Systematics of characteristics of pygmy dipole resonances in middle and heavy atomic nuclei with neutron excess

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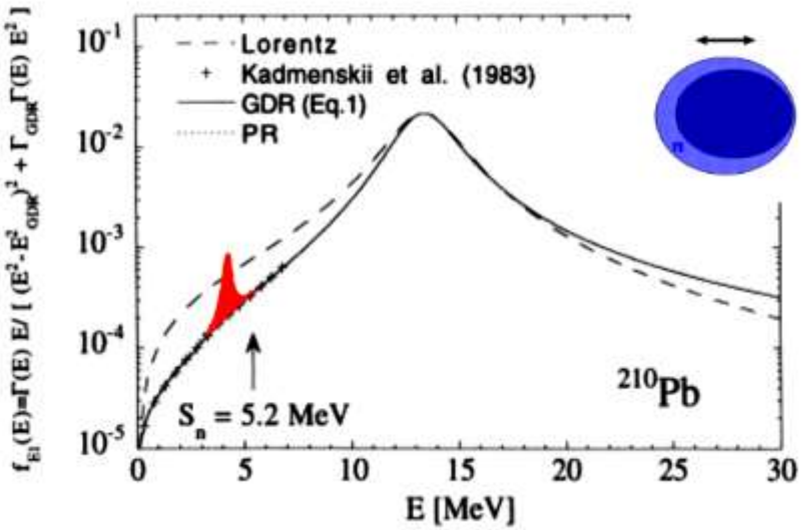
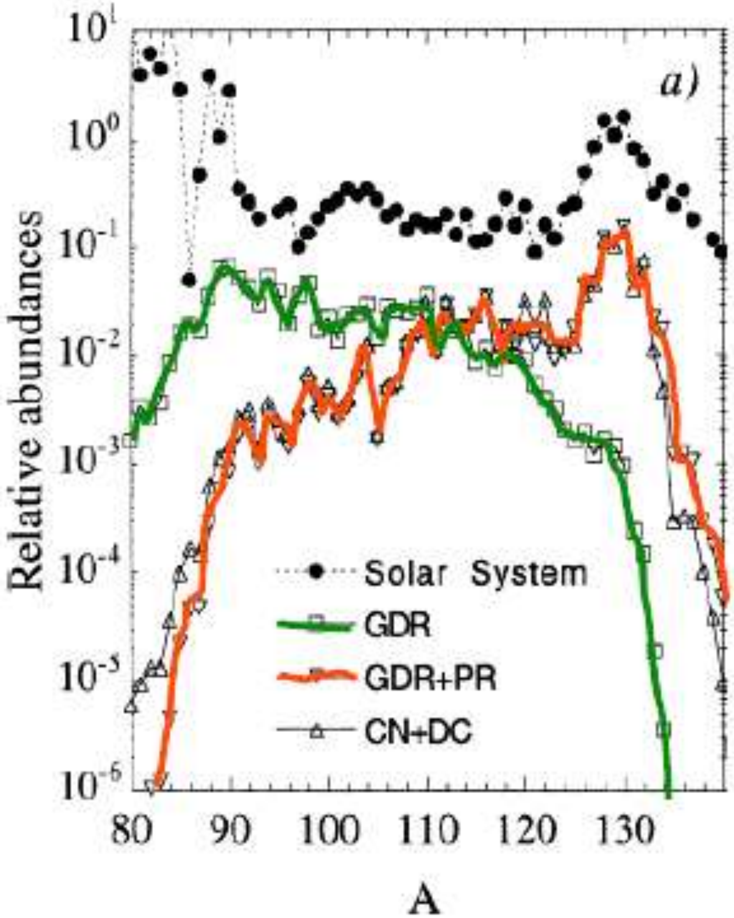
The systematics of energies and contribution to the energy-weighted sum rule of dipole gamma transitions of pygmy dipole resonance (PDR) in middle and heavy nuclei with an excess of neutrons are considered based on combination of macroscopic model of Isacker-Nagarajan-Warner(INW) with the number of near-surface neutrons in proportion to the thickness of neutron skin according to the Pethick- Ravenhall (PR) approach. This model was noted as PR INW.

Report outline:

- Motivation. Pygmy dipole resonances (PDR).
- Description of PDR energies
- Number of neutrons in the skin for neutron-excess nuclei
- Best parameterizations for neutron skin thickness
- Comparison of PDR energy systematics.
- Fraction of the PDR in E1 EWSR
- Conclusions

Motivation.
The effect of pygmy dipole resonance on the abundance of elements





S. Goriely
Radiative neutron captures by neutron-rich nuclei and the r-process nucleosynthesis
Physics Letters B 436 1998. 10–18

Motivation.
TSE – model of two state coupled excited states with response of two damping states (PDR&GDR)

Based on the model of two coupled oscillators [*A. S. Barker, J. J. Hopfield, Phys. Rev. 135(1964)A1732]* :

$$\begin{cases} \ddot{y}_p + \omega_p^2 y_p + \overline{\Gamma}_p \dot{y}_p + \overline{\gamma}(\dot{y}_p - \dot{y}_g) = z_p E \\ \ddot{y}_g + \omega_g^2 y_g + \overline{\Gamma}_g \dot{y}_g - \overline{\gamma}(\dot{y}_p - \dot{y}_g) = z_g E \end{cases} \qquad E \sim \exp(i\omega t) - \text{external field}$$

q_m -characteristics of states: $m = p -> PDR; \;\; m = g -> GDR;$

$\overline{\gamma}$ - amplitude widths for PDR and GDR modes.

Previously proposed for PSF in [*V. Plujko et al. EPJ Web of Conf. 146, (2017) 05014 ; Gorbachenko O.M., et al. Nuclear Physics and Atomic Energy, 24 (1), (2023) 17*]

Response function

$$\boxed{\chi(E_\gamma) = P(E_\gamma; g, p) + P(E_\gamma; p, g)}$$

$$P(E_\gamma;k,l) = \frac{z_k{}^2 + \dfrac{i\, z_k z_l E_\gamma\, \gamma}{E_l^2 - E_\gamma{}^2 + iE_\gamma(\Gamma_l + \gamma)}}{E_k^2 - E_\gamma{}^2 + iE_\gamma(\Gamma_k + \gamma) + \dfrac{\gamma^2 E_\gamma{}^2}{E_l^2 - E_\gamma{}^2 + iE_\gamma(\Gamma_l + \gamma)}}, \quad k,l = g \;\; or \;\; p$$

$$f_{X\lambda}(\varepsilon_\gamma, T) = \frac{L(\varepsilon_\gamma, T)}{3(\pi\hbar c)^2} \cdot \left(-\frac{1}{\pi} \cdot \operatorname{Im}\chi\left(\varepsilon_\gamma, T\right)\right), \quad L(\varepsilon_\gamma, T) \equiv \frac{1}{1-\exp(-\varepsilon_\gamma/T)} \xrightarrow[\varepsilon_\gamma << T_i]{} \frac{T}{\varepsilon_\gamma}, \; L(\varepsilon_\gamma, T=0) = 1$$

$\Gamma_m(E_\gamma) = \Gamma_{m,S} \cdot E_\gamma / E_m$ - SMLO model, $\Gamma(E_\gamma) = \Gamma_{m,c} = const$ -SLO model,

Comparison of different microscopic calculations for describing PDR energies

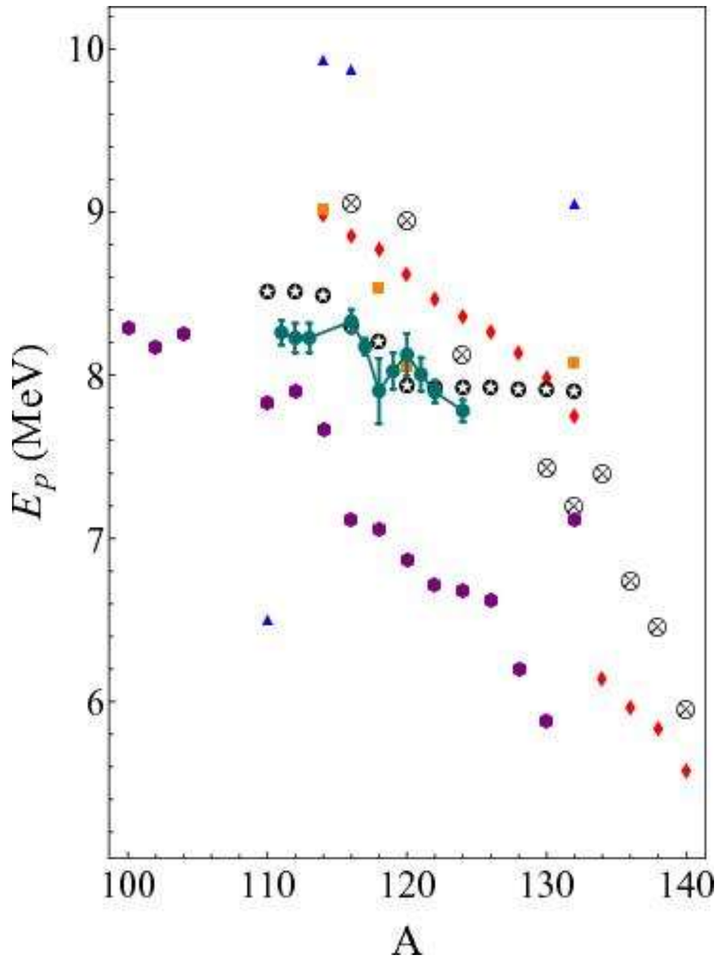
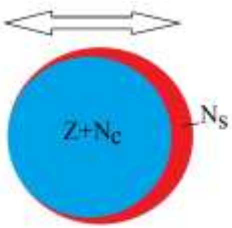


Fig. The energies of the PDR for isotopes of tin ${}^{A}_{50}Sn$ with $N \geq Z$ in dependence on the atomic mass number

The symbols correspond to the microscopic calculations by the following methods the relativistic RPA (RRPA) [2] - square (■), the relativistic quasiparticle RPA (RQRPA) [1] - rhombus (♦), the mean-field (MF) plus the relativistic RPA (MF RRPA) [3] - circle with an asterisk (✪), the quasiparticle-phonon model (QPM)[4] - hexagon (⬢), the elativistic quasiparticle time blocking approximation (RQTBA) [5] - circle with a cross (⊗), self-consistent Hartree-Fock plus continuum RPA approach (HF CRPA) [6] - triangle (▲). The experimental data indicated by the shaded circle (●), which connected by the lines for the better visualization of the data. They are taken from Fig. 2 of Ref. [7] and they correspond to the average values of the PDR energies.

1. N. Paar et al. Phys. Lett. B. 606(3-4) (2005) 288.
2. D. Vretenar et al. Nucl. Phys. A 692 (2001) 496.
3. J. Piekarewicz. Phys. Rev. C. 73 (2006) 044325.
4. N. Tsoneva, H. Lenske, Phys. Rev. C. 77 (2008) 024321.
5. E. Litvinova et al. Phys. Rev. C 79 (2009) 054312.
6. G.Co et al., Phys. Rev. C 87 (2013) 034305.
7. M Markova, P.von Neumann-Cosel, E. Litvinova, Phys. Lett. B 860 (2025) 139216.

Description of PDR energies



The expression of the INW model [1] for the PDR energy can be represented as:

$$E_p = K\left[\frac{1}{3}\frac{Z}{Z+N_c}\right]^{1/2}\left[\frac{6a}{yR_m}\Delta F(R_n,R_p)\right]^{1/2} A^{-1/6}\ , \qquad (1)$$

where $y=\sqrt{5/3}\cdot\Delta r_{np}$ is determined by the thickness of the neutron skin Δr_{np}, $\Delta F(R_n,R_p)=F(R_n,R_p)-F(R_p,R_p)$ from [1], $R_p=R_m-y/2$, $R_n=R_m+y/2$, $R_m=r_m\cdot A^{1/3}$, $r_m=1.25$ fm, $a=0.57$ fm, $N_c=N-N_s$ - number of neutrons in the core, $A_c=Z+N_c$, quantity

$$K=\left[\frac{\hbar^2}{m}\frac{|\kappa_{np}|}{8\pi a r_0^4}\frac{r_m}{r_0}\right]^{1/2}=1.34\cdot\sqrt{|\kappa_{np}|}$$ (MeV), where κ_{np} − effective proton-neutron interaction strength

$$\Delta F=F(R_n,R_p)-F(R_p,R_p),$$

$$F(R_n,R_p)=\frac{1}{4a}\mathrm{csch}^2\left(\frac{y}{2a}\right)\left[\coth\left(\frac{y}{2a}\right)\frac{c_n-c_p}{a}-(c'_n+c'_p)\right], \qquad F(R_p,R_p)=\frac{3R_p^2+(\pi^2-6)a^2}{18a}$$

Here $c_i=(R_i^3+\pi^2a^2R_i)/3$, $c'_i\equiv c'(R_i,a)=\partial c_i/\partial R_i=R_i^2+\pi^2a^2/3$; and R_n, R_p - radii of neutron and proton density distributions from the form of Fermi functions $\rho_j(r)=\rho_{0,j}\cdot\left[1+\exp\left(\left(r-R_j\right)/a_j\right)\right]^{-1}$

[1] P. Van Isacker, M. A. Nagarajan, and D. D. Warner, Physical Review C V. 45, No. 1, R13 (1992)

[2] Goriely S.,. Tondeur F, Pearson J. M., At. Data Nucl Data Tabl. 77, 311 (2001).

Description of PDR energies

In the approximation $R_m >> \pi a$ according to INW model for PDR energy:

$$E_p = K\left[Z/(3(Z+N_c))\right]^{1/2}\left(1-y\cdot R_m/(10a^2)\right)^{1/2} A^{-1/6} \cong. \qquad (2)$$

$$\cong K\left[Z/(3(Z+N_c))\right]^{1/2}\left(1-y\cdot R_m/(20a^2)\right) A^{-1/6}. \qquad (3)$$

PDR energy according to SIS-model [2]:

$$E_{p,SIS} = \frac{2.082}{A^{1/3}}\left[\frac{\hbar^2 8a_{sym}}{m\ r_0^2}\frac{ZN}{A^2}\right]^{1/2}\left[\frac{N_s}{A_c}\frac{Z}{N}\right]^{1/2} = 79\left[\frac{4ZN}{A^2}\right]^{1/2}\left[\frac{N_s}{A_c}\frac{Z}{N}\right]^{1/2} A^{-1/3}\ (\mathrm{MeV})$$

[1] P. Van Isacker, M. A. Nagarajan, and D. D. Warner, Physical Review C V. 45, No. 1, R13(1992)

[2] Y. Suzuki, K. Ikeda, H. Sato. Prog. Theor. Phys. 83, 180 (1990).

Number of neutrons in the skin for neutron-excess nuclei

C.J.Pethick & D.G.Ravenhall (PR) [1] showed that the neutron skin thickness $\delta r_{0,np} = R_{0,n} - R_{0,p}$ and the distance equivalent to the radius of the proton distribution with a sharp edge determines the number of neutrons in the neutron skin N_s:

$$\frac{N_s}{N} = \frac{3\delta r_{o,np}}{R_0} \cong \frac{3y}{R_0} \cong \frac{\sqrt{15}\Delta r_{np}}{R_0}$$

where $R_{0,n}$, $R_{0,p}$ - radius of neutron and proton distributions with a sharp edge, in the leptodemic approximation $R_0 >> a$ value of neutron skin thickness $\delta r_{0,np} \cong \sqrt{5/3}\Delta r_{np}$ coincides with the thickness value $y = R_n - R_p$.

Average constant value of the parameter $r_m = R_m / A^{1/3}$ was from radius fit R_m

$$R_m = \frac{R_n + R_p}{2} \cong \sqrt{\frac{5}{3}} \cdot \left[R_{rms,p} + \frac{\Delta r_{np}}{2} \right],$$

where from [2] $R_{rms,p} = \left[r_1 + \frac{r_2}{A^{2/3}} + \frac{r_3}{A^{4/3}} \right] \cdot A^{1/3}$, $r_1 = 0.9071$ fm, $r_2 = 1.105$ fm,

$r_3 = -0.548$ fm. and $\Delta r_{np} = \alpha_S + \beta_S \cdot I$. We define: $r_m = 1.25 \pm 0.02$ fm

[1] C. J. Pethick, D. G. Ravenhall, NPA, 606(1-2), 173–182(1996).
[2] Angeli I., Marinova K., An update Atomic Data and Nuclear Data Tables 99, 69-95 (2013).

Determinations of the best parameterizations for neutron skin thickness

The neutron skin thickness $\boxed{\Delta r_{np} = R_{rms,n} - R_{rms,p}}$ was considered

$R_{rms,n}, R_{rms,p}$ -root mean square (*rms*) radii for neutron, proton distributions

as a function of neutron excess [1-3] :

$$\boxed{\Delta r_{np} \equiv \Delta r_J = \alpha_J + \beta_J \cdot I,} \qquad I = (N - Z)/A$$

$$\alpha_I = const_1, \qquad \beta_I = const_2$$

It was shown [4] that the expression fits the experimental data better if the coefficients depend on the mass number.

$$\alpha_{II} = A^{1/3}(a_1 + a_2 / A), \qquad \beta_{II} = b_1 A^{1/3}$$

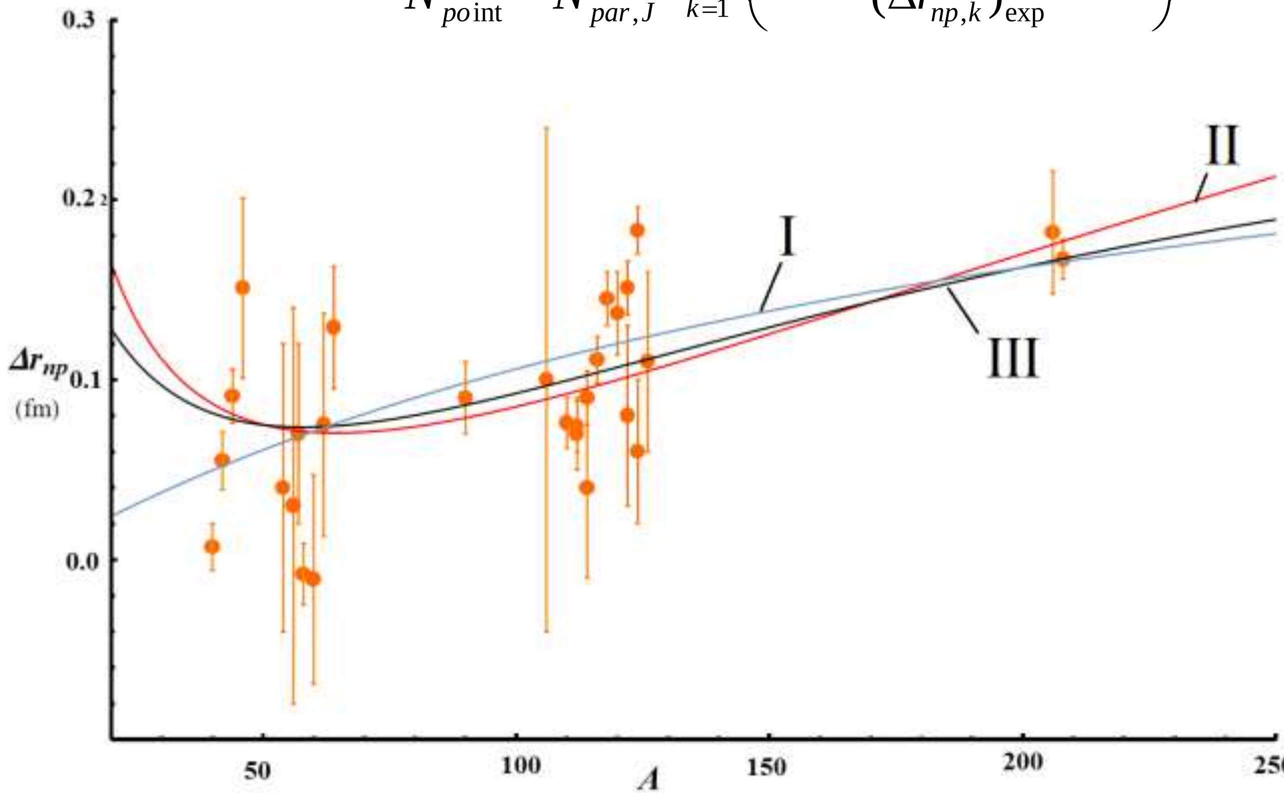
$$\alpha_{III} = A^{1/3}(\bar{a}_1 + \bar{a}_2 A^{2/3} + \bar{a}_3 A^{-1/3}), \qquad \beta_{III} = A^{1/3}(\bar{b}_1 + \bar{b}_2 A^{2/3})$$

[1] Trzcińska A., et al. Phys. Rev. Lett. 87, 082501 (2001).
[2] J. Jastrzebski,A. Trzcinska et al. Int. Jour. Mod. Phys E Vol. 13, No. 1 (2004) 343. https://doi.org/10.1142/S0218301304002168
[3] V. M. Kolomeitz, Sanzhur A. I. Proceedings of the ISTROS 2013 Inter. Conf.., Bratislava 2015, ISBN 978-80-971975-0-6, pp.91 - 102.
[4] Zhang J. T., et al. PRC, Vol.104, 034303 (2021). https://doi.org/10.1103/PhysRevC.104.034303

Systematics fit to experimental and theoretical values

A mixed database was used, including estimated data for even-even nuclei from [1] and data for other nuclei from [2]

$$\chi_J^2 = \frac{1}{N_{point} - N_{par,J}} \sum_{k=1}^{N_{point}} \left(\frac{(\Delta r_{J,k})_{th} - (\Delta r_{np,k})_{\exp}}{(\Delta r_{np,k})_{\exp}} \right)^2$$



[1] J. T. Zhang, et al. PRC, Vol.104, 034303 (2021). https://doi.org/10.1103/PhysRevC.104.034303

[2] J. Jastrzebski,A. Trzcinska et al. Int. Jour. Mod. Phys E Vol. 13, No. 1 (2004) 343. https://doi.org/10.1142/S0218301304002168

Defined systematics :

$$\boxed{\Delta r_{np} \equiv \Delta r_J = \alpha_J + \beta_J \cdot I,}$$

$$\alpha_I = -0.007 \pm 0.012, \quad \beta_I = 0.845 \pm 0.078 \qquad \chi_I^2 = 1.847$$

$$\alpha_{II} = A^{1/3}(a_1 + a_2 / A), \quad \beta_{II} = b_1 A^{1/3} \qquad \chi_{II}^2 = 1.165$$

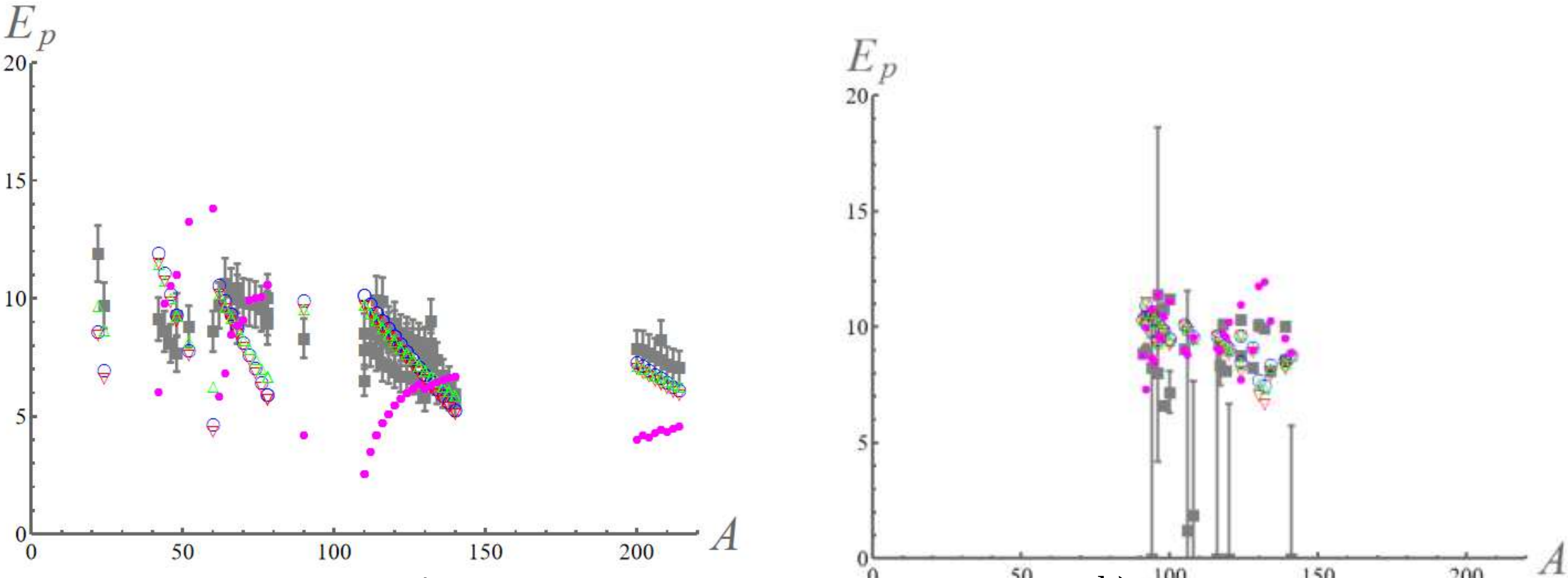
$$a_1 = -0.039 \pm 0.006, \quad a_2 = 1.777 \pm 0.229,$$

$$b_1 = 0.294 \pm 0.025$$

$$\alpha_{III} = A^{1/3}(\overline{a}_1 + \overline{a}_2 A^{2/3} + \overline{a}_3 A^{-1/3}), \quad \beta_{III} = A^{1/3}(\overline{b}_1 + \overline{b}_2 A^{2/3}), \qquad \boxed{\chi_{III}^2 = 0.961}$$

$$\overline{a}_1 = -0.218 \pm 0.073, \quad \overline{a}_2 = 0.003 \pm 0.002, \quad \overline{a}_3 = 0.617 \pm 0.179$$

$$\overline{b}_1 = 0.468 \pm 0.085, \quad \overline{b}_2 = -0.009 \pm 0.005$$



a) b)

Fitting the proportionality constant according to different models based on microscopic calculations[1-7] a) and experimental data b) defined in [8].

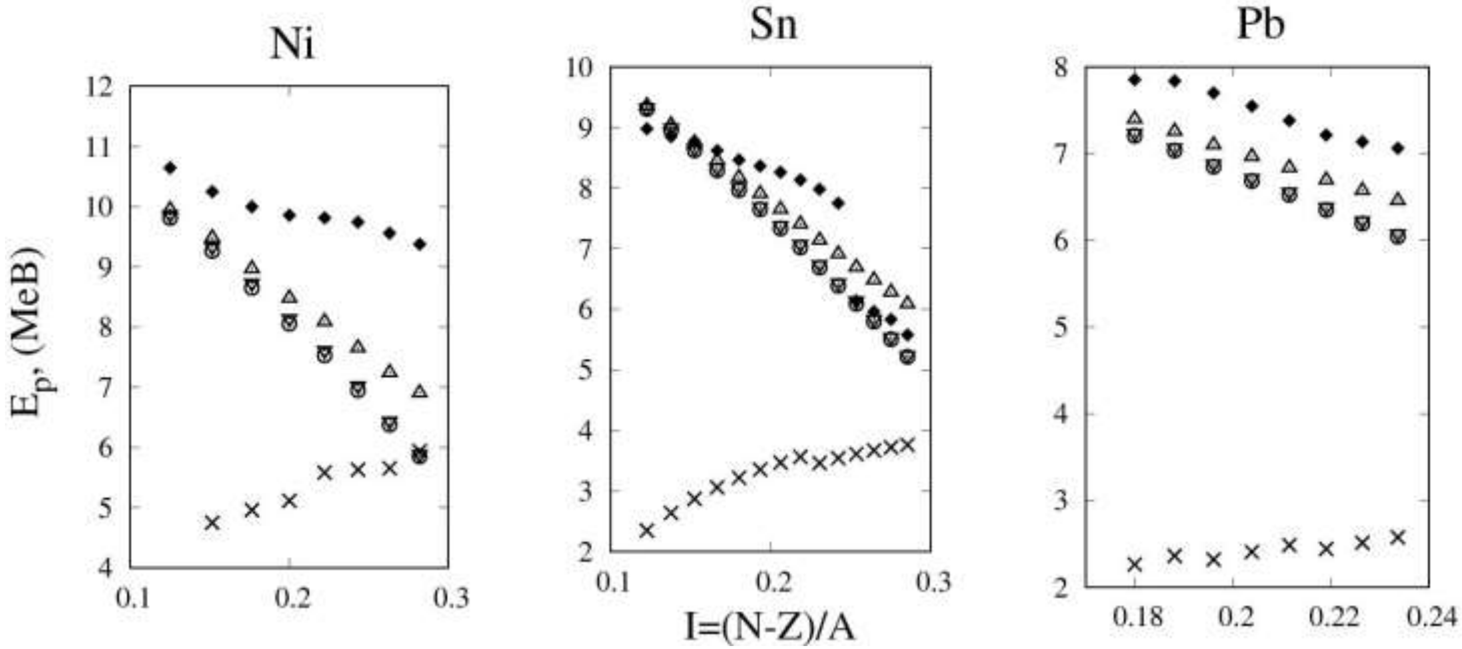
Fitting the constant:

fig. a) $|\kappa_{np}| = 1841 \pm 72.$ fig. b) $|\kappa_{np}| = 2261 \pm 109$

We take $\kappa_{np} = -1828.23\ \ \mathrm{MeV \cdot fm^3}$ from [8].

[1] N. Paar et al. Phys. Lett. B. 606(3-4) (2005) 288.; [2] D. Vretenar et al. Nucl. Phys. A 692 (2001) 496.;
[3] J. Piekarewicz. Phys. Rev. C. 73 (2006) 044325.; [4] N. Tsoneva, H. Lenske, Phys. Rev. C. 77 (2008) 024321.;
[5] E. Litvinova et al. Phys. Rev. C 79 (2009) 054312.; [6] G.Co et al., Phys. Rev. C 87 (2013) 034305.;
[7] M Markova, P. von Neumann-Cosel, E. Litvinova, Phys. Lett. B 860 (2025) 139216.;
[8] Gorbachenko O.M., et al. Nucl. Phys. and Atomic Energy, 24 (1), (2023) 17
[9] Goriely S.,. Tondeur F, Pearson J. M., At. Data Nucl Data Tabl. 77, 311 (2001).

Comparison of PDR energy systematics



Comparison of systematics of pygmy dipole resonance energies for isotopes *Ni*, *Sn*, *Pb* with microscopic calculations of [1] depending on the neutron-proton asymmetry parameter $I = (N - Z)/A$. Definitions:

◆ - calculations based on a microscopic model [1];

× - calculations by SIS model;

⊙ - exact expression (1) according to PR INW model;

▽ - approximate model PR INW (eq. (2));

△ - approximate model PR INW (eq. (3)).

$$E_{p,1} = K\left[\frac{1}{3}\frac{Z}{Z+N_c}\right]^{1/2}\left[\frac{6a}{yR_m}\Delta F(R_n,R_p)\right]^{1/2} A^{-1/6} \quad (1)$$

$$E_{p,2} = K\left[\frac{1}{3}\frac{Z}{Z+N_c}\right]^{1/2}\left(1-\frac{y\cdot R_m}{10a^2}\right)^{1/2} A^{-1/6} \quad (2)$$

$$E_{p,3} = K\left[\frac{1}{3}\frac{Z}{Z+N_c}\right]^{1/2}\left(1-\frac{y\cdot R_m}{20a^2}\right) A^{-1/6} \quad (3)$$

[1] N. Paar et al. Phys. Lett. B. 606(3-4) (2005) 288.

As simplified expressions for the systematics of microscopic calculations, we will use the expressions

$$E_p = e_1\sqrt{Z}A^{-2/3}/\sqrt{1+e_2\Delta r_{np}A^{1/3}} = E_{p,4}\ , \tag{4}$$

$$E_p = d_1\sqrt{Z}A^{-2/3}/\sqrt{1+d_2\Delta r_{np}A^{1/3}} + d_3\left[\Delta r_{np}\right]^{1/2} A^{-5/6} = E_{p,5}\ , \tag{5}$$

Table 1. Parameters in the PDR energies $E_{p,\alpha}$, $\alpha = 1-5$, from comparison and the fit of microscopic calculations ([1-7]) to analytical expressions and the ratio of the mean-square deviations value per one energy for given approach $\bar{\chi}^2_{E_{p,\alpha}} = \chi^2_{E_{p,\alpha}}/N_\Sigma$ to that one for INW model $\bar{\chi}^2_{E_{p,\alpha=1}}$ = 2.38 at $\kappa_{np} = -1828.23\ \ \mathrm{MeV\cdot fm^3}$[8].

PDR energy, equation	Parameters	$\bar{\chi}^2_{E_\alpha}/\bar{\chi}^2_{E_{p,\alpha=1}}$
$E_{p,1}$, (1)	$\kappa_{np} = -1828.23\ \mathrm{MeV\cdot fm^3}$ [1]	1
	$\kappa_{np} = -1930 \pm 78\,\mathrm{MeV\cdot fm^3}$	0.98
$E_{p,2}$, (2)	$\kappa_{np} = -1828.23\mathrm{MeV\cdot fm^3}$[1]	0.98
$E_{p,3}$, (3)	$\kappa_{np} = -1828.23\ \mathrm{MeV\cdot fm^3}$ [1]	0.66
$E_{p,4}$, (4)	$e_1 = 27.3$, $e_2 =$ -0.03	0.38
$E_{p,5}$, (5)	$d_1 = 27.1$, $d_2 = 0.01$, $d_3 = 31.1$	0.37

[1] N. Paar et al. Phys. Lett. B. 606(3-4) (2005) 288.; [2] D. Vretenar et al. Nucl. Phys. A 692 (2001) 496.;
[3] J. Piekarewicz. Phys. Rev. C. 73 (2006) 044325.; [4] N. Tsoneva, H. Lenske, Phys. Rev. C. 77 (2008) 024321.;
[5] E. Litvinova et al. Phys. Rev. C 79 (2009) 054312.; [6] G.Co et al., Phys. Rev. C 87 (2013) 034305.;
[7] M Markova, P. von Neumann-Cosel, E. Litvinova, Phys. Lett. B 860 (2025) 139216.;
[8] Goriely S.,. Tondeur F, Pearson J. M., At. Data Nucl Data Tabl. 77, 311 (2001).

Fraction of the PDR in E1 EWSR

Fraction of the PDR in E1 EWSR, to corresponding fractions of GDR is proportional to the number of neutrons N_s in the skin:

$$\overline{s}_p = \sigma_{\text{int}}(\text{PDR})/\sigma_{\text{int}}(\text{GDR}) = (N_s / N) Z / (A - N_s),$$

With Pethick- Ravenhall approach ($N_s = N \cdot \sqrt{15} \Delta r_{np} / R_0$) this relationship is rewritten as:

$$\overline{s}_p = \frac{\sigma_{\text{int}}(\text{PDR})}{\sigma_{\text{int}}(\text{GDR})} \equiv \frac{m^1_{\text{L}=1}(\text{PDR})}{m^1_{\text{L}=1}(\text{GDR})} = \frac{Z}{A - N \cdot \sqrt{15} \cdot \Delta r_{np} / R_0} \frac{\sqrt{15} \cdot \Delta r_{np}}{R_0} = \overline{s}_{p,1} \,. \qquad (1)$$

$$\frac{\sigma_{\text{int}}(\text{PDR})}{\sigma_{\text{int}}(\text{GDR})} = \frac{m^1_{\text{L}=1}(\text{PDR})}{m^1_{\text{L}=1}(\text{GDR})} = \delta \left(\frac{\Delta r_{np}}{A^{1/3}} \right)^{\gamma} \equiv \overline{s}_{p,sys}, \qquad (2)$$

In the leptodermic approximation and at $N \cong Z \cong A/2$ such expression has the form:

$$s_p = \frac{m^1_{\text{L}=1}(\text{PDR})}{m^1_{\text{L}=1}} = \frac{\sigma_{\text{int}}(\text{PDR})}{\sigma_{\text{int}}} \cong \sqrt{2} \left(\frac{\delta r_{np}}{R_0} \right)^{3/2} = 2.1 \left(\frac{\Delta r_{np}}{R_0} \right)^{3/2} = s_{p,2} \,. \qquad (3)$$

$$\frac{\sigma_{\text{int}}(\text{PDR})}{\sigma_{\text{int}}} = \frac{m^1_{\text{L}=1}(\text{PDR})}{m^1_{\text{L}=1}} = \frac{\overline{s}_{p,sys}}{1 + \overline{s}_{p,sys}} \equiv s_{p,sys}$$

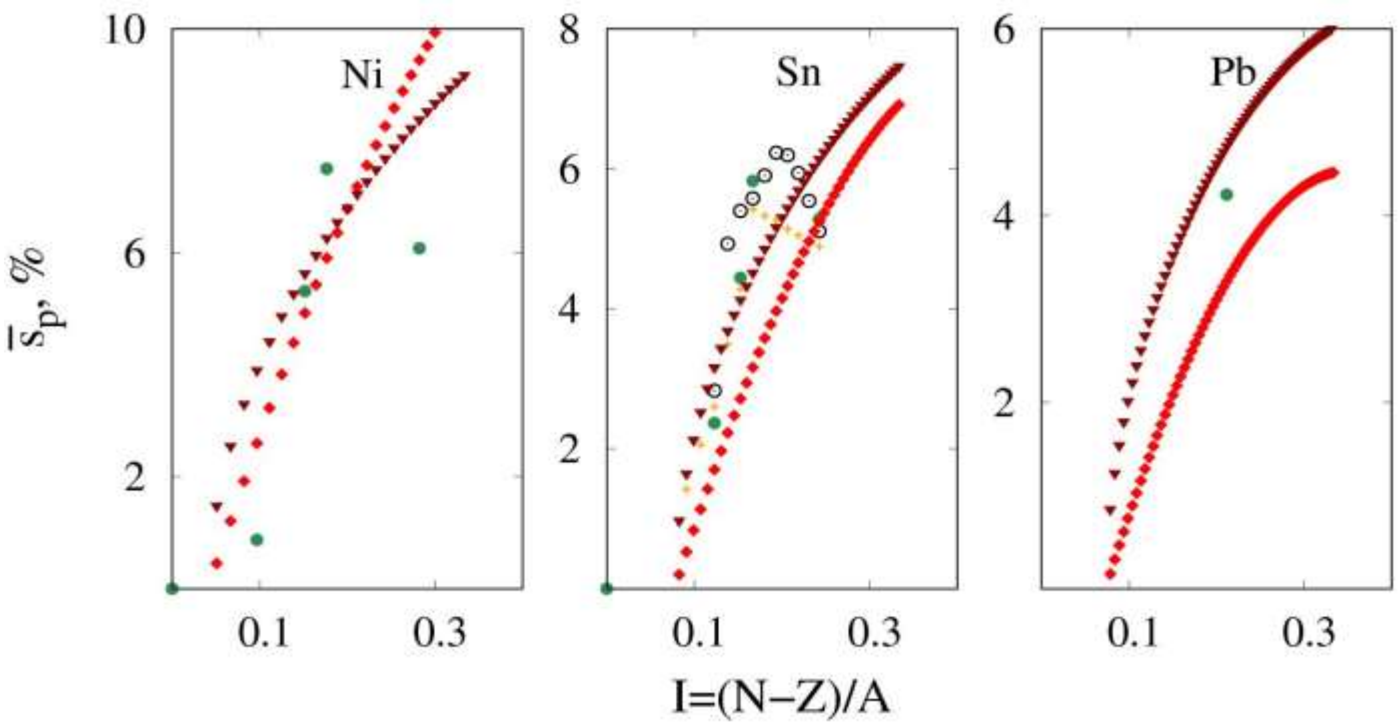


Fig. The ratio $\bar{s}_p = m^1_{L=1}(\mathrm{PDR})/m^1_{L=1}(\mathrm{GDR})$ of the PDR and GDR fractions in E1 EWSR $\bar{s}_{p,1}$ (1), $\bar{s}_{p,2}$ (2) for isotopes of *Ni* , *Sn*, *Pb* as a function of the neutron-proton asymmetry parameter. Denotations: rhombus ◆ - calculation by formula (1), inverted triangle ▼ - systematics (2) with parameters of fitting microscopic calculations, filled circle ● - RRPA results from [1], circle ⊙ - RQRPA [2], crosses + - MF+RRPA [3].

1. D. Vretenar, N. Paar, P. Ring, G.A. Lalazissis, Nucl. Phys. A, 692, 496–517 (2001)

2. N. Paar, P. Ring, T. Nikšić, D. Vretenar, Phys. Rev. C, 67, 034312 (2003)

3. J. Piekarewicz, Phys. Rev. C, 73, 044325 (2006)

Table. Parameters of systematics (13) by the fitting ratio of PDR and GDR fractions in E1 EWSR to the microscopic calculations [1-4], as well as experimental data [5], [6] (shown in brackets). Last column shows the ratio of mean-square deviations per one value of the number $\overline{\chi}^2_{\overline{s}_{p,\alpha}} = \chi^2_{\overline{s}_{p,\alpha}} / N_{\Sigma}$ for indicated parameters to the corresponding mean-square deviation $\overline{\chi}^2_{\overline{s}_{p,1}}$ by the formula (1). The mean squared deviation of the values by the formula (1) from the theoretical calculations (experimental data) is equal to $\overline{\chi}^2_{\overline{s}_{p,1}}$ =2.8 (4.0).

Equation	Parameters	$\chi^2_{\overline{s}_{p,\alpha}} / \chi^2_{\overline{s}_{p,1}}$
$\overline{s}_{p,sys}$, (2)	$\delta = 0.37,\ \gamma = 0.55$ $(\delta = 0.37,\ \gamma = 0.78)$	0.45 (0.37)
$\overline{s}_{p,2}$, (3)	$\delta = 0.435,\ \ \gamma = 1.0$ $(\delta = 0.435,\ \ \gamma = 1.0)$	0.85 (1.45)

[1]. D. Vretenar, N. Paar, P. Ring, G.A. Lalazissis,. Nucl. Phys. A, 692, 496–517 (2001)
[2]. N. Paar, T. Nikšić, D. Vretenar, P. Ring, Phys. Lett. B, 606, 288–294 (2005)
[3]. N. Paar, P. Ring, T. Nikšić, D. Vretenar,. Phys. Rev. C, 67, 034312 (2003)
[4]. J. Piekarewicz, Phys. Rev. C, 73, 044325 (2006)
[5]. D. Savran, T. Aumann, A. Zilges, Progress in Particle and Nuclear Physics, 70, 210–245 (2013).
[6]. M. Markova, P. von Neumann-Cosel, E. Litvinova, Phys. Lett. B, 860, 139216 (2025)

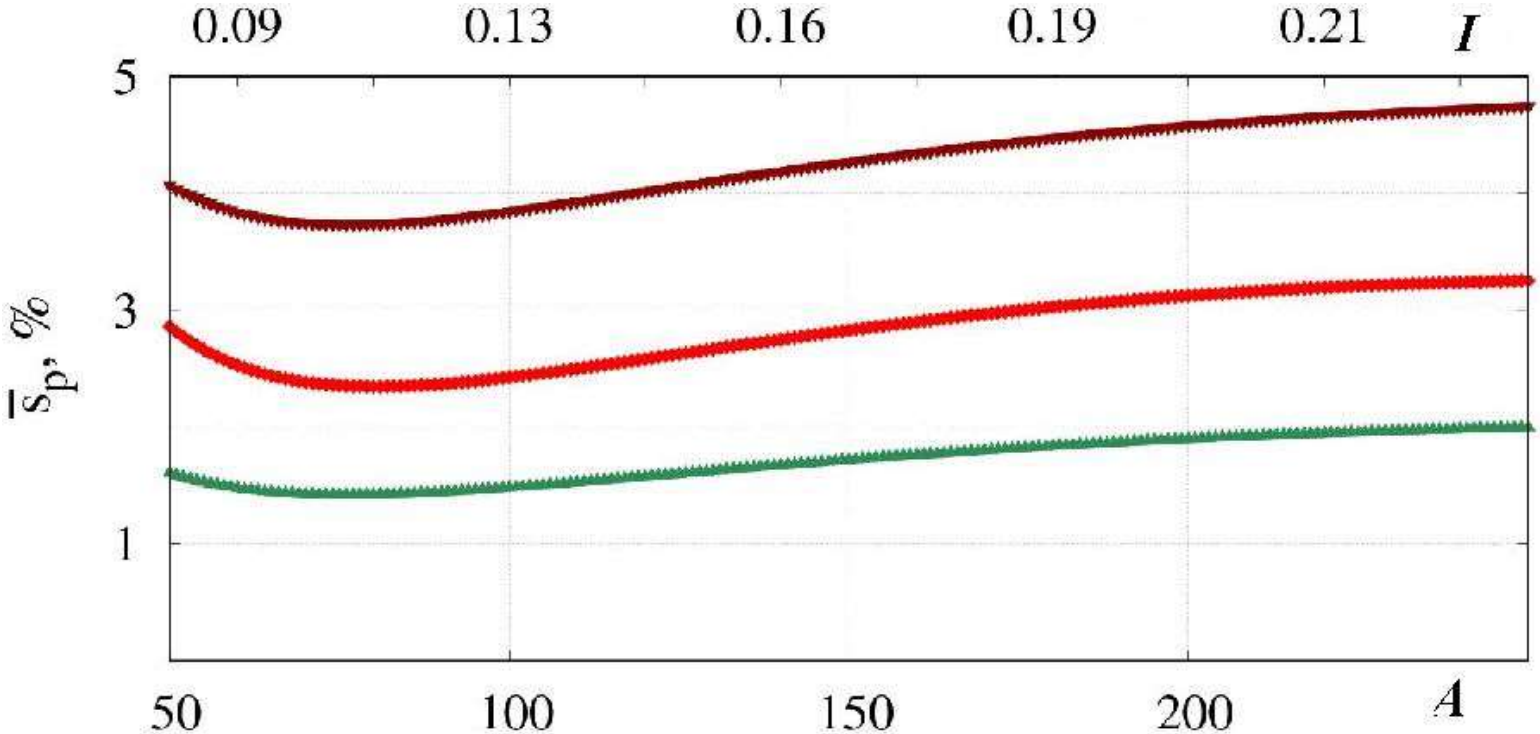


Fig. Ratio $\bar{s}_p$ of the PDR to GDR fractions in E1 EWSR along the beta-stability line.

Denotation: rhombus ♦ - calculation by formula (1), ▲ triangle - systematic (2) of the experimental data ($\delta = 0.37,\ \gamma = 0.78$), inverted triangle ▼- systematics (2) of microscopic the calculations ($\delta = 0.37,\ \gamma = 0.55$). The beta-stability line is defined by Green formula [1-2]: $I = 0.4A/(200 + A)$.

[1]. A.E.S. Green, The Systematics of Nuclear Energies. Phys. Rev., 83, 1248–1249 (1951);

[2]. A.E.S. Green, N.A. Engler, Mass surfaces. Phys. Rev., 91, 40–45 (1952)

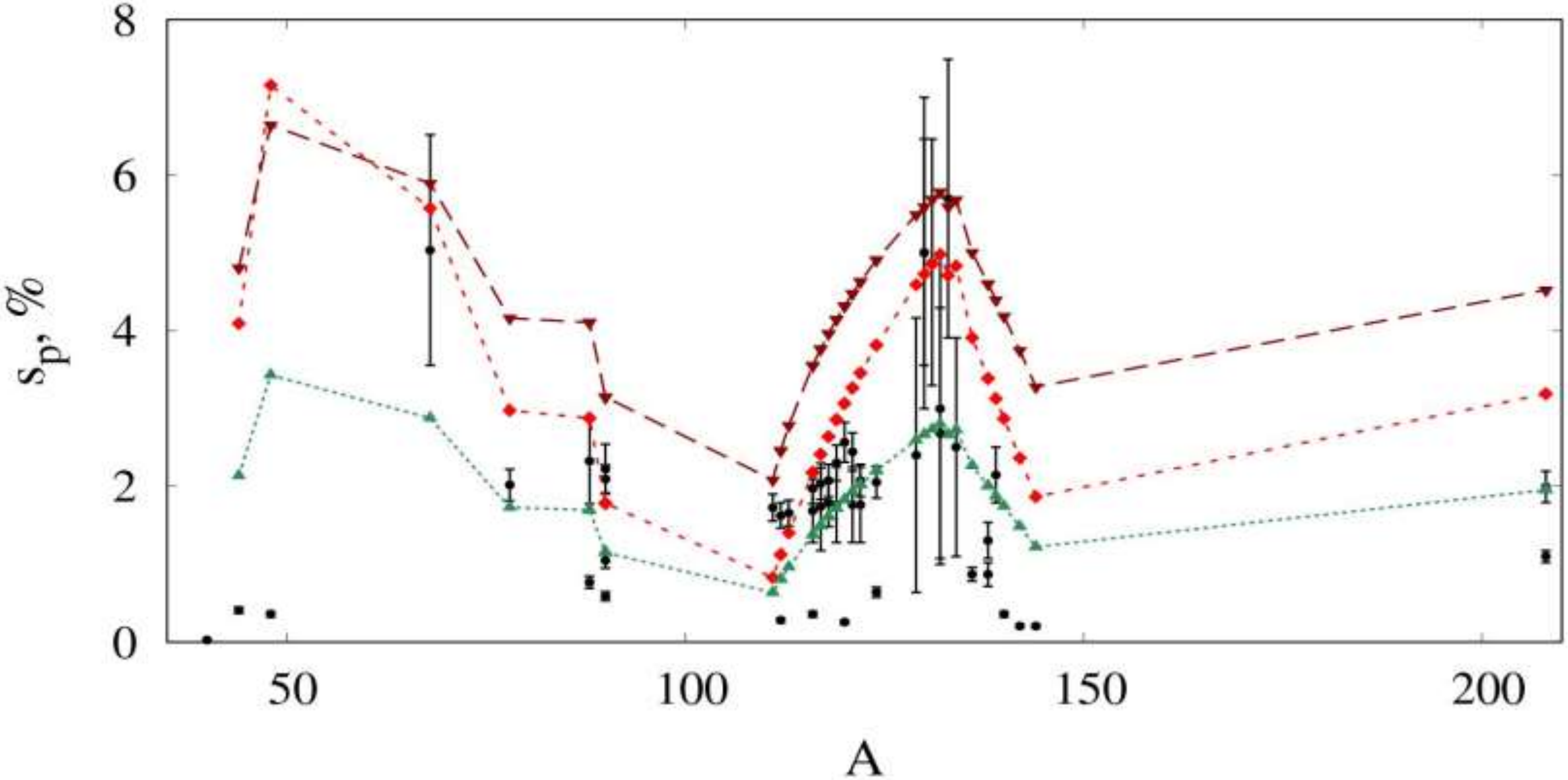


Fig. The comprison of the combination of experimental data from[1-2] for the fractions of the PDR in E1 EWSR with calculations within different approches. Denotations: black points (with or without uncertaintess) - experimental data, rhombus ◆ -$s_{p,1}$ calculation by Eqs.(1), triangle ▲ - systematics (2) with parameters of the experimental data fitting, inverted triangle ▼ - systematics (2) with parameters from the fitting microscopic calculations. The lines are drown for the better visualization of the corresponding symbols.

It can be seen that the fraction of the PDR in E1 EWSR (s_p) does not exceed 6-8%. Its values according to analytical formula (2) is about twice higher than systematics (1) of the experimental data.

[1]. D. Savran, T. Aumann, A. Zilges,. Progress in Particle and Nuclear Physics, 70, 210–245 (2013).
[2]. M. Markova, P. von Neumann-Cosel, E. Litvinova, Phys. Lett. B, 860, 139216 (2025)

Conclusions

1) The macroscopic Isacker-Nagarajan-Warner (INW) model [1] was applied with the number of near-surface neutrons proportional to the neutron skin thickness according to the work of Pesik-Ravenhall [2] (PR INW model). $N_s / N \cong \sqrt{15}\Delta r_{np} / R_0$

2) The neutron skin thickness was approximated by a linear function of the neutron excess parameter $I = (N - Z)/A$ with coefficients that depended on the mass numbers of atomic nuclei.

$$\Delta r_{np} = A^{1/3}(-0.218 + 0.003A^{2/3} + 0.617A^{-1/3}) + A^{1/3}(0.468 - 0.009A^{2/3}) \cdot I, (\mathrm{fm})$$

3) The PDR energies by the PR INW method with the value of the effective proton-neutron interaction strength $\kappa_{np} = -1828.23\ \mathrm{MeV{\times}fm^3}$, from the Skyrm forces MSk7 [3], are generally consistent with microscopic calculations taking into account the uncertainty (~30%) of the microscopic values.

The relation of the PR INW model with the calculation parameters used, gives the formula:

$$E_p = K\left[Z/(3(Z+N_c))\right]^{1/2}\left(1-\sqrt{5/3}\Delta r_{np} \cdot R_m/(20a^2)\right)A^{-1/6}, \quad K = 1.34 \cdot \sqrt{|\kappa_{np}|}\,(\mathrm{MeV})$$

that can be considered as the simplest systematics of microscopic calculations of PDR energies in neutron-excess atomic nuclei with A>~100.

The fraction of the PDR in E1 EWSR does not exceed 6-8% and following systematic can be used

$$\overline{s}_p = \frac{\sigma_{\mathrm{int}}(\mathrm{PDR})}{\sigma_{\mathrm{int}}(\mathrm{GDR})} = \frac{Z}{A/\sqrt{15} - N \cdot \Delta r_{np}/R_0} \frac{\Delta r_{np}}{R_0} = \overline{s}_{p,1}; \text{ or } \overline{s}_{p,sys} = \delta\left(\Delta r_{np}/A^{1/3}\right)^{\gamma}, \text{ with } \delta, \gamma \text{ - fit parameters}$$

to experimental of theoretical data.

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[3] Goriely S.,. Tondeur F, Pearson J. M., At. Data Nucl Data Tabl. 77, 311 (2001).

Thank you for your attention!