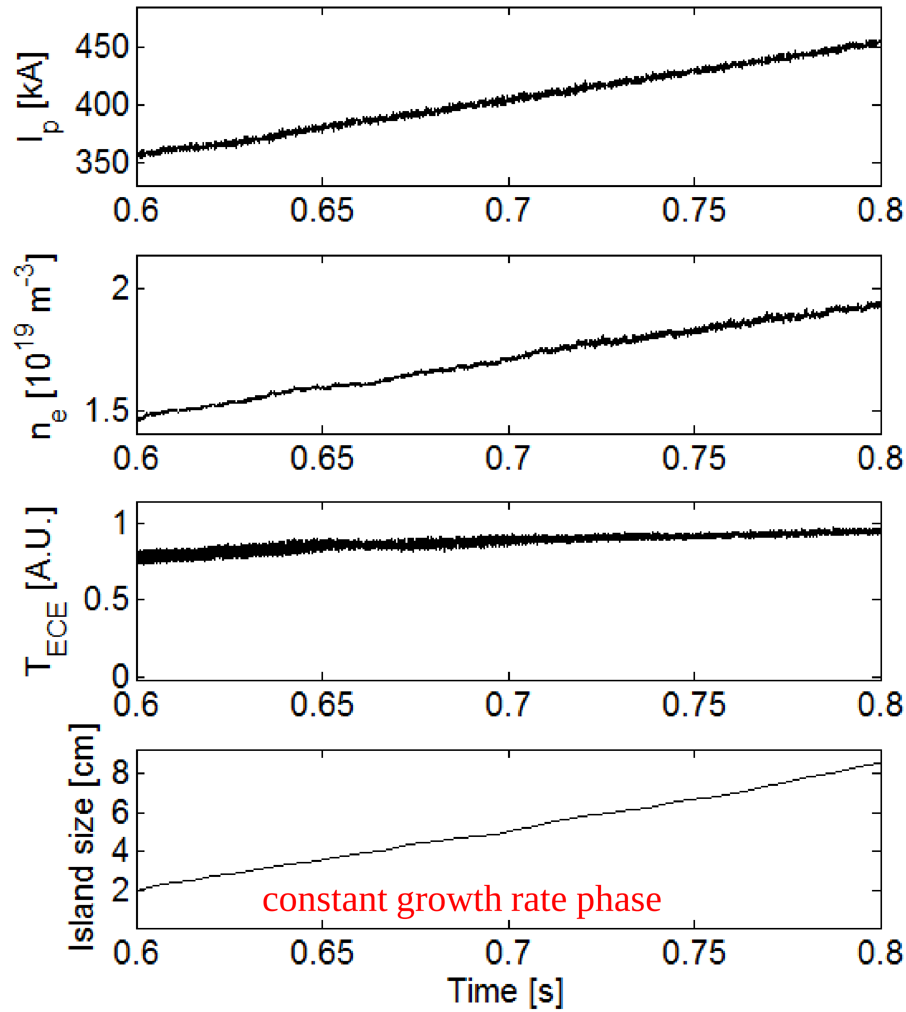
KSTAR plasma # 7318
The L-mode typical diverted plasma with neutral beam injection of 0.6 MW and electron cyclotron resonance heating of 0.3 MW
It has a constant growth rate of island size ($\mathrm{d}w/\mathrm{d}t$ = const) from 0.6—0.8 s

→ $a_1 \frac{\tau_r}{r_s}\frac{\mathrm{d}w}{\mathrm{d}t} \approx r_s\Delta'$

ECE image of m=2 island

at 0.76 s   at 0.8 s





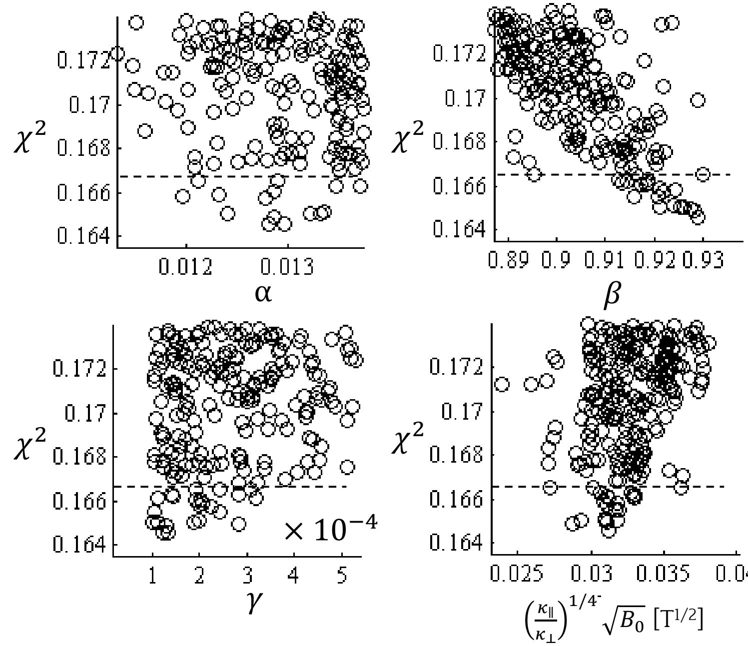
Island size is estimated by Mirnov coil and calibrated with the ECE image

- The Δ' estimation in the KSTAR # 7318

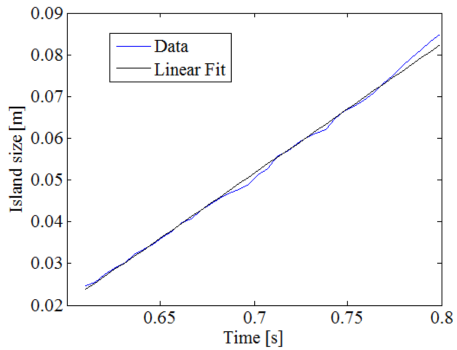
χ^2 fitting between the model $T_e(r, \zeta; \alpha, \beta, \gamma, \frac{\kappa_\parallel}{\kappa_\perp})$ and the ECEI measurement provides

$$r_s\Delta' = 0.52 \pm 0.37$$

Parameter sets whose $\chi^2 < 0.1665$ are selected for the estimation (below the dashed line)



- $\mathrm{d}w/\mathrm{d}t$ in the KSTAR # 7318



Island size was estimated by magnetic fluctuation amplitude measured by Mirnov coil and calibrated with the ECE image within ± 1 cm accuracy

Linear fitting provides $\frac{\mathrm{d}w}{\mathrm{d}t} = 0.322 \pm 0.012$

- a_1 in the KSTAR # 7318

From the Rutherford equation $a_1 \frac{\tau_r}{r_s}\frac{\mathrm{d}w}{\mathrm{d}t} = r_s\Delta'$, $a_1 = 0.26 \pm 0.16$ is obtained.

Spitzer transverse resistivity $\eta_\perp = 1.03 \times 10^{-4} Z \ln\Lambda\, T^{-3/2}$ [Ωm] is used for $\tau_r = \frac{\mu_0 r_s^2}{\eta_\perp} \sim 1.56$.

This coefficient a_1 can be applied to the plasma whose magnetic geometry is similar to # 7318.

Theoretical a_1 estimation with the cylindrical plasma assumption is 0.82

Summary and Discussion

- Method to estimate parameters Δ', w_c, and a_1 of modified Rutherford equation is developed

- Obtain the form of the Rutherford equation for the KSTAR plasma

$$0.26 \frac{\tau_r}{r_s} \frac{\mathrm{d}w}{\mathrm{d}t} = r_s \Delta'$$

The coefficient $a_1 = 0.26$ can be used for a_2 determination in the modified Rutherford equation

$$0.26 \frac{\tau_r}{r_s} \frac{\mathrm{d}w}{\mathrm{d}t} = r_s \Delta' + a_2 \sqrt{\epsilon} \frac{\beta_\theta}{w} \frac{L_q}{L_p} \frac{w}{w^2 + (w_c)^2} + \cdots$$

- The obtained Rutherford equation will be checked with the M3D-C1 simulation

Levenberg–Marquardt Algorithm (LMA): the most standard multi-parameter fit algorithm

data points (t_i, y_i) → $\boldsymbol{y}$

model function values with given parameters $\boldsymbol{p}$ → $\hat{\boldsymbol{y}}(t_i;\boldsymbol{p})$

goodness-of-fit (chi-squared error) → $\chi^2(\boldsymbol{p}) = \frac{1}{2}\sum_{i=1}^{m}\left[\frac{y(t_i)-\hat{y}(t_i;\boldsymbol{p})}{\omega_i}\right]^2$

goal : find $\boldsymbol{p}$ which minimizes $\chi^2(\boldsymbol{p})$

$\boxed{\text{initial } \boldsymbol{p}}$

⇩

1. gradient decent method

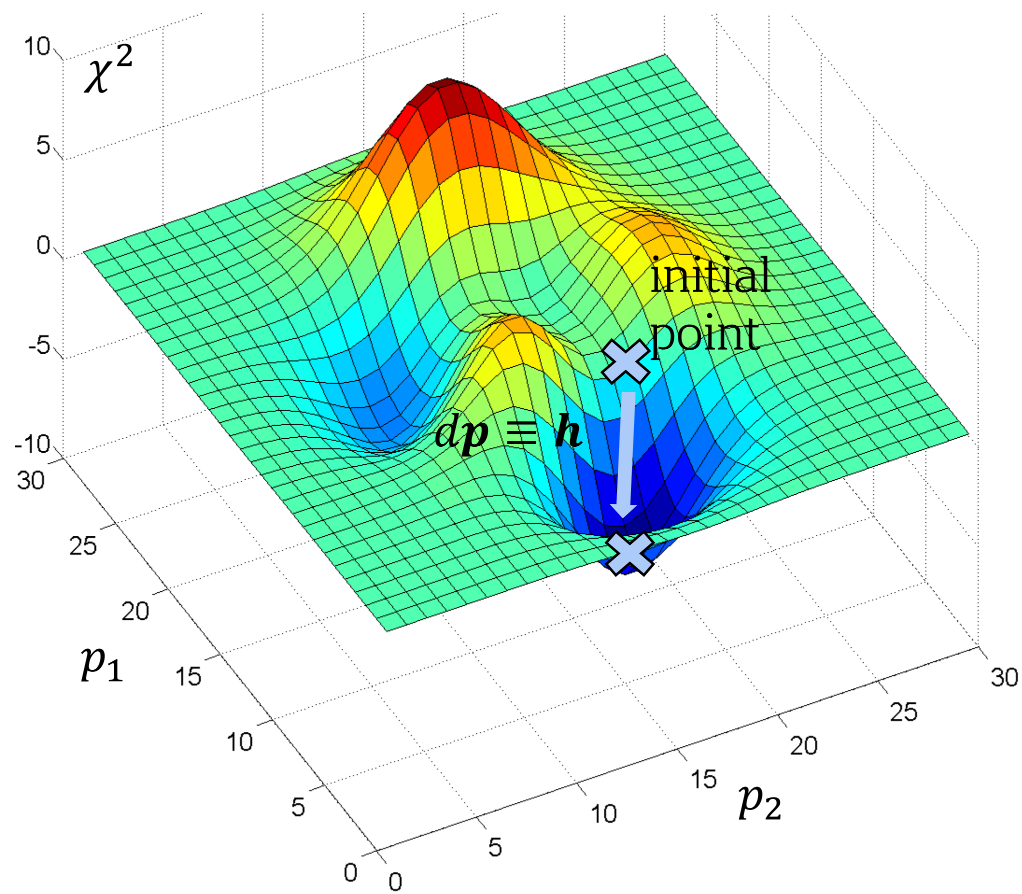
update $\boldsymbol{p}$ by $\boldsymbol{p} = \boldsymbol{p} + \boldsymbol{\epsilon}(-\nabla\chi^2)$

1. Gauss–Newton method

update $\boldsymbol{p}$ by $\boldsymbol{p} = \boldsymbol{p} + \boldsymbol{h}$ where $\frac{\partial\chi^2}{\partial\boldsymbol{h}} = \boldsymbol{0}$

⇩

$\boxed{\text{final } \boldsymbol{p}}$



EC Emission profile

Radial Natural line width : relativistic broadening or Doppler broadening, ⋯ + re-absorption process

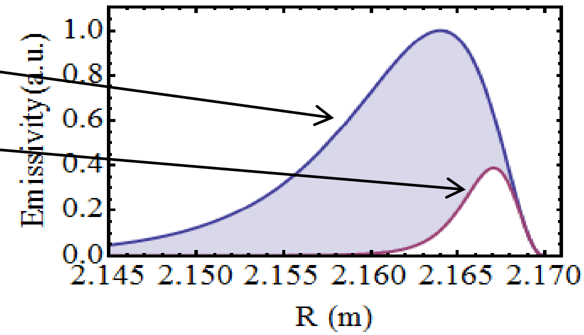
$$\frac{1}{N^2}\frac{j_\omega}{\alpha_\omega} = \frac{\omega^2}{8\pi^3 c^2} kT_e$$

Relativistic broadening + re-absorption: (assumed $n_e = 10^{19}\ \mathrm{m^{-3}}$ and $T_e = 500$ eV)

Emission profile for $f_0 = 2\ f_{ce} \approx 90$ GHz

Re-absorption reduces the width

$w_{rel} \approx 0.5$ cm



Instrument broadening:

Frequency bandwidth of each channel = 0.7 GHz → $w_{\mathrm{instr}} \sim 1.5$ cm

Doppler effect due to finite beam size:

$$\Delta\omega_D \approx \frac{2\sqrt{2}\log 2}{w_{\mathrm{beam}}} v_T \rightarrow w_D < 0.5\ \mathrm{cm}$$

$$j_\omega(R) = j_\omega(R) e^{-\tau(R)}$$

$$\tau(R) = \int_R^\infty \alpha_\omega(R) dR$$

Vertical : Gaussian-like response (designed by optics)

I. H. Hutchinson, "Principles of Plasma Diagnostics", 2nd ed., Cambridge Press (2005)
*M. Bornatici et al., Nucl. Fusion, **23**, 9 (1983)*
*C. Watts et al., Rev. Sci. Instrum., **75**, 10 (2004)*