From Total Neutron Cross Sections to Nuclear Charge Radii

Tables and systematics

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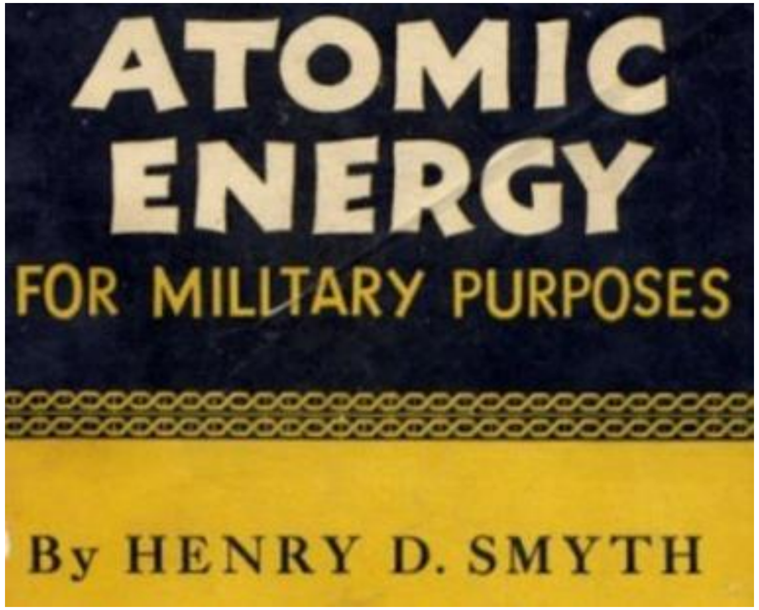
IEAEA, Vienna
27 – 30, January, 2025

War and Peace



1945: The ‚*Smyth Report*'

National nuclear research institutes

Quest for uranium

1954: Research Institute for Physics, Debrecen

Basic research

Search for U in coal-mines

Health physics

1957: International Atomic Energy Agency

Peaceful purposes of nuclear energy

IAEA

Total neutron cross sections

1965: J. Csikai: Neutron physics group, $E_n \sim 14$ MeV cross sections.

Reactions by activation method. $\boldsymbol{\sigma_T}$ by transmission method.

1967: → Institute of Experimental Physics, University of Debrecen

IAEA(NDS): Research Contract, neutron generator.

Thank you Joe Schmidt! Thank you Joe Dolnićar!

1968: $\boldsymbol{\sigma_T(^{12}C) < \sigma_T(^{9}Be)}$ **!** (Nucl. Phys., **A119**, 525) Z, N = 6, ... magic?

1969: Correlation of **matter radii** with binding energy (Phys.Lett. **B29**, 36)

Black nucleus formula: $\sigma_{BN} = 2\pi(R_m+\lambda\!\!\!^{-})^2 \rightarrow \boldsymbol{R_m}$

1971: Visit to Ioffe Institute: 1 week: total neutron cross sections;

3 weeks: K_α *Isotope Shifts* $\rightarrow \delta \langle r^2 \rangle_{ch}$ + other $\mathbf{e^-}$ methods!

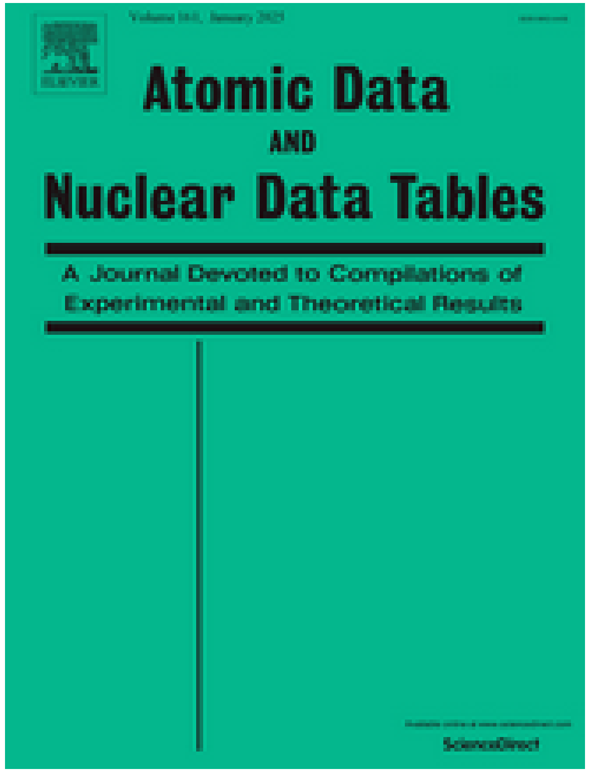
1974: Atomic Data and Nuclear Data Tables, **14**, special issue 5 – 6:

p.: 479: DeJager, et al.: Electron scattering ...

p.: 509: Engfer, et al.: Muonic atoms ...

p.: 605: Boehm, et al.: K_α Isotope Shifts ...

p.: 613: Heilig, et al.: Optical Isotope Shifts ...



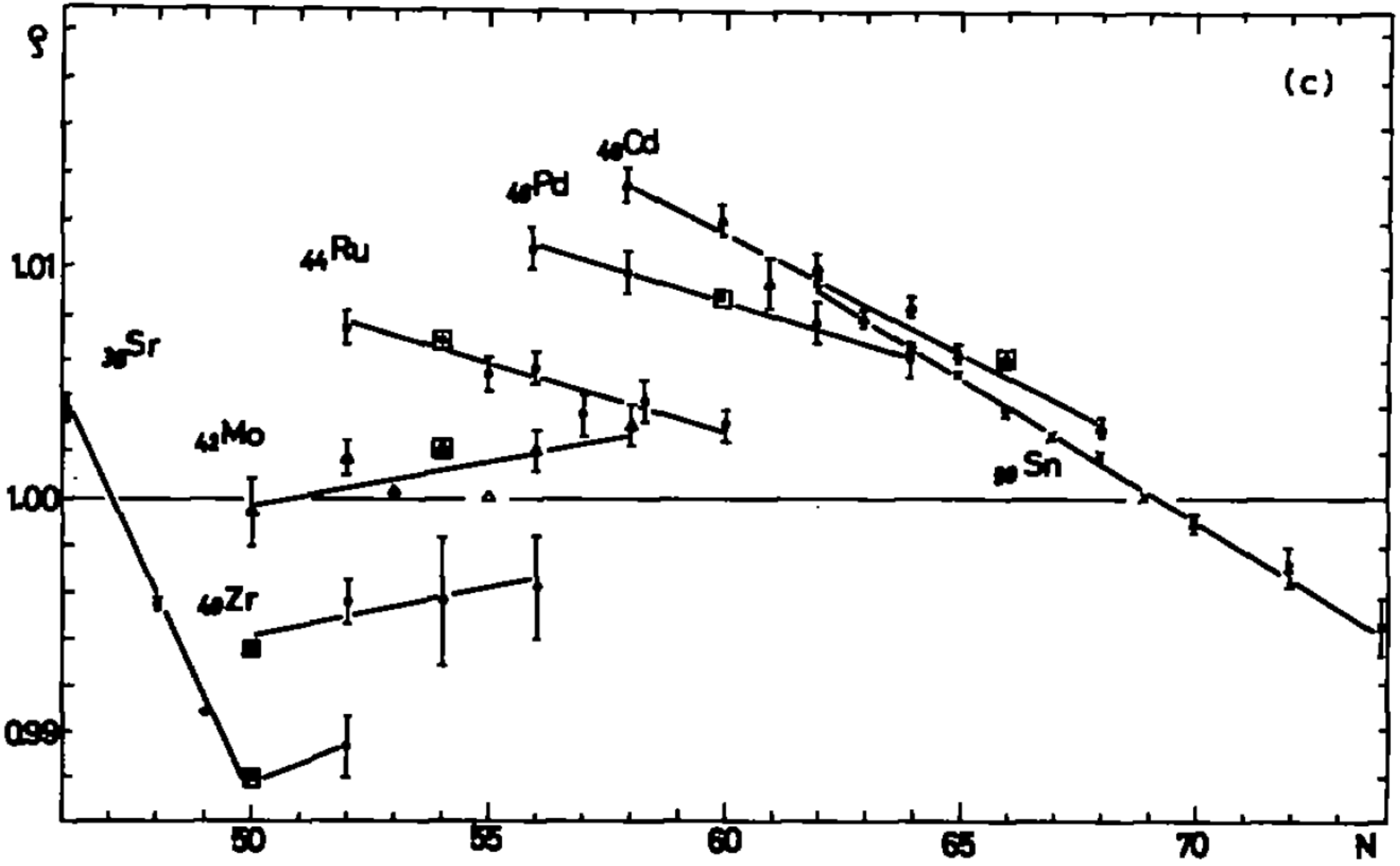
What is the physics behind these data?

$\rho = R/[(r_0 + r_1/A^{2/3} + r_2/A^{4/3}).A^{1/3}]$ liquid drop + surface thickness

(*Elton*)

N = 50:

Shell effect!

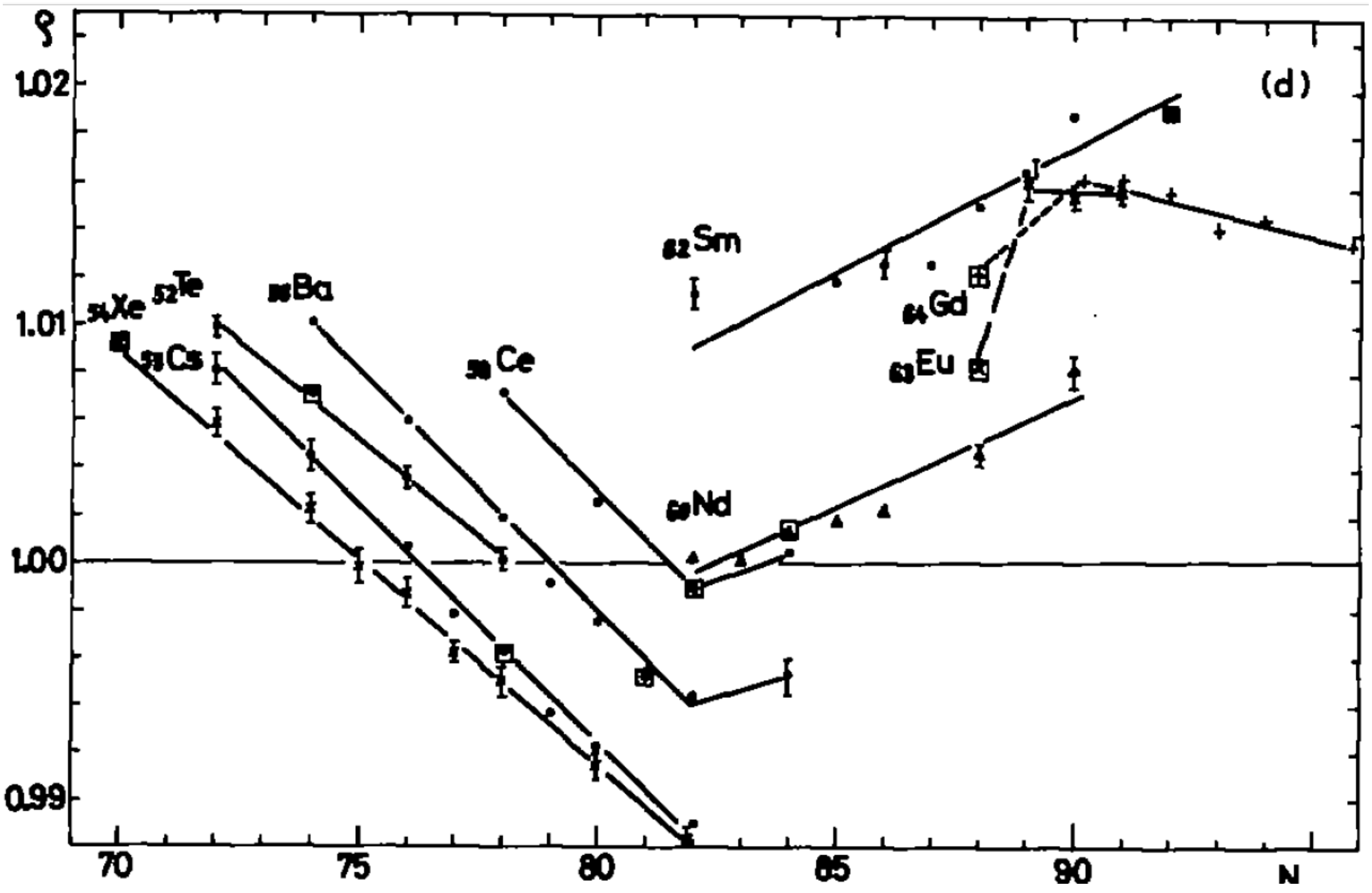


N = 82:

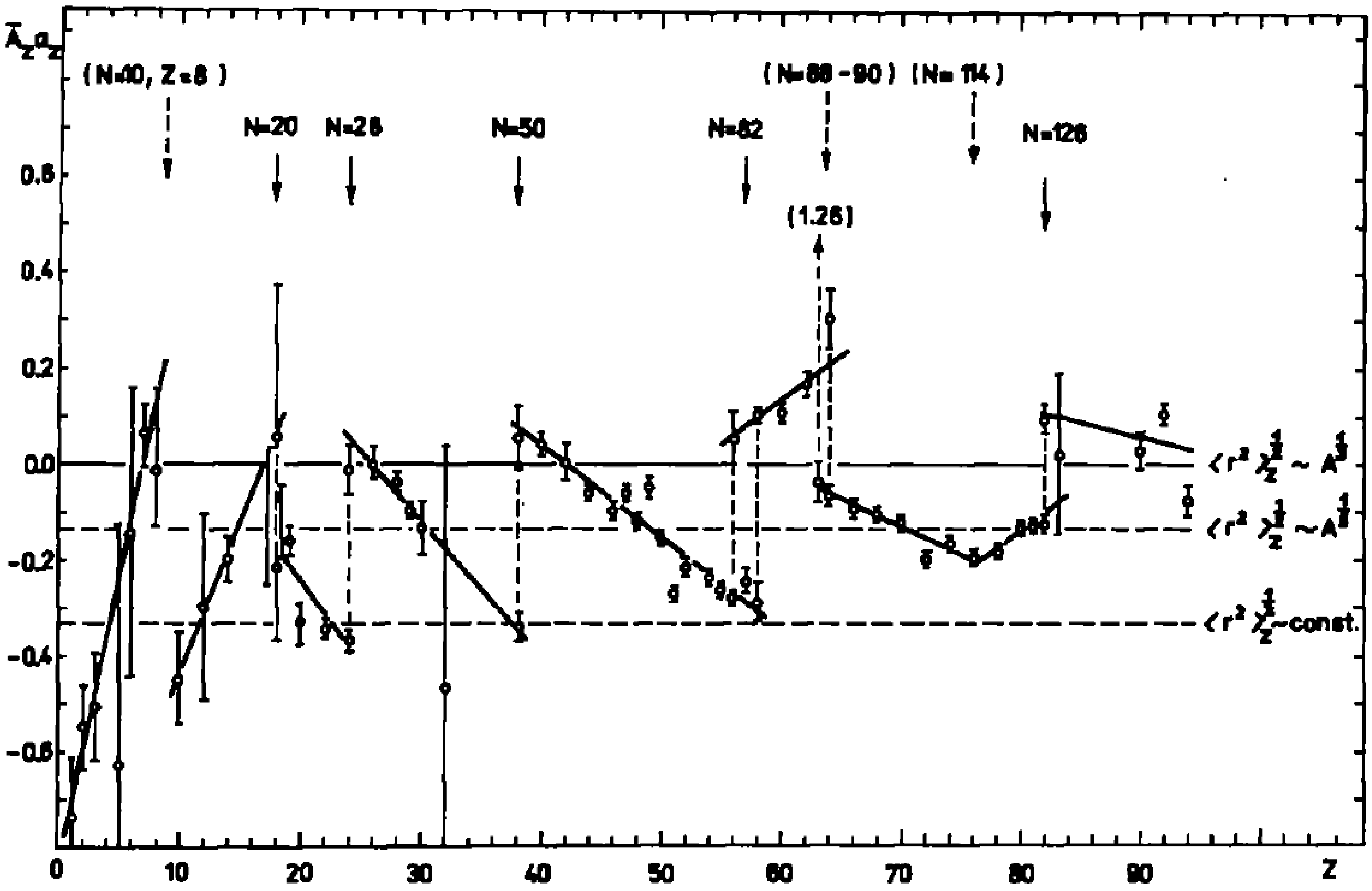
Shell effect!

N ~ 90:

Deformation effect!



a_Z: slope of the lines for the element. **A_Z**: Average mass number



a_Z: slope of the lines for the element. **A_Z**: Average mass number

1977: (Nucl. Phys. **A288**, 480) :

Isotopic series: shell effects, deformation effects,

odd-even effects ~ 6×10^{-4}.

Data?

1978: (ATOMKI Közlemények = Reports, **20**, 1)

Isotonic, isobaric and iso-symmetric series: effects

+ Appendix: r**ms charge data table**!

Table I. (1978)

rms charge radii and their normalized values ρ

Element	Mass number	rms radius (fm)	Normalized rms radius ρ	References
$_1H$	1	0.810±0.009	0.5975±0.0066	2,13,17,18,44
	2	2.057±0.046	1.1660±0.0261	2,13,19
	3	1.700±0.051	0.8756±0.0263	2,13
$_2He$	3	1.869±0.020	0.9626±0.0103	2,13,45
	4	1.663±0.013	0.8073±0.0063	2,20,21,22,23
$_3Li$	6	2.505±0.020	1.1228±0.0090	2,13,14,23,24,25
	7	2.405±0.020	1.0456±0.0087	2,13,14,23,26
$_4Be$	9	2.512±0.012	1.0380±0.0050	2,13,14,22,23,25,26
$_5B$	10	2.461±0.100	0.9950±0.0404	2,14
	11	2.388±0.042	0.9463±0.0166	2,13,14,25,27,28

Comparison to theory: normalized *rms* radii (ρ)

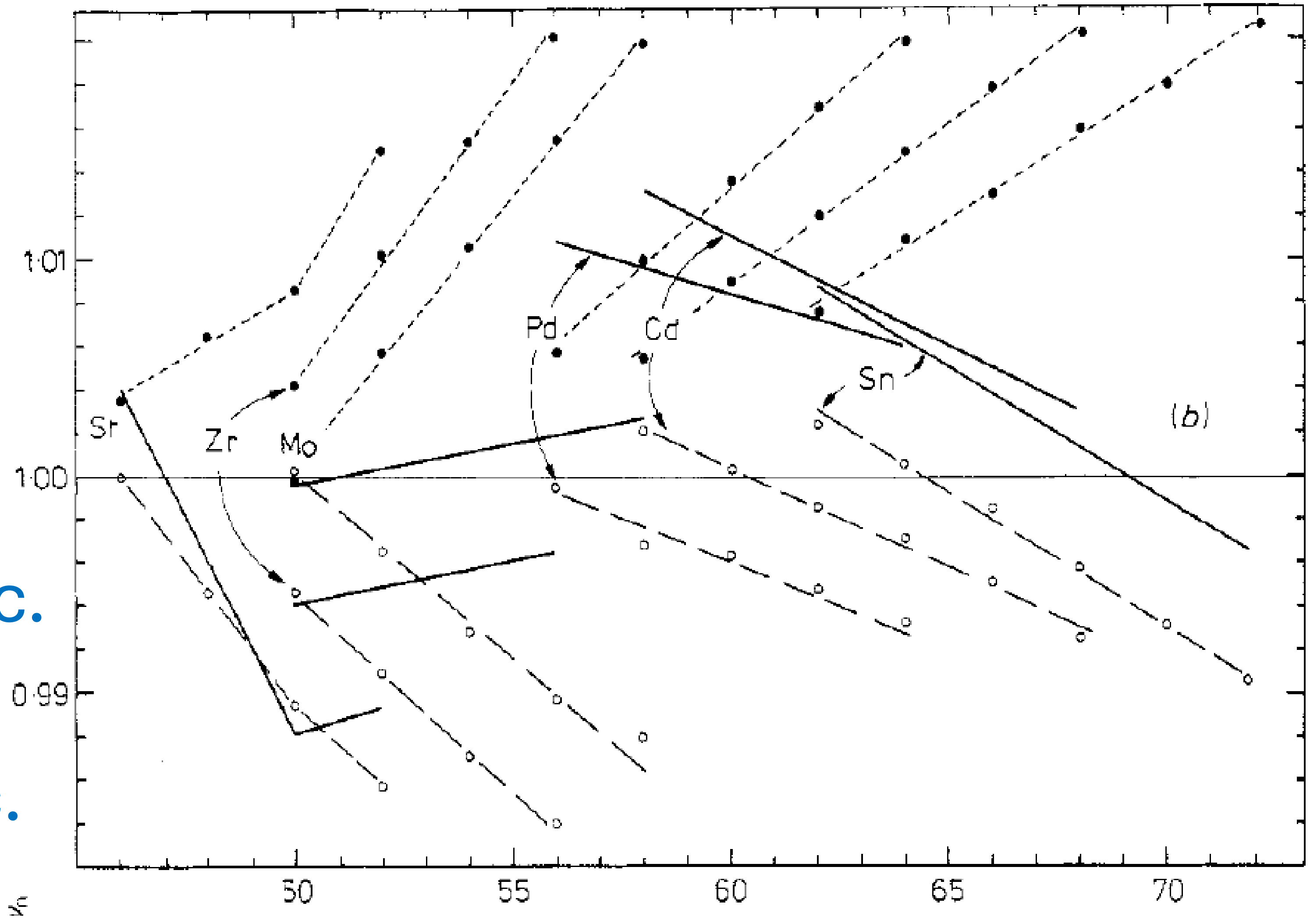
(1980: Journ. Phys. **G6**, 303)

Lombard, et al., Orsay

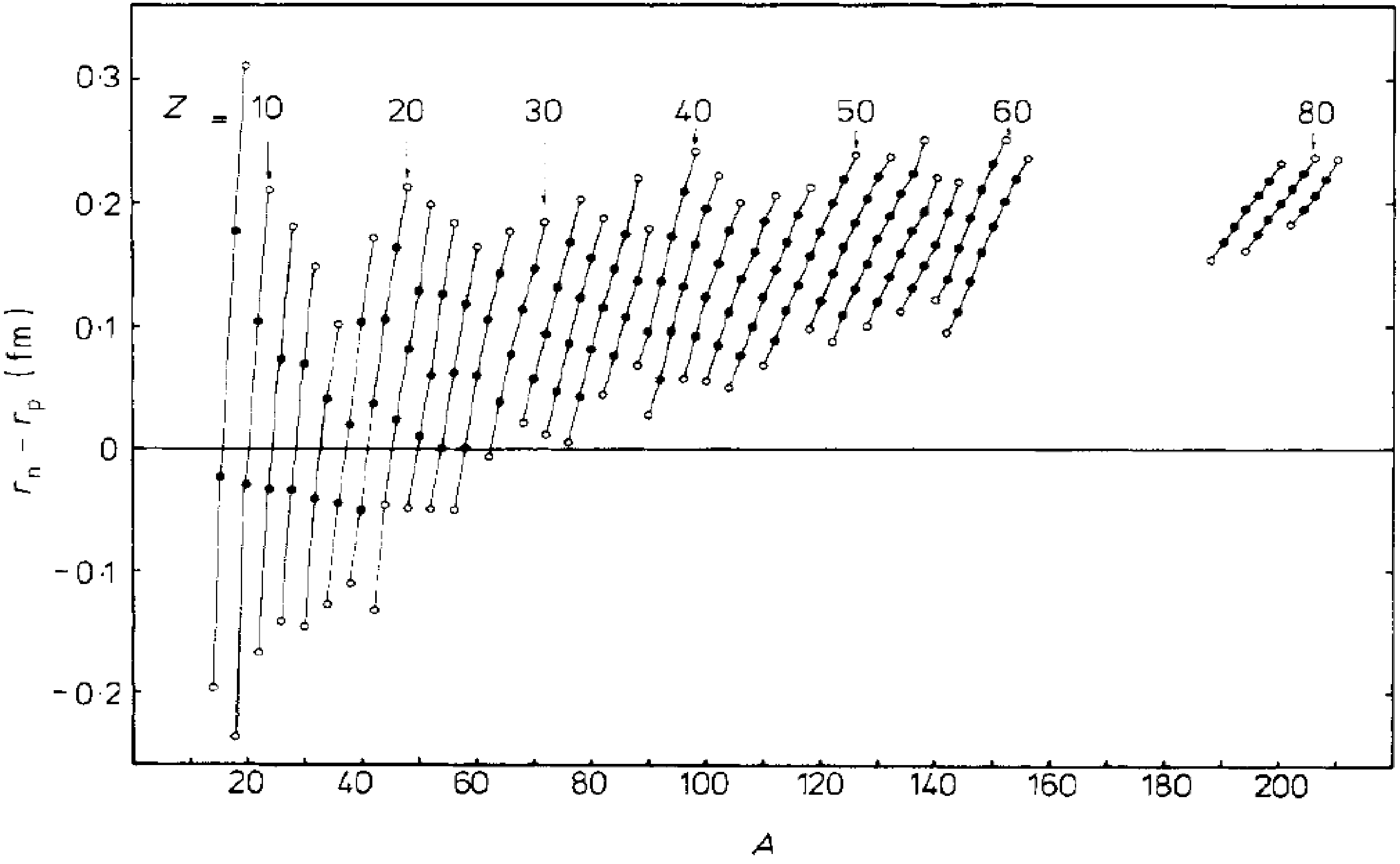
Full lines: ~ experiment.

Open circles: charge calc.

Full circles: neutron calc.



Neutron skins calculated



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1987: At. Data Nucl. Data T. **36**, 495: DeVries: Electron scattering

At. Data Nucl. Data T. **37**, 455: Aufmuth: Optical Isotope Shifts

Data sources!

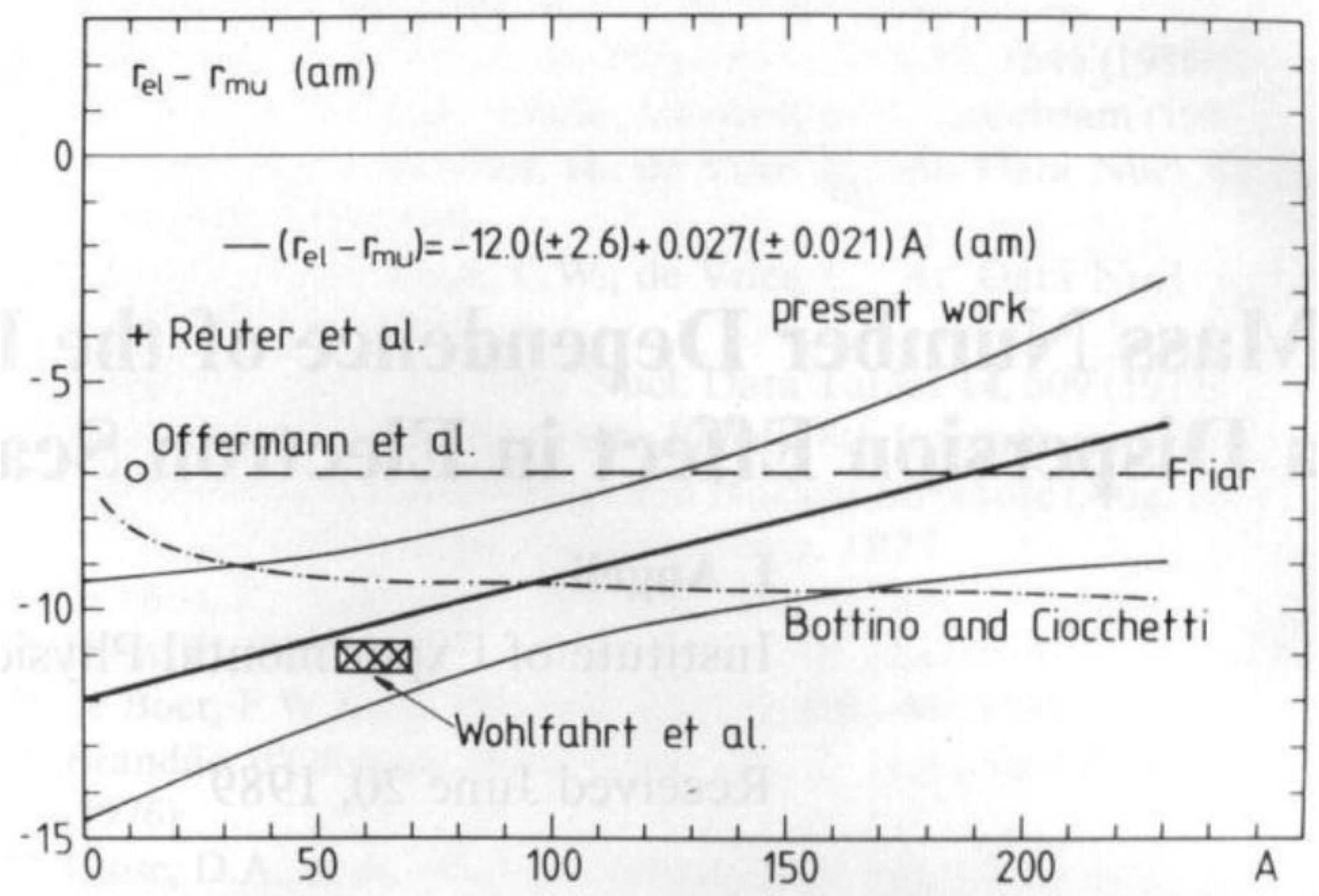
1989: Do r_{el} and r_{mu} measure the same quantity?

Difference ($r_{el} - r_{mu}$):

85 nuclei, mean: **-9.3(1.5) am**.

R(A) ≈ -12 + 0.03×A fm.

Negative, decreasing with **A**



(Zeitschrift für Physik, **A334**, 377)

1991: Evaluation procedure for nuclear *rms* charge radii (Acta Phys. Hung. **69**, 233)

Search for discrepant data: „*outliers*".

Different data screening and averaging procedures tested.

How to form a weighted mean and its uncertainty for a data group?

Conclusion: - „Final results are less sensitive to the actual procedure of evaluation than to **the selection of input data.**"

(*Cohen, Taylor*).

Illustrative example from the input file
RMS charge radii (fm). Updated: 01–Jul–90

1 H 1	17 el	0 mu			
0.7400	.2400	Ho55	el	X	Included in Ho56. Old
0.7700	.1000	Ch56	el	X	Included in Ho56. Old
0.7700	.1000	Ho56	el	X	Contains Ho55, Ch56, MA56. Old
0.8000	.0400	Ho58	el	X	Old
0.7500	.0500	Bu61	el	X	Included in Th72
0.8400	.0400	Le62	el	X	Included in Th72
0.8200	.0400	Be63	el	X	Outlier
0.8600	.0300	Du63	el	X	Included in Si80
0.8000	.0250	Fr66	el	X	Included in Th72
0.8100	.0200	Ak72	el	X	Outlier
0.8500	.0200	Th72	el	X	Analysis of Bu61, Le62, and Fr66
0.9000	.0300	Bo74	el	X	Included in Bo75a, Si80
0.8100	.0400	Mu74	el	X	Included in Bo75a, Si80
0.8400	.0200	Bo75	el	X	Included in Bo75a, Si80
0.8700	.0200	Bo75a	el	X	Analysis of Du63, Bo74, Mu74, Bo75
0.8400	.0500	An77	el		From Lamb shift!
0.8620	.0120	Si80	el		Contains Du63, Bo74, Mu74, Bo75

Table II. 1991

(Acta Phys. Hung. **69**, 233)

"1990-best-values" of RMS charge radii
Input data updated: 01–Jul–90

Z	El	A	R	dR
0	n	1	-0.3475	.0063
1	H	1	0.8608	.0115
		2	2.1168	.0058
		3	1.7723	.0401
2	He	3	1.9419	.0438
		4	1.6733	.0010
3	Li	6	2.5741	.0440
		7	2.4221	.1000
4	Be	9	2.5304	.0122
5	B	10	2.4315	.0491
		11	2.4171	.0240
6	C	12	2.4826	.0015
		13	2.4635	.0035
		14	2.5122	.0213

1994: At. Data Nucl. Data Tables, **56**, 133: Nadjakov: Systematics ...

1995: At. Data Nucl. Data Tables, **60**, 177: Fricke: Tables ...

1995: RETIREMENT

IAEA(NDS) Research Contract

Thank you *Hans Lemmel!*

1998: Table III. (Acta Phys. Hung. New Series, **8**, 23)

Comparison. Refined method (**FOR**TRAN). Simple method (**EXC**EL)

$$\frac{|R_{FOR} - R_{EXC}|}{dR_{F+E}}$$

91% within ±½ combined error

Z	El	A	R_{FOR}	dR_{FOR}	R_{EXC}	dR_{EXC}	Rel. diff.	R_{F+E}	dR_{F+E}
0	n	1	−0.1201	0.0034	−0.1186	0.0041	0	−0.1194	0.0038
1	H	1	0.8521	0.0069	0.8520	0.0070	0	0.8521	0.0070
		2	2.1352	0.0064	2.1352	0.0065	0	2.1352	0.0064
		3	1.7591	0.0356	1.7591	0.0363	0	1.7591	0.0359
2	He	3	1.9373	0.0296	1.9408	0.0240	0	1.9390	0.0268
		4	1.6758	0.0026	1.6757	0.0028	0	1.6758	0.0027
3	Li	6	2.5521	0.0311	2.5522	0.0333	0	2.5522	0.0322
		7	2.3952	0.0506	2.3953	0.0514	0	2.3952	0.0510
4	Be	9	2.5180	0.0114	2.5180	0.0119	0	2.5180	0.0116
5	B	10	2.4278	0.0492	2.4277	0.0499	0	2.4277	0.0496
		11	2.4059	0.0291	2.4060	0.0294	0	2.4059	0.0293
6	C	12	2.4704	0.0023	2.4705	0.0023	0	2.4704	0.0023
		13	2.4625	0.0036	2.4625	0.0037	0	2.4625	0.0037
		14	2.4978	0.0126	2.4966	0.0165	0	2.4972	0.0146
7	N	14	2.5519	0.0083	2.5520	0.0087	0	2.5520	0.0085
		15	2.6094	0.0085	2.6095	0.0094	0	2.6094	0.0089
8	O	16	2.7061	0.0084	2.6995	0.0068	0.5	2.7028	0.0076

1999: **Table IV.** (IAEANDS_indc-hun-0033)

Similar to **III.** but:

Data search updated to May 1999.

Contains full input data tables, and background materials.

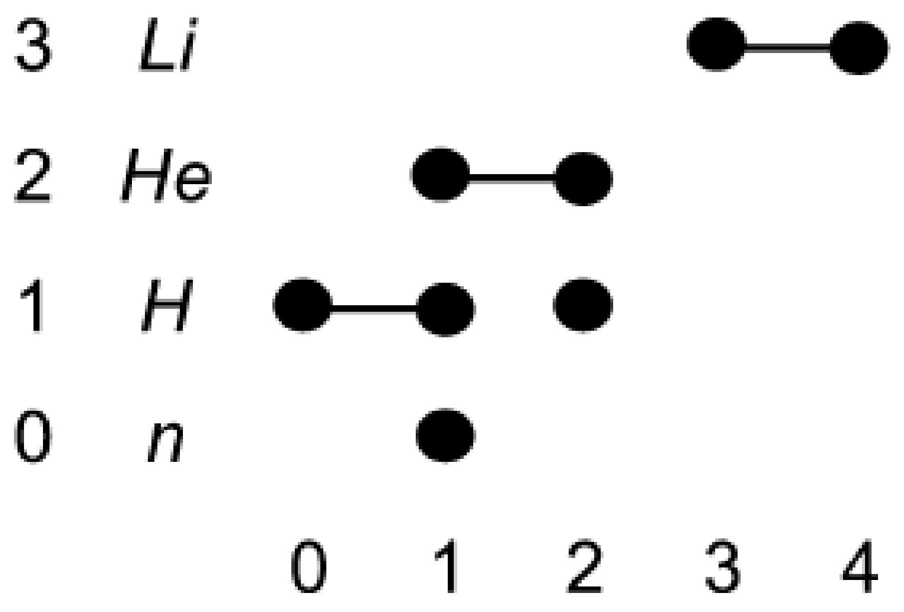
Presented also in electronic files.

Chapter with easy-to-use formulae added.

Table V. 2004: Consistent set of nuclear rms charge radii

(Atomic Data and Nuclear Data Tables, **87**, 185)

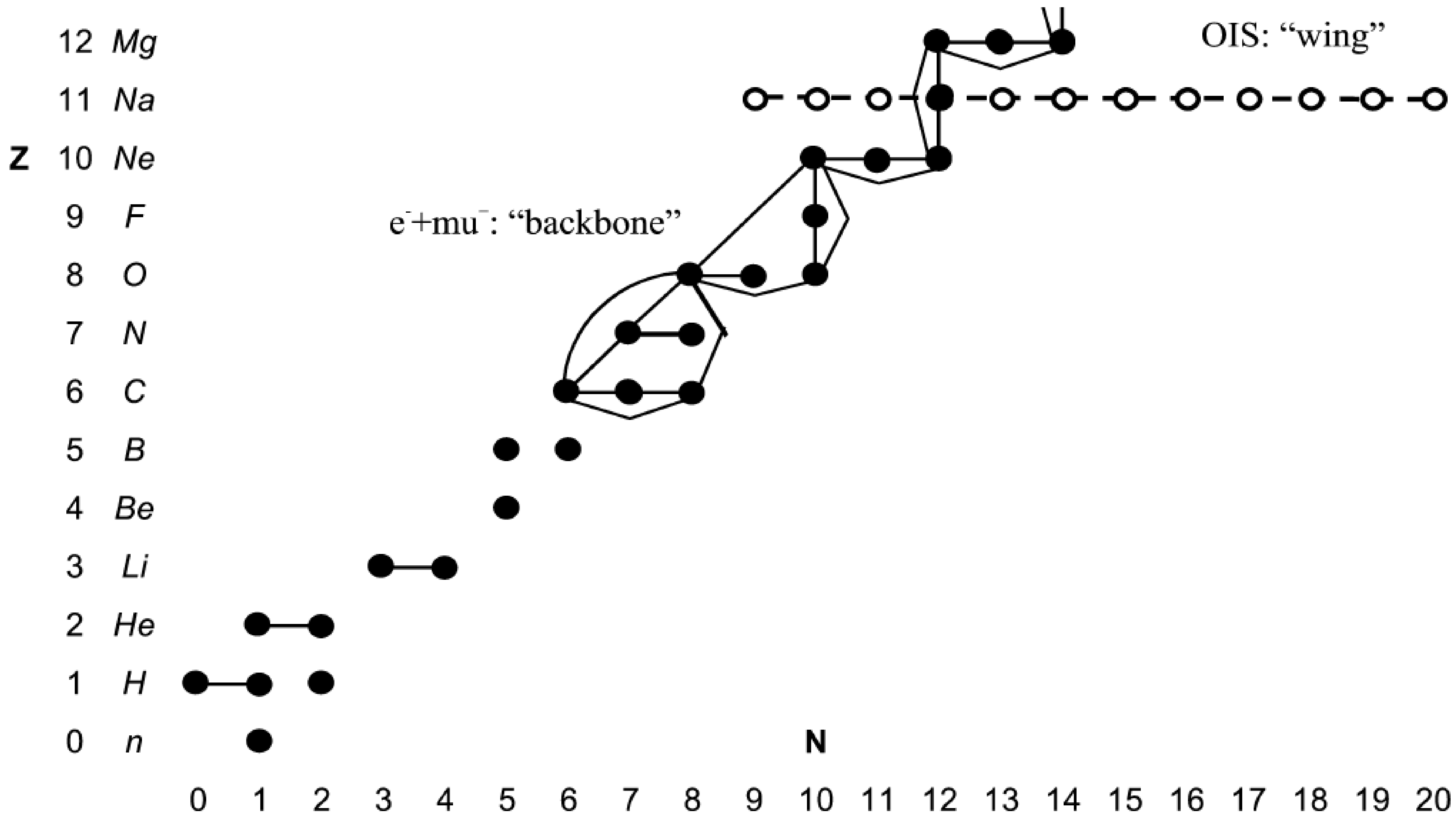
Measured differences used for constraint by least squares;



Redundancy improves accuracy:

Output errors are less than input!

The least precise value benefits
most of the constraint.



OIS: "wing"
e⁻+mu⁻: "backbone"
Z
12 Mg
11 Na
10 Ne
9 F
8 O
7 N
6 C
5 B
4 Be
3 Li
2 He
1 H
0 n
N
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

References for the Tables

Differences between radii of neighboring nuclei (not isotopes) from electron scattering and muonic atom X-rays

Fr92	G.Fricke, et al.: *Phys. Rev.* **C45** (1992) 80, Table III	O, F, Ne, Na, Mg, Al, Si
Fr95	G.Fricke, et al.: *At. Data Nucl. Data Tables*, **60** (1995) 177, Table VII	Many; compilation
He89	J.Herberz: *Ph.D. thesis*, *Univ. Mainz*, KPH **6/89** (1989)	O, F, Na, Ne, Mg, Al, Si
Vr87	H.de Vries, et al.: *At. Data Nucl. Data Tables*, **36** (1987) 495, Table III	Many; compilation.
Wo80	H.D.Wohlfahrt, et al.: *Phys. Rev.* **C22** (1980) 264	Ti, Cu Zn
Wo81	H.D.Wohlfahrt, et al.: *Phys. Rev.* **C23** (1981) 533, Table VI	K,Ca,Sc,Ti,V,Cr,Mn,Fe

Differences between isotopes from optical isotope shifts

Ah85	S.A.Ahmad, et al.: *Z. Physik*, **A321** (1985) 35	Eu
Ah88	S.A.Ahmad, et al.: *Nucl. Phys.*, **A483** (1988) 244	Ra
Al79	E.Alvarez, et al.: *Physica Scripta*, **20** (1979) 141	Xe
Al83	G.D.Alhazov, et al.: *Zhurn.Exp.Teor. Fiz. Letters*, **37** (1983) 231	Eu
Al85	G.D.Alhazov, et al.: *Izv. Ak. Nauk SSSR*, *Ser. Fiz.*, **49** (1985) 24	Sm, Eu
Al85a	G.D.Alhazov, et al.: *Tez. Dok. XXXV. Sov. Leningrad* (1985)	Tm
Al86a	G.D.Alhazov, et al.: *Yadernaya Fizika*, **44** (1986) 1134	Sm, Eu
Al87	G.D.Alhazov, et al.: *Tez. Dokl. XXXVII. Soveshch.* (1987) 96	Nd, Sm
Al88	G.D.Alkhazov, et al.: *Nucl. Phys.*, **A477** (1988) 37	Tm
Al88a	G.D.Alkhazov, et al.: *Pisma v Zs.E.T.F.*, **48** (1988) 373	Gd
Al89	G.D.Alkhazov, et al.: *Nucl. Phys.*, **A504** (1989) 549	Ho
Al90	G.D.Alkhazov, et al.: *Z. Phys.*, **A337** (1990) 367	Tb
Al90a	G.D.Alkhazov, et al.: *Z. Phys.*, **A337** (1990) 257	Eu
An82	A.Andl, et al.: *Phys. Rev.*, **C26** (1982) 2194	Ca

2005: Nobel Peace Prize!

International Atomic Energy Agency

and

Mohamed ElBaradei

Congratulations!

2007: Moments of the 2p-Fermi charge distribution

(Acta Phys. Deb., **41**, 59)

Moments $\langle r^m \rangle$ and isotopic differences $\delta\langle r^m \rangle$ for $m = 1$ to 10

Diffusity $\boldsymbol{a}$ assumed to be constant.

$\boldsymbol{\delta\langle r^m \rangle}$ for even $\boldsymbol{m}$ are given in terms of $\boldsymbol{\delta\langle r^2 \rangle}$

Useful parameter: $\beta = \pi.a/c$ introduced

<r^m>

$$\langle r^m\rangle = a^m \frac{F_{m+2}(k)}{F_2(k)} = \frac{3}{m+3} c^m \times$$

$$\Bigg\{1 + \left[\frac{(m+3)(m+2)}{3!} - 1\right]\beta^2 + \left[\frac{7}{3}\frac{(m+3)(m+2)(m+1)m}{5!} - \right.$$

$$\left.\frac{(m+3)(m+2)}{3!} + 1\right]\beta^4 + \left[\frac{31}{3}\frac{(m+3)(m+2)\ldots(m-1)(m-2)}{7!}\right.$$

$$\left. - \frac{7}{3}\frac{(m+3)(m+2)(m+1)m}{5!} + \frac{(m+3)(m+2)}{3!} - 1\right]\beta^6$$

$$+ \left[\frac{381}{5}\frac{(m+3)\ldots(m-3)(m-4)}{9!} - \frac{31}{3}\frac{(m+3)\ldots(m-2)}{7!}\right.$$

$$\left. + \frac{7}{3}\frac{(m+3)\ldots m}{5!} - \frac{(m+3)(m+2)}{3!} + 1\right]\beta^8$$

$$+ \left[\frac{2555}{3}\frac{(m+3)\ldots(m-5)(m-6)}{11!}\right.$$

$$\left. - \left(\frac{381}{5}\frac{(m+3)\ldots(m-3)(m-4)}{9!} - \ldots - 1\right)\right]\beta^{10}$$

$$+ \left[\frac{1414477}{105}\frac{(m+3)\ldots(m-7)(m-8)}{13!}\right.$$

$$\left. - \left(\frac{2555}{3}\frac{(m+3)\ldots(m-5)(m-6)}{11!} + \ldots + 1\right)\right]\beta^{12} + \ldots\Bigg\}$$

Even *m*

$$\langle r^2 \rangle = \frac{3}{5}c^2\left(1+\frac{7}{3}\beta^2\right) = \frac{3}{5}c^2 + \frac{7}{5}(\pi a)^2$$

$$\langle r^4 \rangle = \frac{3}{7}c^4\left(1+6\beta^2+\frac{31}{3}\beta^4\right)$$

$$\langle r^6 \rangle = \frac{1}{3}c^6\left(1+11\beta^2+\frac{239}{5}\beta^4+\frac{381}{5}\beta^6\right)$$

$$\langle r^8 \rangle = \frac{3}{11}c^8\left(1+\frac{52}{3}\beta^2+\frac{410}{3}\beta^4+\frac{1636}{3}\beta^6+\frac{2555}{3}\beta^8\right)$$

$$\langle r^{10} \rangle = \frac{3}{13}c^{10}\left(1+25\beta^2+\frac{926}{3}\beta^4+\frac{46714}{21}\beta^6+\frac{910573}{210}\beta^8 + \frac{19447}{210}\beta^{10}\right)$$

Odd m

$$
\begin{aligned}
\langle r \rangle &= \frac{3}{4}c\left[1+\beta^2-\frac{8}{15}\beta^4(1-\beta^2+\beta^4-\beta^6+\beta^8-\ldots)\right] \\
&= \frac{3}{4}c\left[1+\beta^2-\frac{8}{15}\frac{\beta^4}{1+\beta^2}\right] \\
\langle r^3 \rangle &= \frac{1}{2}c^3\left[1+4\beta^2+3\beta^4-\frac{32}{21}\frac{\beta^6}{1+\beta^2}\right] \\
\langle r^5 \rangle &= \frac{3}{8}c^5\left[1+\frac{25}{3}\beta^2+\frac{73}{3}\beta^4+17\beta^6-\frac{128}{15}\frac{\beta^8}{1+\beta^2}\right] \\
\langle r^7 \rangle &= \frac{3}{10}c^7\left[1+14\beta^2+84\beta^4+226\beta^6+155\beta^8-\frac{2560}{33}\frac{\beta^{10}}{1+\beta^2}\right] \\
\langle r^9 \rangle &= \frac{1}{4}c^9\left[1+21\beta^2+210\beta^4+1154\beta^6+3037\beta^8+2073\beta^8\right. \\
&\qquad \left.-\frac{1415168}{1365}\frac{\beta^{12}}{1+\beta^2}\right] \qquad (20)
\end{aligned}
$$

δ<r^m>

In

δ<r^2>

$$\delta\langle r^4\rangle = \frac{25}{14}\frac{A_1+A_2}{A_2-A_1}\left(\delta\langle r^2\rangle\right)^2 + \frac{30}{7}(\pi a)^2\delta\langle r^2\rangle$$

$$\delta\langle r^6\rangle = \frac{125}{48}\left(\frac{A_1+A_2}{A_2-A_1}\right)^2\left(\delta\langle r^2\rangle\right)^3 + \frac{275}{18}(\pi a)^2\frac{A_1+A_2}{A_2-A_1}\left(\delta\langle r^2\rangle\right)^2 + \frac{239}{9}(\pi a)^4\delta\langle r^2\rangle$$

$$\delta\langle r^8\rangle = \frac{625}{176}\left(\frac{A_1+A_2}{A_2-A_1}\right)^3\left(\delta\langle r^2\rangle\right)^4 + \frac{1625}{44}(\pi a)^2\left(\frac{A_1+A_2}{A_2-A_1}\right)^2\left(\delta\langle r^2\rangle\right)^3 + \frac{5125}{33}(\pi a)^4\frac{A_1+A_2}{A_2-A_1}\left(\delta\langle r^2\rangle\right)^2 + \frac{8180}{33}(\pi a)^6\delta\langle r^2\rangle$$

$$\delta\langle r^{10}\rangle = \frac{15625}{3328}\left(\frac{A_1+A_2}{A_2-A_1}\right)^4\left(\delta\langle r^2\rangle\right)^5 + \frac{15625}{208}(\pi a)^2\left(\frac{A_1+A_2}{A_2-A_1}\right)^3\left(\delta\langle r^2\rangle\right)^4 + \frac{57875}{104}(\pi a)^4\left(\frac{A_1+A_2}{A_2-A_1}\right)^2\left(\delta\langle r^2\rangle\right)^3 + \frac{583925}{273}(\pi a)^6\frac{A_1+A_2}{A_2-A_1}\left(\delta\langle r^2\rangle\right)^2 + \frac{910573}{546}(\pi a)^8\delta\langle r^2\rangle$$

2010: Calculation of Fermi parameters from charge moments

(Acta Phys. Deb. **44**, 6)

Fricke: ADNDT, 60, (1995) 177. Table IX:

Experimental moments: $<r^2>^{1/2}$, $<r^4>^{1/4}$, $<r^6>^{1/6}$ for 20 nuclei.

Fermi parameters ***c*** and ***a*** calculated for 20 nuclei.

Uncertainty estimated.

Method and program described.

Table VI. 2013: Table of experimental nuclear ground state charge radii (Atomic Data and Nuclear Data Tables, **99**, 69)

Atomic Data and Nuclear Data Tables, **56** (1994) 133: Nadjakov, et al. [12]

Updated by K. Marinova

Atomic Data and Nuclear Data Tables, **87** (2004) 185: Angeli. [13]

Updated by I. Angeli

Combined: $R = R_{av} = 0.5\times(R[12] + R[13])$ Not independent!

$\Delta R_{tot} = \max(\Delta R[12], \Delta R[13], 0.5\times|R[12] - R[13]|)$

4% differ more than $1\times\Delta R_{tot}$ underlined in Table1.

2015: Correlaton of charge radii with other nuclear observables

(J. Phys. **G42**, 055108)

Isotonic series:

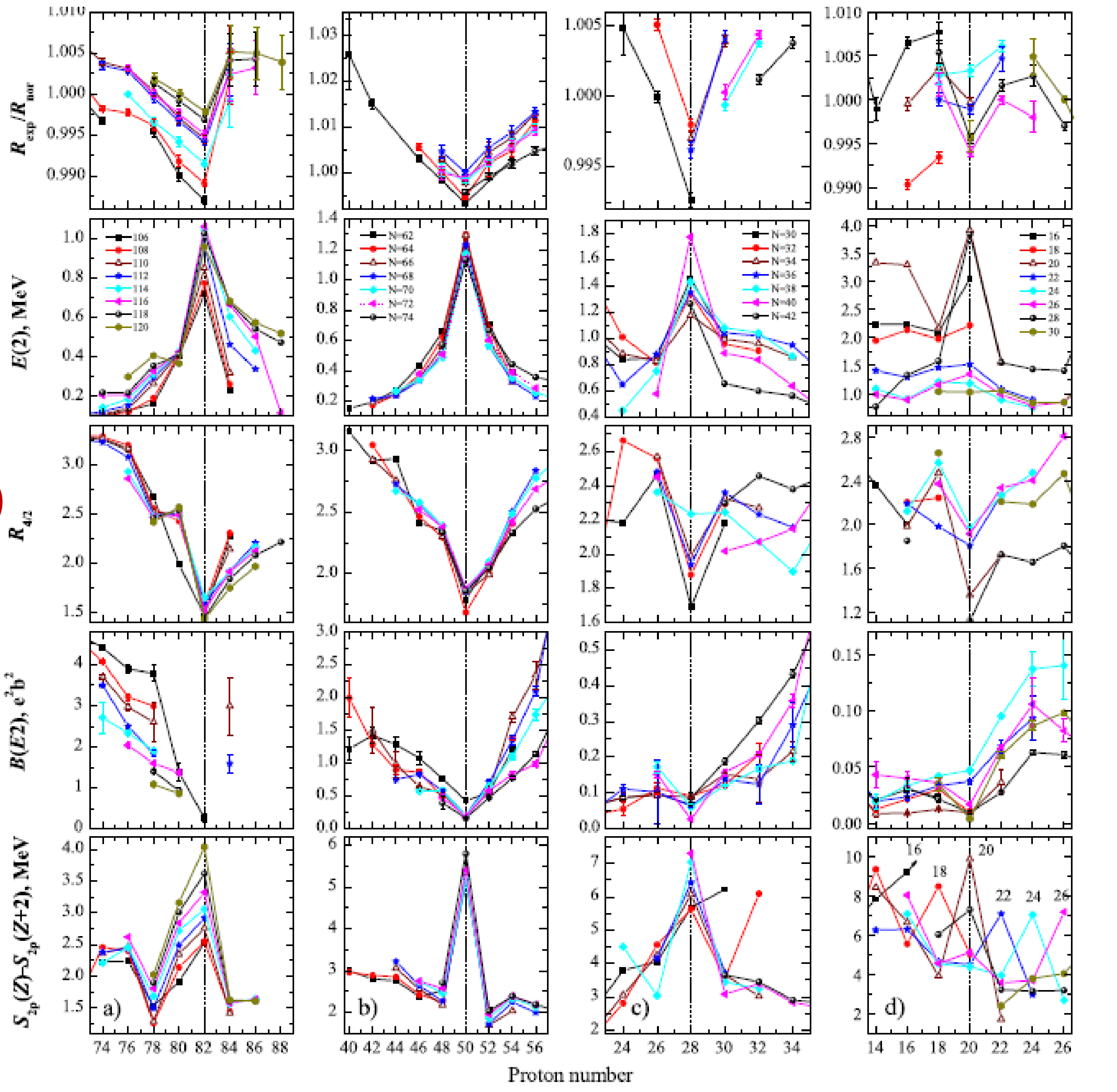
R_{exp}/R_{nor}, $R_{nor} \sim A^{1/2}$ (ADNDT 2004)

$E_1(2+)$

$R_{4/2} = E_1(4^+)/E_1(2^+)$

$B(E2)\uparrow$

$S_{2p}(Z) - S_{2p}(Z+2)$



THANK YOU, KRASSIMIRA!

2021:The quest for the proton charge radius

[p.31. in: Gribov-90 Memorial Volume, World Scientific]

(arXiv: 2103.17101v1 [physics.hist-ph] 29 Mar 2021)

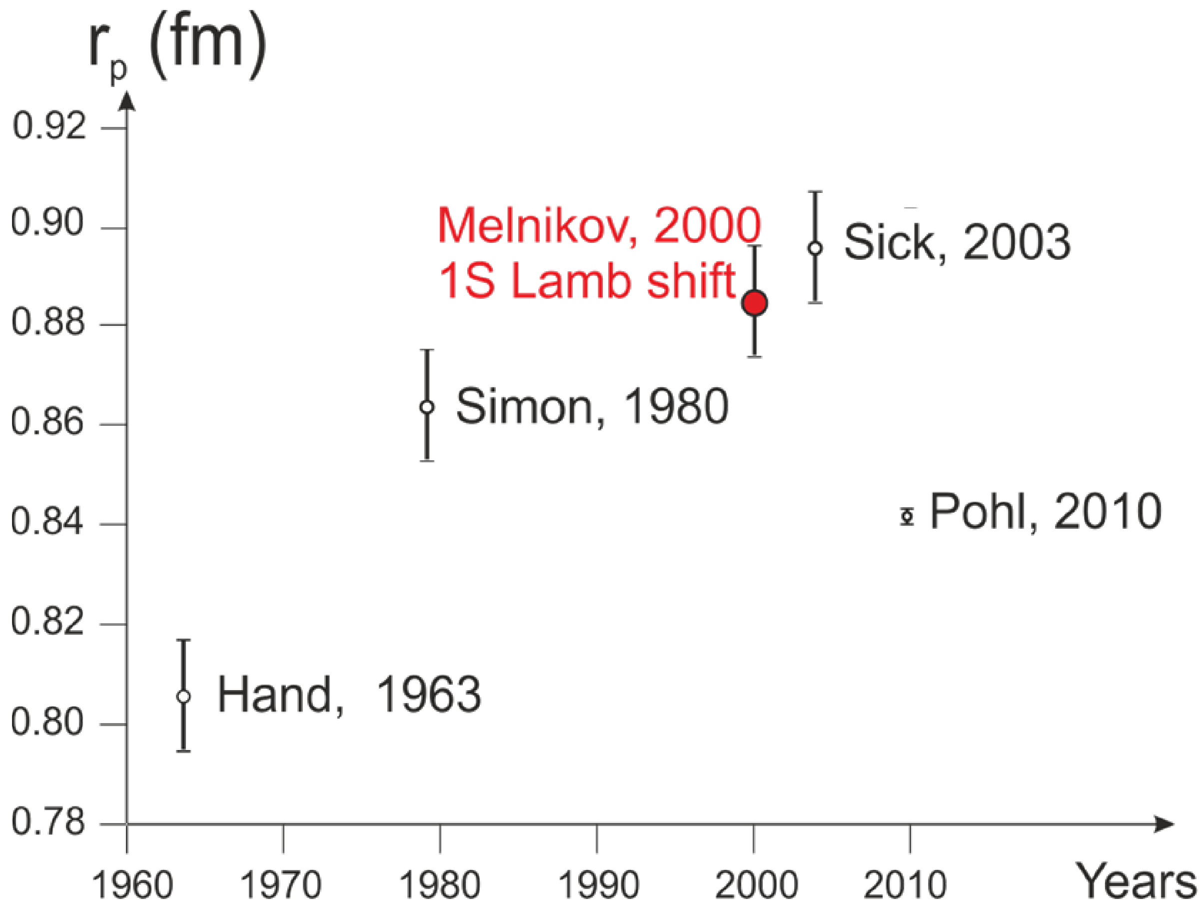
1963, Hand: Evaluation

1980, Simon: Electron scatt.

2000, Melnikov: 1S Lamb shift

2010: Pohl: Muonic H Lamb shift

„Proton radius puzzle!"



Present and future

1) *Personal: not able to continue!*

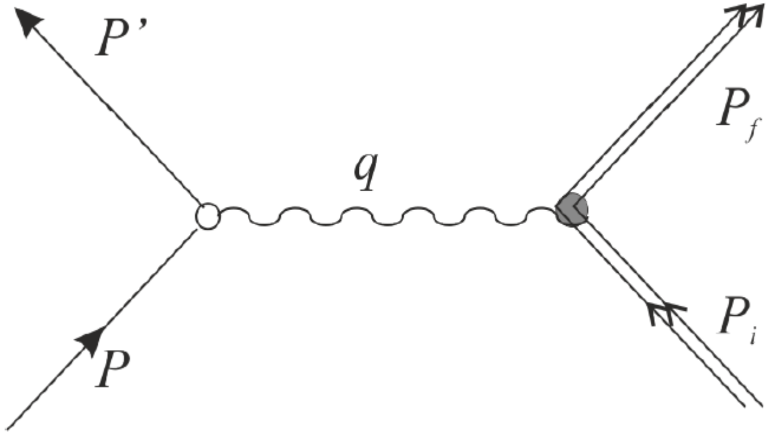
→ ***Endre TAKÁCS***

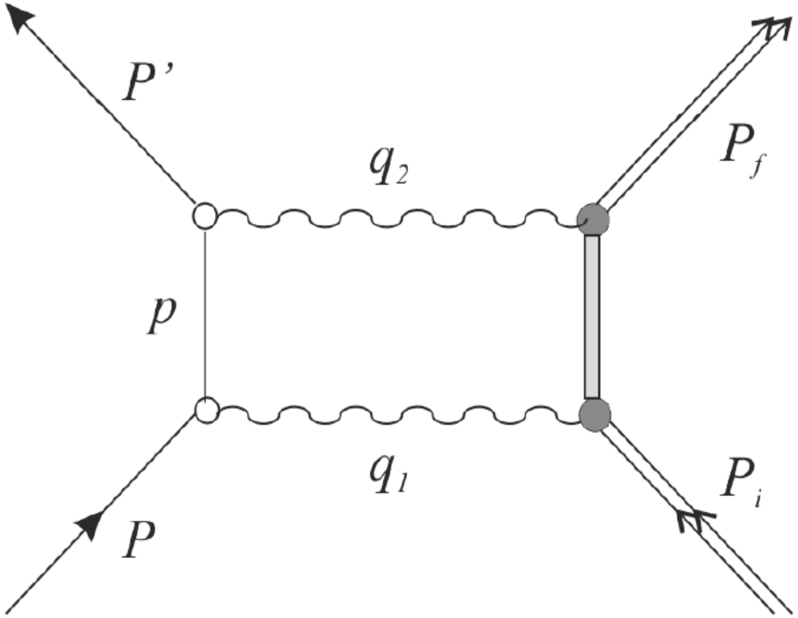
2) Problems

3) Future: neutron radii?

PROBLEMS

Dispersion correction?





1929_ProcRoySoc_124_1929_425_Mott_ElScatt	2023. 03. 26. 22:03	Adobe Acrobat do...	2 302 KB
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Theory ≈ 40 papers

1954, Lewis: *Thesis, Univ. Michigan* (unpublished)

1955, Schiff: *PhysRev.* **98** (1955) 756.

„Small but not negligible…"

1959, Krall: *PhysRev*. **115** (1959) 457.

„The change in $\mathbf{R_0}$ deduced from experiment when dispersive effects are included:

^{2}H: -0.01 fm. **^{4}He: -0.08 fm.** **^{12}C: -0.05 fm."**

1966, Bottino: *NuclPhys.* **89** (1966) 192.

„Dispersion effects of the **order of 1/Z** are found **at the diffraction minima** of the form factors …"

1971, Bottino: *PhysLett*. **34B** (1971) 187.

„The experimental determination of $<r^2>$ by means of a purely static model yields an **overestimate**. The corrections in percentage:

^{2}H: -1.5 %, ^{12}C: -1.0 %.”

1972, Bottino: *NuclPhys.* **A178** (1972) 593.

„The static interpretation leads to an **underestimate** of the radii. Dispersive effects **decrease roughly as *1/A***.”

^{2}H: 1.3 %, ^{4}He: 0.7 %, ^{12}C: 0.6 %, ^{40}Ca: 0.5 %.

1975, DeForest: *PhysLett.* **58B** (1975) 397.

„Dispersion corrections are presently the greatest theoretical obstacle to precision analysis of elastic electron scattering data.”

„These effects depend on the complete dynamical structure of the nucleus and **are not well understood** at the present time.”

1980: Friar:*Proc.Int.School, Ariccia,* (1979) 143.

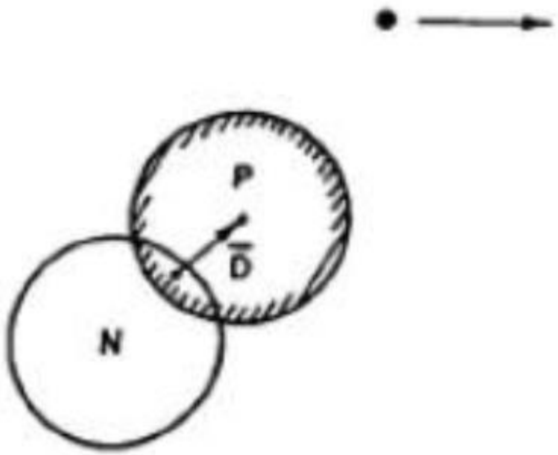
- „Two of the least understood and more complicated subjects in the field of electron scattering are the topics of **dispersion** and recoil corrections."

„Different formulations of the dispersion effect make comparison of calculated results very difficult."

„A **static nucleus** presents the same shape at each instance of time.”

(a)

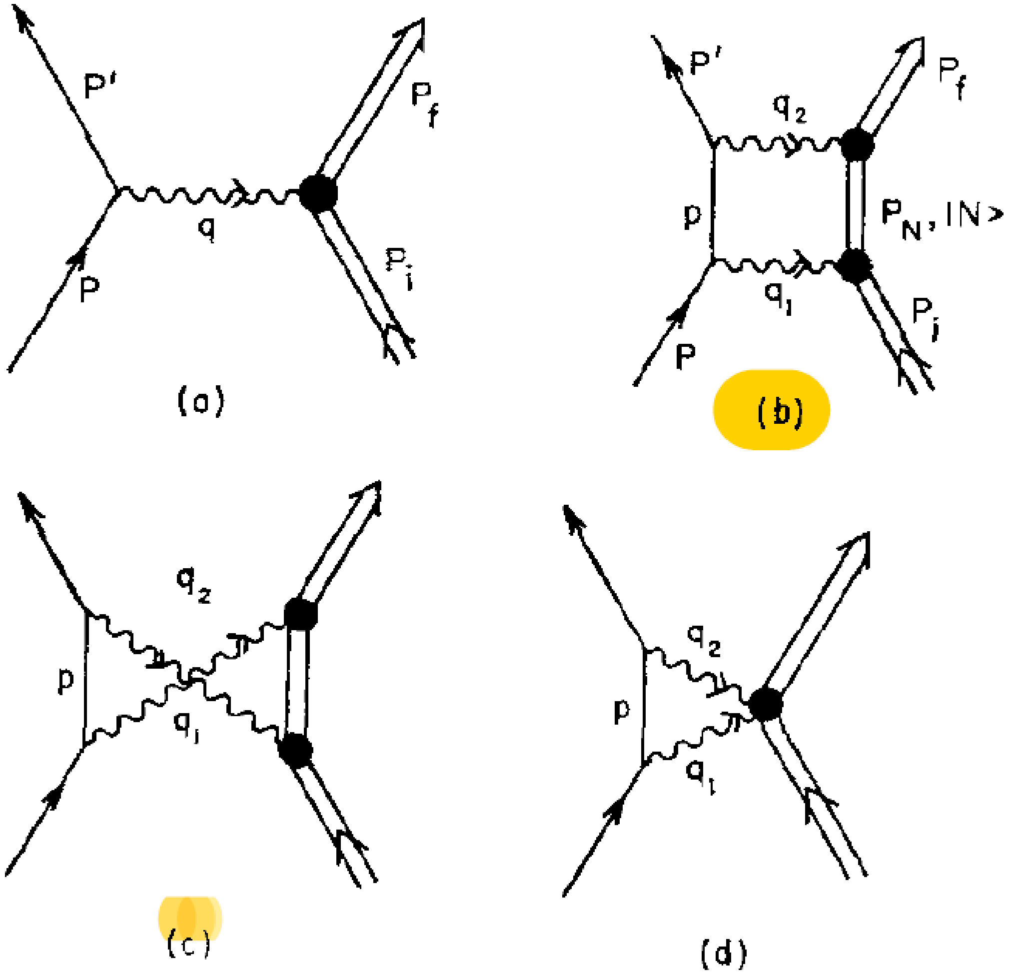
„The behavior of an actual nucleus is determined by the **induced dipole moment D**, and the **characteristic frequencies** of oscillations by **D**.”



[See papers on: „*Quasi-deuteron model*”]

Dispersion effects are attractive and react on the electron

These **1b and 1c contributions are just the dispersion corrections** to elastic scattering.



The fitted rms radius is then given in terms of
the „real" radius $<r^2>^{1/2}$ by the approximate relationship:

$$<r^2>^{1/2}_{eff} \approx <r^2>^{1/2} + \Delta r$$

For ^{16}O with approximations, an upper bound on the dispersion effect:

$$\Delta r = -7.10^{-3} \text{ fm.}$$

For ^{12}C where exact numerical calculations have been performed:

$$\Delta r = -(2\text{-}3).10^{-3} \text{ fm.}"$$

Experiment ≈ 30 papers

1969, Madsen: *PhysRev.* **17** (1969) 1122.

ANOMALOUS ELECTRON SCATTERING FROM Nd^{142}

„Dispersion corrections should be applied to our data."

1970, Sick: *NuclPhys.* **A150** (1970) 631.

„The presence of dispersion effects appears to be necessary to explain the cross sections of ^{12}C and ^{16}O in the sharp first diffraction minimum."

1982, Reuter: *PhysRev.* **C26** (1982) 806.

Theoretical results differ, - **even in the sign of the effect!**

Applying a crude estimate for dispersion corrections,

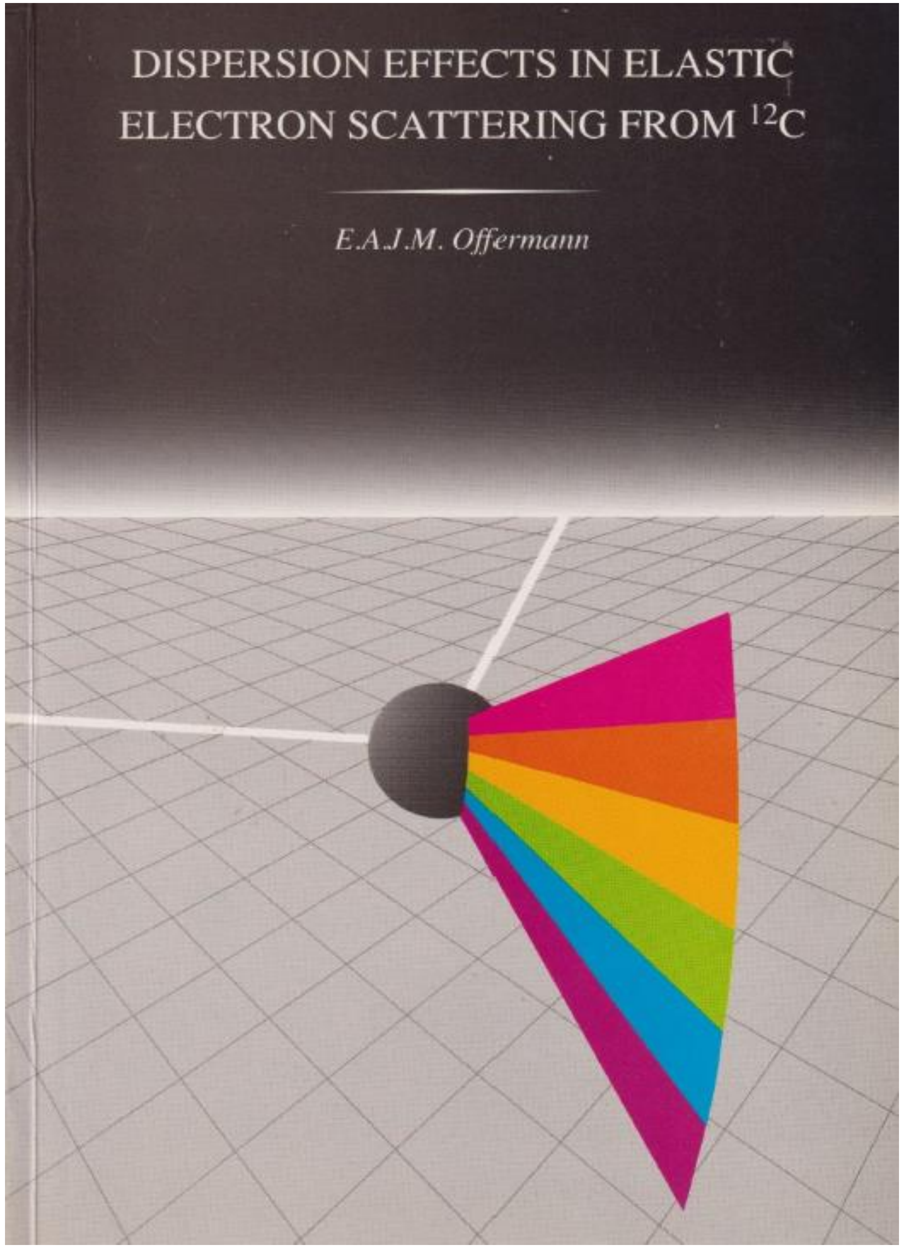
increases the rms radius of ^{12}C by 4 am.

1988, Offermann: *Thesis, University of Amsterdam* (1988).

„DISPERSION EFFECTS IN ELASTIC ELECTRON SCATTERING FROM *^{12}C*"

The dispersion corrections **increase** the *^{12}C* rms radius with:

+0.007 (+0.007/-0.011) fm.



1989: Mass Number Dependence of the Difference ($r_{el} - r_{mu}$).

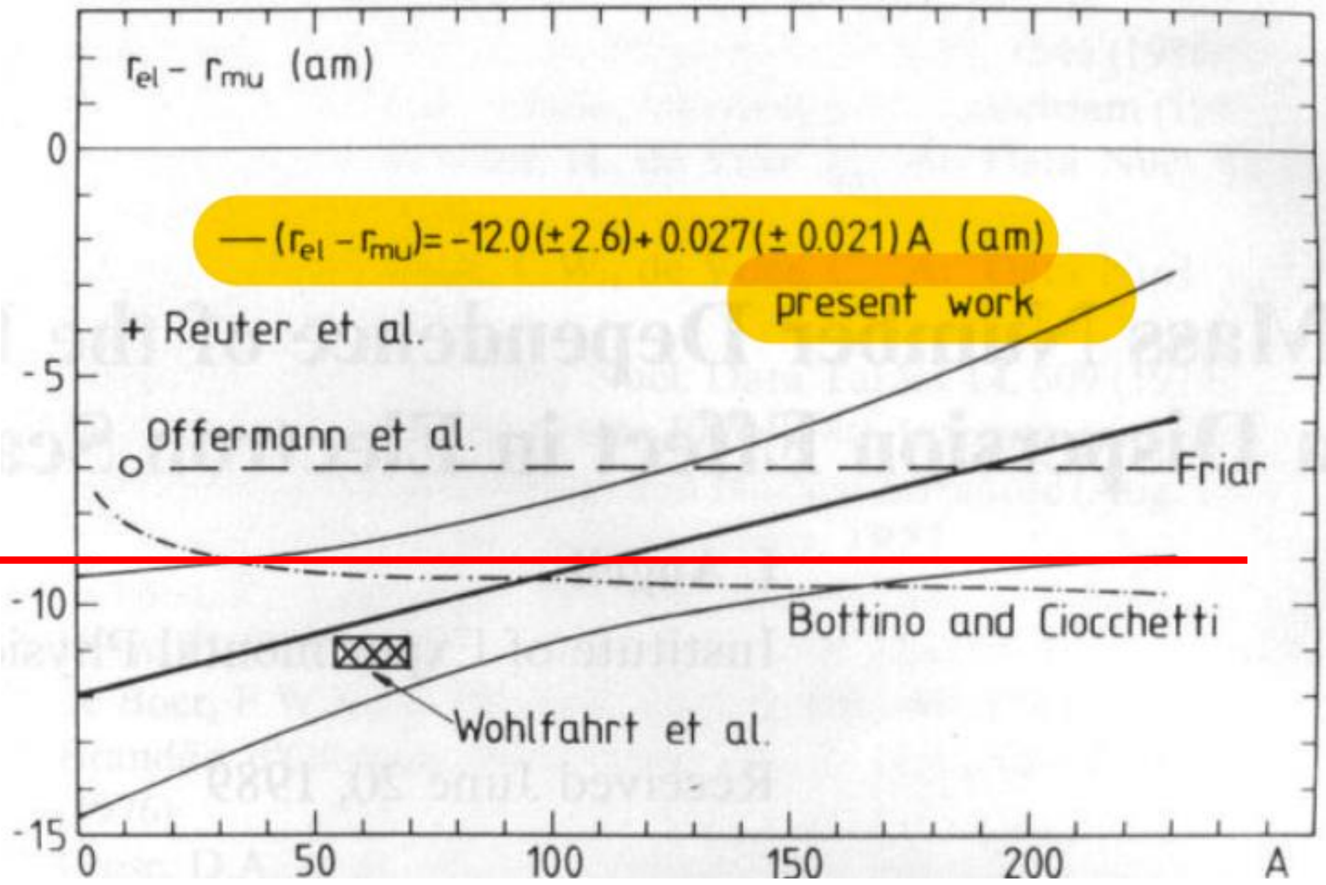
A Dispersion Effect in Electron scattering? (*Zeitschrift für Physik*, **334,** 377)

„The weighted average of ($r_{el} - r_{mu}$) differences for 85 nuclides:

-9.3 (1.5) am."

Average:

-9.3 (±1.5) am



Acta Physica Hungarica 69 (3–4), pp. 233–247 (1991)

EVALUATION PROCEDURE FOR NUCLEAR RMS CHARGE RADII* **

I. ANGELI

In this way, RMS radii were collected. To take into account dispersive effects in electron scattering, a correction term [40]

$$dR_{dc} = 12.0 - 0.27A(\pm 0.025)\ (\text{am})$$

was added to RMS radii measured by fast electron scattering. At the beginning, 1998: Herrmann: *EurPhysJ.* **A2** (1998) 29.

„For the deuteron charge radius our dispersion corrections lead to **a decrease of**

^{2}H: -0.003 fm."

„In [Bottino 1972] **the sign** of the correction to the rms-radius **seems to be wrong**."

The true rms-radius is smaller than the one obtained by analyzing the scattering data in a static framework."

INTERNATIONAL ATOMIC ENERGY AGENCY

INDC(HUN)-033
Distr.: ND+G

INDC INTERNATIONAL NUCLEAR DATA COMMITTEE

TABLE OF NUCLEAR ROOT MEAN SQUARE CHARGE RADII*

The ten-year-old *"deuteron radius discrepancy"* [Kl86, Wo94] seems to have been solved by [Si96 and Si98] applying the Coulomb distortion correction. In our previous paper [An98] a dispersion correction of +0.008 fm was also added estimated from early theoretical calculations [Bo72, Fr79], the only experimental result for ^{12}C [Of88, Of91] and a systematic analysis of the R_{el}-R_{mu} differences [An89]. However, a recent, precise calculation [He98] resulted in a net dispersion correction of – 0.003 fm. Therefore, *in the present work no dispersion correction was applied*.

Normalizations to proton

1987: At. Data Nucl. Data Tables, **36**, 495. deVries: Electron scattering

Nucleus	model	$\langle r^2\rangle^{1/2}$ [fm]	c or a [fm]	z or α [fm]	w	q-range [fm^{-1}]	ref.	remarks
n*		0.3359(36)				0	Kr73	1
		0.3455(26)				0	Ko76	1
^1H*	MI	0.85(2)				0.33 - 1.42	Th72	a,2
	MI	0.84(1)				0.36 - 11.50	Ho76	3
	MI	0.862(12)				0.36 - 1.18	Si80	4
^2H*	MI	2.095(6)				0.22 - 0.71	Be73a	a,h,5
	MI	2.116(6)				0.21 - 0.77	Si81	h,6
^3H*	FB	1.68(3)				0.51 - 2.83	Be84	†,b,7
	SOG	1.76(4)				0.55 - 4.79	Ju85	$,8
^3He*	SOG	1.844(45)				0.59 - 4.47	MC77	$
	FB	1.877(19)				0.18 - 10.1	Re84b	†,9
	MI	1.976(15)				0.45 - 1.92	Ot85	a,h,10

h) • Measurement relative to ^1H.

h in 53 remarks!

1963, Hand: Cross section evaluation

$\langle r^2 \rangle^{1/2} = 0.805\ (11)$ fm.

1980, Simon: Electron scattering

$\langle r^2 \rangle^{1/2} = 0.862\ (12)$ fm.

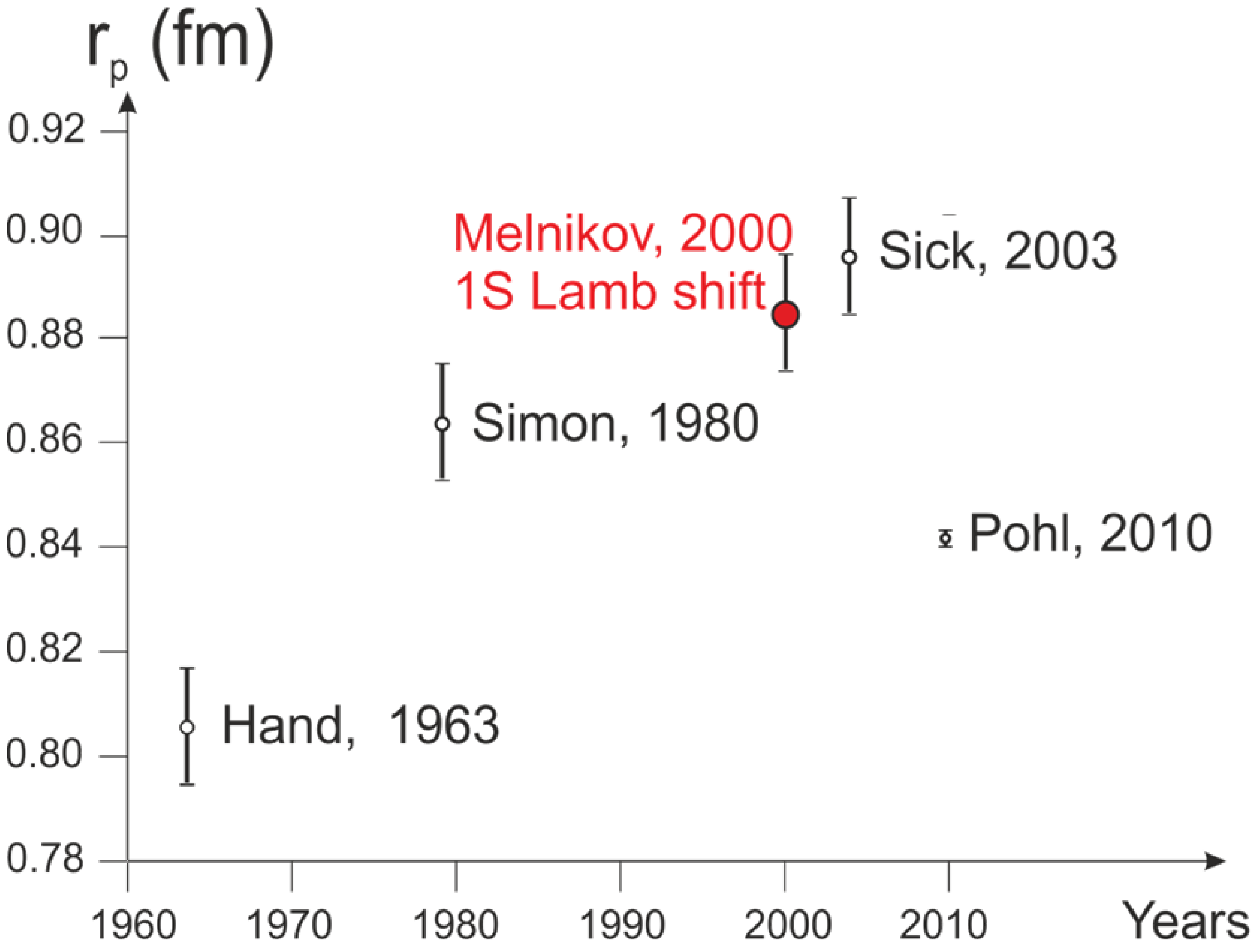
2000, Melnikov: 1S Lamb shift

$\langle r^2 \rangle^{1/2} = 0.883\ (14)$ fm.

2010: Pohl: Muonic H Lamb shift

$\langle r^2 \rangle^{1/2} = 0.84184\ (67)$ fm.

„Proton radius puzzle!"



What to do?

Renormalize the original data? If they exist? Who?

Nature, **575**, (2019) 174: Xiong, ... Electron scattering!

$\mathbf{r_p = 0.831\,(7)_{stat}\,(12)_{syst}\ fm.}$

Estimate common systematic error? How?

Other ideas? ....

The proton radius in charge radii

To the attention of authors and referees!

$r_c^2 \approx r_p^2 + \langle r^2 \rangle_p \ldots$ Friar, Negele: Adv. Nucl. Phys. **8**, (1975) 219.

The second term depends on time!

1963: $\langle r^2 \rangle^{1/2}$ = 0.805 (11)

1980: $\langle r^2 \rangle^{1/2}$ = 0.862 (12) fm.

Nucl. Phys. **A624** (1997) 349: $r_c^2 \approx r_p^2 + \mathbf{0.64\ fm^2}$

2000: $\langle r^2 \rangle^{1/2}$ = 0.883 (14) fm.

2010: $\langle r^2 \rangle^{1/2}$ = 0.84184 (67) fm.

Braz. Journ.Phys. **54** (2024) 105: $r_c^2 \approx r_p^2 + \mathbf{0.64}$ (without unit!)

Phys. Rev. **C109** (2024) 054323: $r_c^2 \approx r_p^2 + \mathbf{(0.8\ fm)^2}$

Future?

Weak and strong

Nucleus = protons + neutrons

Electric charge	= 1	= 0
Weak charge	≈ 0	≈ 1

Beta-decay, *Fermi* → *„weak interaction"*!

Electrons have electric and weak charges.

→ *neutron-distribution determination*!

Two simultaneous interactions.

How to separate them?

The weak interaction scatters

electrons of different polarization differently!

The electromagnetic interaction does not

make difference to polarization.

→ Scattering of polarized electrons!

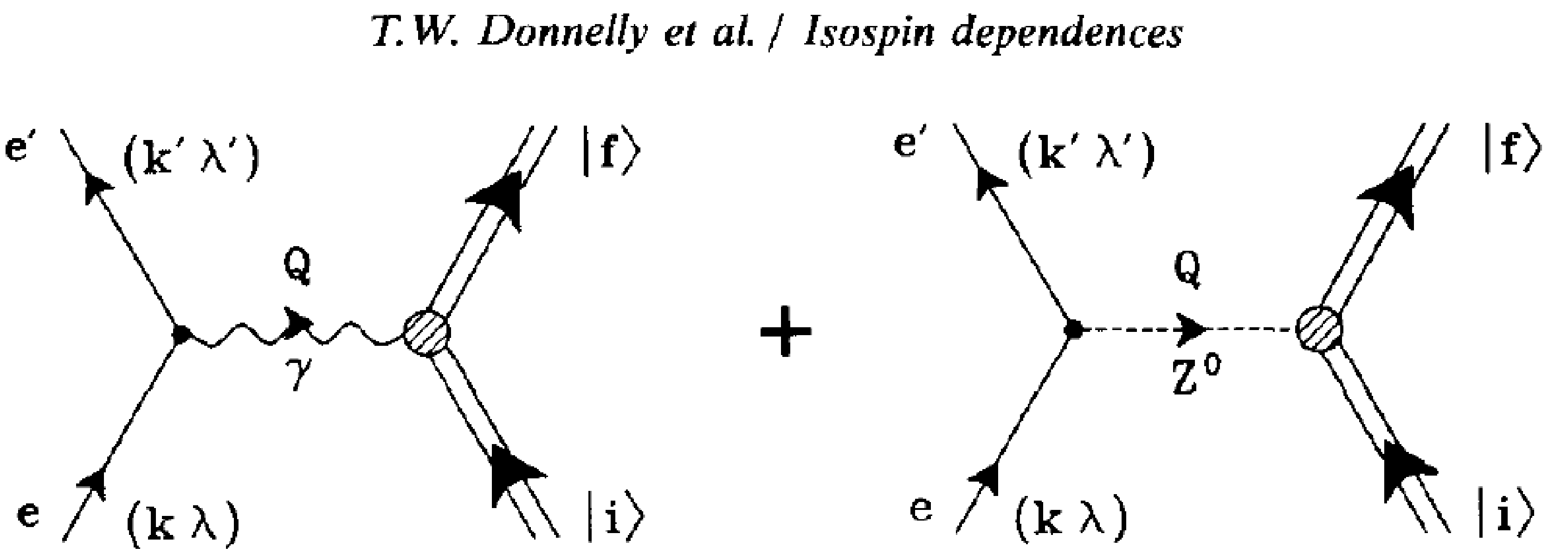
Laser beam of circular polarization $\rightarrow$

$\rightarrow$ *GaAs* semiconductor crystal, photo-effect

$\rightarrow$ **electrons take over the (linear) polarization**!

Scattering $\rightarrow$ *„right"* (R) electron beam,

scattering $\leftarrow$ *„left"* (L) electron beam.



Simultaneous elektromagnetic and

weak interactions with sattering amplitudes

a_E a_{WR} a_{WL}

The cross sections are:

$$\sigma_R = (a_E + a_{WR})^2 = a_E^2 + 2\,a_E a_{WR} + a_{WR}^2$$

$$\sigma_L = (a_E + a_{WL})^2 = a_E^2 + 2\,a_E a_{WL} + a_{WL}^2$$

$$\sigma_R - \sigma_L \approx 2\,a_E \times (a_{WR} - a_{WL})$$

I*nterference* - through $\boldsymbol{a_E}$ - renders observation possible!

„Parity Violating Asymmetry"

$$A_{PV} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} \approx \frac{G_F Q^2}{4\pi\alpha\sqrt{2}} \frac{F_W(Q^2)}{F_{ch}(Q^2)}$$

contains the weak (≈ neutron) form factor $F_W(q)$

G_F *Fermi-constant,*

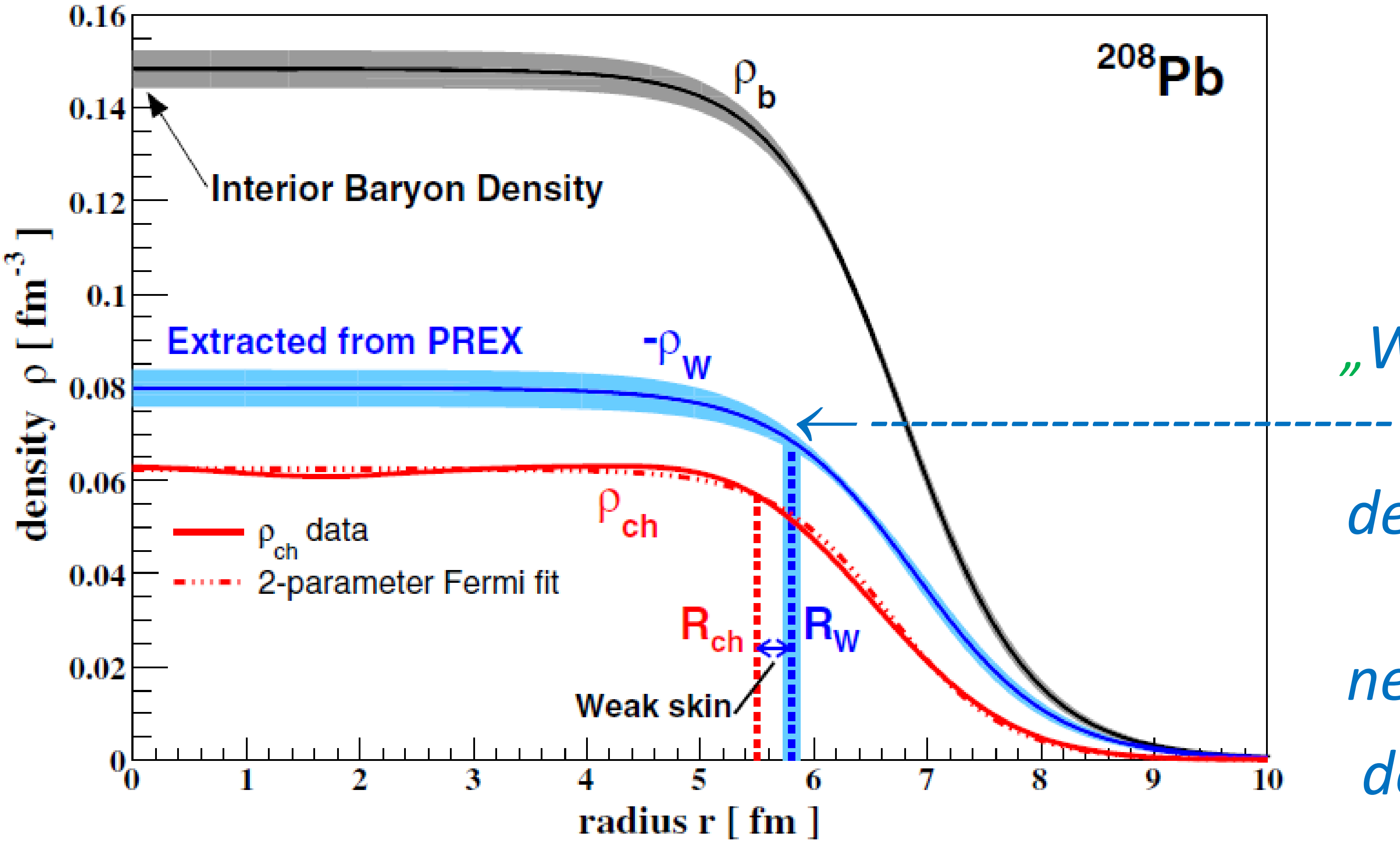
Q momentum transfer,

$F_{ch}(q)$ known.

Cross section ratio eliminates systematic errors!

Phys.Rev.Lett. **126** (2021) 172502*:* Adhikari, et al. …

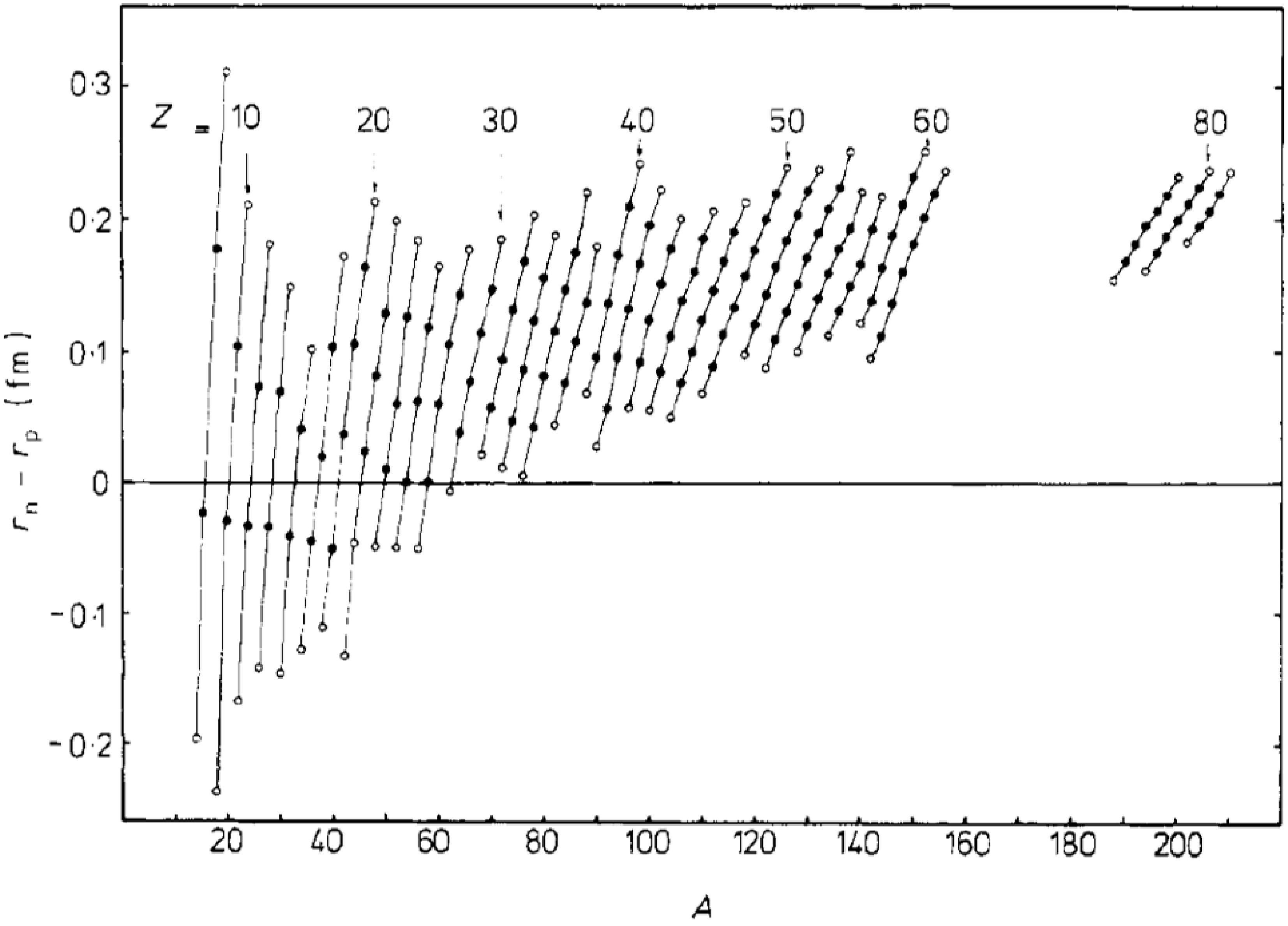
Jefferson Laboratory.



„Weak density" ≈ neutron-density

$A_{PV} \rightarrow R_n - R_p$ *„neutron skin"* ≈ **0.28 (± 0.07) fm.**

J.Phys. **G6** (1980) 303.



Sic itur ad astra!

For ^{208}Pb, from asymmetry

$\rightarrow$ $R_n - R_p$ *„neutron-skin”* $\approx$ **0.28 fm**.

$\rightarrow$ ***S*** *symmetry energy,* $\rightarrow$ $\boldsymbol{L \sim \frac{\partial S}{\partial \rho}}$

$\rightarrow$ $\boldsymbol{R_{NS}}$ *radius of neutron star*!

„Giant nucleus” held together by gravitation.

10% protons

Reading

Experiments:

Phys. Rev. Letters, **108** (2016) 112502.: Abrahamyan, ...

Phys. Rev. Letters, **126** (2021) 172502.: Adhikari, ...

Review, short:

Nuclear Physics News, **34** (2024) 34.: Mammei, ...

Reviews, detailed:

Frontiers in Physics, 11/1 (2016) 111301.: Souder, ...

Annual Rev. Nucl. Part. Phys. **74** (2024) 321.: Mammei, ...

Thank you for your attention!