

Recent improvements in the theory of heavy muonic atoms and their influence on nuclear radii

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of Nuclear Charge Radii

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MAX-PLANCK-GESELLSCHAFT



Outline

Historical introduction

Latest improvements

Preliminary results

Double checking

Additional problems



Fig: <https://www.shackelfordfuneraldirectors.com/>

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How did it start: The Bible

Atomic Data and Nuclear Data Tables 99 (2013) 69–95



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Atomic Data and Nuclear Data Tables

journal homepage: www.elsevier.com/locate/adt



Table of experimental nuclear ground state charge radii: An update

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ABSTRACT

The present table contains experimental root-mean-square (*rms*) nuclear charge radii R obtained by combined analysis of two types of experimental data: (i) radii changes determined from optical and, to a lesser extent, K_{α} X-ray isotope shifts and (ii) absolute radii measured by muonic spectra and electronic scattering experiments. The table combines the results of two working groups, using respectively two different methods of evaluation, published in ADNDT earlier. It presents an updated set of *rms* charge radii for 909 isotopes of 92 elements from ${}^1\text{H}$ to ${}^{96}\text{Cm}$ together, when available, with the radii changes from optical isotope shifts. Compared with the last published tables of R -values from 2004 (799 ground states), many new data are added due to progress recently achieved by laser spectroscopy up to early 2011. The radii changes in isotopic chains for He, Li, Be, Ne, Sc, Mn, Y, Nb, Bi have been first obtained in the last years and several isotopic sequences have been recently extended to regions far off stability, (e.g., Ar, Mo, Sn, Te, Pb, Po).

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Shortly about muons and muonic atoms

Cute leptons source:
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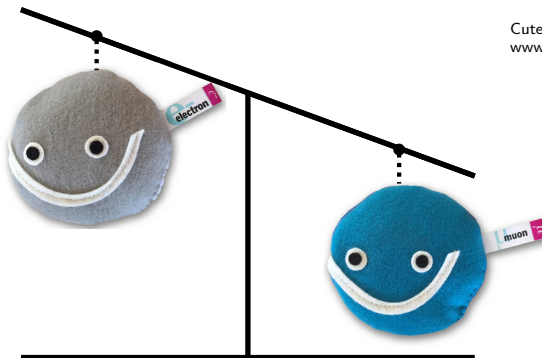
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- Same interaction laws, same formulas

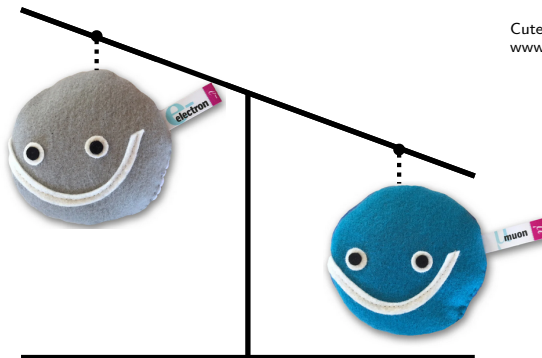
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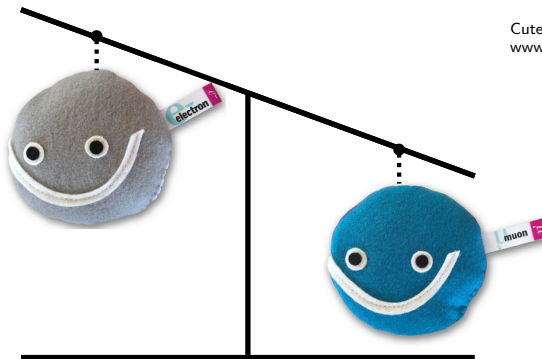
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Shortly about muons and muonic atoms



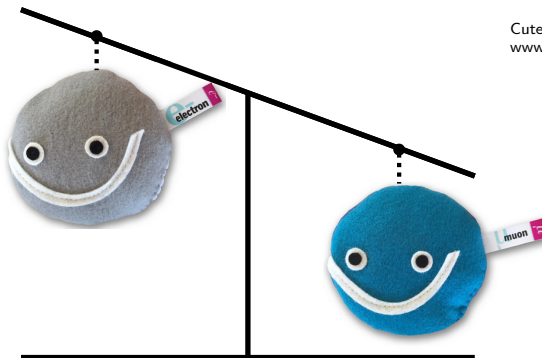
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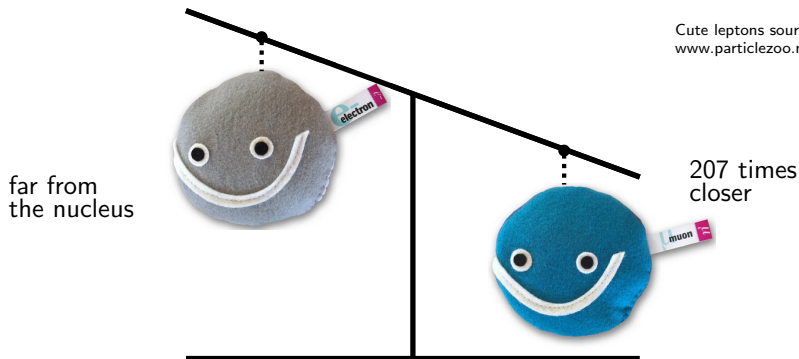
far from
the nucleus



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Shortly about muons and muonic atoms

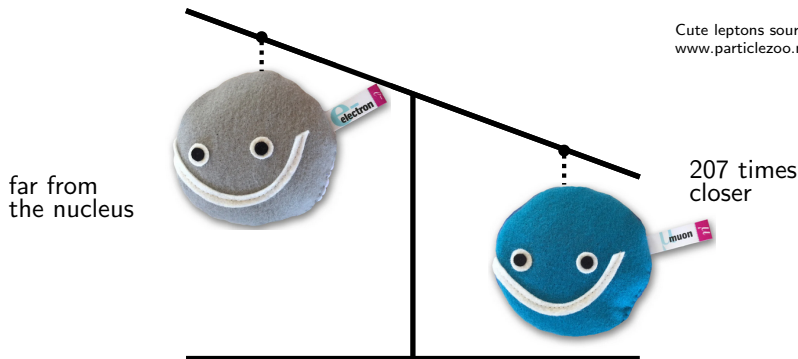
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- Same interaction laws, same formulas
- Heavy sibling of electron; $m_\mu \approx 207m_e$
- Lifetime 0.1-2.2 μs , capture and decay $10^{-12} - 10^{-9}$ s
- Always hydrogen like, some electrons are far away
- Sensitive to nuclear structure

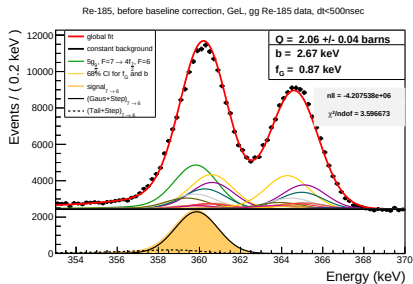
2016: restart of muonic physics



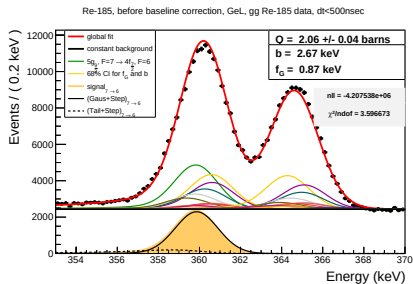
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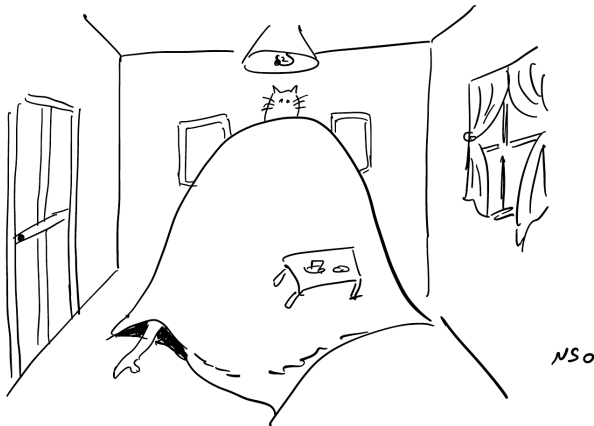
2016: restart of muonic physics



2016: restart of muonic physics



2022: Discovery of muonic fine-structure anomaly



[1] Yamazaki *et al.*, PRL **42**, 1470 (1979)

[2] Phan *et al.*, PRC **32**, 609 (1985)

[3] Bergem *et al.*, PRC **37**, 2821 (1988)

[4] Piller *et al.*, PRC **42**, 182 (1990)

What is the fine-structure anomaly

Very poor fit of experimental data for $2p_{3/2,1/2} \rightarrow 1s_{1/2}$
for muonic ^{90}Zr , $^{112-124}\text{Sn}$, ^{208}Pb

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→ nuclear-polarization corrections as variable parameters



the root of the problem

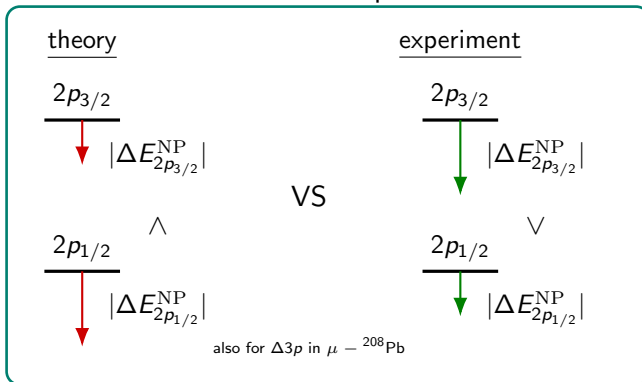
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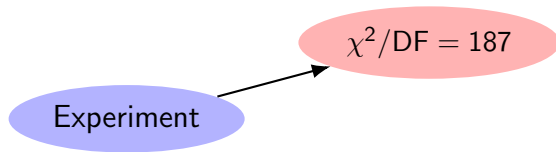


One of the best: ^{208}Pb

Experiment

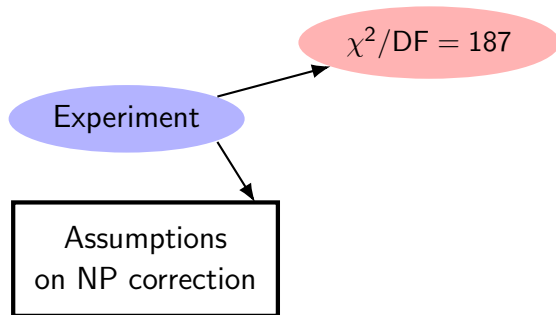
[1] Bergem *et al.*, PRC **37**, 2821 (1988)

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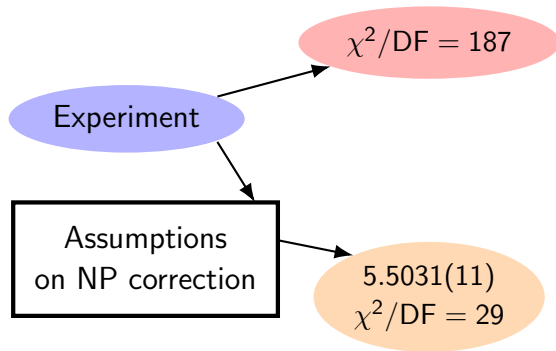
[1] Bergem *et al.*, PRC **37**, 2821 (1988)

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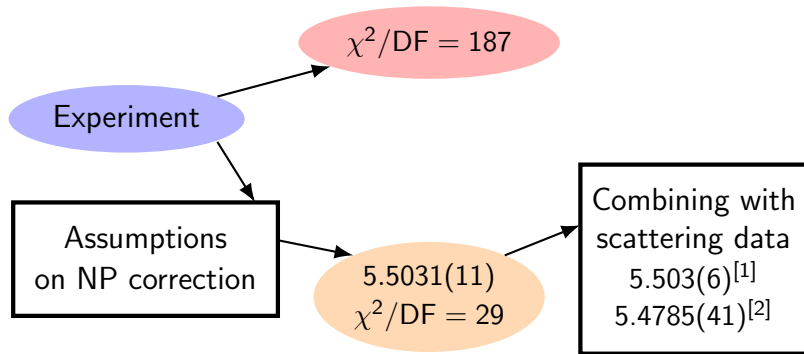
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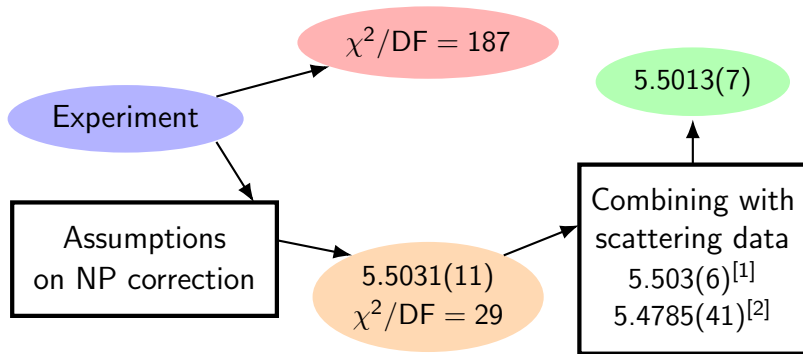
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[1] Bergem *et al.*, PRC **37**, 2821 (1988)

[2] Fricke and Bernhardt, At. Data Nucl. Data Tables **60**, 177 (1995)

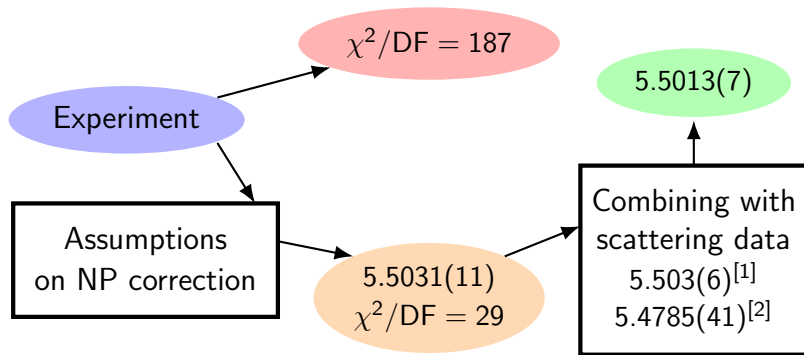
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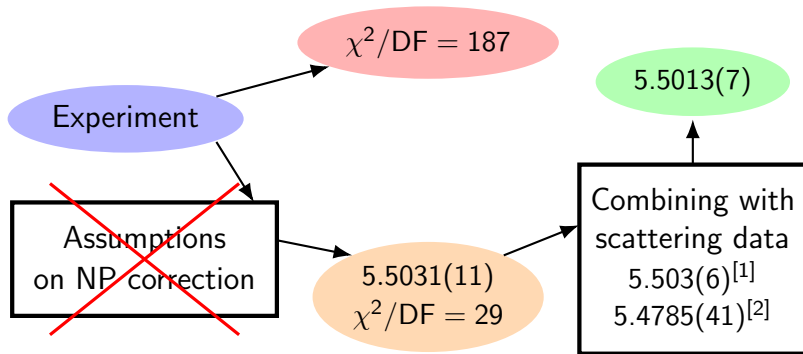
One of the best: ^{208}Pb rms=5.5012(13)



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[2] Fricke and Bernhardt, At. Data Nucl. Data Tables **60**, 177 (1995)

[3] Valuev *et al.*, PRL **128** 203001 (2022)

Nuclear polarization effect

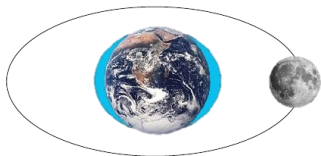


Image source: www.universetoday.com

$$V_{\text{Coul}}(r) = -\frac{\alpha Z}{r}$$

$$V_{\text{ext}}(r) = -\alpha \int \frac{\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d\mathbf{r}'$$

$$V_{\text{NP}}(r) = -\alpha \sum_Z \frac{1}{|\mathbf{r} - \mathbf{r}_{\text{Ni}}|}$$

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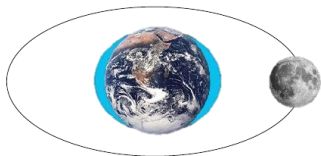


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$$H = H_N + \alpha \mathbf{p} + \beta m_\mu + V(\mathbf{r}, \mathbf{r}_{\text{Ni}})$$

$$\Delta E_I = \sum_{nN}^I \frac{\langle aA | \Delta V | nN \rangle \langle nN | \Delta V | aA \rangle}{E_{aA} - E_{nN}}$$

Nuclear polarization effect

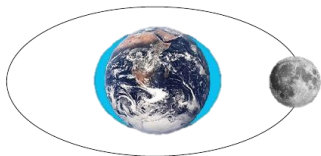


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Nuclear polarization effect

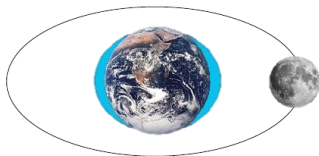


Image source: www.universetoday.com

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Light muonic atoms: $|a\rangle \rightarrow \delta(0)$

Heavy highly charge ions: $|A\rangle \rightarrow \delta(R)$

Transverse part of muon-nucleus interaction

$$H = H_N + \alpha \mathbf{p} + \beta m_\mu + V(\mathbf{r}, \mathbf{r}_{N_i})$$

$\Downarrow$

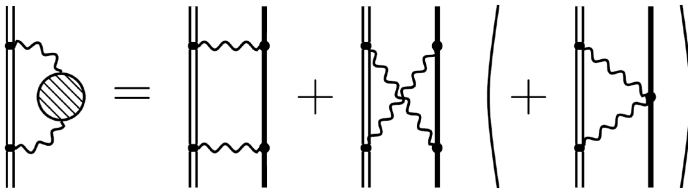
$$H = H_N + \alpha(\mathbf{p} - e\mathbf{A}(\mathbf{r}, \mathbf{r}_{N_i})) + \beta m_\mu + V(\mathbf{r}, \mathbf{r}_{N_i})$$

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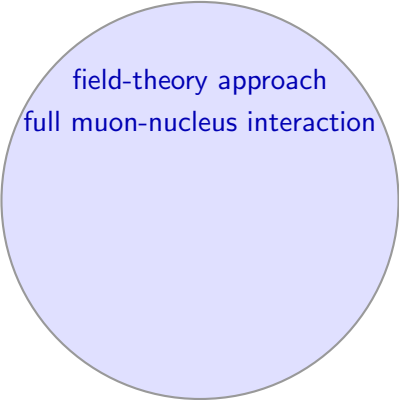
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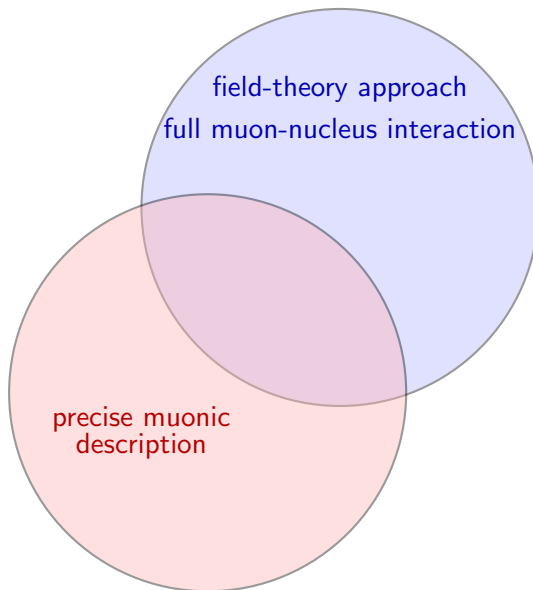
$$\begin{aligned} \Delta E_{\text{NP}}^{\text{L}} &= -i(4\pi\alpha)^2 \sum_{i'I'} \iint \frac{d\mathbf{q} d\mathbf{q}'}{(2\pi)^6} \int \frac{d\omega}{2\pi} \frac{D_{\mu\xi}(\omega, \mathbf{q}) D_{\zeta\nu}(\omega, \mathbf{q}') \langle iI | j_m^\mu(-\mathbf{q}) J_N^\xi(\mathbf{q}) | i'I' \rangle \langle i'I' | J_N^\zeta(-\mathbf{q}') j_m^\nu(\mathbf{q}') | iI \rangle}{(\omega + \omega_m - iE_{i'}\epsilon)(\omega - \omega_N + i\epsilon)}, \\ \Delta E_{\text{NP}}^{\text{X}} &= +i(4\pi\alpha)^2 \sum_{i'I'} \iint \frac{d\mathbf{q} d\mathbf{q}'}{(2\pi)^6} \int \frac{d\omega}{2\pi} \frac{D_{\mu\xi}(\omega, \mathbf{q}) D_{\zeta\nu}(\omega, \mathbf{q}') \langle iI | j_m^\mu(-\mathbf{q}) J_N^\xi(\mathbf{q}) | i'I' \rangle \langle i'I' | J_N^\zeta(-\mathbf{q}') j_m^\nu(\mathbf{q}') | iI \rangle}{(\omega + \omega_m - iE_{i'}\epsilon)(\omega + \omega_N - i\epsilon)}, \\ \Delta E_{\text{NP}}^{\text{SG}} &= -i(4\pi\alpha)^2 \sum_{i'} \iint \frac{d\mathbf{q} d\mathbf{q}'}{(2\pi)^6} \int \frac{d\omega}{2\pi} \frac{D_{\mu\xi}(\omega, \mathbf{q}) \delta^{\xi\zeta} D_{\zeta\nu}(\omega, \mathbf{q}') \langle i | j_m^\mu(-\mathbf{q}) | i' \rangle \langle i' | j_m^\nu(\mathbf{q}') | i \rangle \langle I | \rho_N(\mathbf{q} - \mathbf{q}') | I \rangle}{(\omega + \omega_m - iE_{i'}\epsilon) m_p}. \end{aligned}$$

Our goal

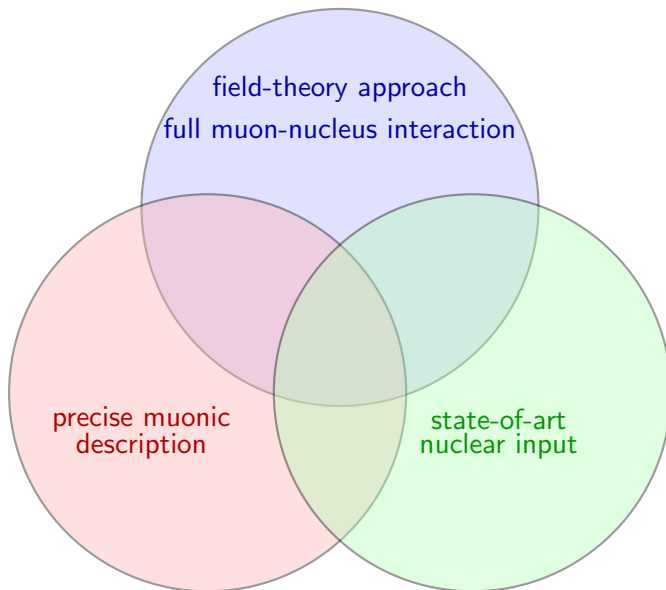


field-theory approach
full muon-nucleus interaction

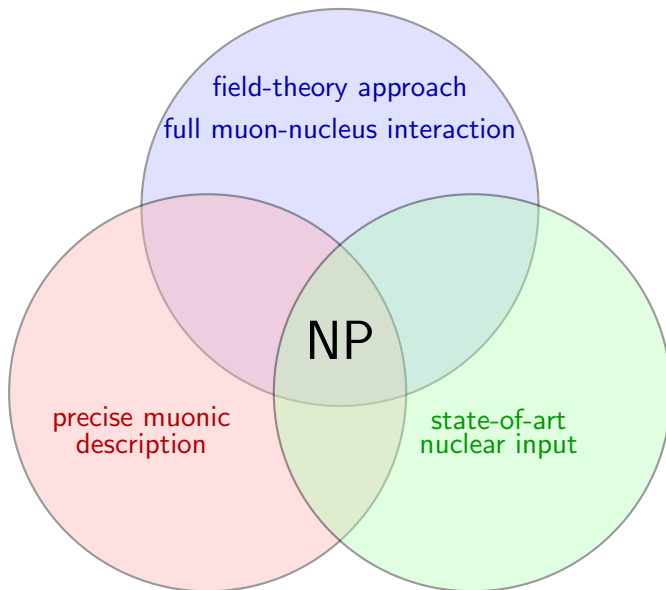
Our goal



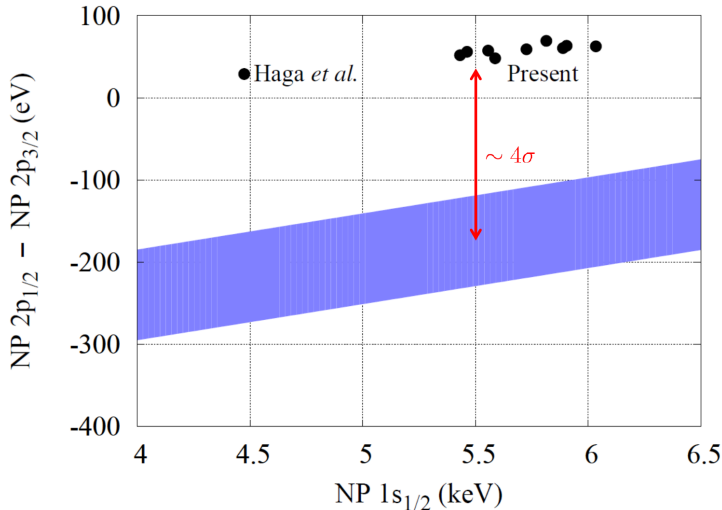
Our goal



Our goal



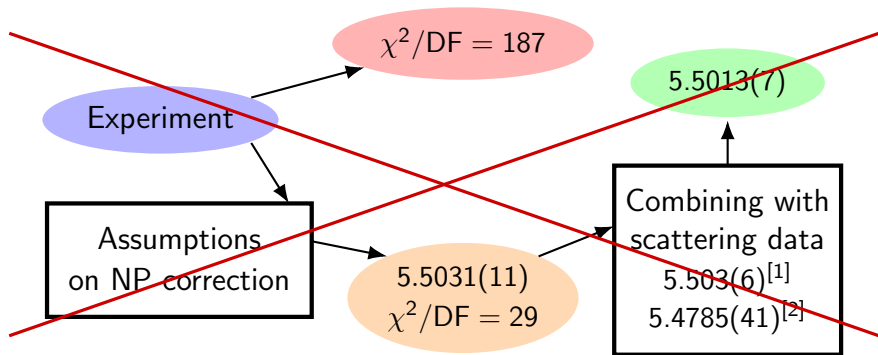
Nuclear polarization correction ^{208}Pb



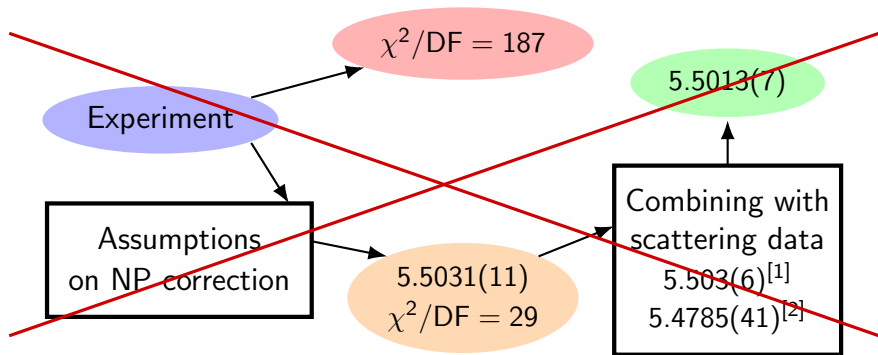
around 150 eV, or 4σ standard deviations gap

Valuev *et al.*, PRL **128** 203001 (2022)

One of the best: ^{208}Pb rms=5.5012(13)?



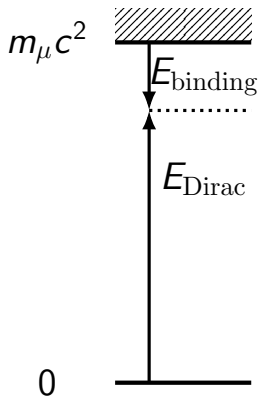
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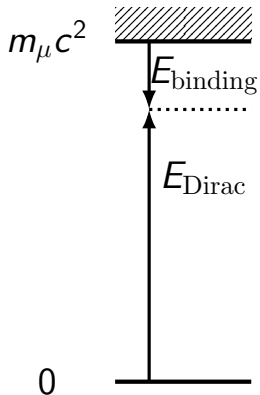
Total reevaluation of all QED and nuclear effects is needed

Dirac value and nuclear size

- Muons are close to the nucleus, relativistic $\rightarrow$ Dirac equation

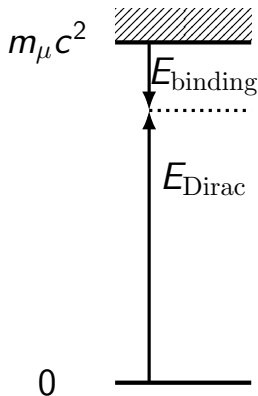


Dirac value and nuclear size



- Muons are close to the nucleus, relativistic $\rightarrow$ Dirac equation
- $m_\mu c^2 \approx 100 \text{ MeV}$
 $E_{\text{Dirac}} \approx 80 \text{ MeV}$
 $E_{\text{binding}} \approx 20 \text{ MeV}$

Dirac value and nuclear size



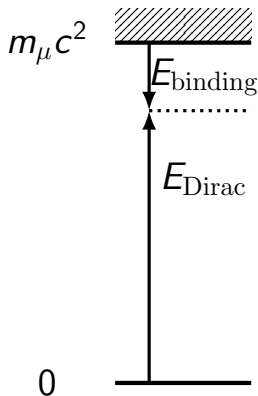
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- Extended nucleus: sphere,

$$V_{\text{Sph}}(r) = \begin{cases} a + br^2; & r \leq R \\ -\frac{Z\alpha}{r}; & r \geq R \end{cases}$$

Dirac value and nuclear size



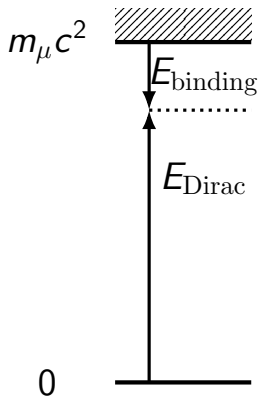
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 $E_{\text{Dirac}} \approx 80 \text{ MeV}$
 $E_{\text{binding}} \approx 20 \text{ MeV}$

- Extended nucleus: sphere, Fermi,

$$\rho_{a,c}^F(r_{\mu}) = \frac{N}{1 + e^{(r-c)/a}}$$

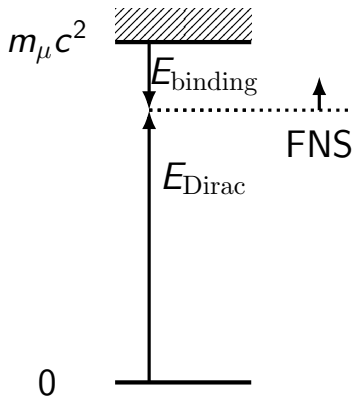
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 $E_{\text{Dirac}} \approx 80 \text{ MeV}$
 $E_{\text{binding}} \approx 20 \text{ MeV}$
- Extended nucleus: sphere, Fermi, deformed Fermi

$$\rho_{a,c,\beta}(r_\mu, \vartheta_\mu) = \frac{N}{1 + e^{[r - c(1 + \beta Y_{20}(\vartheta_\mu))]/a}}$$

Dirac value and nuclear size



- Muons are close to the nucleus, relativistic $\rightarrow$ Dirac equation

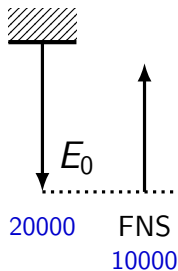
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- Extended nucleus: sphere, Fermi, deformed Fermi

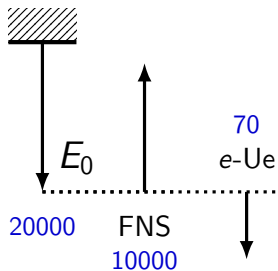
$$\rho_{a,c,\beta}(r_\mu, \vartheta_\mu) = \frac{N}{1 + e^{[r - c(1 + \beta Y_{20}(\vartheta_\mu))]/a}}$$

- $\Delta E_{\text{FNS}} \approx 10 \text{ MeV}$

QED effects:



QED effects: Uehling

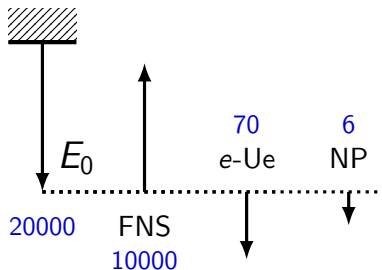


$$V_{\text{Ue}}(r) = -\frac{2\alpha Z\alpha}{3r} \int_0^\infty dr' r' \rho(r') \left[\chi_2(2|r-r'|) - \chi_2(2|r+r'|) \right]$$

$$V_{\text{Ue}}(r) = V_{\text{point}}(r) \left[\frac{2\alpha}{3\pi} \chi_1(2r) \right]$$

$$V_{\text{Ue}}(r) \approx V_{\text{Fermi}}(r) \left[\frac{2\alpha}{3\pi} \chi_1(2r) \right]$$

QED effects: Nuclear polarization

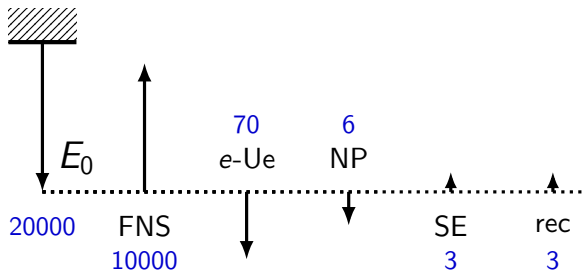


Improvements:

- field-theory approach, including transverse part
- state-of-art muonic and nuclear input, model dependence
- $0^+, 1^-, 2^+, 3^-, 4^+, 5^-$ and 1^+ excitation modes
- $4252 \text{ eV} \rightarrow 5712 \text{ eV}$

Valuev *et al.*, PRL **128** 203001 (2022)

QED effects: Self energy and recoil



- rigorous QED calculations
- $\Delta E_{SE} = 3270(160)^{[1]}, 3373^{[2]} \text{ eV} \rightarrow 3225(15)^{[3]} \text{ eV}$
- $\Delta E_{rec} = 385^{[4]*} \text{ eV} \rightarrow 3902^{[5]} \text{ eV}$

[1] Cheng *et al.*, PRA **17**, 489 (1978)

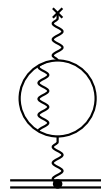
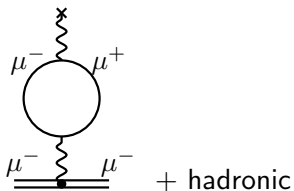
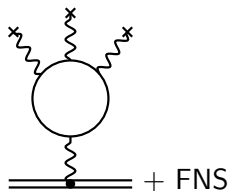
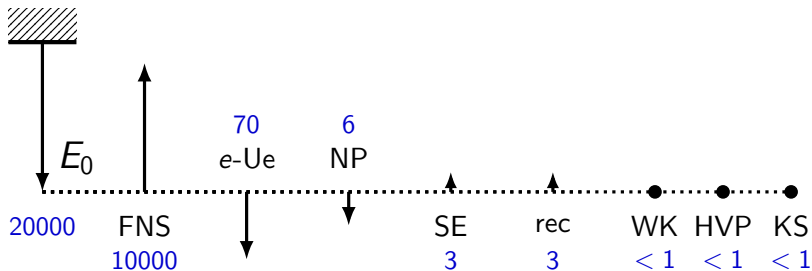
[2] Haga *et al.*, PRC **75**, 044315 (2007)

[3] Oreshkina, PRR **4**, L042040 (2022)

[4] Bergem *et al.*, PRC **37**, 2821 (1988)

[5] Yerokhin and Oreshkina, PRA **108**, 052824 (2023)

QED effects: sub-leading



in preparation

Results for ^{208}Pb RMS extraction PRELIMINARY

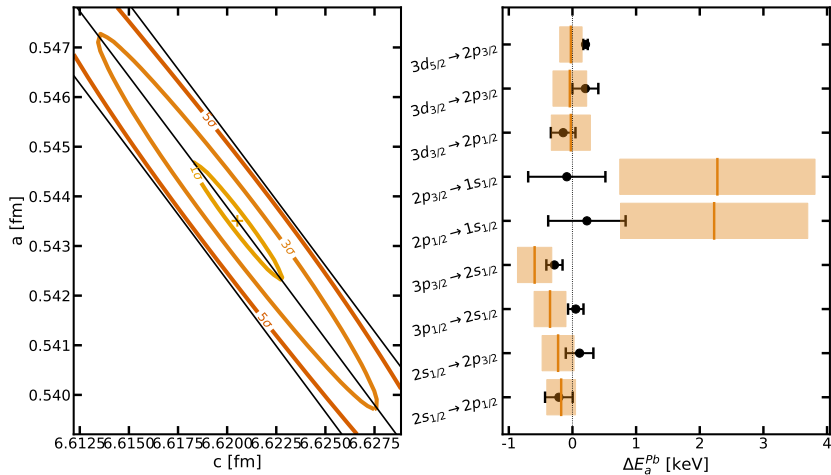


Fig: Konstantin Beyer

Results for ^{208}Pb RMS extraction PRELIMINARY

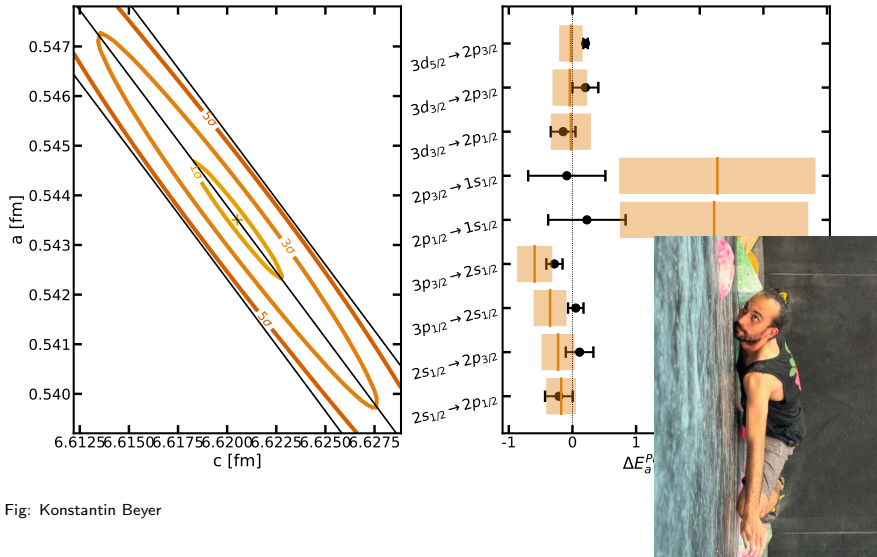
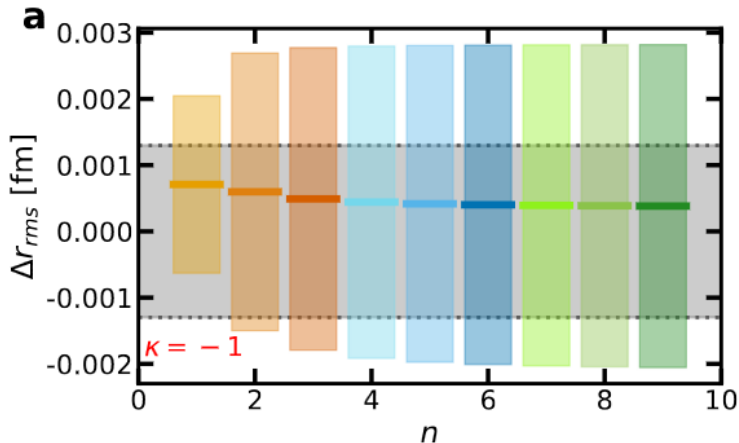


Fig: Konstantin Beyer

Model dependence PRELIMINARY



The difference between the Fermi-equivalent radius and the tabulated value. The solid colourful line is the mean value over all SKYRME distributions.

Fig: Konstantin Beyer

Outline

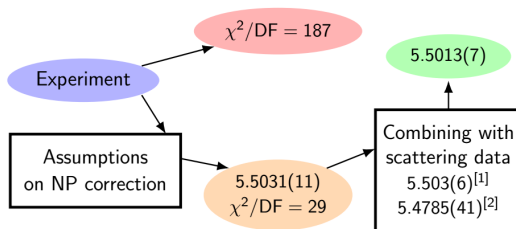
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Theory uncertainties

- Completely ignored in previous studies



Theory uncertainties

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- Important for fitting and for final errorbars



Theory uncertainties

- Completely ignored in previous studies
- Important for fitting and for final errorbars
- Adding uncertainties (improving the results) will lead to visibly worse outcome



Barrett radii

$$E_{if}^{\text{FNS}} \approx C \int d^3r \rho_c(\mathbf{r}) r^k e^{-\alpha_B r} \equiv B_{if}(C, k, \alpha_B)$$

α_B is commonly fixed to be the same for all transitions

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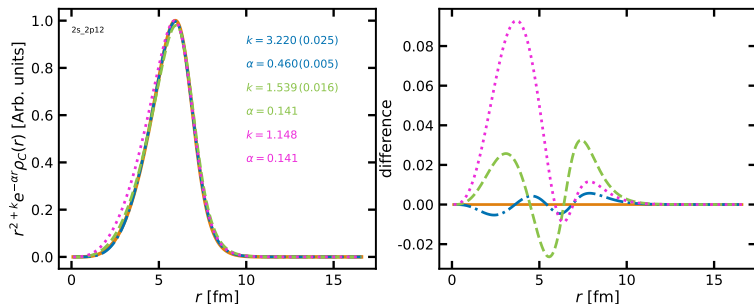
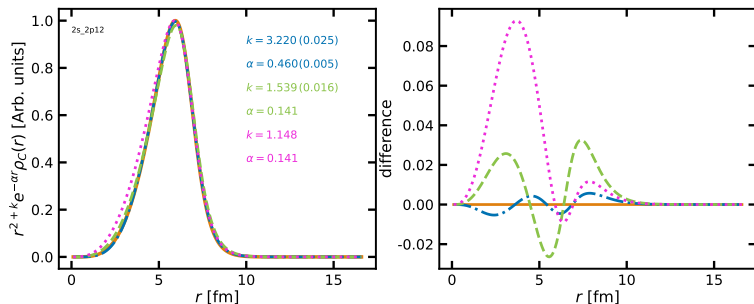


Fig: Konstantin Beyer

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$$R_B = 6.752 / 6.954 / 6.881$$

Fig: Konstantin Beyer

V_2 parameter

$$V_2 \equiv \frac{r_{\text{rms}}^e}{B_{if}^e(k, \alpha_B)} = \frac{r_{\text{rms}}^\mu}{B_{if}^\mu(k, \alpha_B)} \quad \Rightarrow \quad r_{\text{rms}}^\mu = B_{if}^\mu(k, \alpha_B) V_2^e$$

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$$\frac{\delta V_2^e}{V_2^e} = \sqrt{\left(\frac{\partial V_2^e}{\partial r_{\text{rms}}^e} \frac{\delta r_{\text{rms}}^e}{V_2^e} \right)^2 + \left(\frac{\partial V_2^e}{\partial B_{if}^e} \frac{\delta B_{if}^e}{V_2^e} \right)^2} + 2 \frac{\partial V_2^e}{\partial r_{\text{rms}}^e} \frac{\partial V_2^e}{\partial B_{if}^e} \frac{\text{cov}(r_{\text{rms}}^e, \delta B_{if}^e)}{(V_2^e)^2}$$

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- For the uncertainty estimation cov matrix is needed

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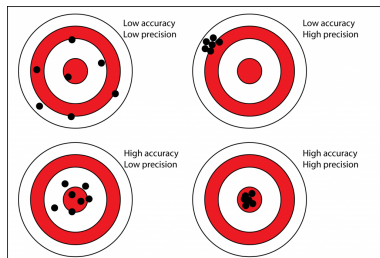
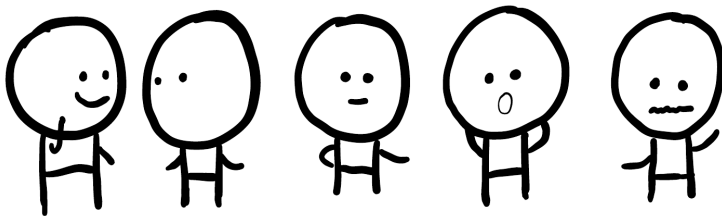


Fig source:

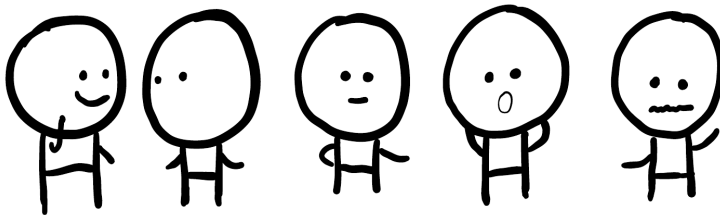
<https://www.portaspecs.com/precision-and-accuracy/>

Errors propagation over time



Errors propagation over time

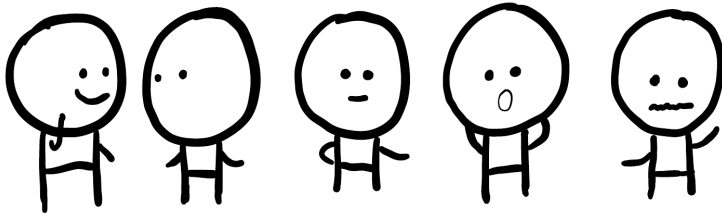
On the
current
level of
accuracy



Errors propagation over time

On the
current
level of
accuracy

an
adequate
method

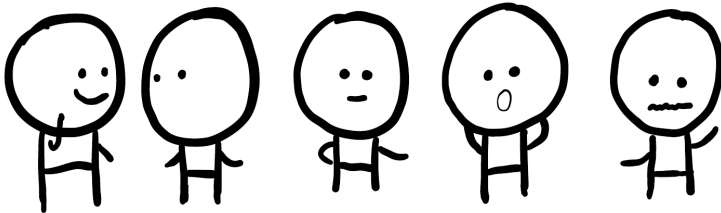


Errors propagation over time

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good
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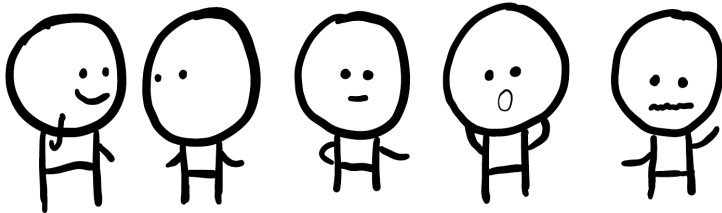
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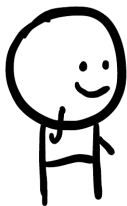
good
approximation

state-of-
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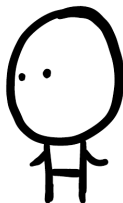


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precise
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Errors propagation over time

Volume 33B, number 6

PHYSICS LETTERS

23 November 1970

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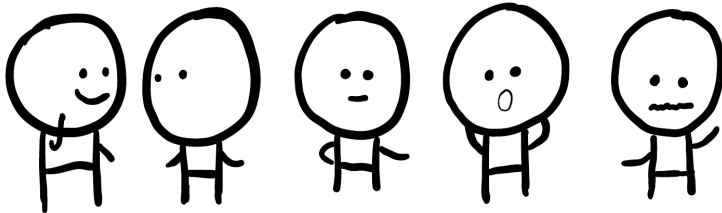
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MODEL-INDEPENDENT PARAMETERS OF THE NUCLEAR CHARGE
DISTRIBUTION FROM MUONIC X-RAYS

R. C. BARRETT

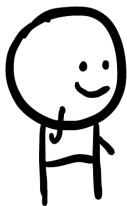
University of Surrey, Guildford, UK

Received 4 October 1970

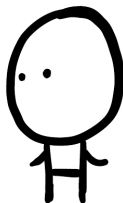


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“A method described in details in [1]” (dozens of citations)

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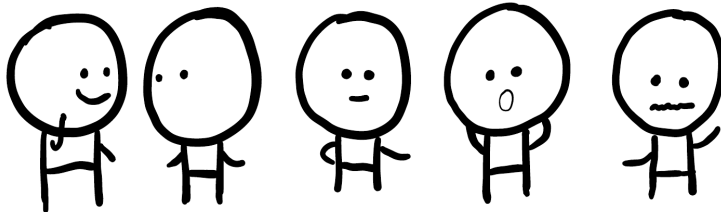
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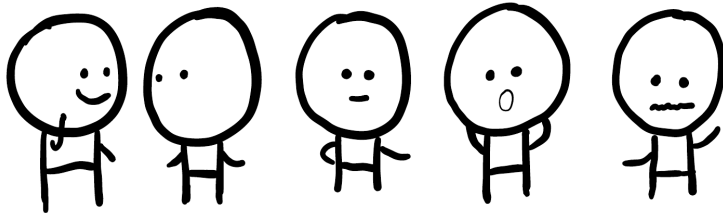
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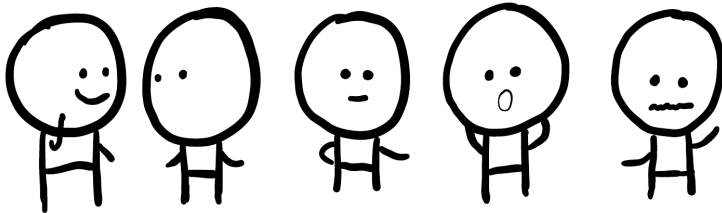
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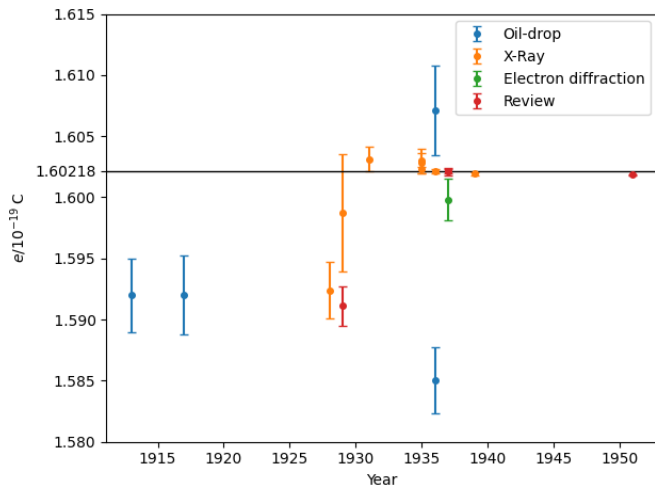
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[2] “applying method presented in [3]”

Cargo Cult Science by RICHARD P. FEYNMAN (1974)

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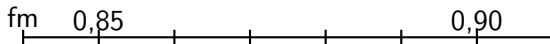
A scatter plot of electron charge measurements as suggested by Feynman, using papers published from 1913–1951 (from wikipedia)

More recent: Proton radius puzzle

$$1\text{fm} = 10^{-15}\text{m} = 0,000\,000\,000\,000\,001\text{ m}$$

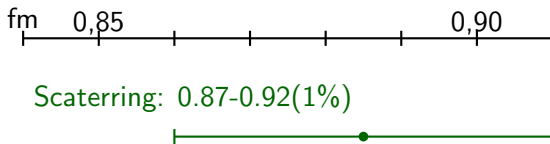
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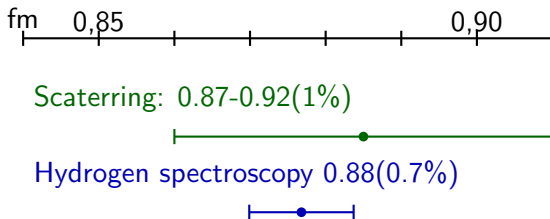
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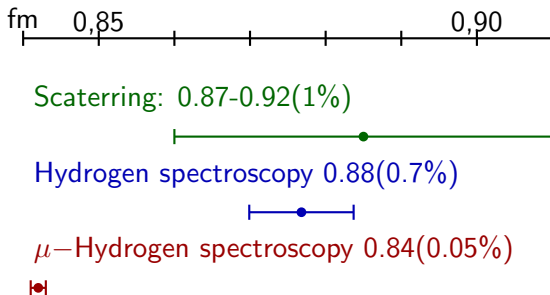
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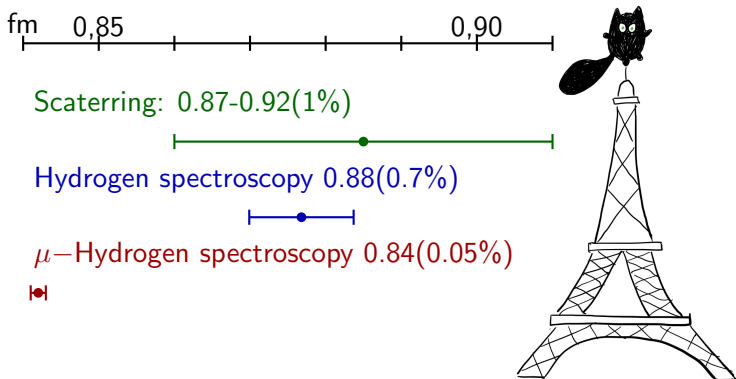
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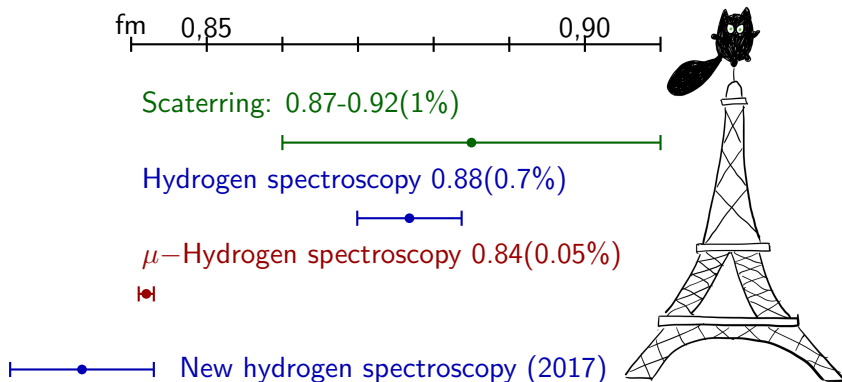
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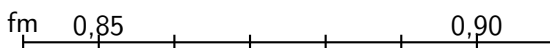
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Scattering: 0.87-0.92(1%)

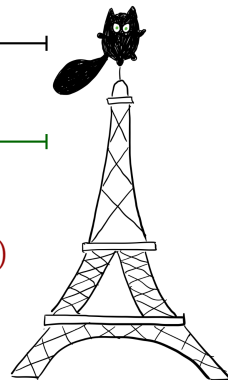
Hydrogen spectroscopy 0.88(0.7%)

μ -Hydrogen spectroscopy 0.84(0.05%)

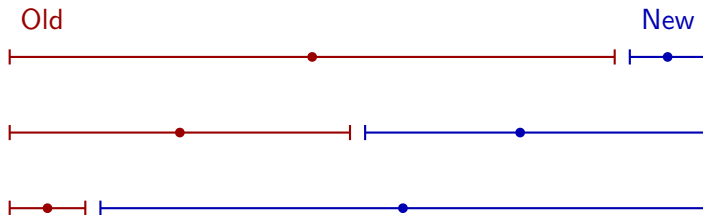


New hydrogen spectroscopy (2017)

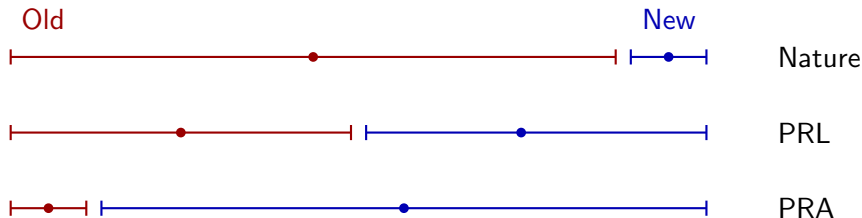
New scattering (2019)



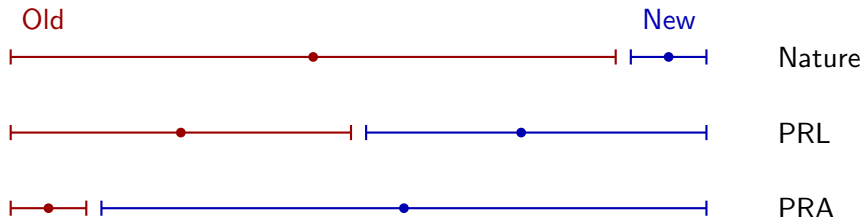
New vs Old: Win or Lose?



New vs Old: Win or Lose?

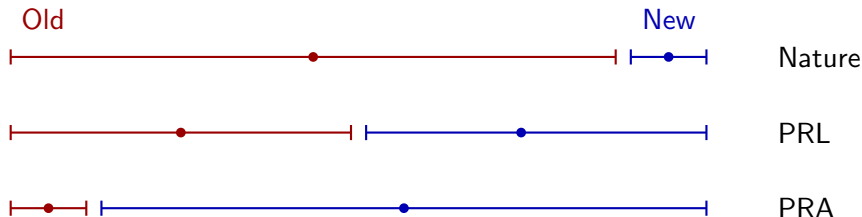


New vs Old: Win or Lose?



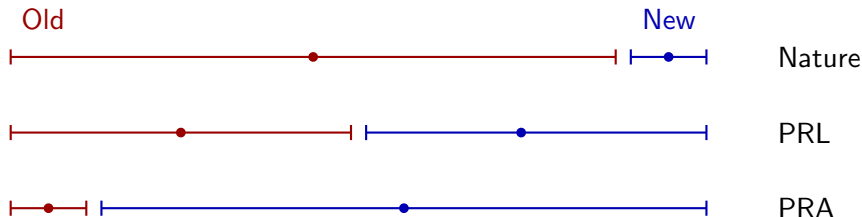
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New vs Old: Win or Lose?



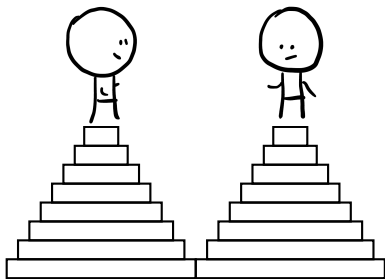
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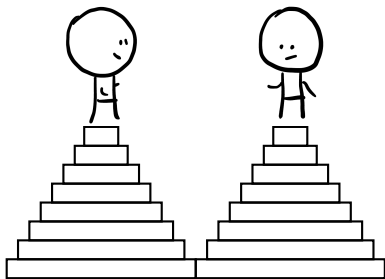


- Only improvements are welcomed
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- Be brave, but it can cost you

Interdisciplinarity and reproducibility

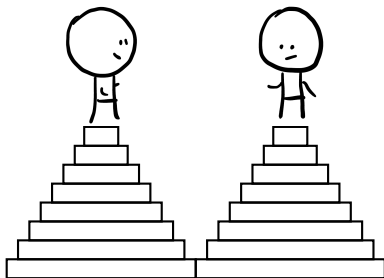


Interdisciplinarity and reproducibility



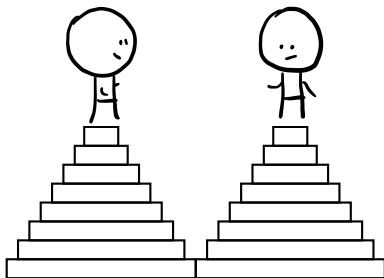
- Complexity grows exponentially $\Rightarrow$ narrow specializations

Interdisciplinarity and reproducibility



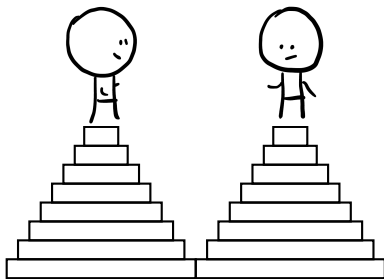
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Interdisciplinarity and reproducibility



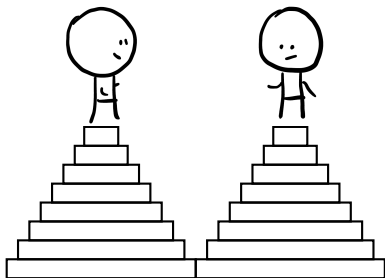
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Interdisciplinarity and reproducibility



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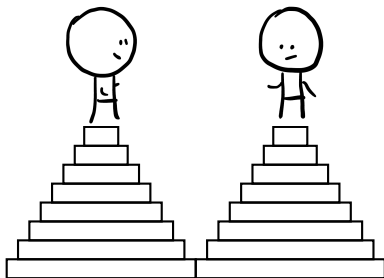
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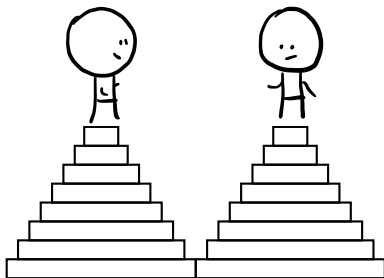
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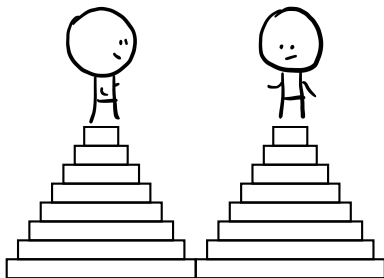
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- Constants: changing with time!
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- Hidden raw data
- Omitted details of methods/experiments

Conclusions and outlook

if I will ignore it



maybe it will go away

Conclusions and outlook

if I will ignore it



maybe it will go away



Conclusions and outlook

if I will ignore it



maybe it will go away

