

# Sensitivity analysis and optimization of multi-scale models for microstructural evolution in metal materials under neutron irradiation

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## 1) Introduction

Irradiation damage in tungsten

Multi-scale modeling of irradiation effects

Uncertainty quantification & sensitivity analysis

## 2) Method

The OKMC model

Sensitivity analysis

Surrogate model

## 3) Sensitivity Analysis

Sensitivity at 733K and 0.02dpa

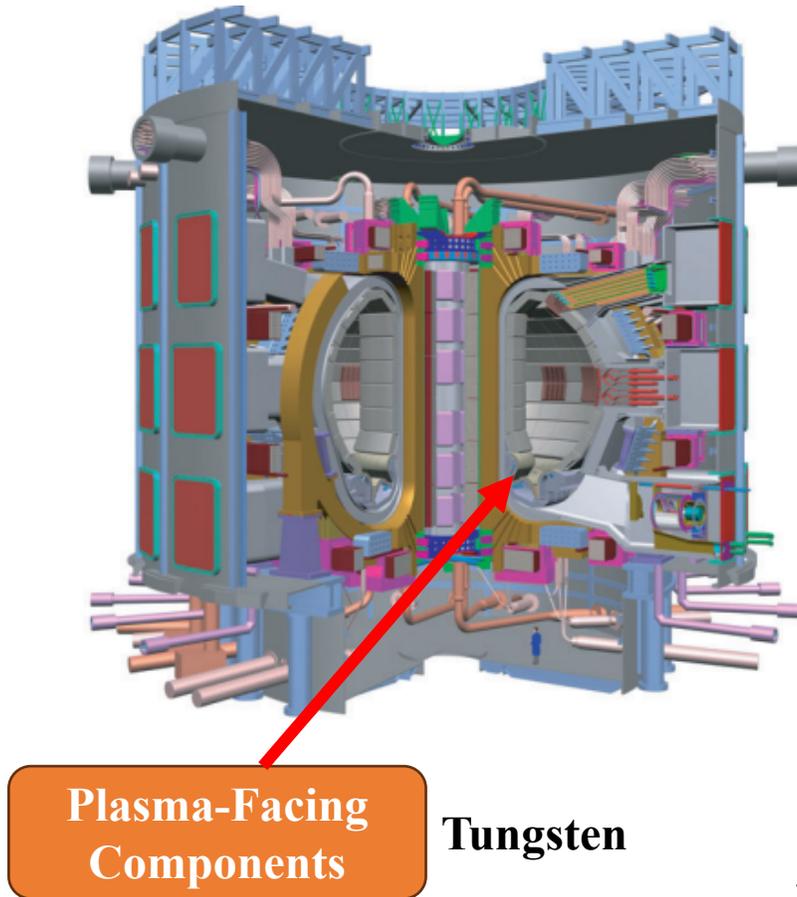
Factors affecting the sensitivity potentially

## 4) Surrogate Models

Polynomial Chaos Expansion

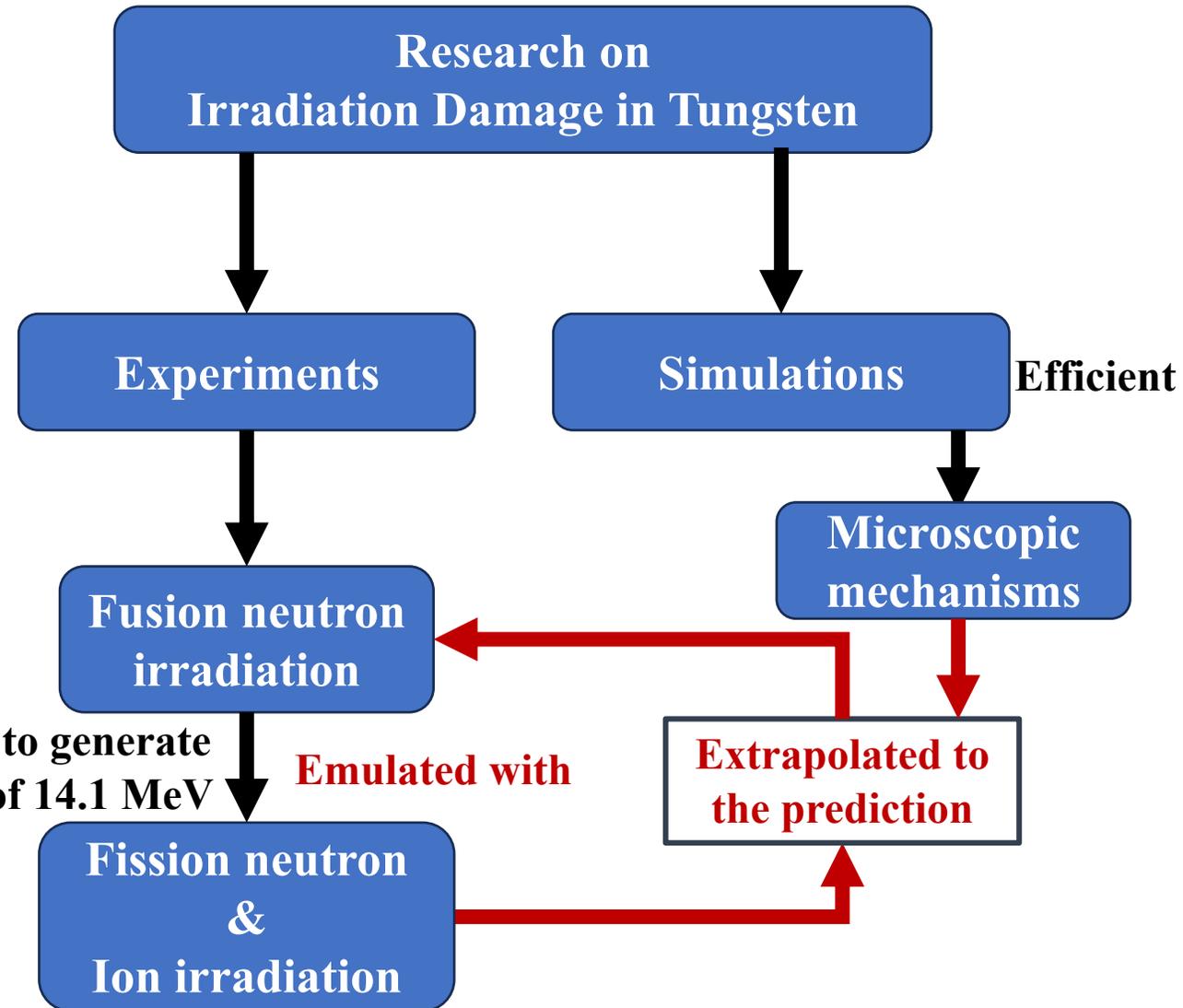
Artificial Neural Network

## 5) Conclusion & Fututure Work

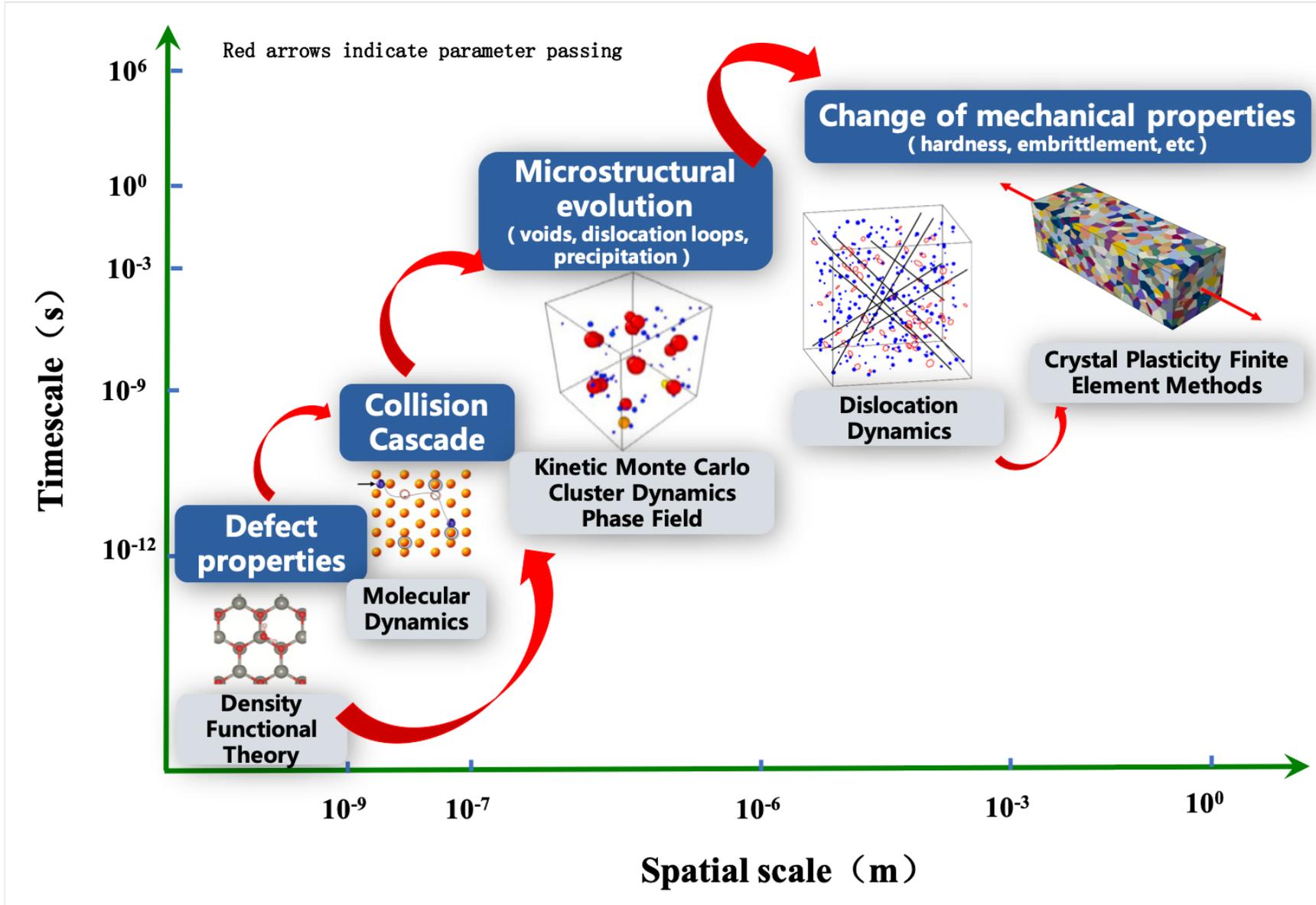


- High flux of H and He ions
- Intense neutron irradiation

Lack of devices to generate fusion neutron of 14.1 MeV



# Multi-scale modeling of irradiation effects



Google Scholar fusion tungsten simulation and modeling

Articles About 55,100 results (0,12 sec) **55,100 results**

Any time  
Since 2025  
Since 2024  
Since 2021  
Custom range...

Recent advances in **modeling** and **simulation** of the exposure and response of **tungsten to fusion energy conditions**  
[J.Marian](#), [CS Becquart](#), [C.Domain](#), [SL Dudarev](#)... - ... **Fusion**, 2017 - iopscience.iop.org  
... -the-art in materials **simulations** of W in **fusion** environments and highlight ... **modeling** and **simulation** have produced. Often, the **simulation** paradigm within which computational **modeling** ...  
☆ Save 77 Cite Cited by 150 Related articles All 14 versions

Sort by relevance  
Sort by date

[PDF] Recent advances in computational materials **modeling** of **tungsten** as plasma-facing material for **fusion** energy applications

- Thousands of related research work (Google Scholar: 55100 results).
- Most of them use DFT or MD methods, forming a solid foundation for mesoscale methods such as Object kinetic Monte Carlo (OKMC) and Cluster Dynamics (CD).

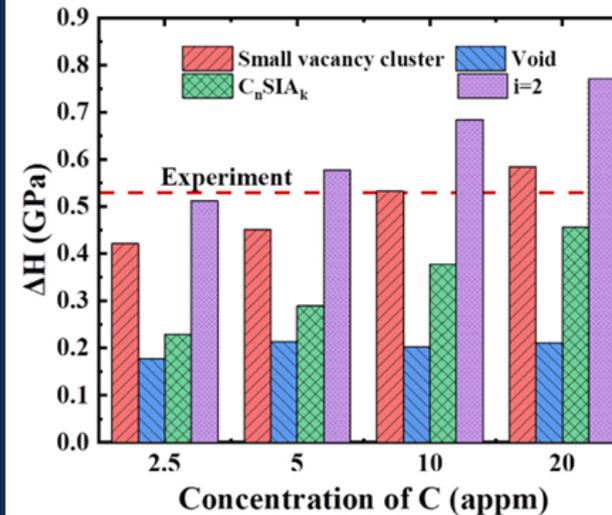
## OKMC

Simulate defect diffusion and microstructure evolution processes

Defect size, concentration

Microstructural morphology

Can be easily compared with experimental findings

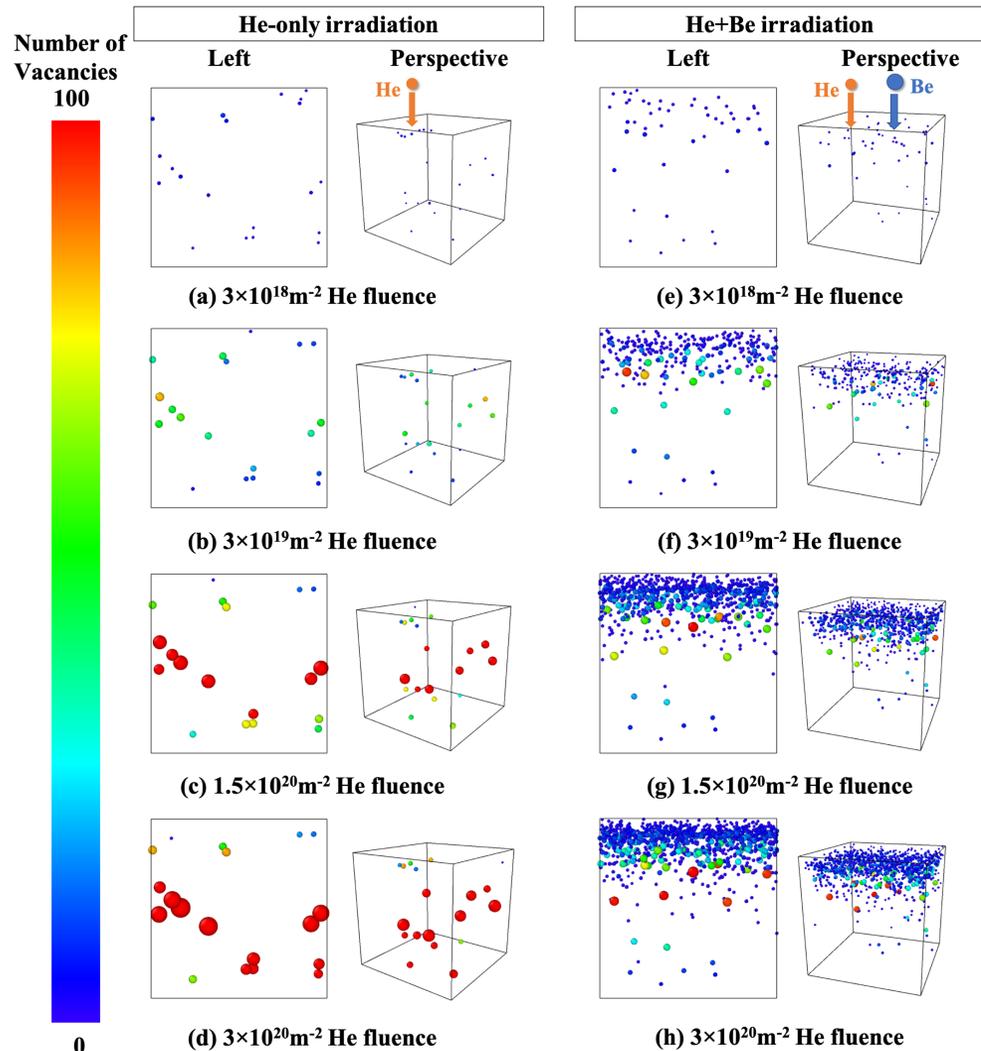


Niu, Y. Z., et al. JNM (2023)

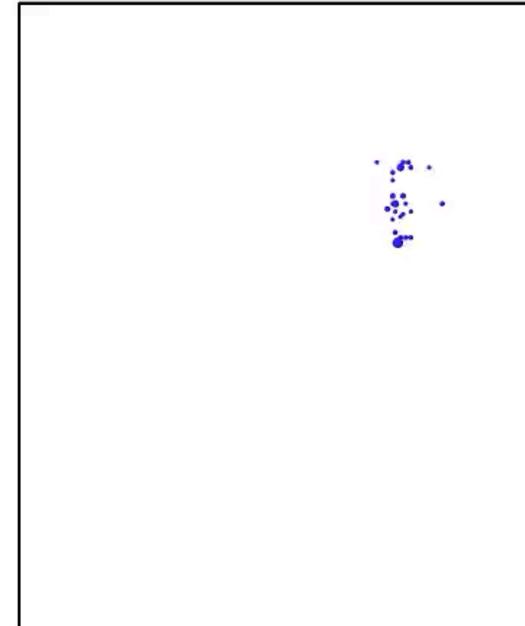
Information of microstructures can be fed to further predict the change of mechanical properties..

**OKMC bridges the simulation of microscopic mechanisms with the macroscopic observations.**

## Suppression of He-induced damage by Beryllium

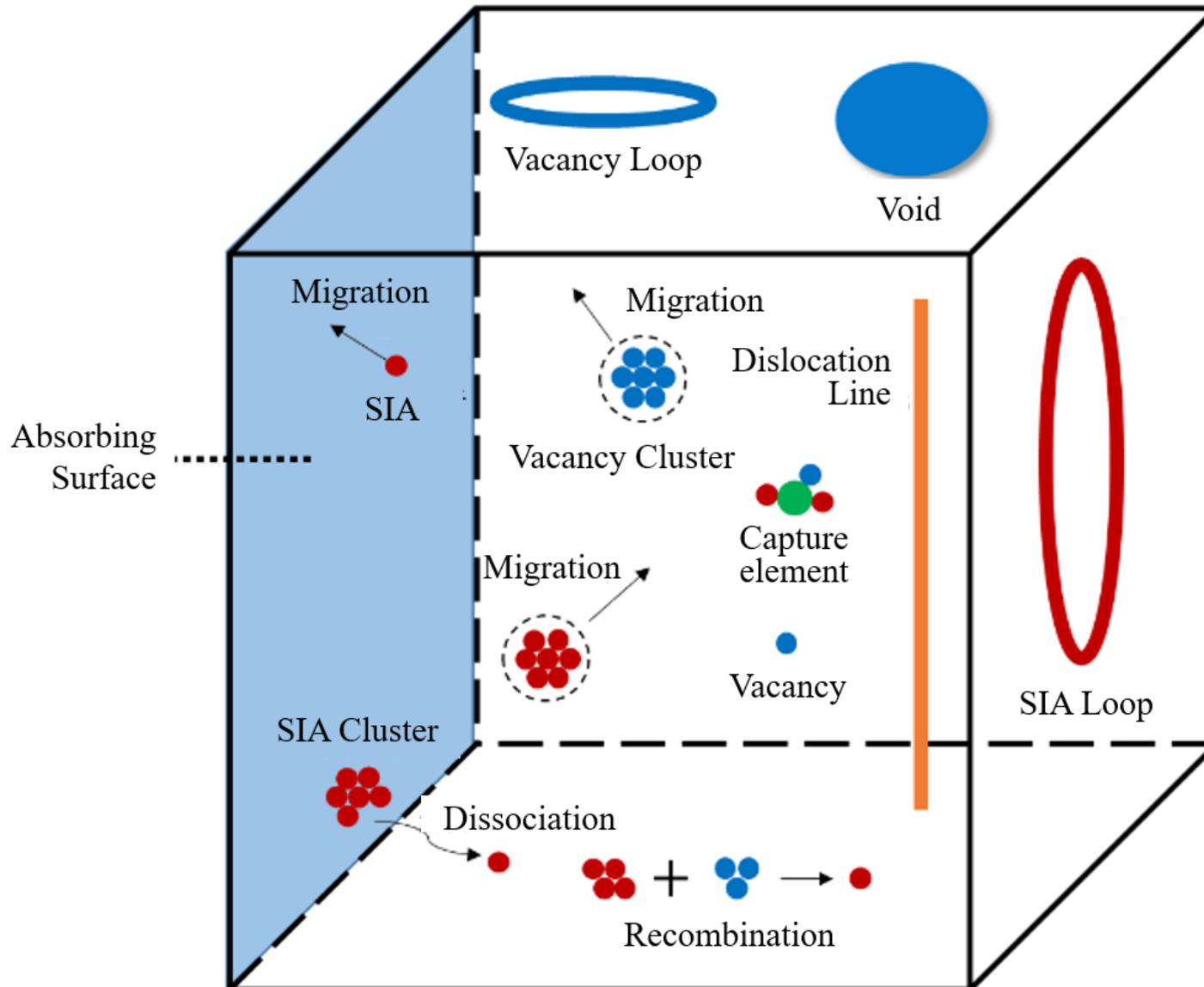


## Void lattice formation



- [1] Zhou et al., Nuclear Fusion. 64 (2024) 106021.
- [2] Li et al., Acta Materialia. 219 (2021) 117239.

# The OKMC model



## Self-Interstitial Atom

- ↓
- SIA
  - Vacancy

## Thermal-activated

Based on **probability**

- Migration** Defects **move** to neighboring locations.
- Rotation** Defects change the current migration **direction**.
- Dissociation** Individual defects **emit** from clusters.

## Non-thermal-activated

Based on **capture radius**

- Aggregation** Defects or clusters **combine** to form larger clusters.
- Recombination** Dissimilar defects or clusters combine and **heal**.
- Capture** Defects are **absorbed** by capture elements.
- Annihilation** Defects are **removed** when moving to absorbing surfaces.

## Uncertainty

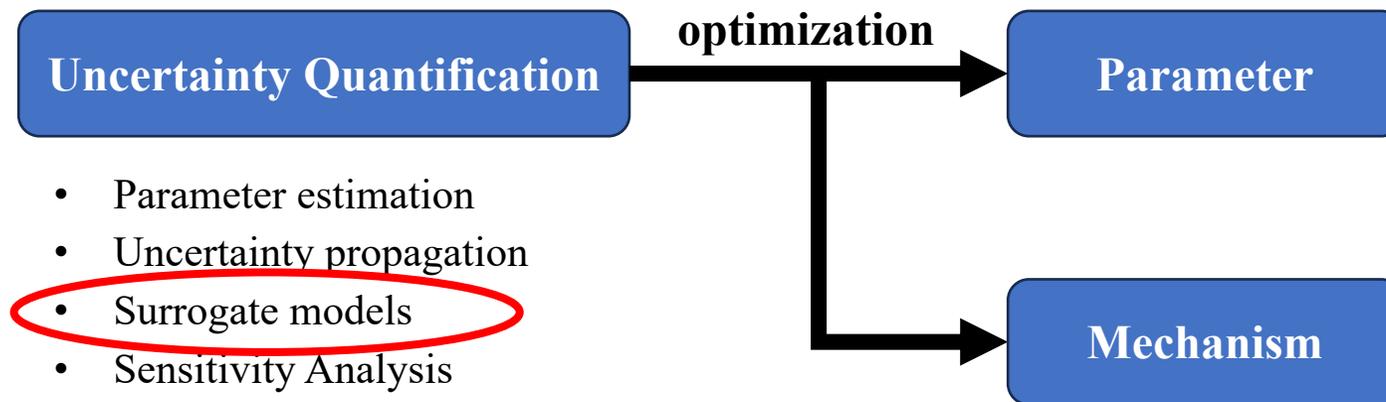
$E_a, \nu_0$   
Migration energies  
Binding energies  
Rotation energies

*Stochastic events*  
based on probabilities

e.g. Monovacancy migration energy : 1.68eV, 1.50eV, 1.97eV ...  
Divacancy binding energy : **-0.41eV, -0.06eV, +0.42eV, +0.7 eV ...**  
Probability of dissociation : **1.44e-5, 2.17e-10 ...**

Different outputs in several simulations with same probabilities.

## Sensitivity Analysis

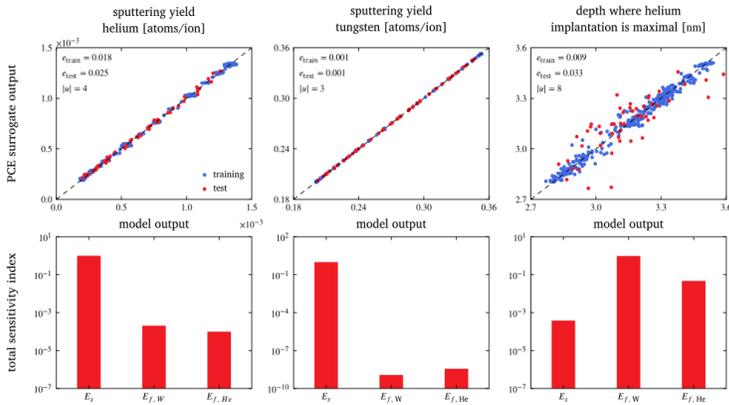


- Identify sensitive parameters
- Perform calibration

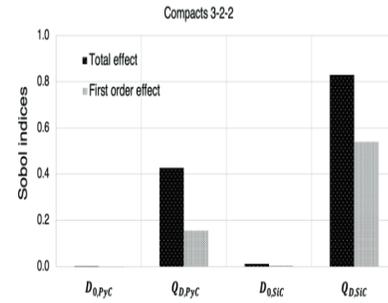
- The stochastic mechanism is crucial to the OKMC model.

**Reduce calculational costs of parameter calibration**

## PCE surrogate model + Sobol' indices ( PCE: Polynomial Chaos Expansion )



Pieterjan Robbe, et al. (2023)



Seok Bin Seo, et al. (2023)

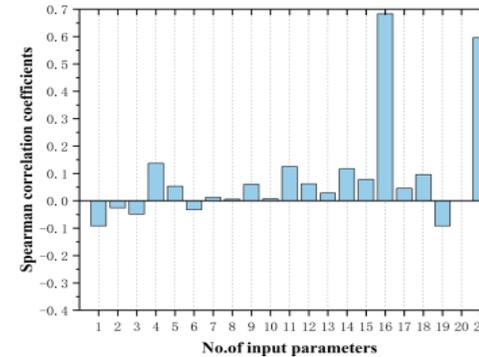
There have been attempts in nuclear materials field to employ the approach.

### Characteristics

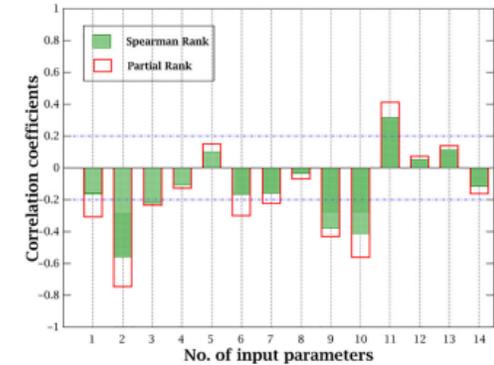
A large number of training samples

The increase of uncertainty

## LHS + Spearman correlation coefficient ( LHS: Latin Hypercube Sampling )



C. Peng, et al. (2023)



Ye Yang, et al. (2021)

The method was applied in many studies on severe accidents and thermal-hydraulics in fission reactors.

### Characteristics

More universal / Broad applicability

Relatively low computational cost

Robbe, P., Blondel, S., Casey, T. A., Lasa, A., Sargsyan, K., Wirth, B. D., & Najm, H. N. (2023). Global sensitivity analysis of a coupled multiphysics model to predict surface evolution in fusion plasma–surface interactions. *Computational Materials Science*, 226, 112229.

Seo, S. B., & Wirth, B. D. (2023). Sensitivity analysis of cesium and strontium release from TRISO particle under irradiation and high temperature conditions. *Nuclear Engineering and Design*, 408, 112333.

Peng, C., et al. (2023, May). Best Estimate Plus Uncertainty Analysis of a Pressurizer Surge Line Break LOCA on China's Advanced PWR. In *Proceedings of the 23rd Pacific Basin Nuclear Conference, Volume 2: PBNC 2022, 1-4 November, Beijing & Chengdu, China* (pp. 490-505). Singapore: Springer Nature Singapore.

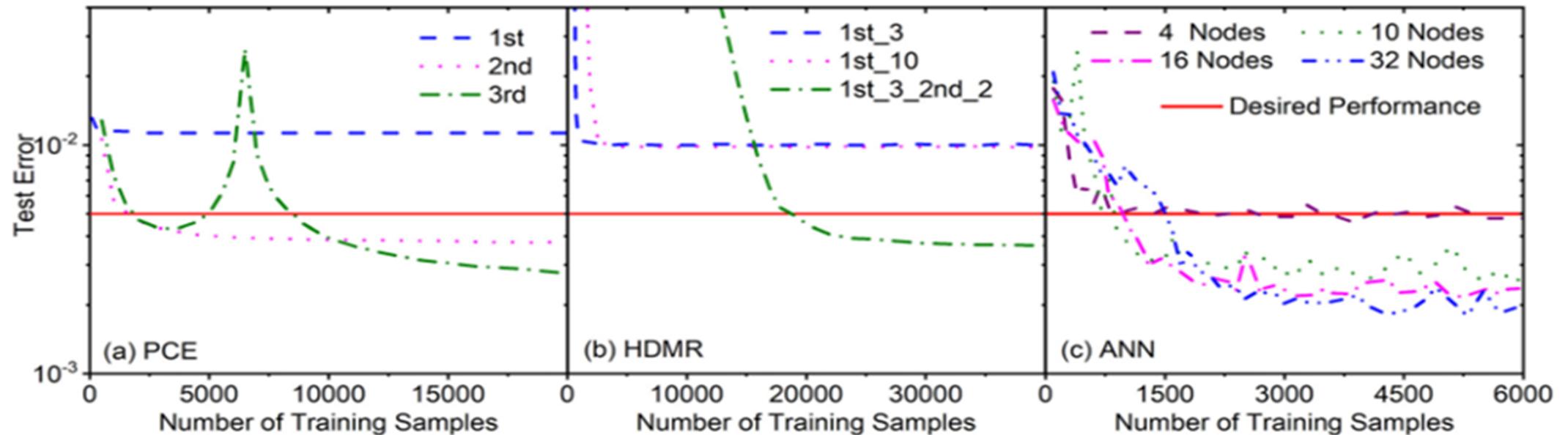
Yang, Y., Deng, C., & Yang, J. (2021). Best estimate plus uncertainty analysis of a small-break LOCA on an advanced Generation-III pressurized water reactor. *International Journal of Energy Research*, 45(8), 11916-11929.

In an inverse UQ study of combustion kinetic models, the test errors of three typical surrogate models are compared.

## Polynomial Chaos Expansion

## High Dimensional Model Representation

## Artificial Neural Network



J. Wang, et al. (2020)

PCE & ANN

HDMR

High convergence speed

High accuracy

Convergence speed and accuracy are mutually exclusive.

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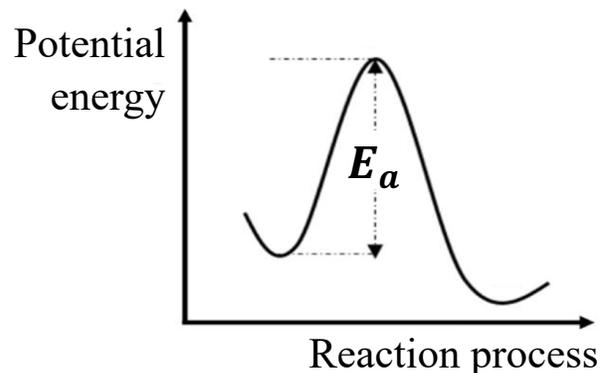
## How to calculate the probabilities/rates ?

### Arrhenius formula

$$R = \nu_0 \times \exp\left(-\frac{E_a}{k_B T}\right)$$

- $R$  : rates of events
- $\nu_0$ : attempt frequency
- $E_a$ : **activation energy**
- $k_B$ : Boltzmann constant
- $T$ : temperature

Transition-state theory



$E_a \uparrow$  ,  $R \downarrow$

$T \uparrow$  ,  $R \uparrow$

## What outputs need attention ?



# Sensitivity analysis: flow chart

## Identification & Characterization of Uncertainties

Migration energies  
Binding energies  
Rotation energies  
Capture radiuses  
etc.

**Probability estimation**

**Filter the parameters to be analyzed**

## Simulation of OKMC

### Facility of the experiments

High Flux Isotope Reactor (HFIR)  
at Oak Ridge National Laboratory (ORNL)

The thermal and cold neutrons produced by HFIR are used to study physics, chemistry, materials science, engineering, and biology.

### The outputs of OKMC

- Average size of Vac clusters
- Number density of Vac clusters
- Average size of SIA clusters
- Number density of SIA clusters

## Generation of Samples

**Latin Hypercube Sampling (LHS)**

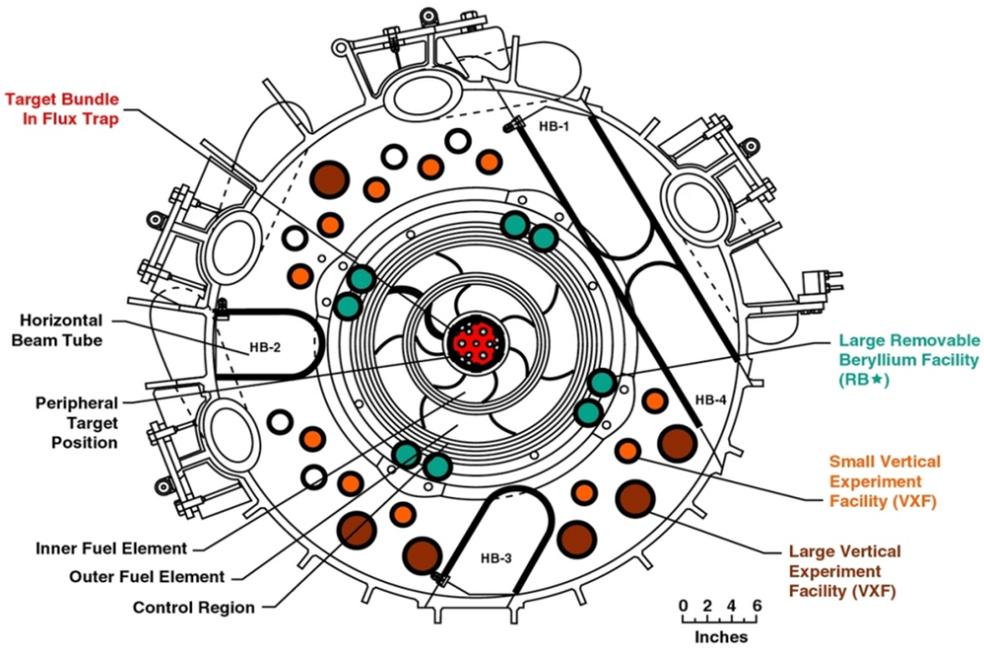
A method creating **more representative samples**

## Sensitivity Analysis

### Potential factors

**Spearman correlation coefficient**

- Range of values
- Multiparameter
- Irradiation dose
- Temperature



High Flux Isotope Reactor (HFIR)



Irradiation test samples

Fission reactor	Material Type	Irradiation temperature [°C]	Dose [dpa]		
HFIR (mixed neutron spectrum) [21-23,37]	Single crystal	~90	0.02		
		~90	0.39		
		460	0.02		
		797	0.092		
		700	0.44		
		770	1.80		
		1100	0.47		
		800	1.50		
		700	2.80		
		600	0.15		
JMTR (mixed neutron spectrum) [13,17]	Polycrystalline	800	0.15		
		400	0.17		
Joyo (Fast reactor) [8,11]	Polycrystalline	531	0.44		
		538	0.96		
		583	0.47		
		740	0.40		
		750	1.54		
		756	0.42		
		900	1.67		
		800	1.25		
		HFR (mixed neutron spectrum) [38]	Single crystal	600	0.2
				800	0.2
1200	0.2				
BR2 (mixed neutron spectrum) [24,39,40]	Polycrystalline	600	0.18		
		800	0.2		
		1200	0.2		
ITER grade	Cold rolled pure polycrystalline	600	0.18		
		800	0.2		
		1200	0.2		

733K, 0.02dpa

# Sensitivity analysis: flow chart

## Identification & Characterization of Uncertainties

Migration energies  
Binding energies  
Rotation energies  
Capture radiuses

**Probability estimation**

**Parameters to be analyzed**

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A method creating **more representative samples**

## Sensitivity Analysis

### Potential factors

**Spearman correlation coefficient**

- Range of values
- Multiparameter
- Irradiation dose
- Temperature

## Parameters to be analyzed

Physical Meaning	Symbol	Reference value	Range
Capture radius of vacancy	$R_V$	1	*(0.8~1.2)
Capture radius of SIA	$R_W$	1	*(0.8~1.2)
Migration energy of monovacancy	$E_{m,1V}$	1.68 eV	+(-0.1~0.1) eV
Migration energy of divacancy	$E_{m,2V}$	1.44 eV	+(-0.1~0.1) eV
Migration energy of tri-vacancy	$E_{m,3V}$	0.83 eV	+(-0.1~0.1) eV
Binding energy of divacancy	$E_{b,2V}$	-0.12 eV	+(-0.1~0.1) eV
Binding energy of tri-vacancy	$E_{b,3V}$	-0.0636 eV	+(-0.1~0.1) eV
Migration energy of SIA	$E_{m,W}$	0.023 eV	+(-0.01~0.01) eV
Rotation energy of SIA	$E_{r,W}$	0.38 eV	+(-0.1~0.1) eV

## Settings

- Number of samples: 60;
- Number of recalculations for one sample: 15.

## Expansion

$$y = F(\mathbf{X}) = \sum_{\alpha \in I^d} c_{\alpha} \Psi_{\alpha}(\mathbf{X})$$

- $y$ : model outputs, QoI
- $F(\mathbf{X})$ : Functional relationship
- $I^d$ : d-dimensional parameter space
- $c_{\alpha}$ : PCE coefficient
- $\Psi_{\alpha}(\mathbf{X})$ : orthogonal multivariate basis function

## Truncation

$$P = \binom{d+p}{p} = \frac{(d+p)!}{d! p!}$$

- $P$ : number of polynomial terms
- $p$ : highest order of polynomials

## Calculation of PCE coefficients

### Intrusive methods

Galerkin projection

- Requiring modifications of the numerical code  OKMC

### Non-intrusive methods

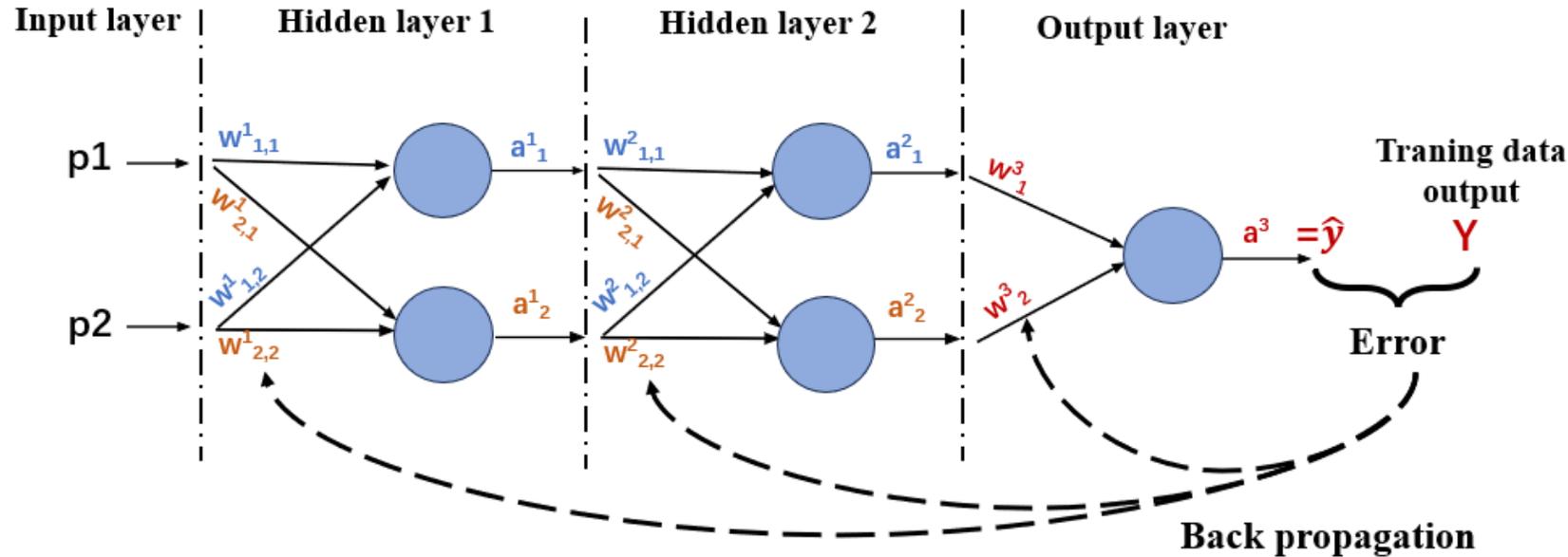
Spectral projection

- Gaussian quadrature rules, Smolyak sparse grid

Regression

- Least squares methods, compressed sensing methods

## Multilayer Perception trained with Backpropagation Algorithm



- (1) Randomly assign the weights and compute the network solution;
- (2) Compute the error between the outputs of network and training data, and back propagate the weights to each layer;
- (3) Re-assign the weights to avoid the same error, and so on do iterations;
- (4) Obtain the optimal weight matrix as the best approximation.

### Settings

- **Activation function:** the hyperbolic tangent,  $a = \tanh(n)$
- **Training method:** Levenberg-Marquardt algorithm, a second-order Quasi-Newton optimization method

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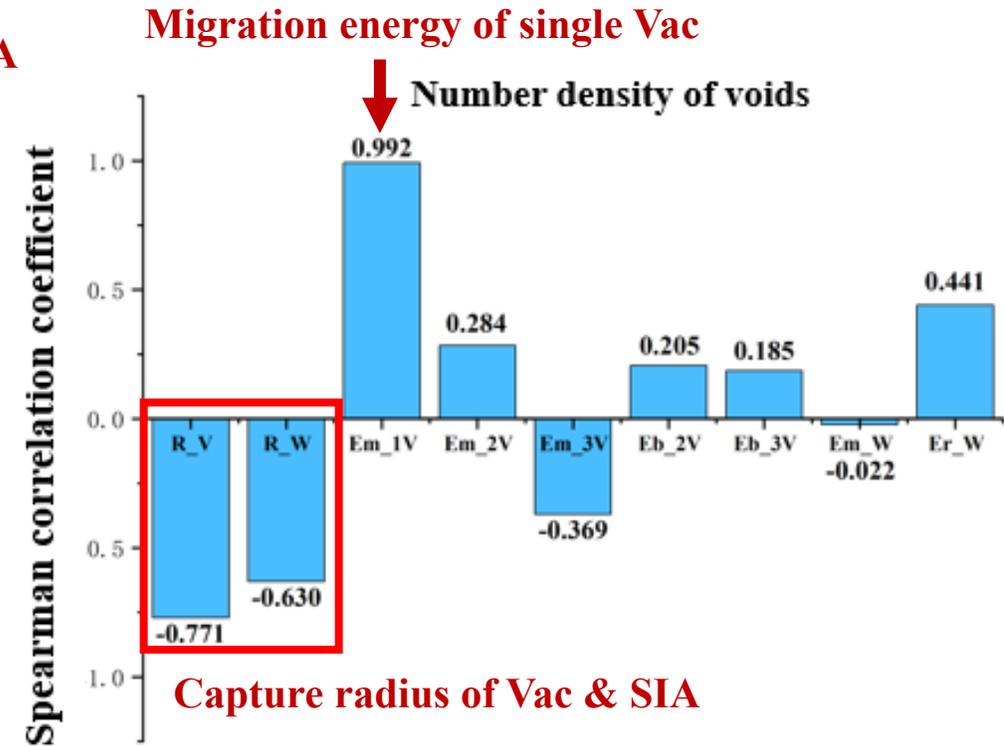
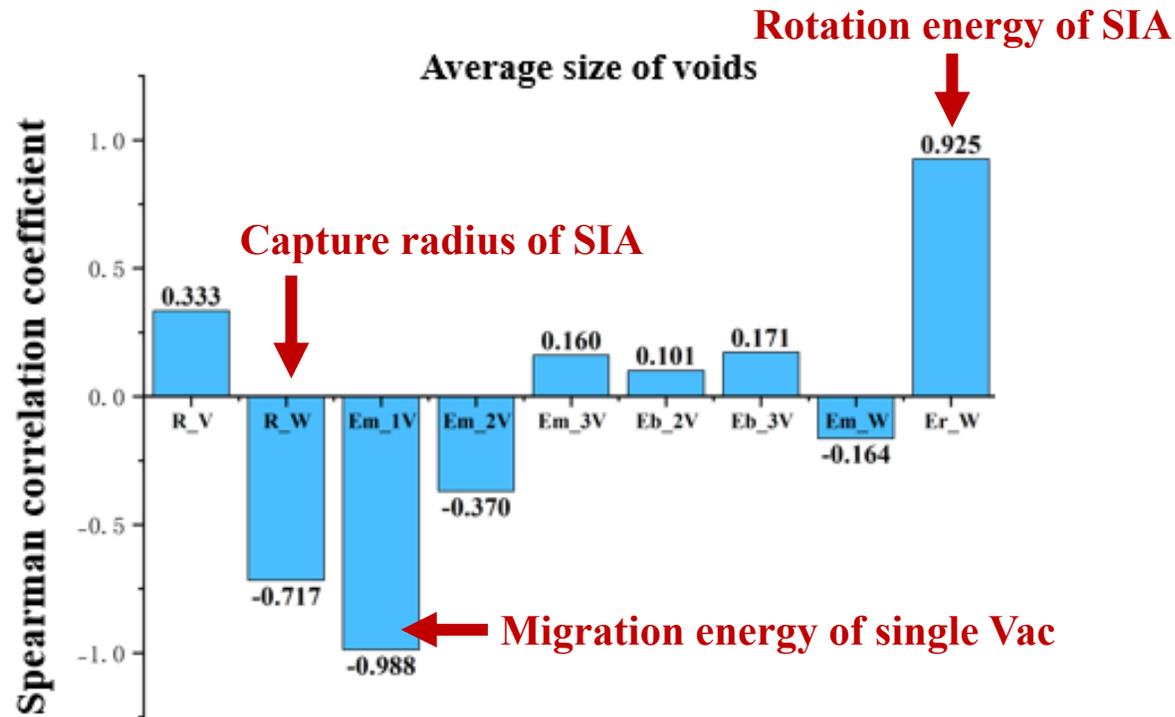
Factors affecting the sensitivity potentially

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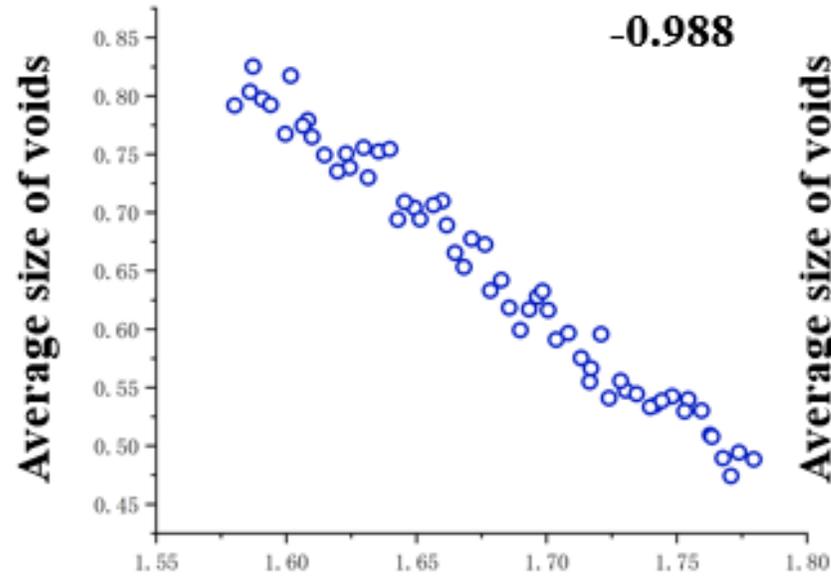
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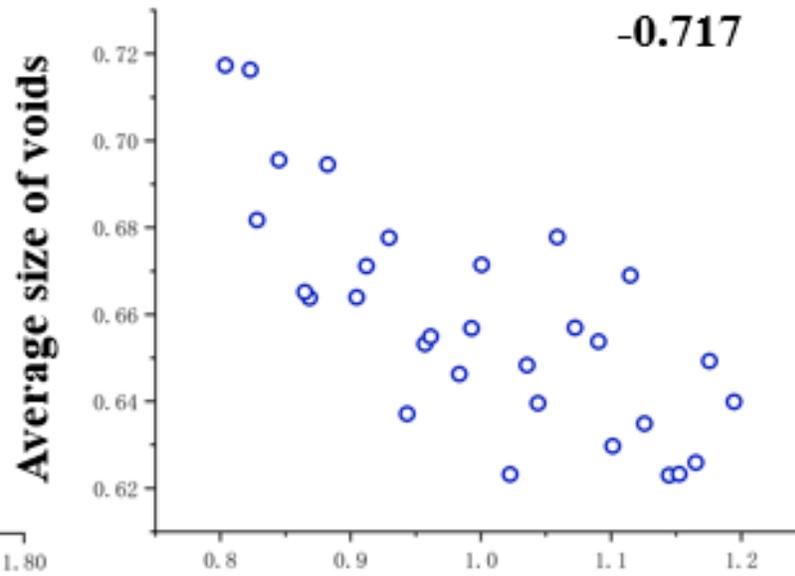
# Sensitivity at 733K and 0.02dpa



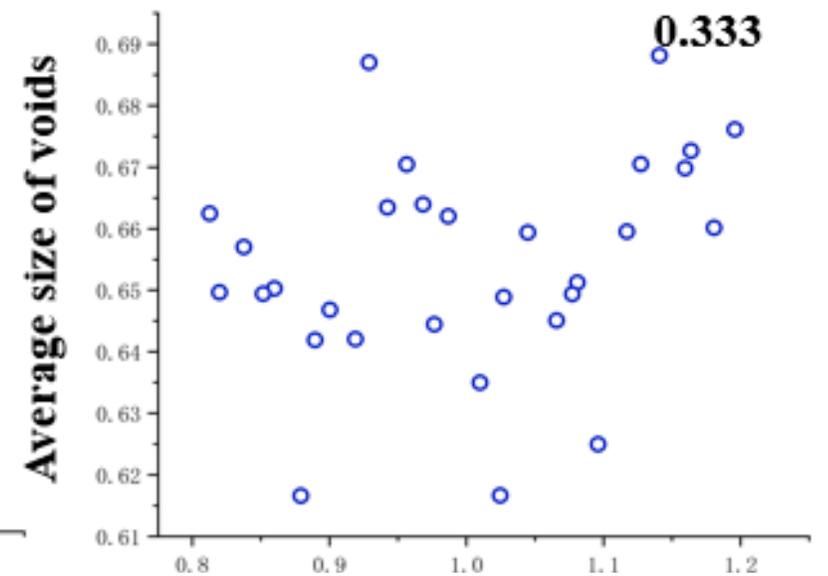
# Sensitivity at 733K and 0.02dpa



Migration energy of Single Vac



Capture radius of Vac

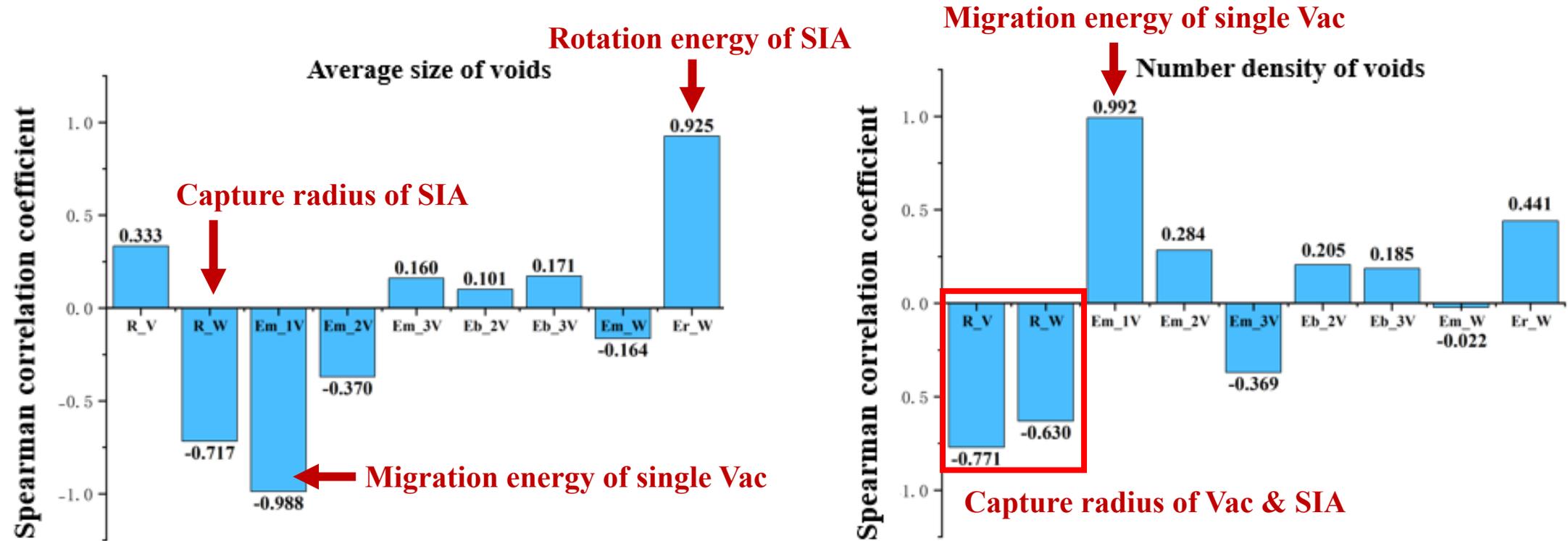


Capture radius of SIA

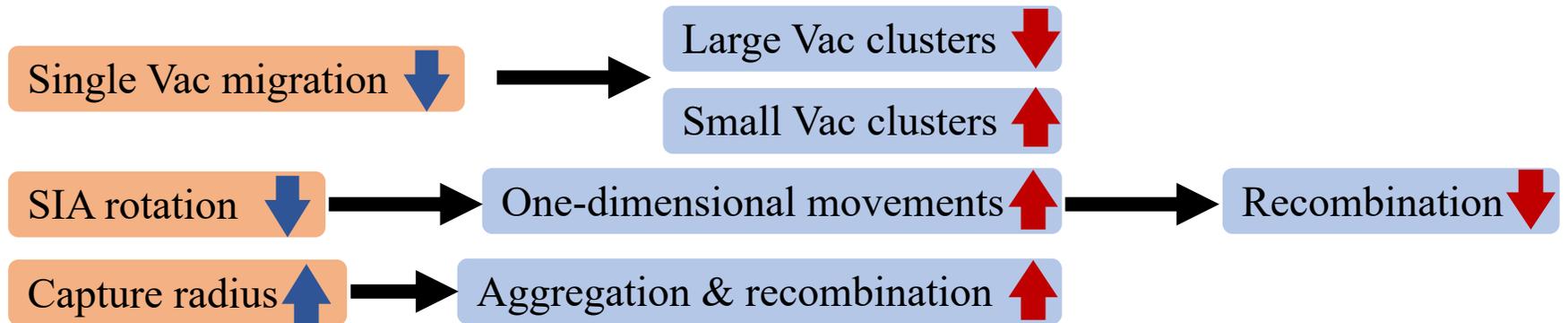
A distinct monotonic trend

Dispersed points

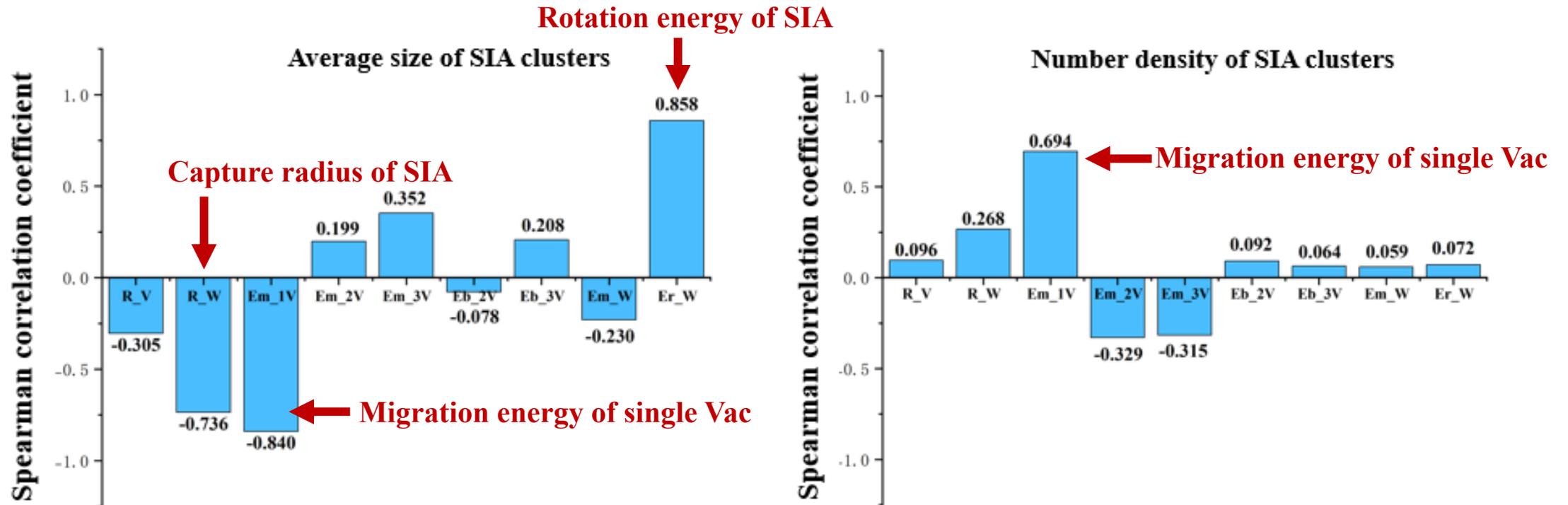
# Sensitivity at 733K and 0.02dpa



$$R = v_0 \times \exp\left(-\frac{E_a}{k_B T}\right)$$



# Sensitivity at 733K and 0.02dpa



Four sensitive parameters

Migration energy of single Vac

Rotation energy of SIA

Capture radius of Vac

Capture radius of SIA

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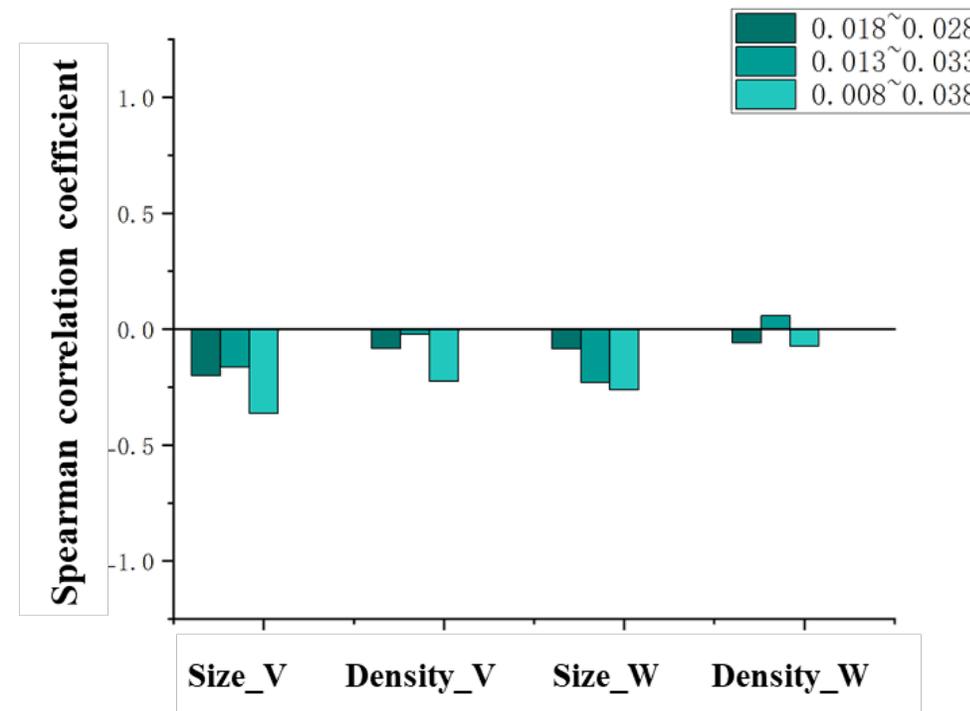
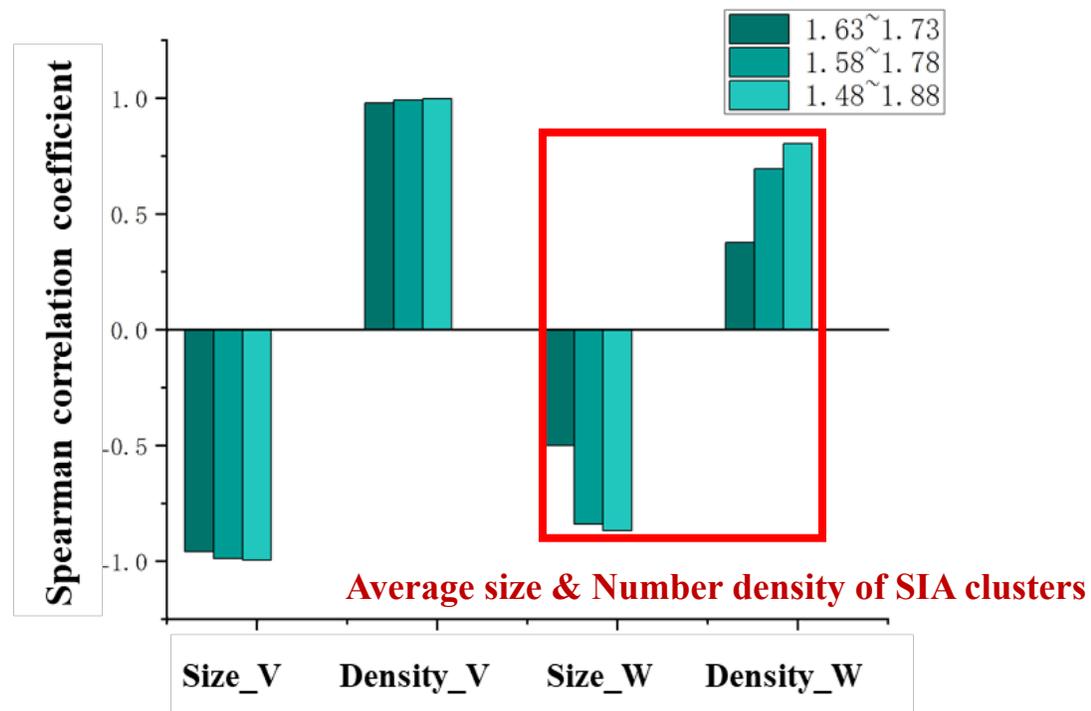
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# Potential factor: different ranges of values

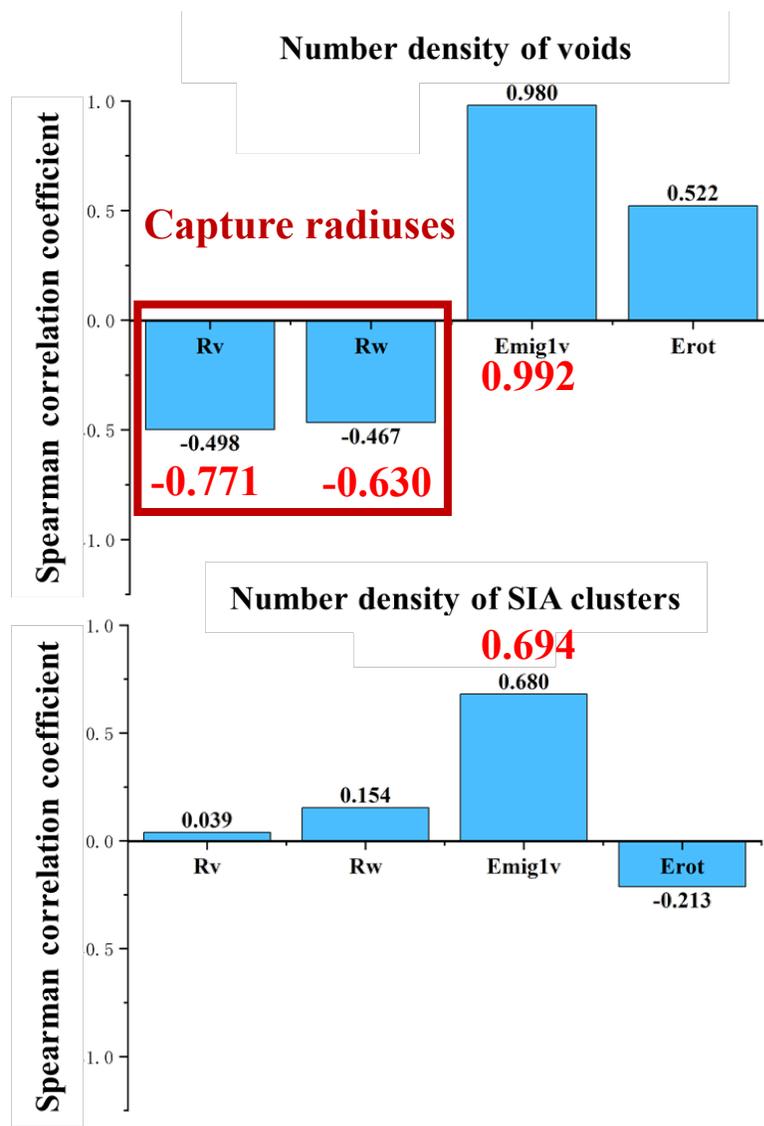
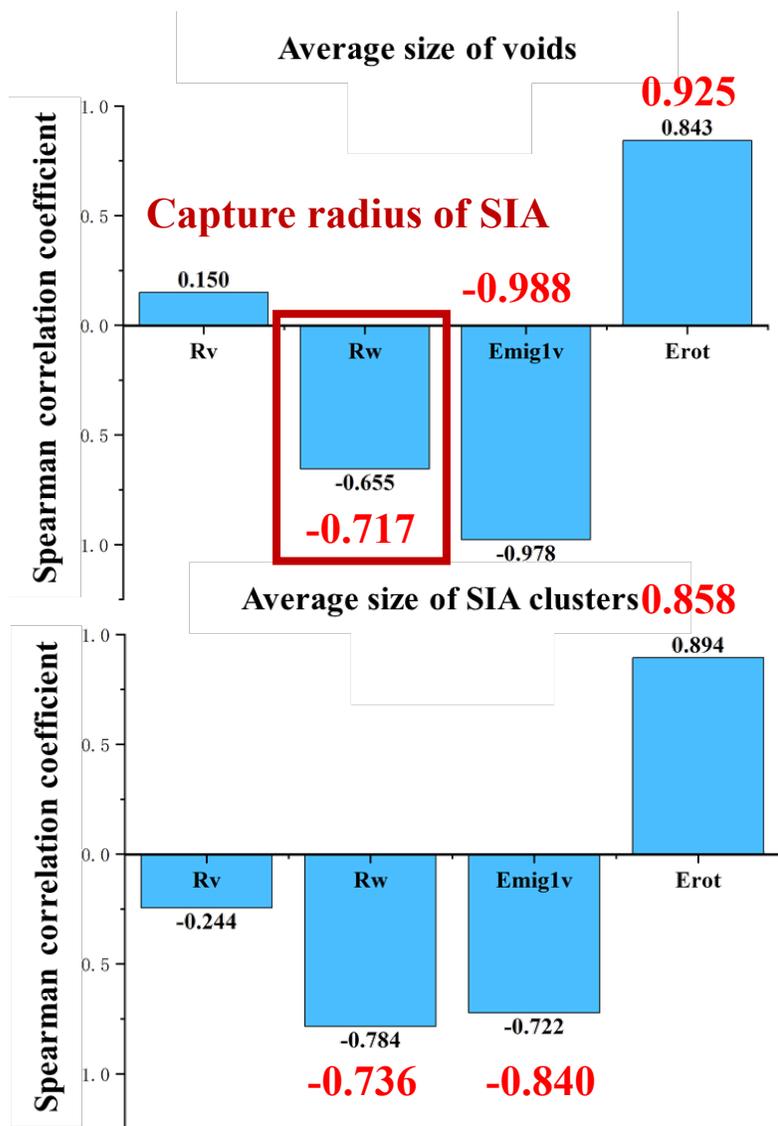
We selected one high-sensitivity parameter (**Migration energy of single Vac**, <left>) and one low-sensitivity parameter (**Migration energy of SIA**, <right>) from the previous analysis results for further study.



Samples concentrating within a smaller range may affect the accuracy of the analysis results.

No clear trends or patterns.

# Potential factor: multiparameter analysis



We selected **the four sensitive parameters** as the target parameters for multi-parameter analysis.

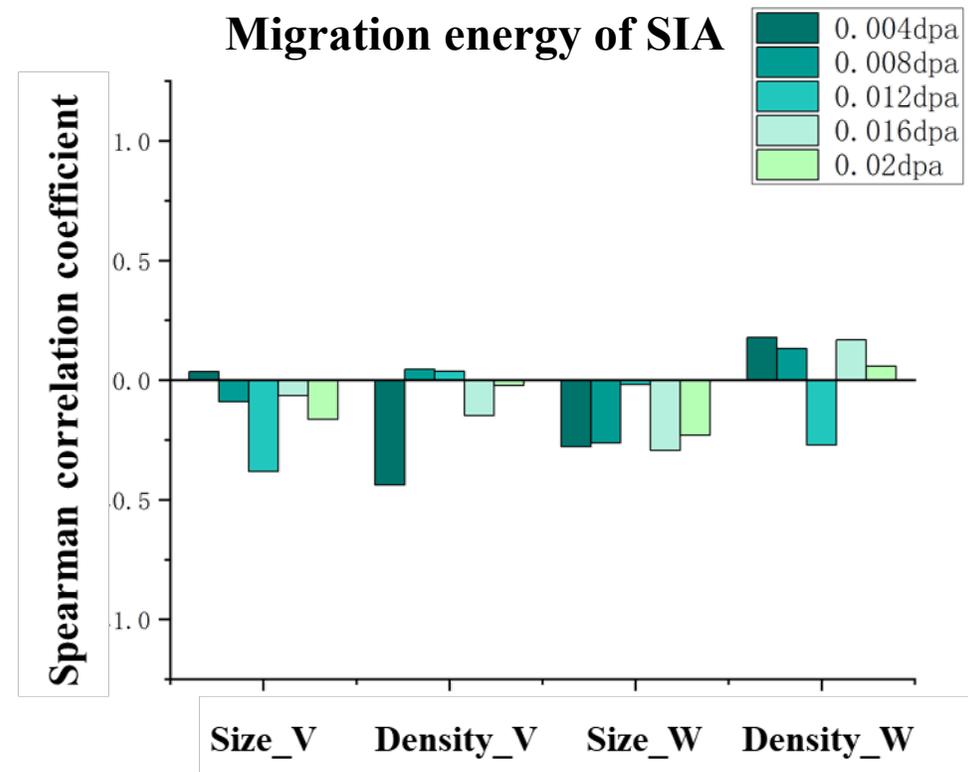
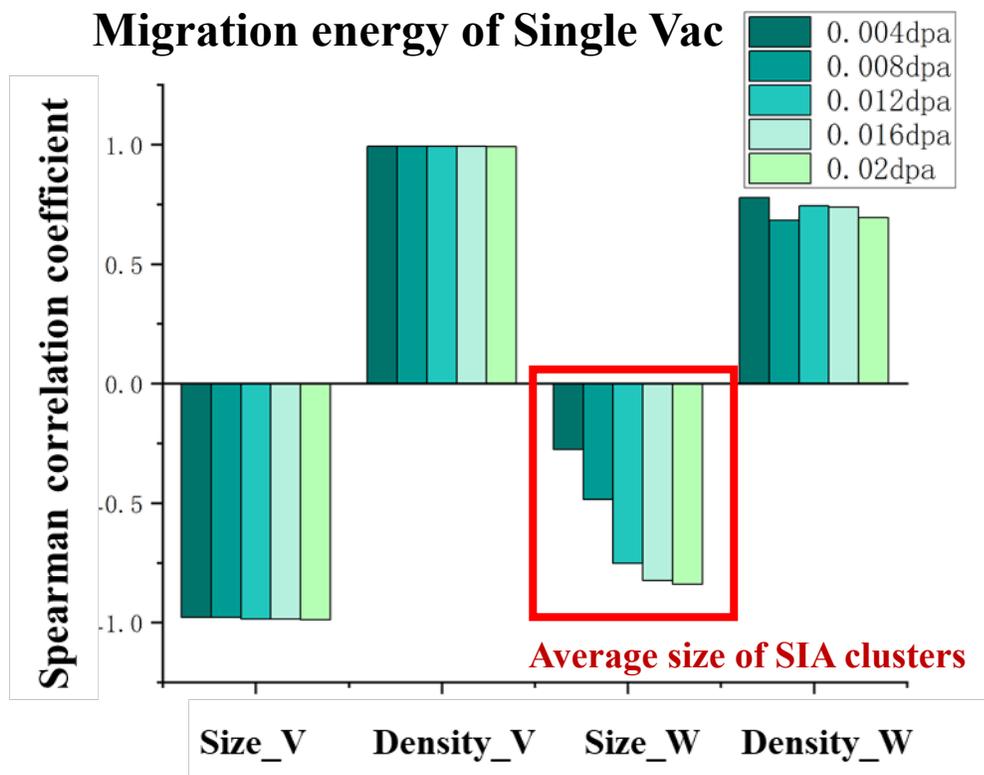
The sample size is 300.

The numbers in red are the sensitivity for single-parameter analysis.

Sensitivity of **capture radiuses** is **reduced**.

**No other substantial impact** identified for now.

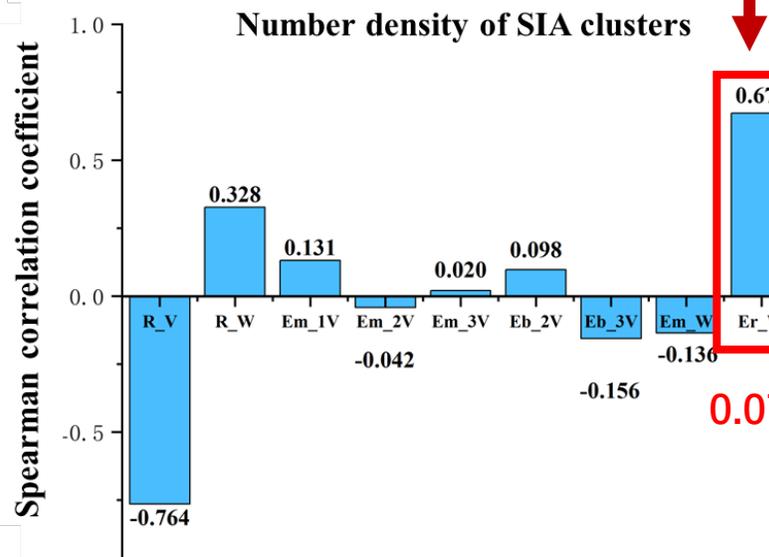
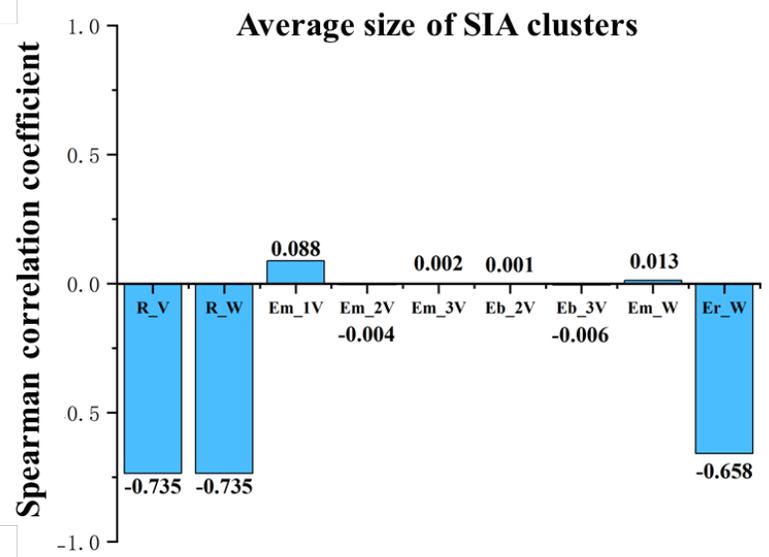
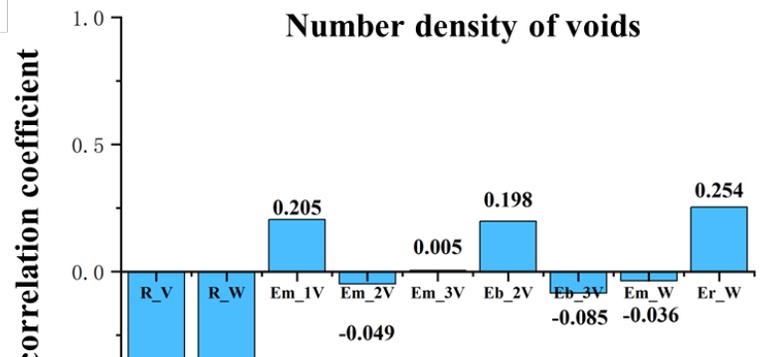
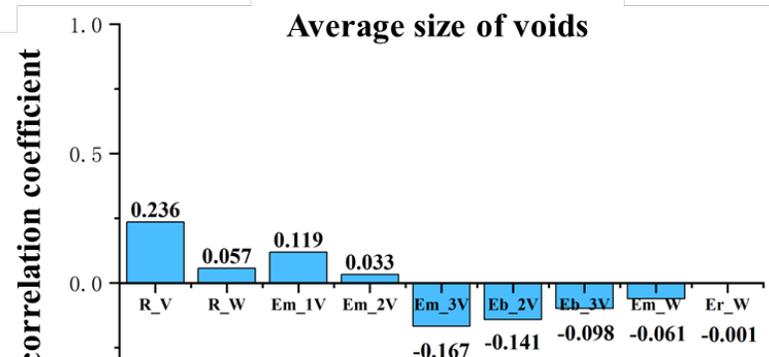
# Potential factor: different irradiation doses



At low irradiation doses, recombination may be insufficient and the aggregation of SIAs dominates.

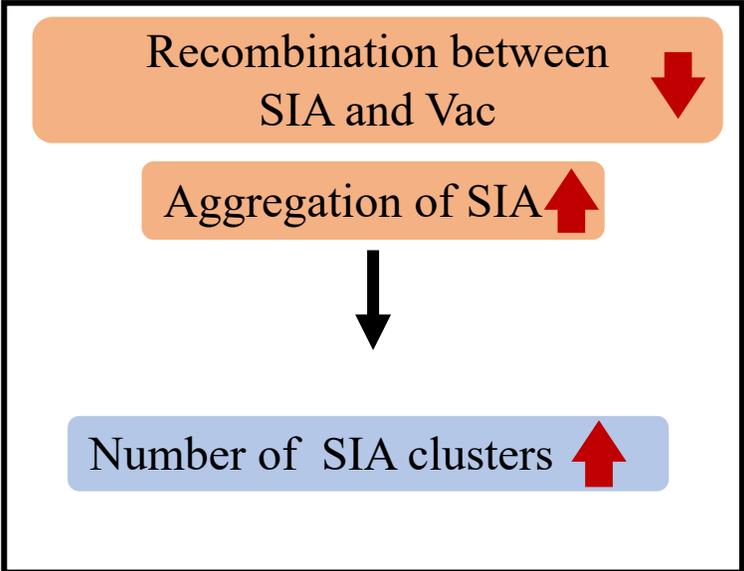
No clear trends or patterns.

# Potential factor: different temperatures



363K, 0.004dpa

Vacancy migration events are almost nonexistent.



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## Input variables

Migration energy of single Vac

Rotation energy of SIA

Capture radius of Vac

Capture radius of SIA

## Settings of PCE surrogate models

Method	Order/ level	Number of training samples	Number of test samples
Linear regression	5	126	
Gaussian quadrature	4	256	30
Sparse grid	4	705	

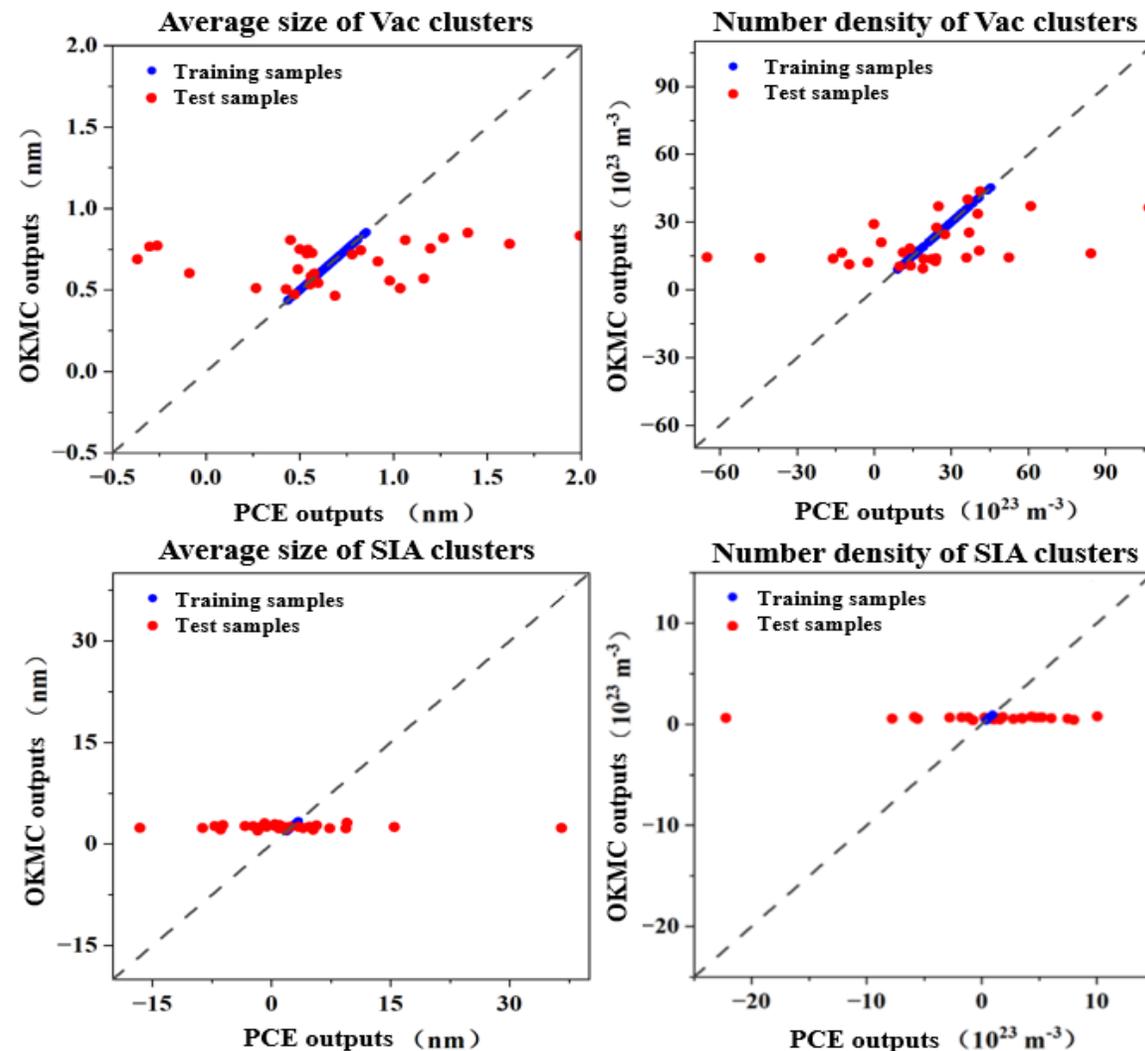
Minimization with least squares

Limited training samples

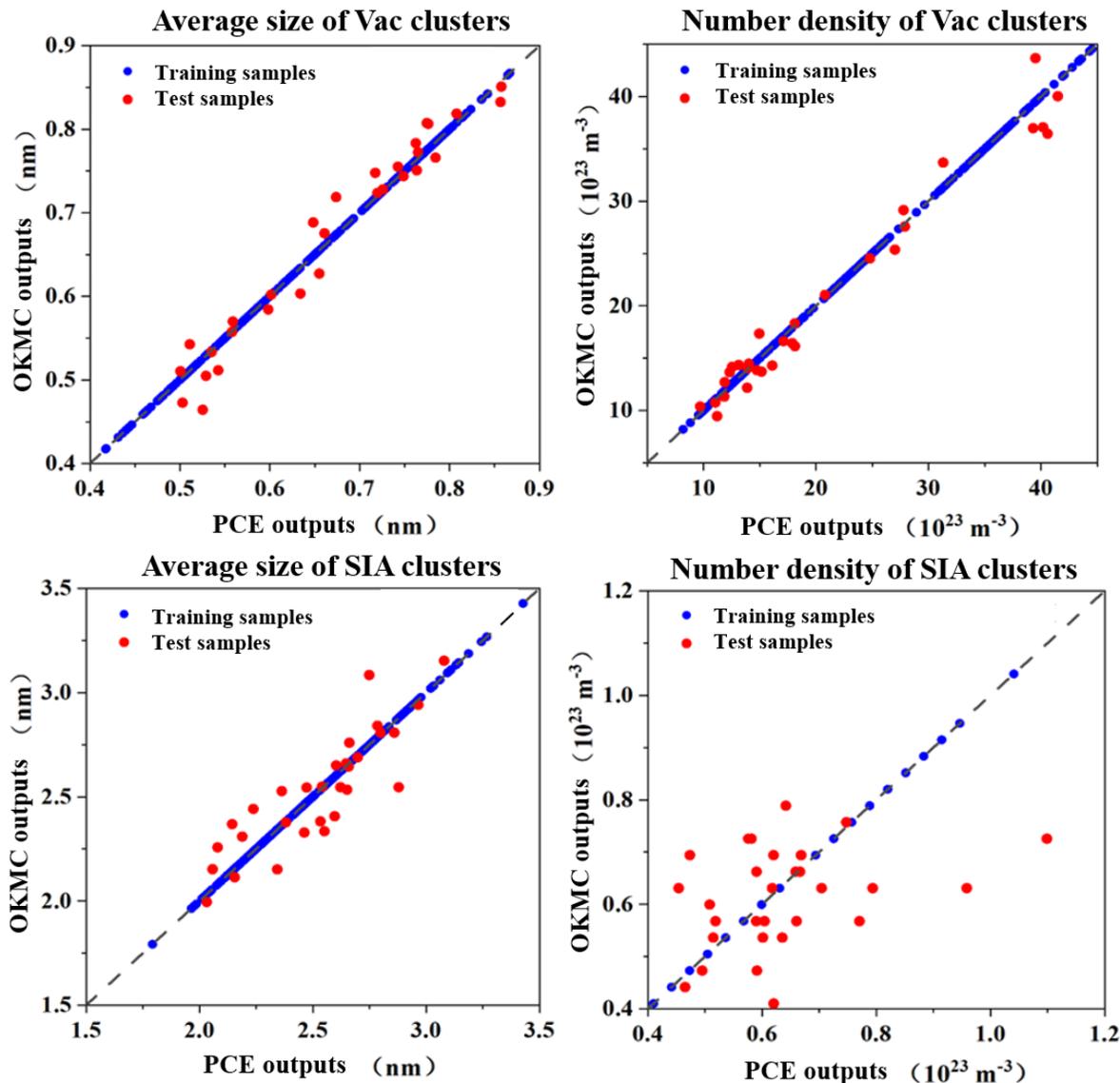
Multicollinearity of basis functions

Noise-induced overfitting

## Linear regression



## Gaussian quadrature



Gaussian-quadrature PCE results match OKMC outputs better.

Smoothing and global averaging in the process of integral computation

Training samples that are **more uniformly distributed** in the parameter space

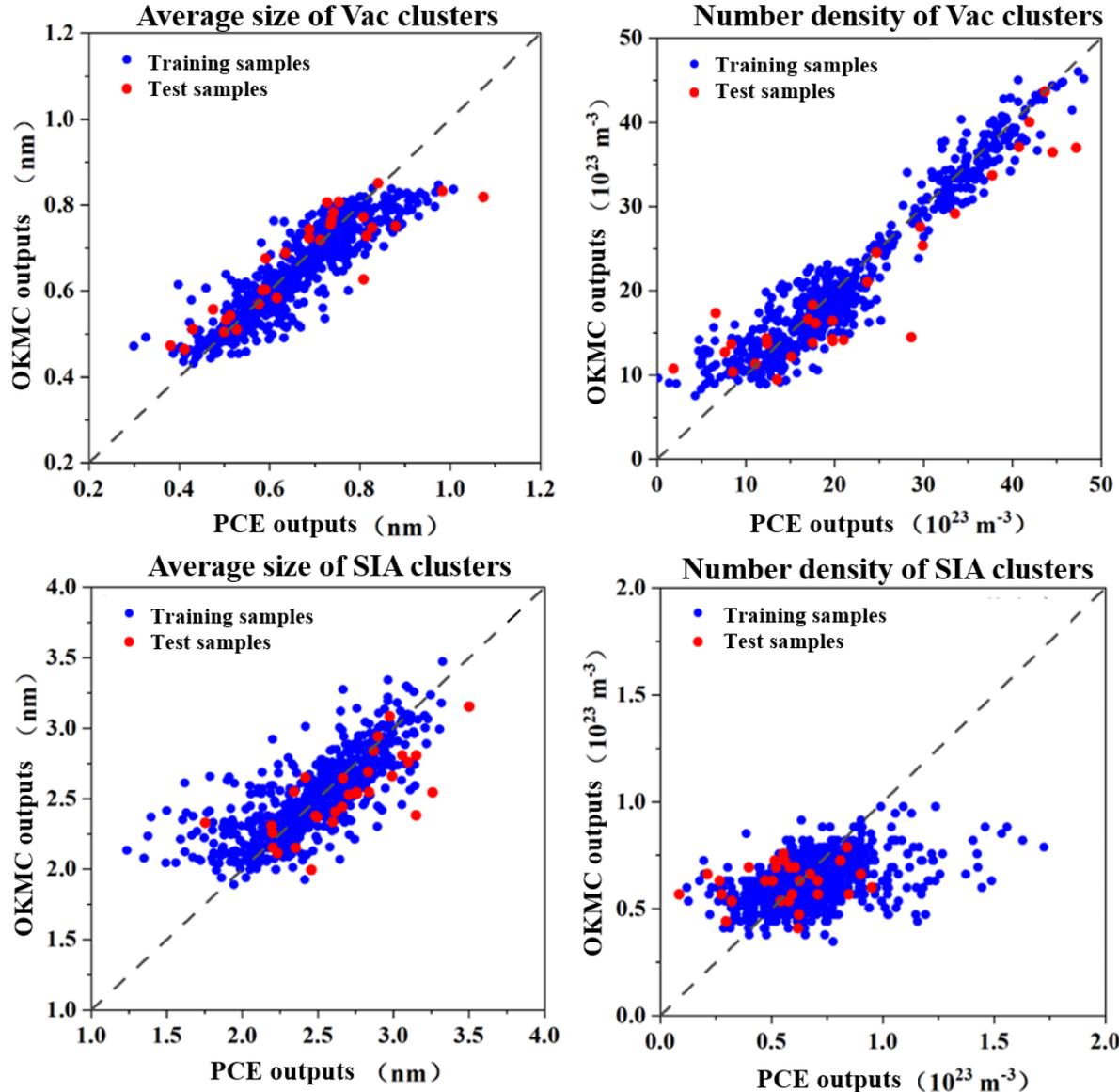


Reducing interference from data noise

### Limitation:

Due to the **sample size**, it is difficult to build surrogate models that include more input parameters.

## Sparse grid



Sparse-grid PCE results basically match the OKMC outputs, but some of the data are biased.

Lower uniformity of training sample distribution

Larger training sample size

Lack of knowledge of the overall characteristics of training samples

Increased noise interference

Partial data bias

**Limitation:**

Less effectiveness in dealing with data noise

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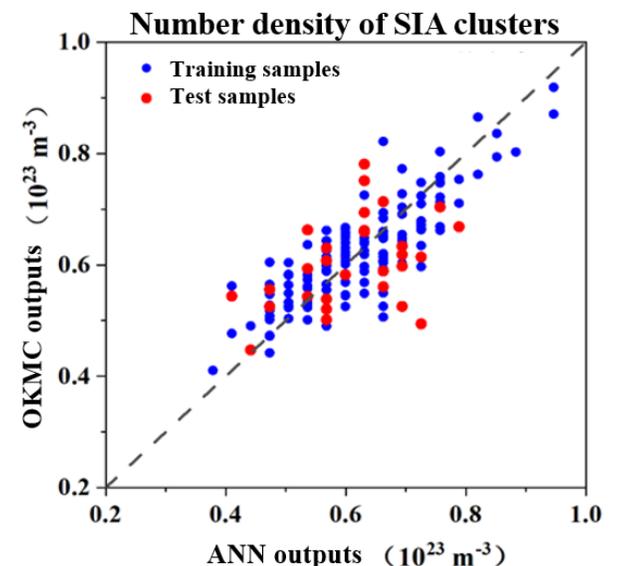
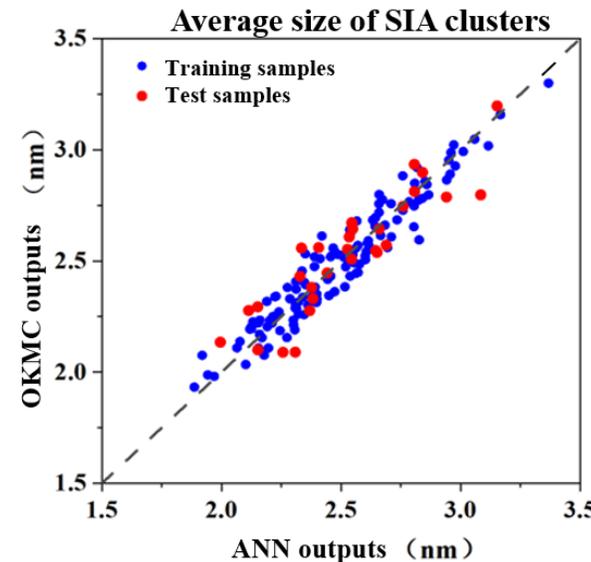
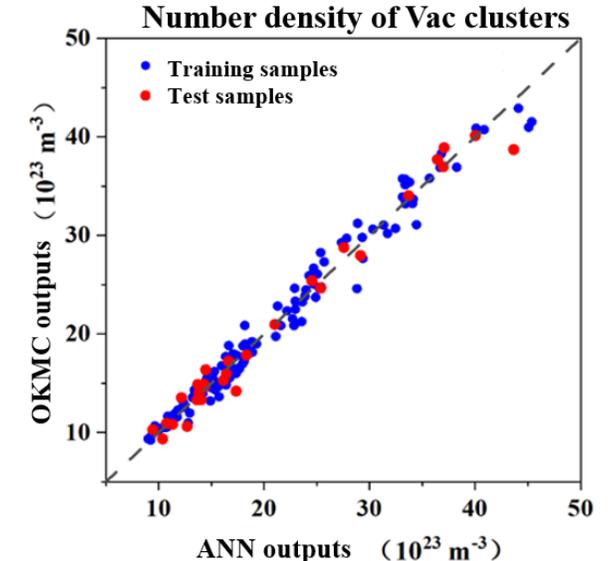
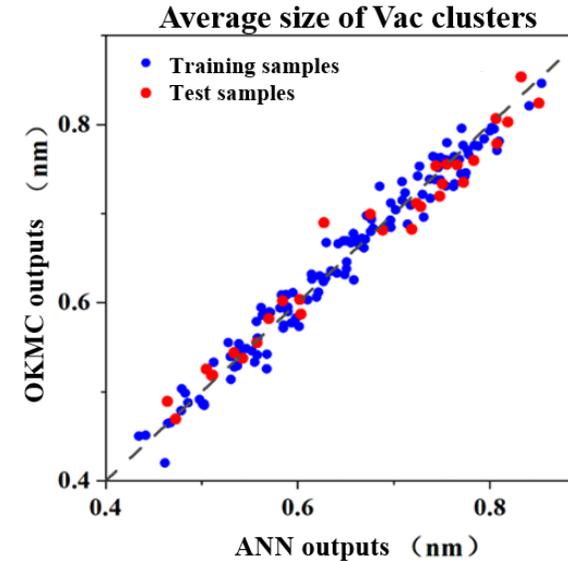
## Settings of ANN surrogate models with 4 inputs and 4 outputs

Number of hidden layers	1	2	3
Number of nodes in each layer	10	8,8	6,6,6
	16	12,12	8,8,8

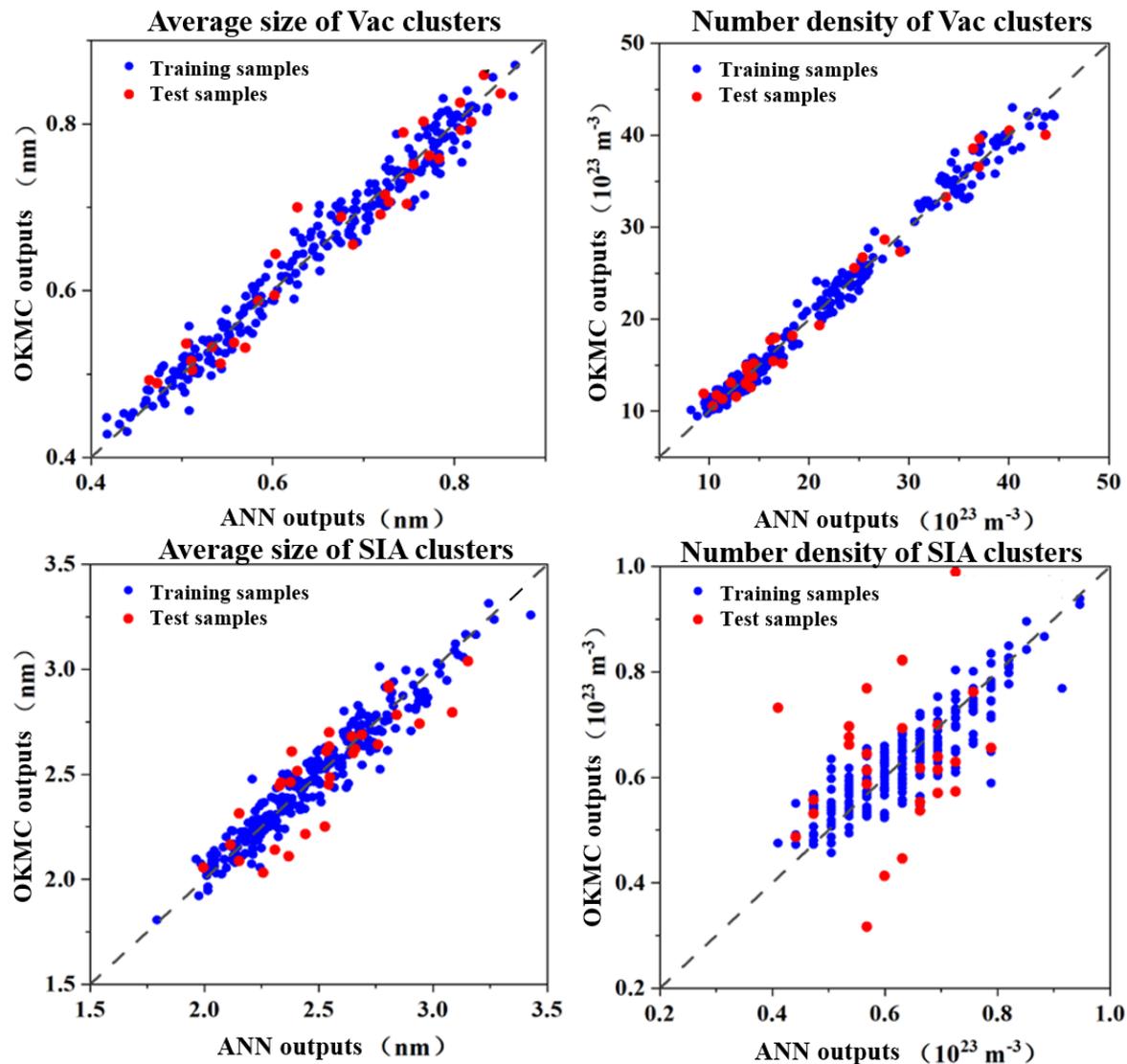
Six different surrogate models were built for each of the three sets of training samples in PCE part.

- ANN outperforms PCE in overall surrogate performance.
- MLP [ 4, 10, 4 ] and [ 4, 6, 6, 6, 4 ] have better performance in ANN surrogate models.
- **MLP [ 4, 10, 4 ]** is more computationally efficient than MLP [ 4, 6, 6, 6, 4 ].

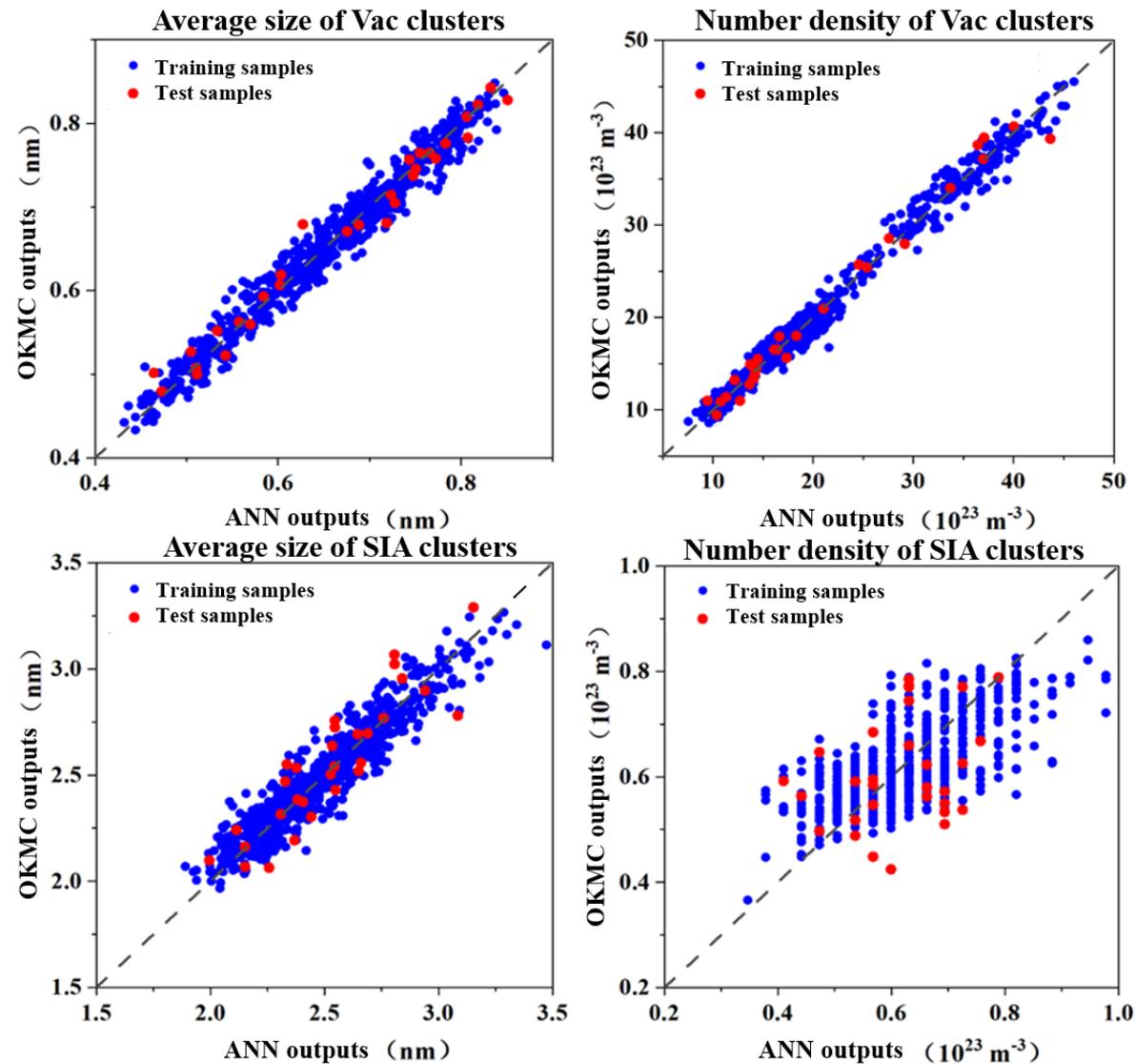
## MLP [ 4, 10, 4 ] with 126 samples



## MLP [ 4, 6, 6, 6, 4 ] with 256 samples



## MLP [ 4, 10, 4 ] with 705 samples



## 1) Introduction

Nuclear fusion in renewables-based system  
Irradiation damage in tungsten  
Multi-scale modeling of irradiation effects  
Uncertainty quantification & sensitivity analysis

## 2) Method

The OKMC model  
Sensitivity analysis

## 3) Sensitivity Analysis

Sensitivity at 733K and 0.02dpa  
Factors affecting the sensitivity potentially

## 4) Surrogate Models

Polynomial Chaos Expansion  
Artificial Neural Network

## 5) Conclusion & Future Work

To **better characterize the parameters** of OKMC and **optimize the model prediction**, we performed a **sensitivity analysis** of OKMC and assessed 4 **potential factors** that may affect the results of sensitivity.

- **Four sensitive parameters** with significant impacts on the OKMC simulation results were identified:

**Migration energy of single Vac**

**Rotation energy of SIA**

**Capture radius of Vac**

**Capture radius of SIA**

- ***Four potential factors***

**Range of value**



**Reasonable**

**Multiparameter**



**If computing resources are limited**

**Irradiation dose**



**Not too low**

**Temperature**



**Activation of specific processes**

To **reduce the computational costs** of inverse uncertainty quantification, two types of **surrogate models** constructed for OKMC are analyzed and assessed in terms of method setup, surrogate performance, and error calculation.

- **Simple structure ANN models with fewer hidden layers** are preferred for building OKMC surrogate models.

Capable of reducing the interference of data noise

Have stable performance

- It is difficult for PCE models to fundamentally solve the **overfitting problem**. The Gaussian PCE model has a good surrogate performance. However, its obvious defects in high-dimensional problems cannot be ignored.

## Future Work

- ▶ To reduce uncertainty, we need to **characterize** the sensitive parameters **more precisely**.

**Inverse Uncertainty Quantification**

Bayesian approach + MCMC method

Experimental data



Surrogate model

**Parameter calibration**

*Thank you for your attention!*

