Semi-empirical model for fission product yields

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Technical Meeting on Nuclear Data Needs for Antineutrino Spectra and their Applications
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FPY and antineutrino

- **Fission Product Yields (FPY) and β-decay**
  - Fission produces various fragments, some of which undergo β-decay
  - The emitted antineutrinos carry information about the fission process

- **How FPY affects antineutrino predictions**
  - Accurate FPY data improves reactor antineutrino spectrum calculations
  - Uncertainties in FPY lead to discrepancies in antineutrino flux predictions

Why FPY matters

» Discrepancies in decay heat calculations

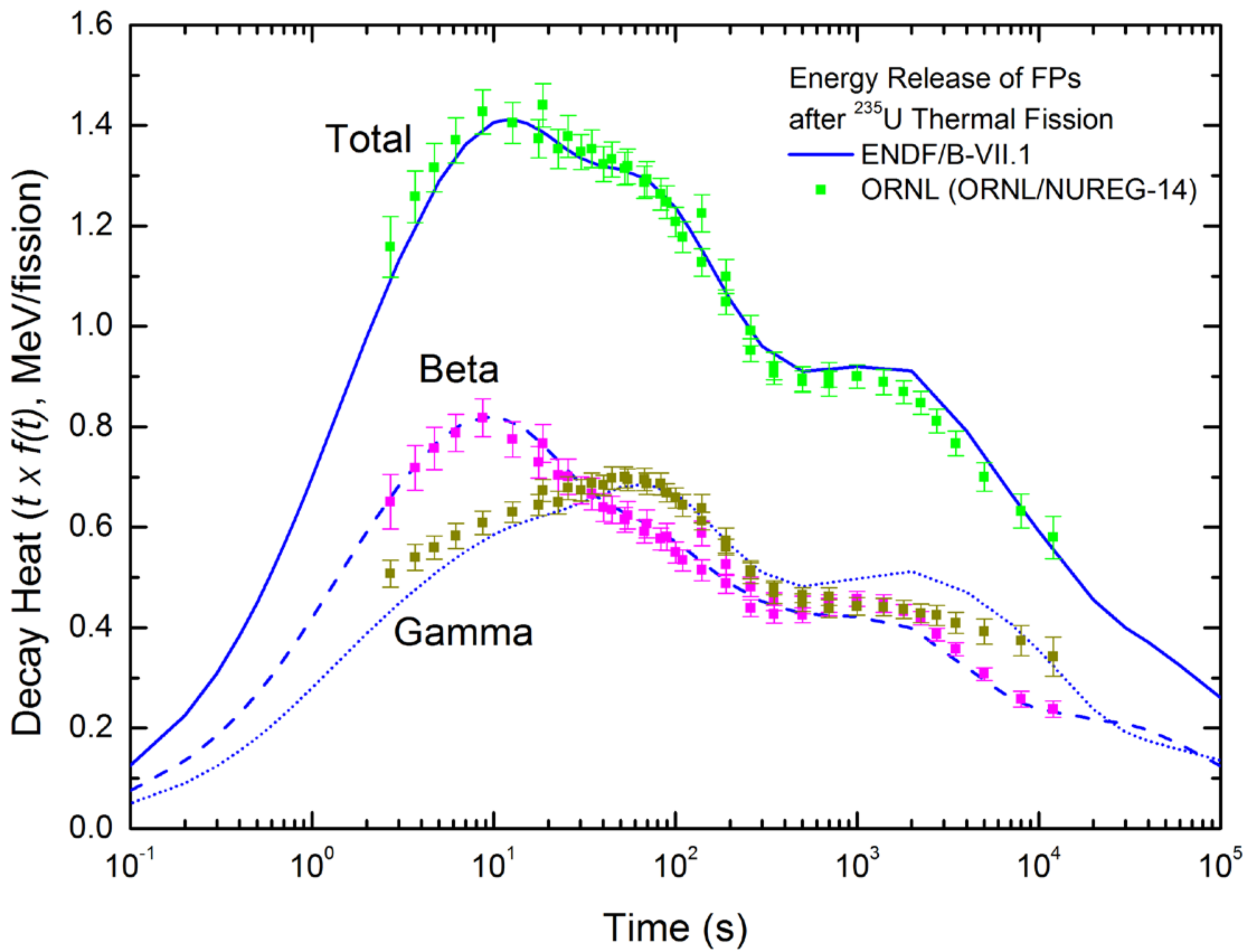
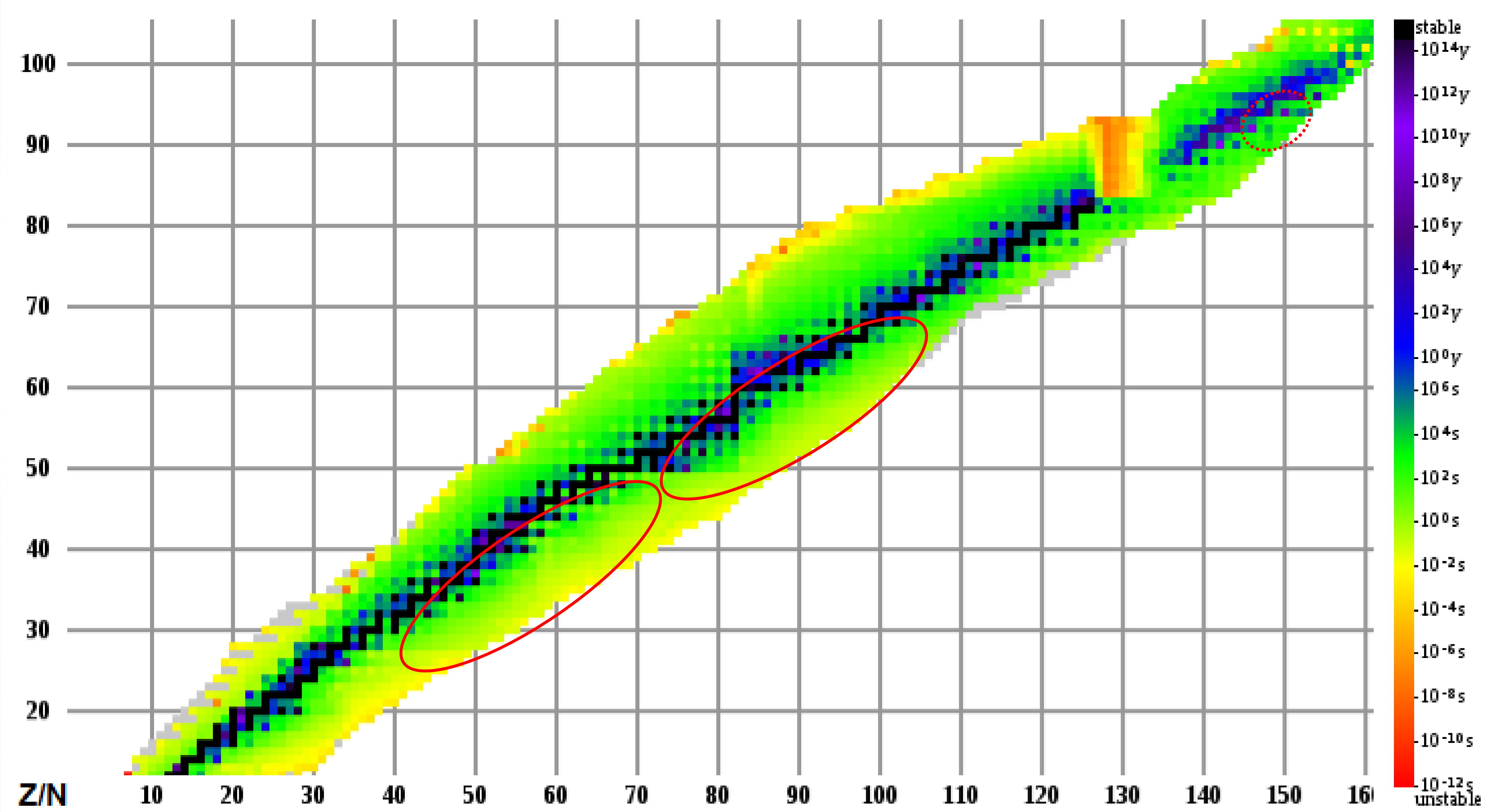


Chart of nuclide



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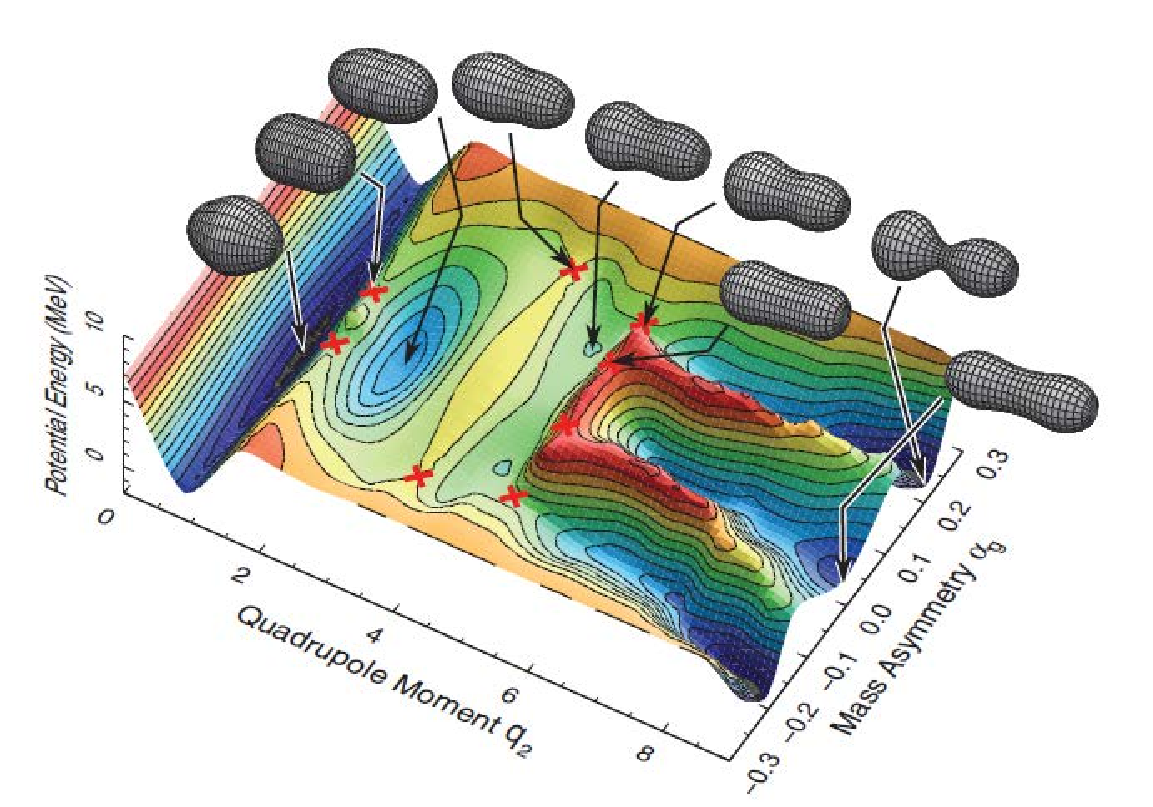
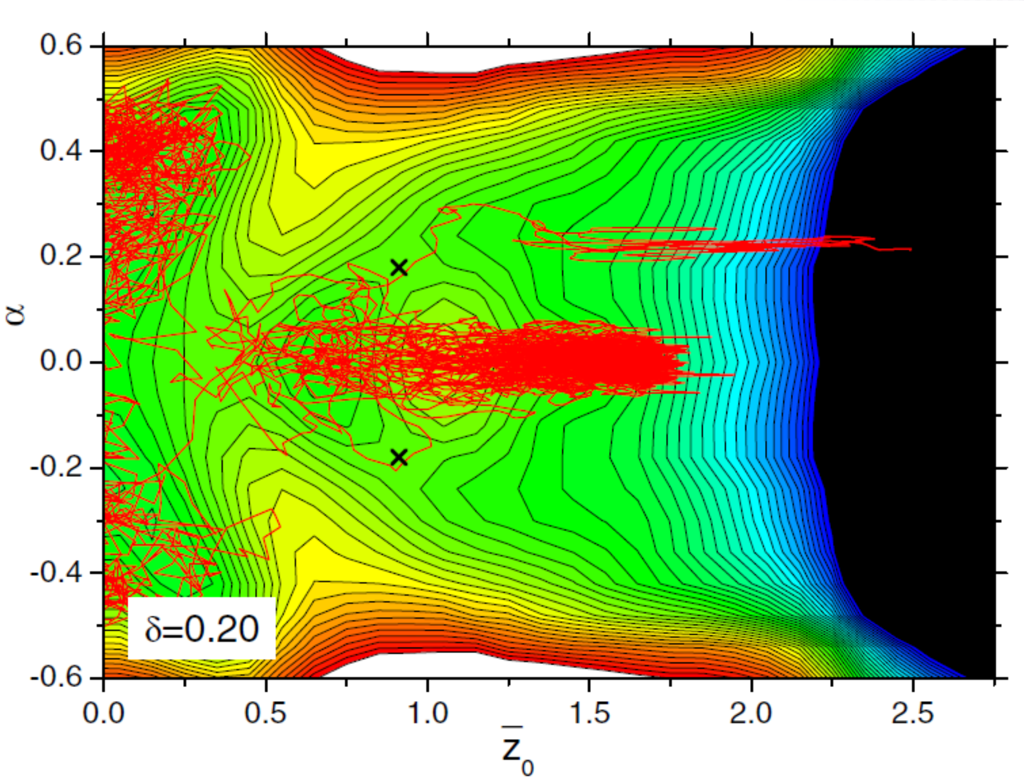
Theoretical model of nuclear fission

» A stochastic approach to fission



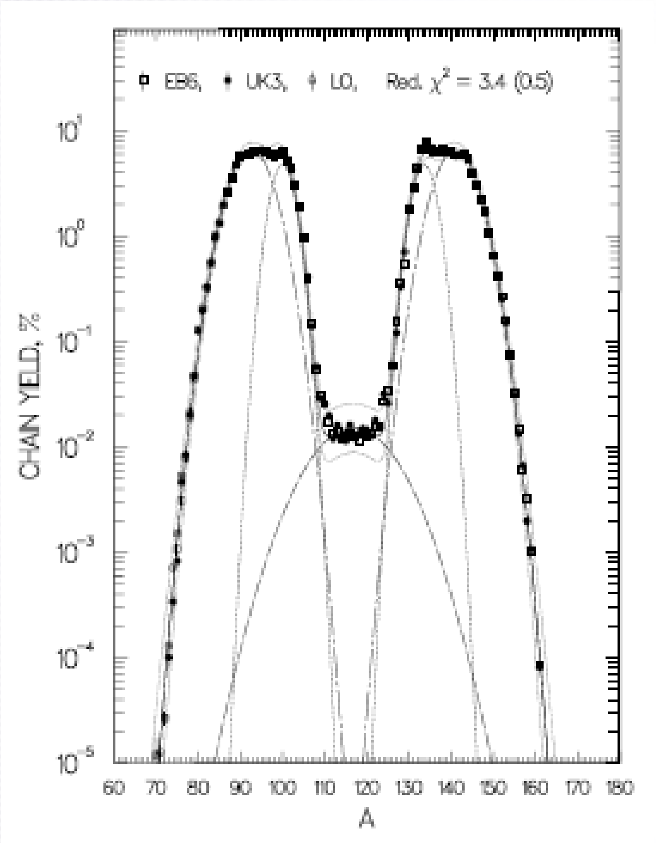
T. Ichikawa *et al*., Phys. Rev. C **86**, 024610 (2012).



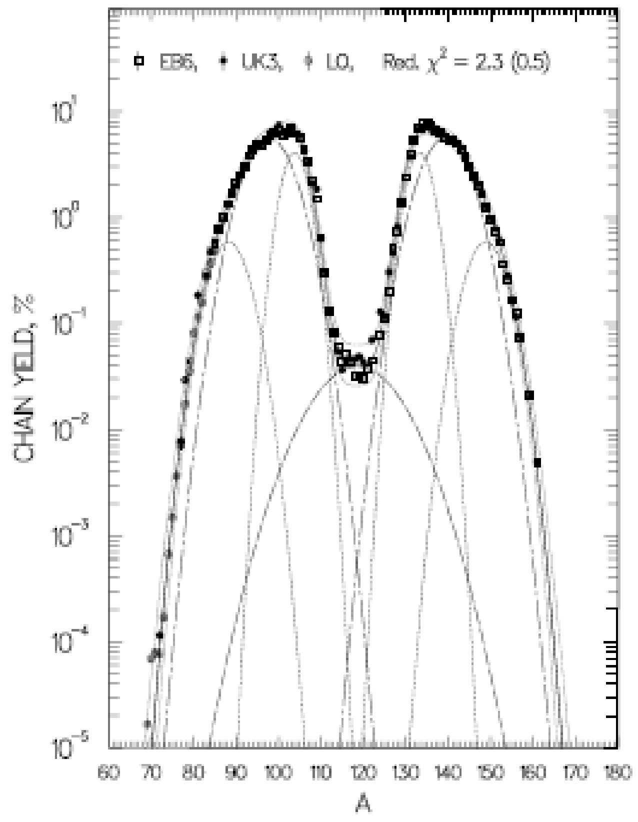
Y. Aritomo, S. Chiba, and F. Ivanyuk, Phys. Rev. C **90** (2014)

Empirical model of nuclear fission

Multi Gaussian



U235T



Pu239T

$$Y(A) = \sum_{i=1}^{7} Y_i e^D / \sigma_i \sqrt{2\pi}$$

$$D = -(A - \bar{A} + \Delta_i)^2 / 2\sigma_i^2$$

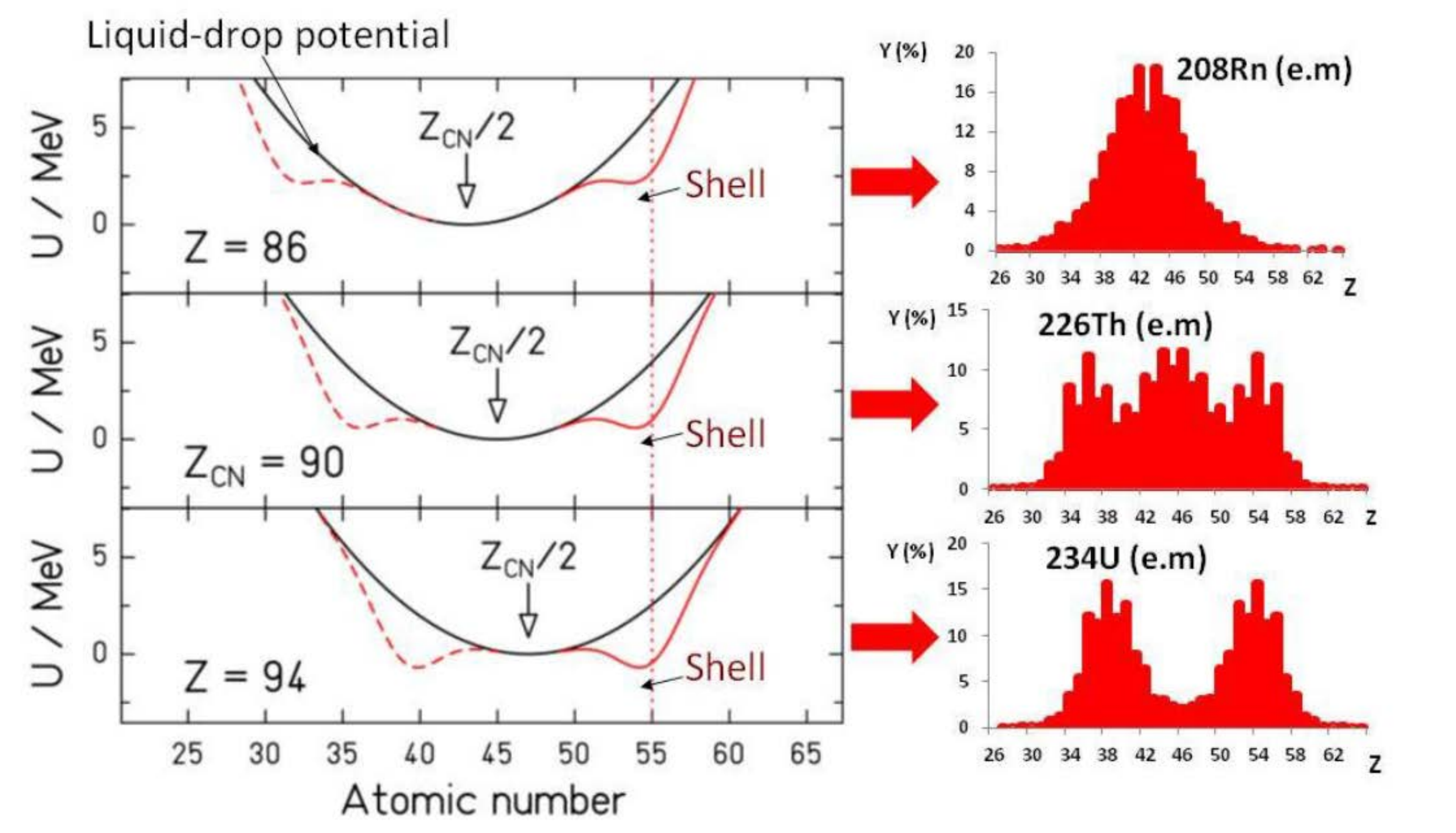
A.C.Wahl, Los Alamos report LA-13928 (2002).

Semi-empirical model

- Semi-empirical approach
  - Based on theoretical understanding
  - Enhanced through empirical modeling

- Why semi-empirical model?
  - Low computational cost
  - Sufficient accuracy
  - Predictive power

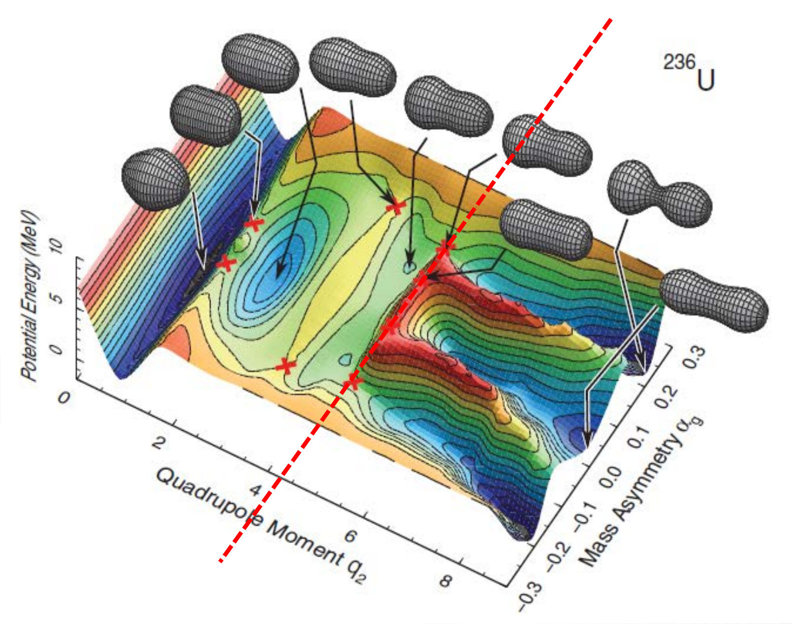
How potential energy shapes affect FPY



Jurado, B., and K. H. Schmidt. "Status of the general description of fission observables by the GEF code." (2014).

Our modeling strategy

- Our modeling approach
  - Simplify the fission process
  - Replace complex calculations with an empirical model
    - Empirical fission barrier
    - Embed fission dynamics into the barrier
    - Parameter fitting to experimental data



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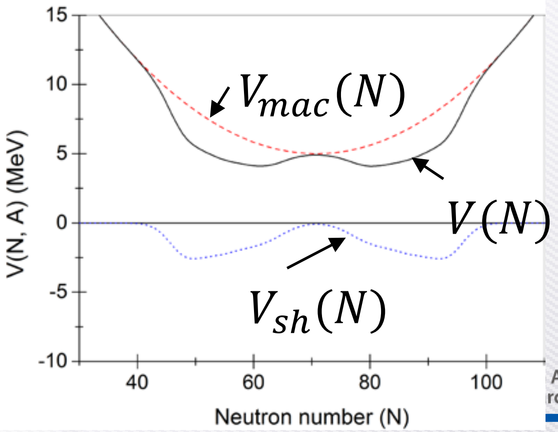
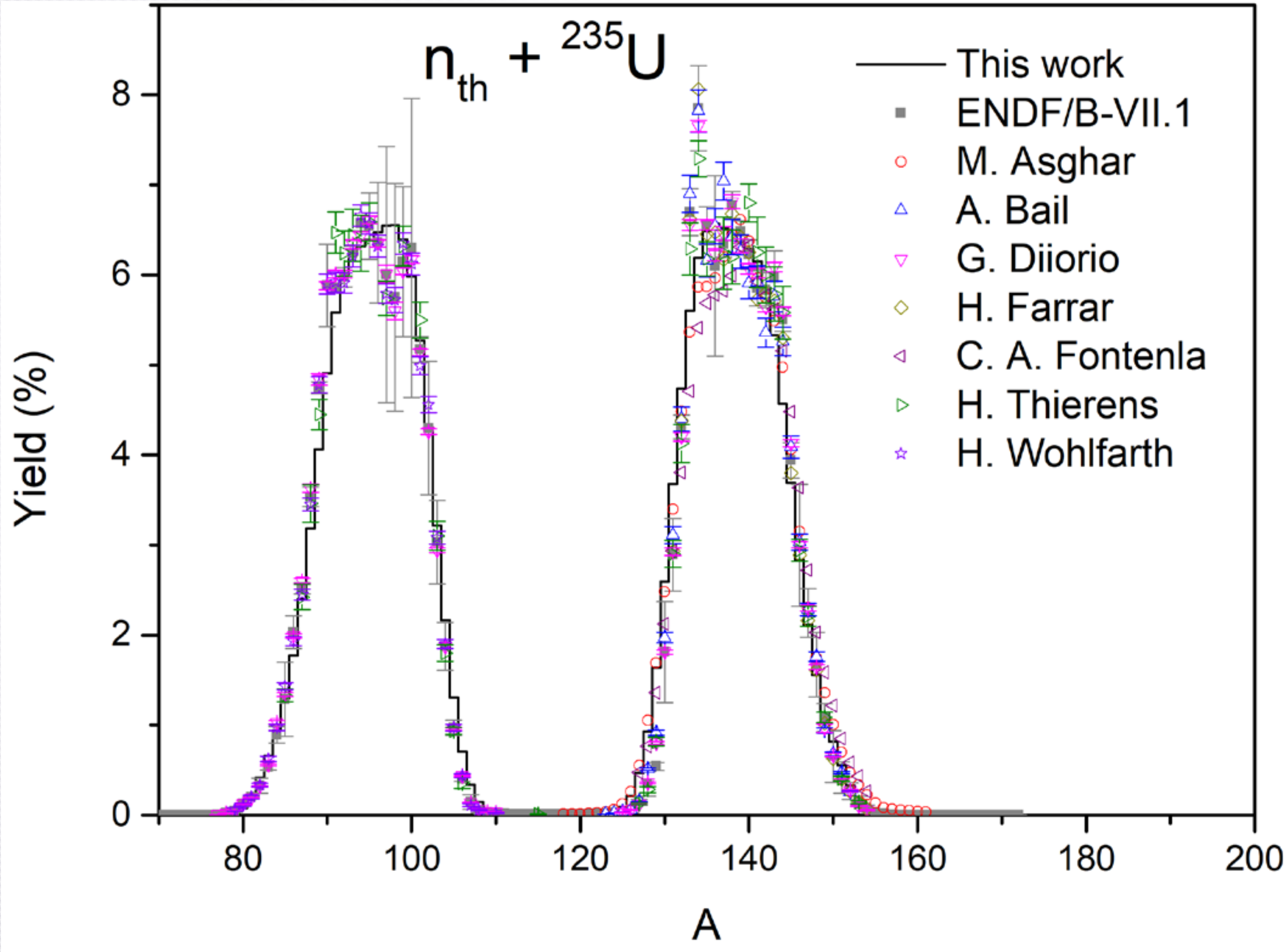
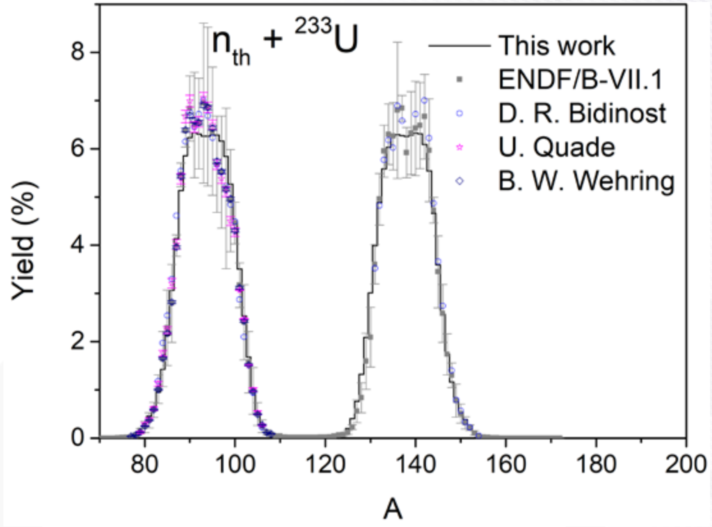
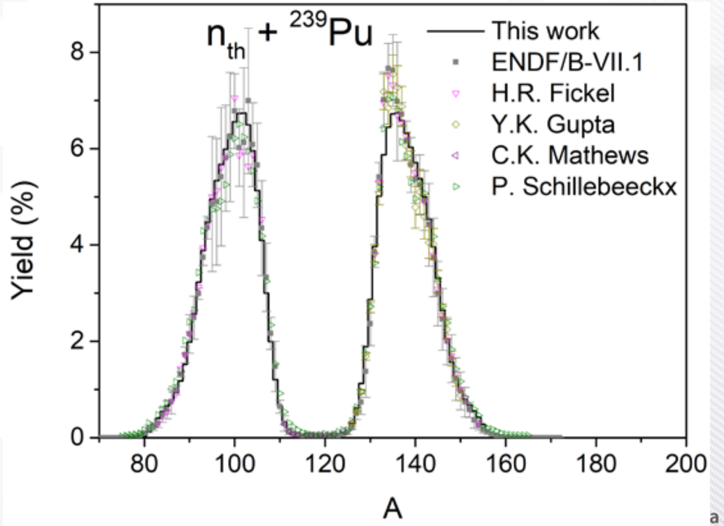
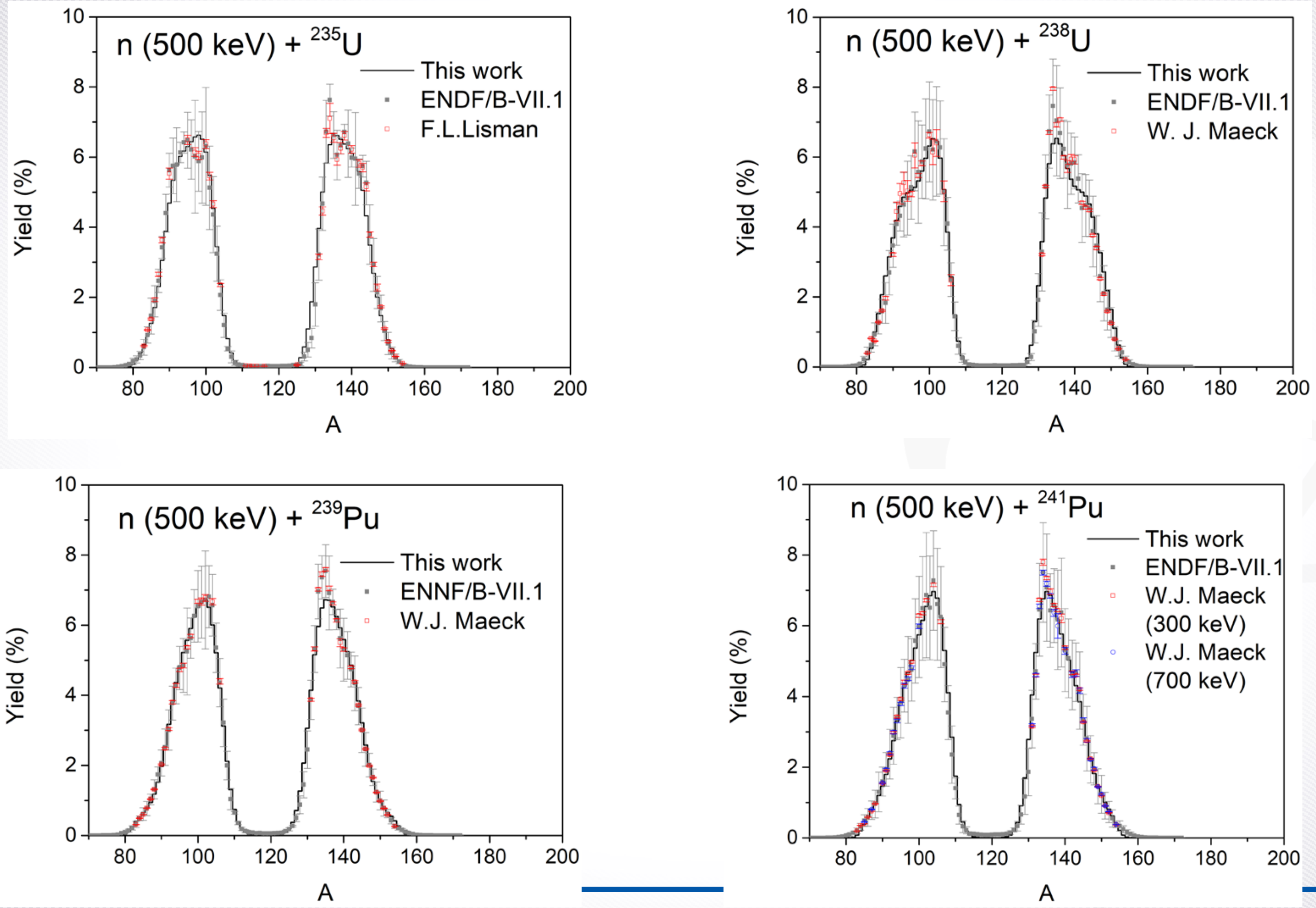


Calculation results (post-FPY)









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Applying the model to pre-neutron FPY data

- Previously applied to post-neutron FPY
  - Fairly complete datasets are available (e.g., ENDF)
  - Pre-neutron FPY data are much more limited in comparison

- Pre-neutron FPY represents the fission process up to scission
  - Post-neutron FPY includes effects of neutron evaporation of nascent fragments

- Our semi-empirical model is well-suited for pre-neutron FPY
  - Pre-neutron FPY is symmetric, and our model naturally reflects that shape

Pre-/Post-neutron FPY and neutron multiplicity

» Nuclear fission process

10^{-21} – 10^{-20} s .... 10^{-18} 10^{-14} s — — — 10^{-3} s t ⟶ ∞

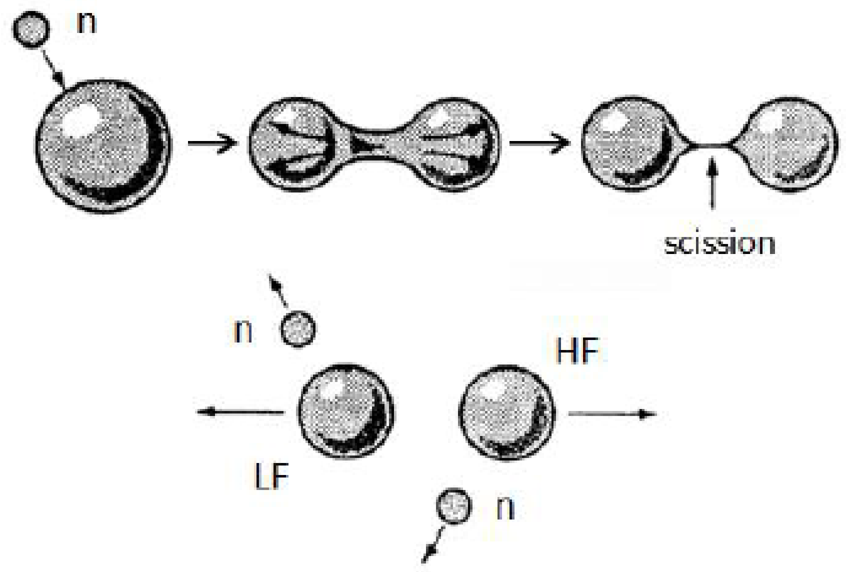
saddle to scission

Light charged particles
Scission neutrons

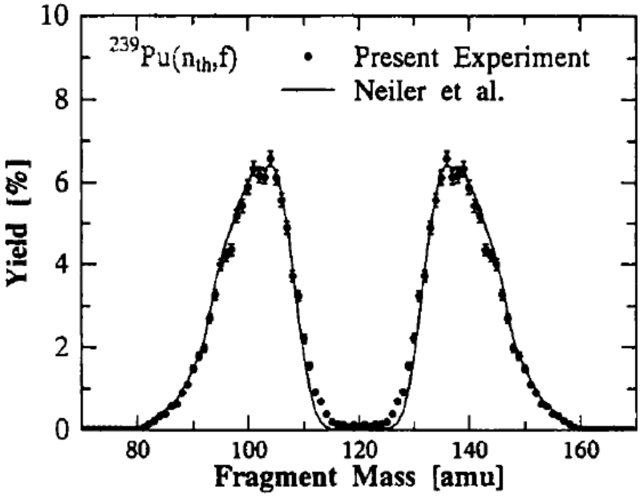
prompt **Neutrons**

prompt **Gammas**

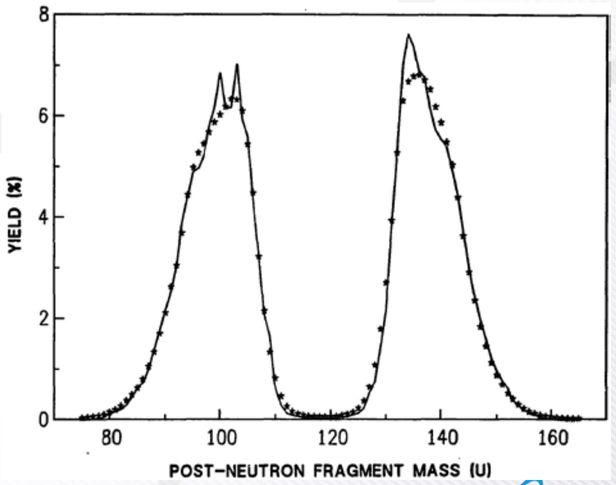
β-decay: betas and antineutrinos, delayed neutrons, gammas and fission



F. Gönnenwein, LANL, FIESTA 2014



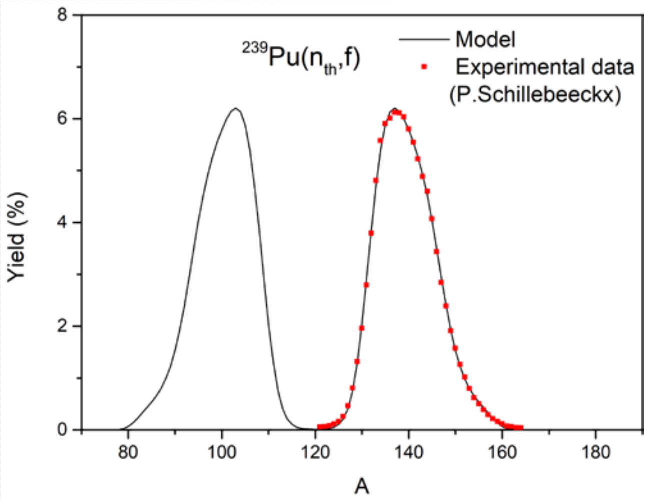
Fission fragment(J. Nucl. Sci. Technol. 32, 404 (1995))



Fission product(Phys. A 545, 623 (1992))

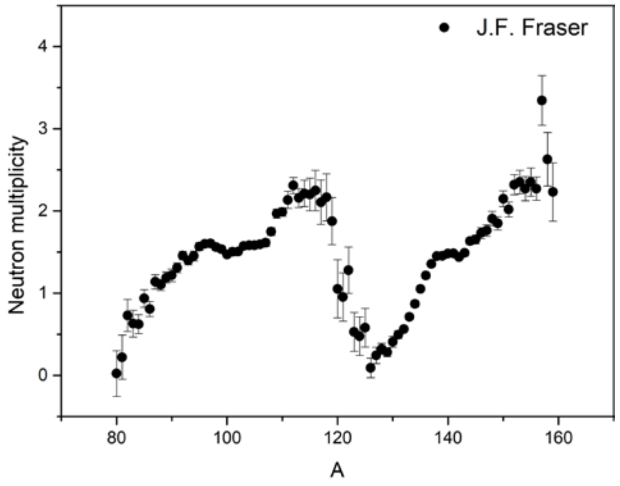
Pre-/Post-neutron FPY and neutron multiplicity

- **Post-neutron FPY is calculated by combining pre-neutron FPY and neutron multiplicity**



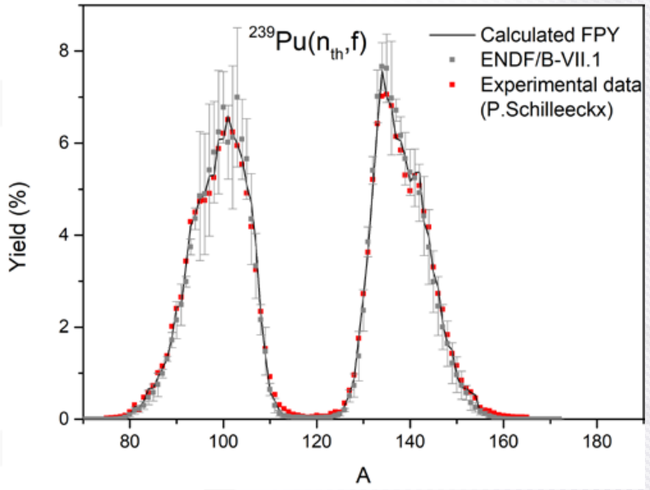
Pre-neutron emission fission product yield

+



Neutron multiplicity of each fragment mass

$$\nu(A^*) = \sum_{n=0,1,2\ldots} nP_n(A^*)$$



Post-neutron emission fission product yield

Pre-/Post-neutron FPY and neutron multiplicity

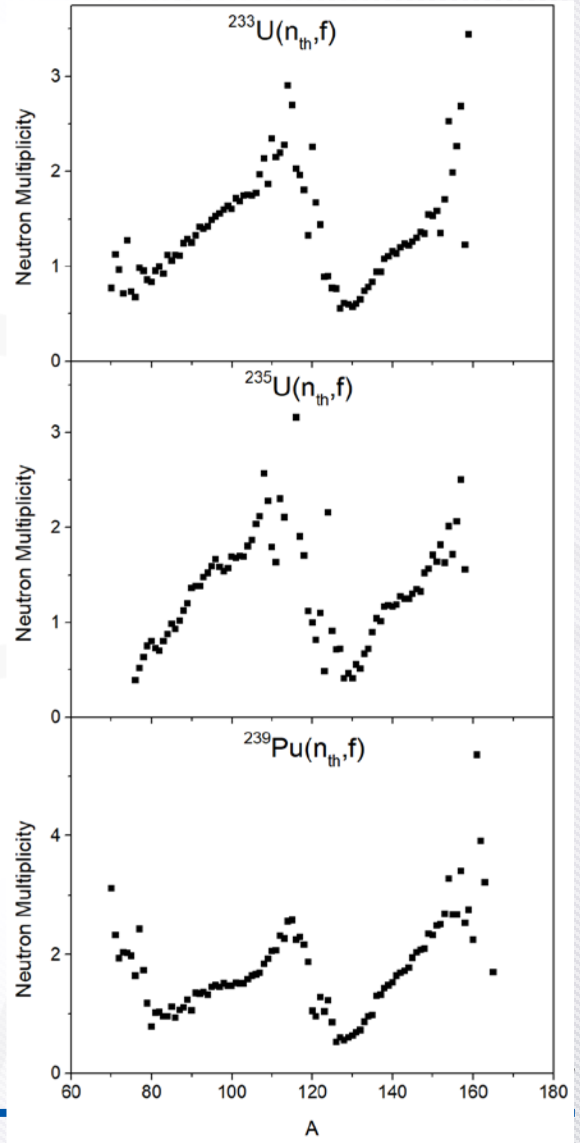
Neutron multiplicity

$$\nu(A^*) = \sum_{n=0,1,2\ldots} nP_n(A^*)$$

The experimental data of neutron multiplicity

Gaussian distribution

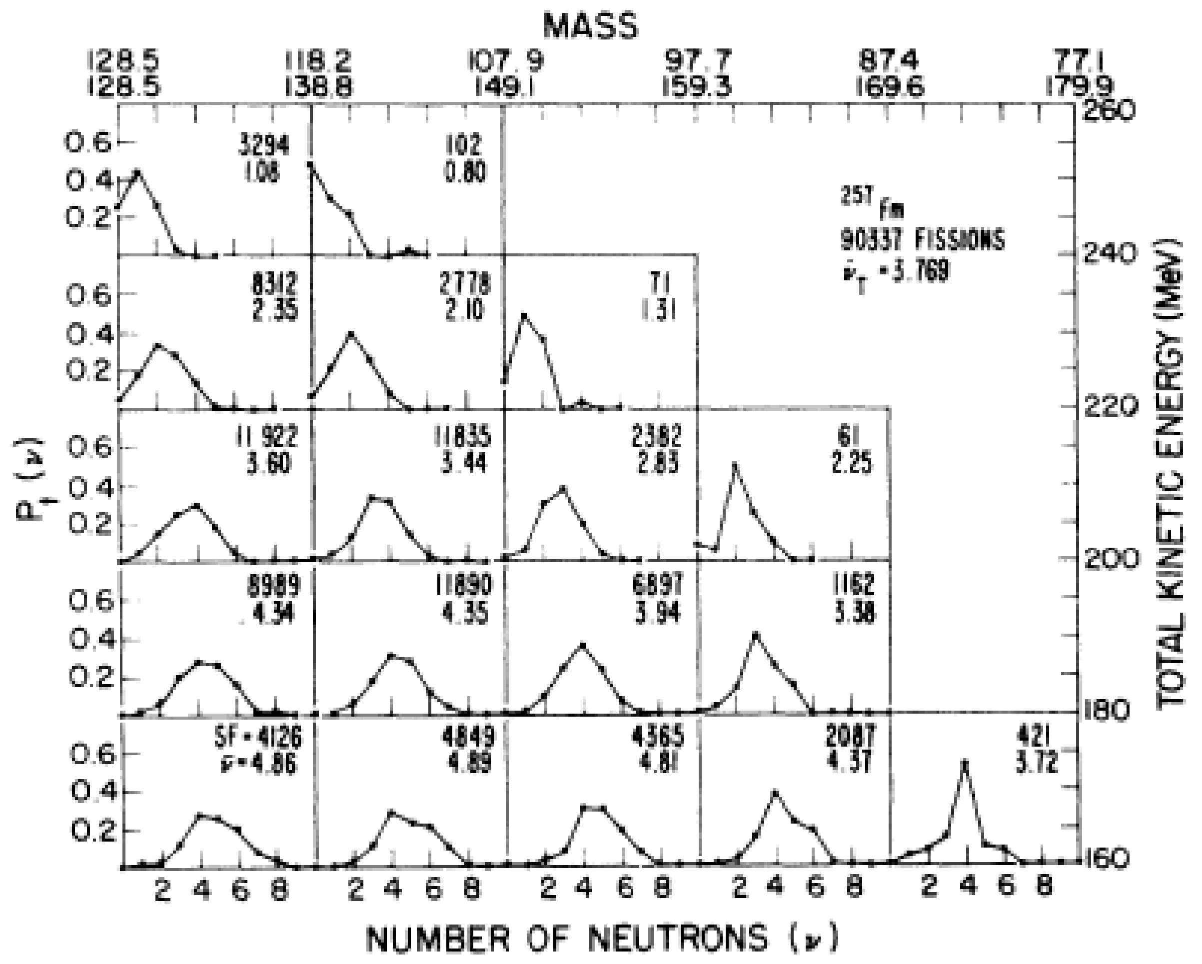
with normalization condition $\sum P_n(A^*) = 1$



Probability distribution of neutron emission

P_n of spontaneous fission of ^{257}Fm

(D.C. Hoffman et al., Phys. Rev. C 21, 637 (1980))



Poisson distribution

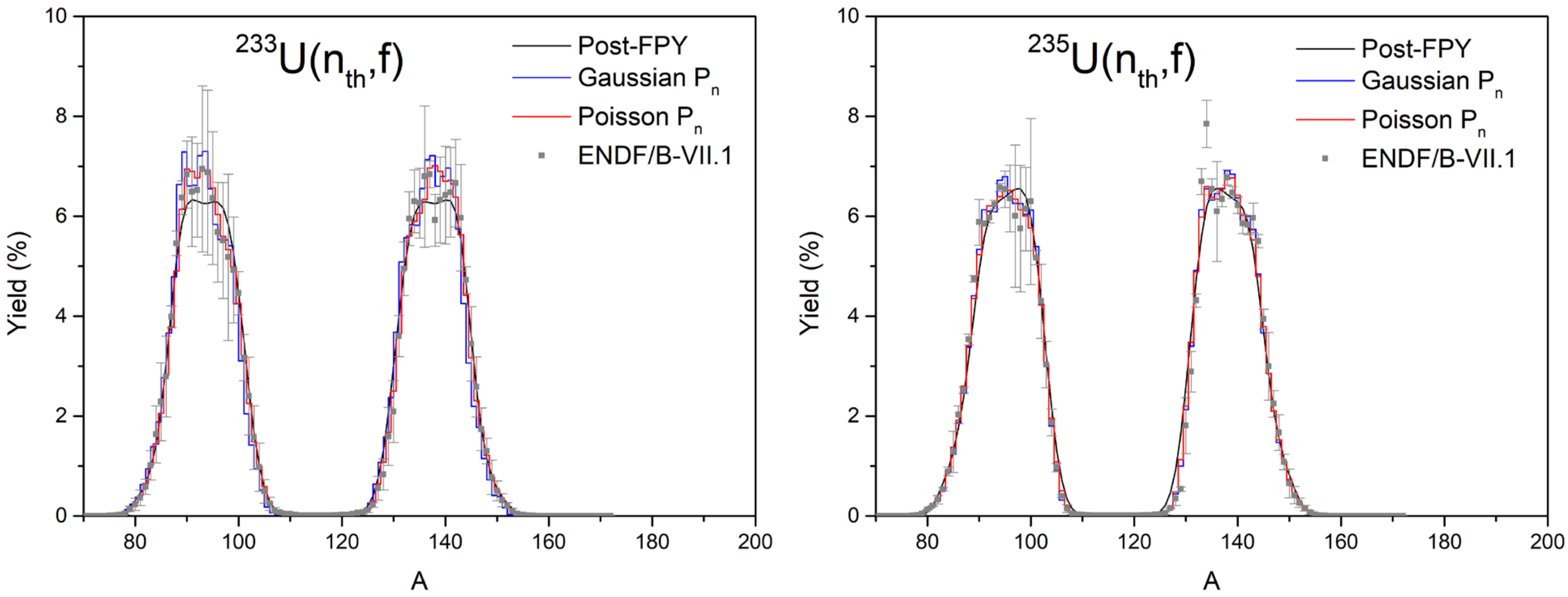
or

Gaussian distribution

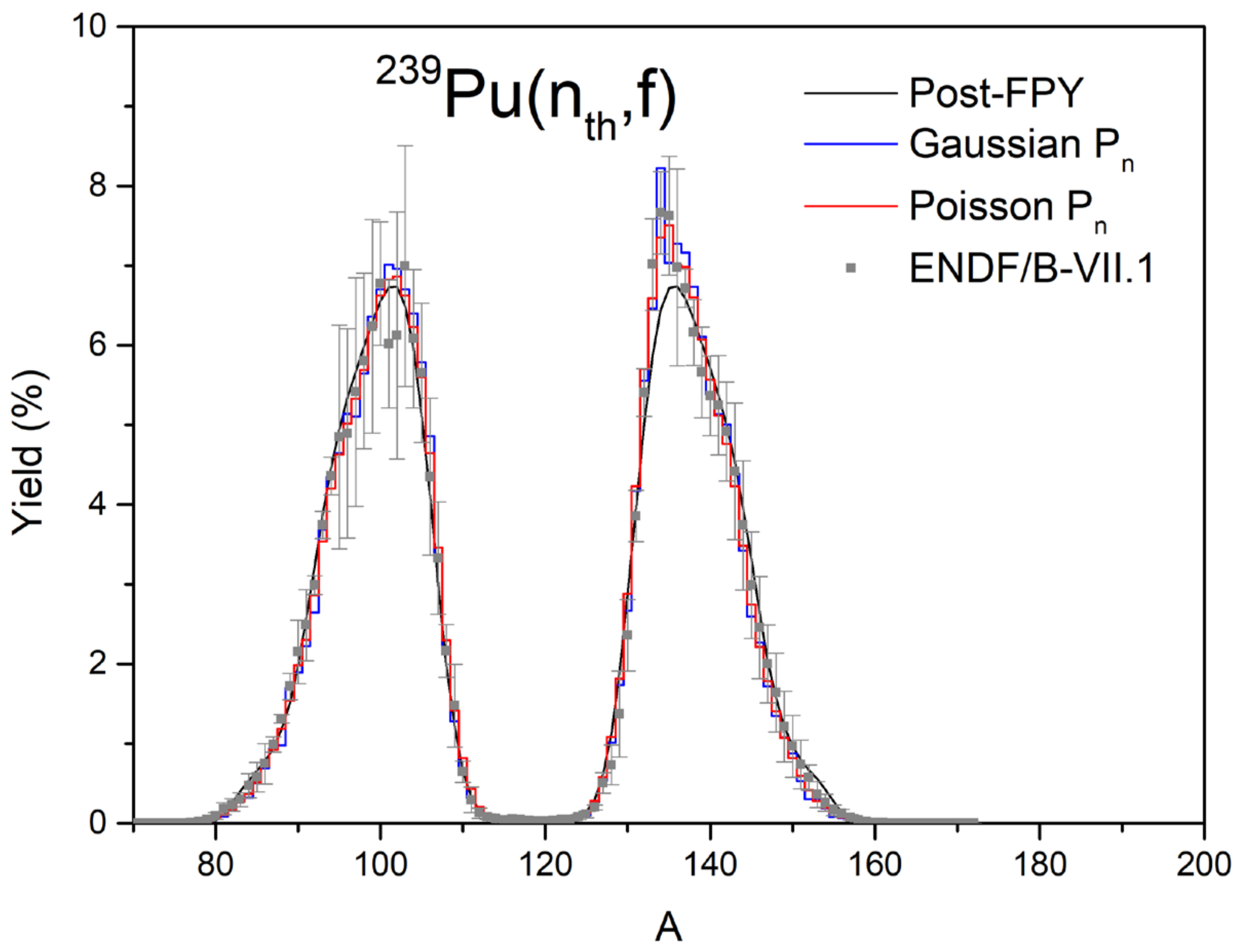
or

…something else?

Calculation results (pre-FPY)

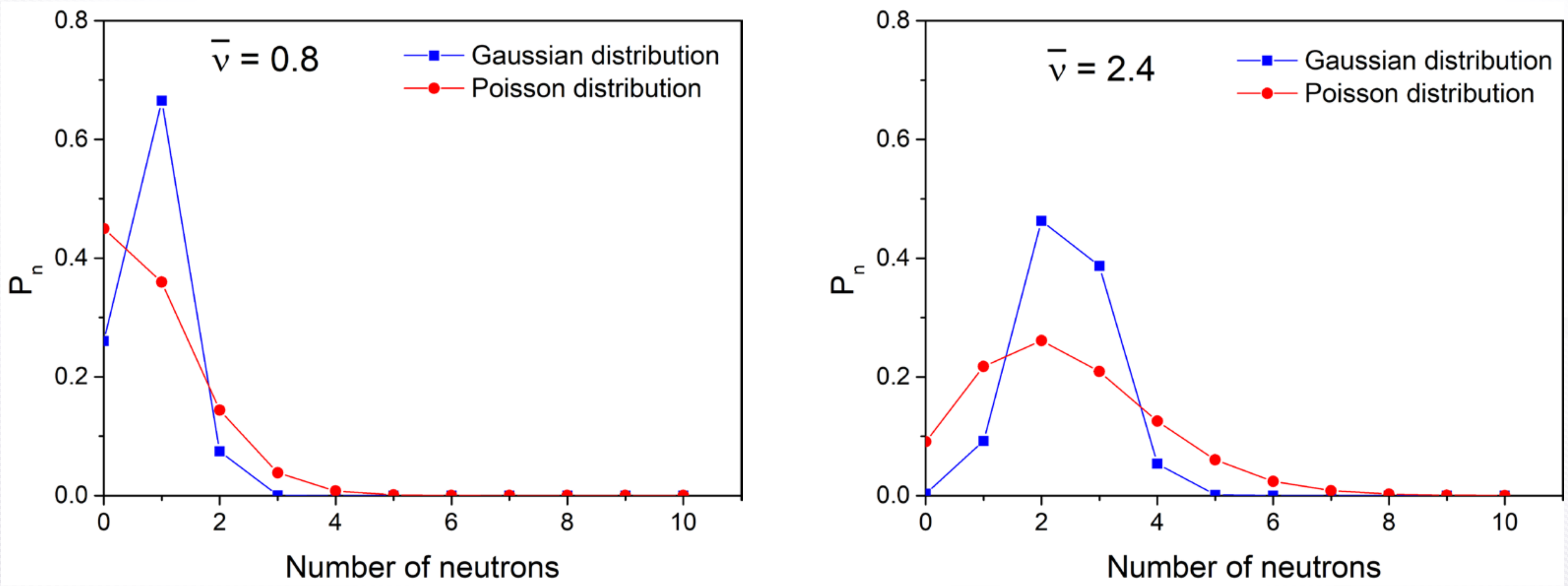


Calculation results (pre-FPY)



Gaussian vs Poisson

» Comparison of P_n calculated by using the Gaussian distribution and the Poisson distribution



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Gaussian vs Poisson

Degree of agreement between the calculation results and the ENDF

Calculated the degree of agreement with ENDF/B-VII.1

- $< \Delta Y^2 > \equiv \frac{1}{n} \sum_{k=1}^{n} (Y_k - \bar{Y}_k)^2$
- $\chi^2 \equiv \frac{1}{n} \sum_{k=1}^{n} \left(\frac{Y_k - \bar{Y}_k}{\Delta Y_k} \right)^2$

Fission	$< \Delta Y^2 >$		χ^2	
	Gaussian P_n	Poisson P_n	Gaussian P_n	Poisson P_n
$^{233}U(n_{th},f)$	0.0727	0.0512	1.0545	0.7460
$^{235}U(n_{th},f)$	0.0619	0.0686	9.0896	10.632
$^{239}Pu(n_{th},f)$	0.0672	0.0463	1.5585	0.8331
Average	0.0673	0.0554	3.9009	4.0704

Summary

- Proposed a semi-empirical model fission product yields and applied it to both post- and pre-neutron emission data
- For post-neutron FPY, the model showed good agreement with experimental data, but has limitations in describing the fission process
- Converting pre-neutron FPY to post-neutron FPY requires the probability distribution of neutron emission for each fragment mass
- We used Gaussian and Poisson distribution to model neutron emission probabilities

THANK YOU

Backup

Description of model

- Compound nucleus are considered as micro-canonical ensemble

- Assume that the mass yield distribution is determined by the level density at fission barrier
  - From the functional form of level density of Fermi-gas model $\rho(E^*) \propto \exp\left(2\sqrt{\tilde{a}E_{int}}\right)$ fission yield can be expressed as

$$Y(N,A;E) \approx \exp\left(2\sqrt{\tilde{a}\left(E^* - V(N,A)\right)}\right)$$

  - Internal excitation energy at fission barrier $E_{int} = E^* - V(N,A)$

- Shell structure of fission products play important role in determine the mass distribution
  - $N_{in} = 82$, $N_{out} \sim 88$
  - Express V as a function of N
  - Unchanged charge distribution

Potential energy at fission barrier

Fission barrier $V(N,A) = V_{mac}(N) + V_{sh}(N)\exp(-\gamma\epsilon(N))$

Macro potential

$$V_{mac}(N) = C_{mac}\left(N - \frac{\tilde{N}_{CN}}{2}\right)^2 + V_0$$

Shell correction

$$V_{sh}(N) = C_{in}\left[\exp\left(\frac{(N-N_{in})^2}{\sigma_{in}^2}\right) + \exp\left(\frac{(N-\overline{N}_{in})^2}{\sigma_{in}^2}\right)\right]$$
$$+ C_{out}\left[\exp\left(\frac{(N-N_{out})^2}{\sigma_{out}^2}\right) + \exp\left(\frac{(N-\overline{N}_{out})^2}{\sigma_{in}^2}\right)\right]$$

Damping term

$$\epsilon(N) = E^* - [V_{mac}(N) + V_{sh}(N)]$$

where

$\tilde{N}_{CN} \equiv N_{CN} - \nu$ and $\overline{N}_j \equiv \tilde{N}_{CN} - N_j\ (j = in, out)$

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