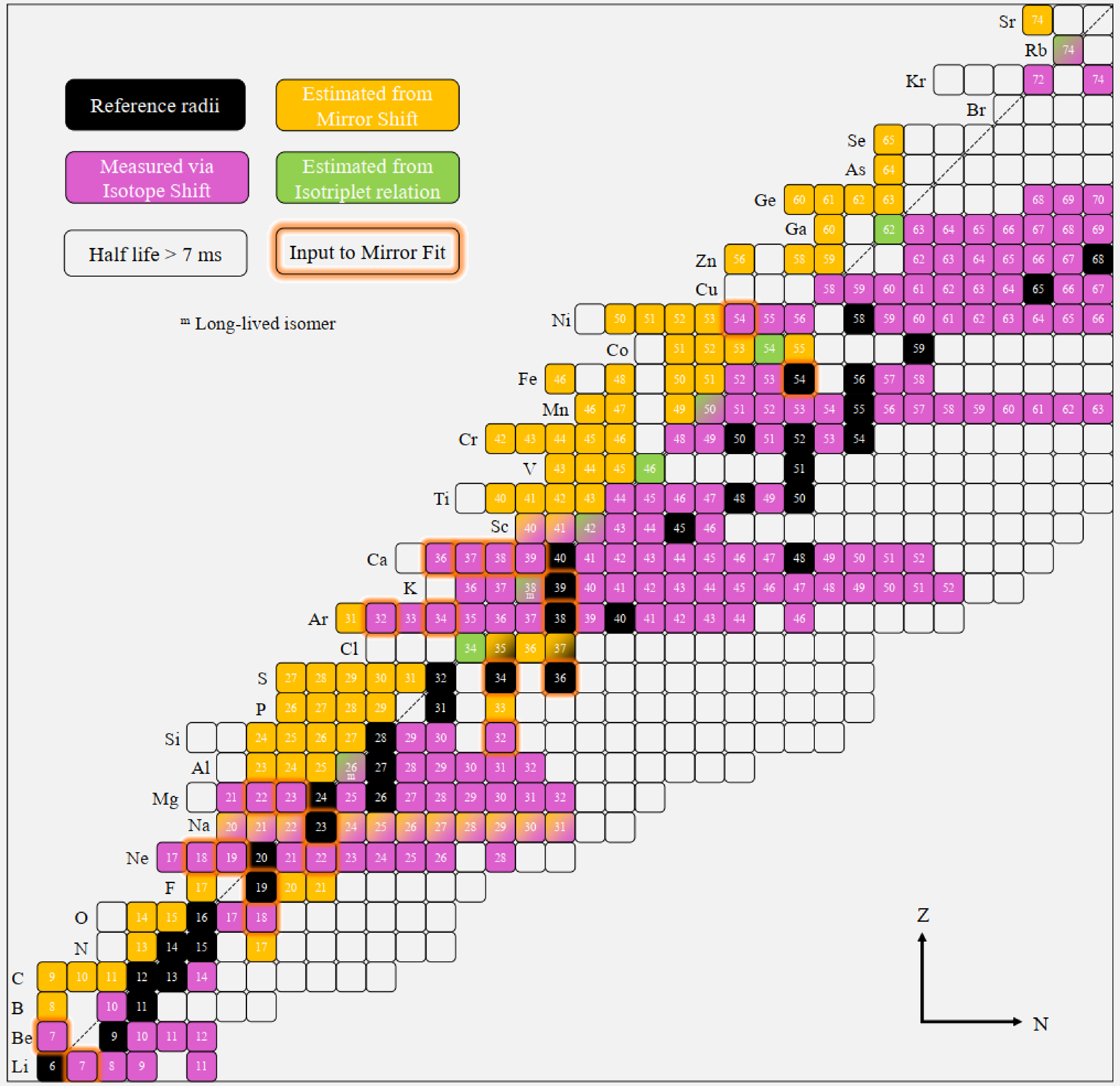
Vision & Precision in charge radii determinations

arXiv:2409.08193



Ben Ohayon | Technion IIT | boahyon@technion.ac.il

Technical meeting at the IAEA, January 26-30, 2025

Precision ⟷ Vision

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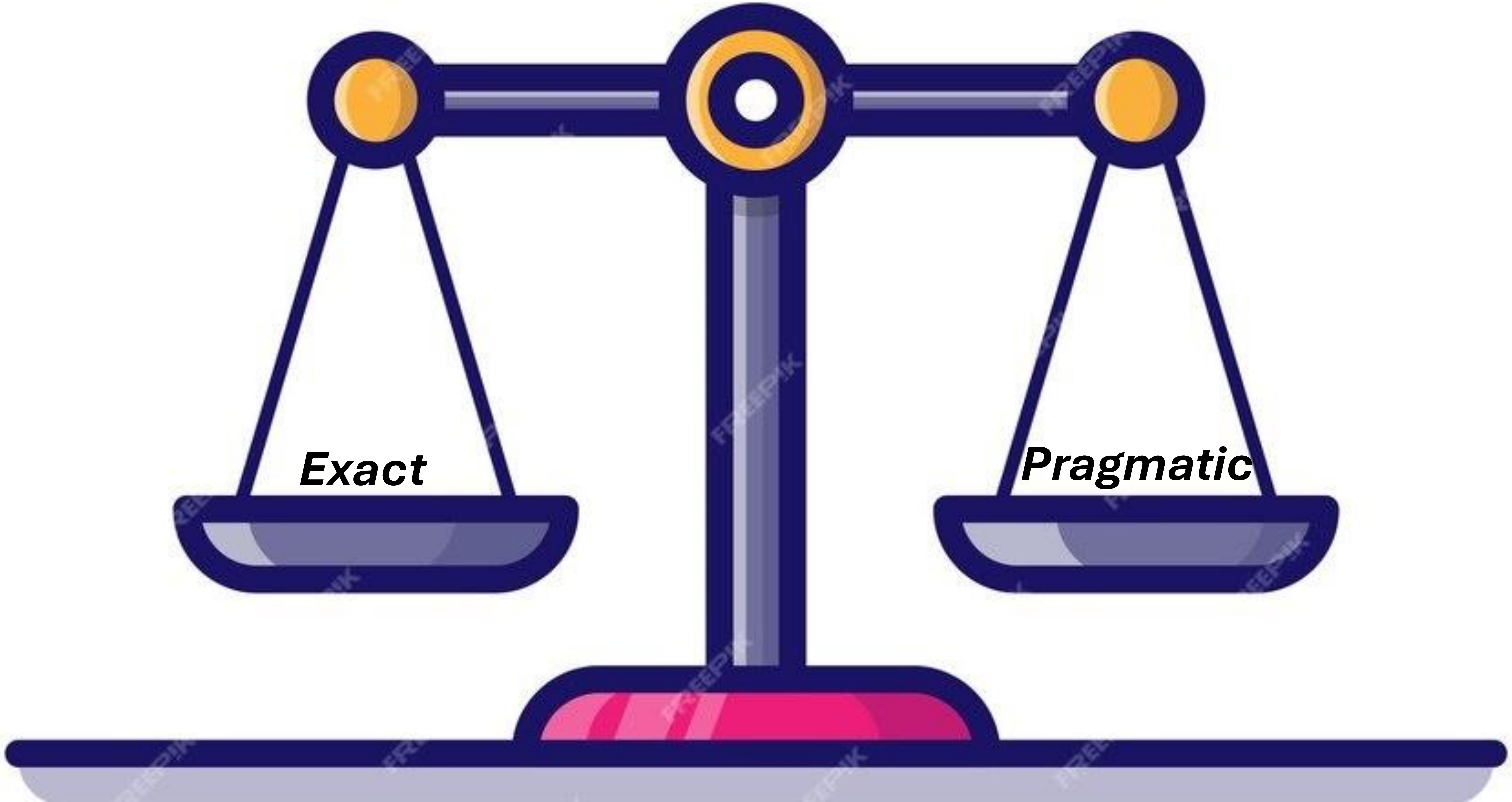

- Increased demand from nuclear physics (e.g. V_{ud})

(MG)

- Laser spectroscopy going from strength-to-strength

(DY, XY, KF, WN, …)

- ‘New’ kids on the block: H-like, He-like, Na-like ions

g-factor (FH) Laser (WN) EUV spec. (AT)

- Revival of muonic atoms (experiment+theory)

(NO, MG)

Vision: “AME” for nuclear charge radii, “NCRE”?

- Increased demand from nuclear physics (e.g. V_{ud})

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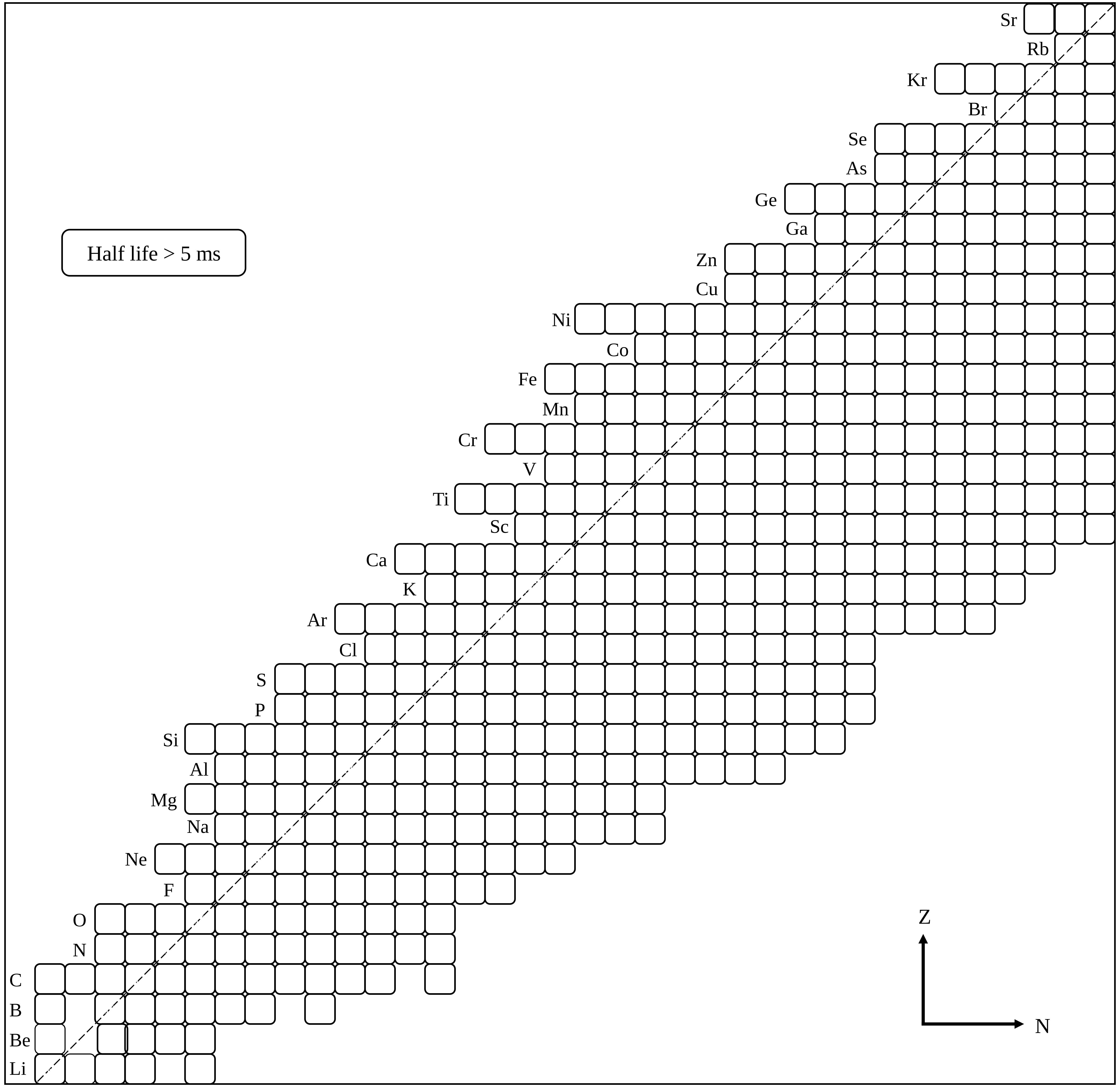
In this talk:

- Overview: Reference-radii
- Focus on light muonic atoms
- Reanalysis project(s):
  from isotope shifts to charge radii

Where do charge radii come from?



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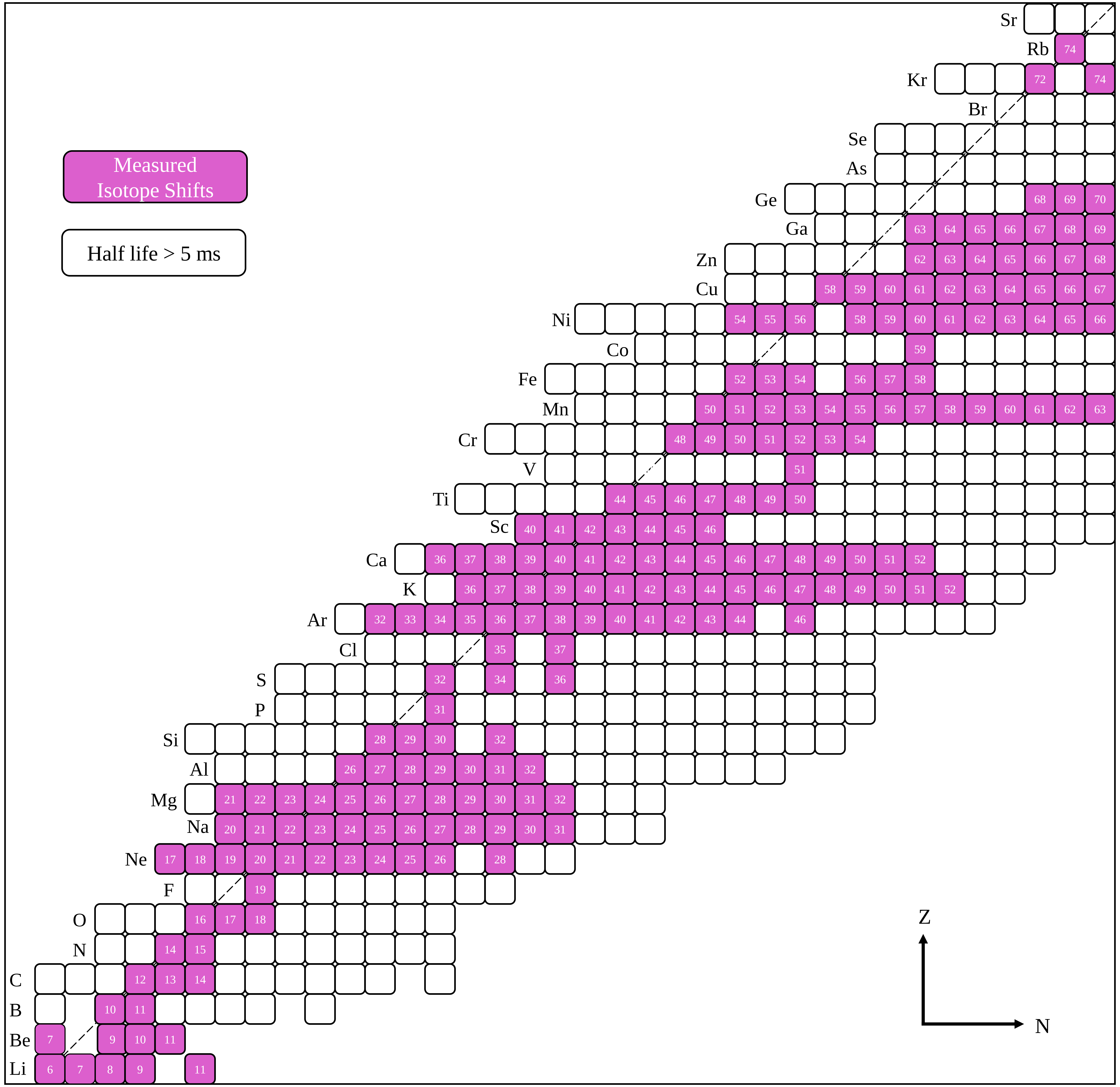
Where do charge radii come from?

Extraction of MS radius difference from measurements

$$\delta\nu_{A,A'} \approx \left(\frac{1}{M_{A'}} - \frac{1}{M_A}\right) K + F \delta r^2_{A,A'}$$



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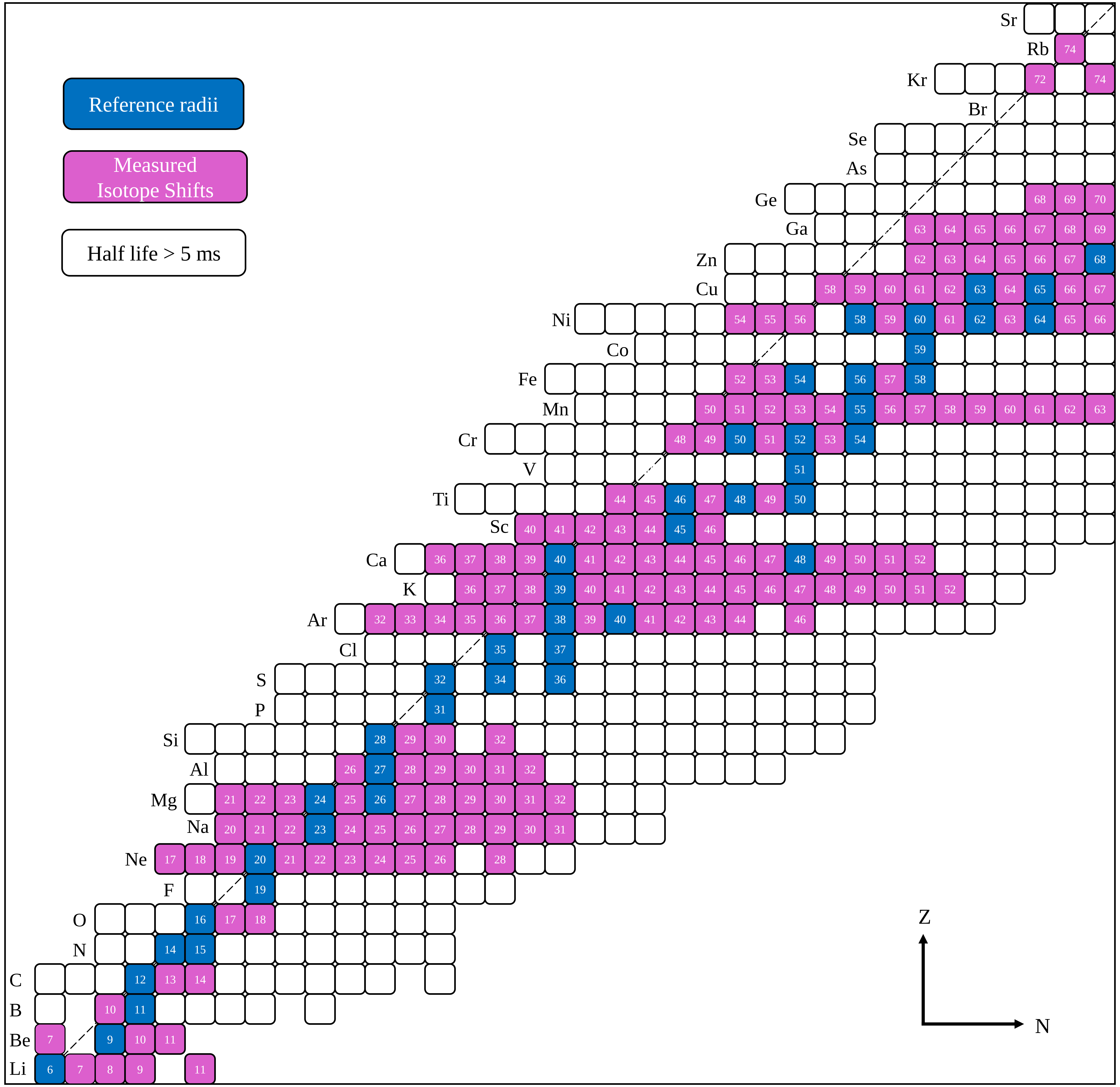
Reference radii connect MS differences with absolutes

$$r^2_{A'} = r^2_A + \delta r^2_{A,A'}$$



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$$\delta\nu_{A,A'} \approx \left(\frac{1}{M_{A'}} - \frac{1}{M_A}\right)\boldsymbol{K} + \boldsymbol{F}\delta r^2_{A,A'}$$

Atomic factors, either calculated or extracted from reference radii (King Plot).

Reference radii connect MS differences with absolutes

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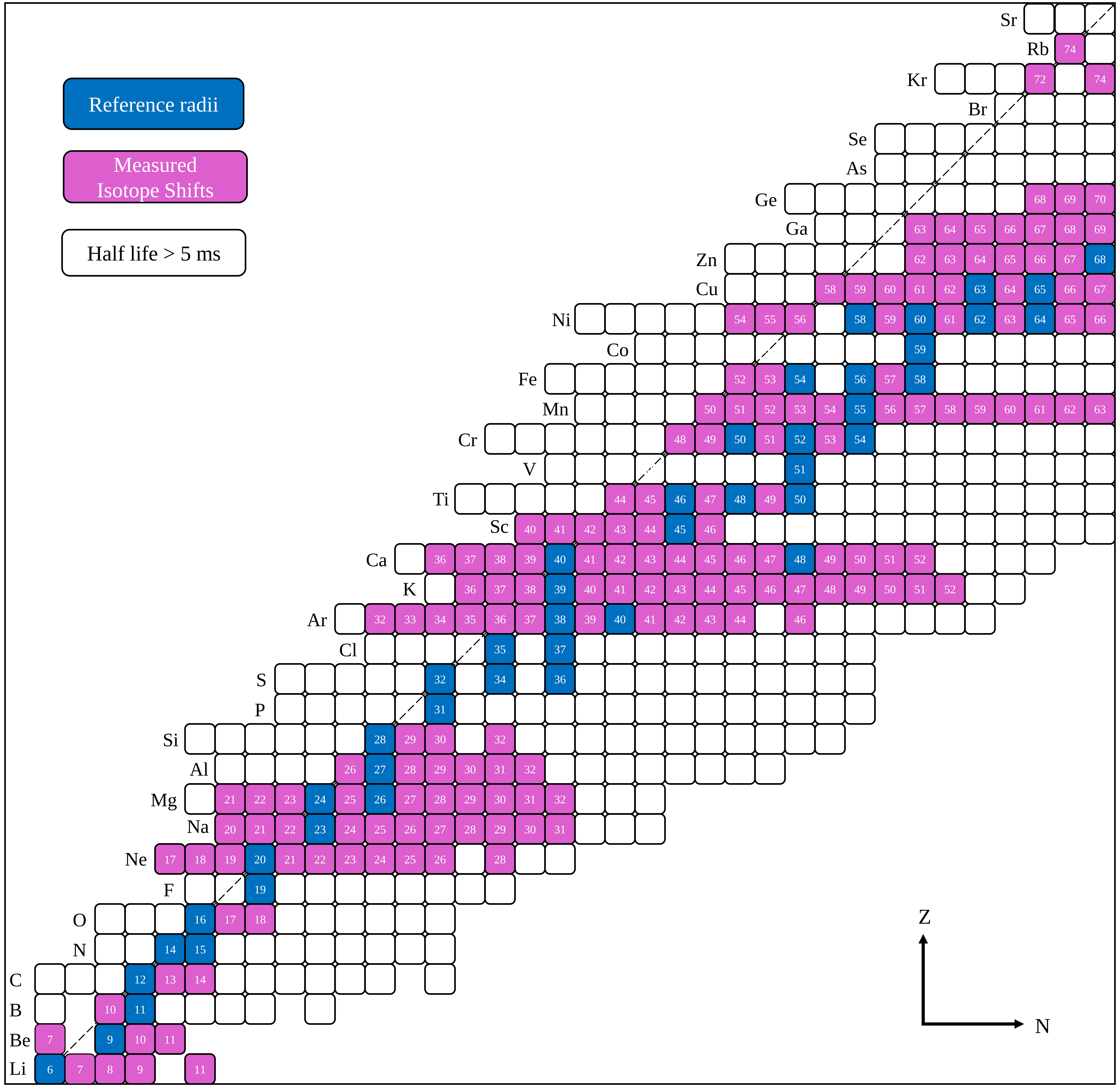
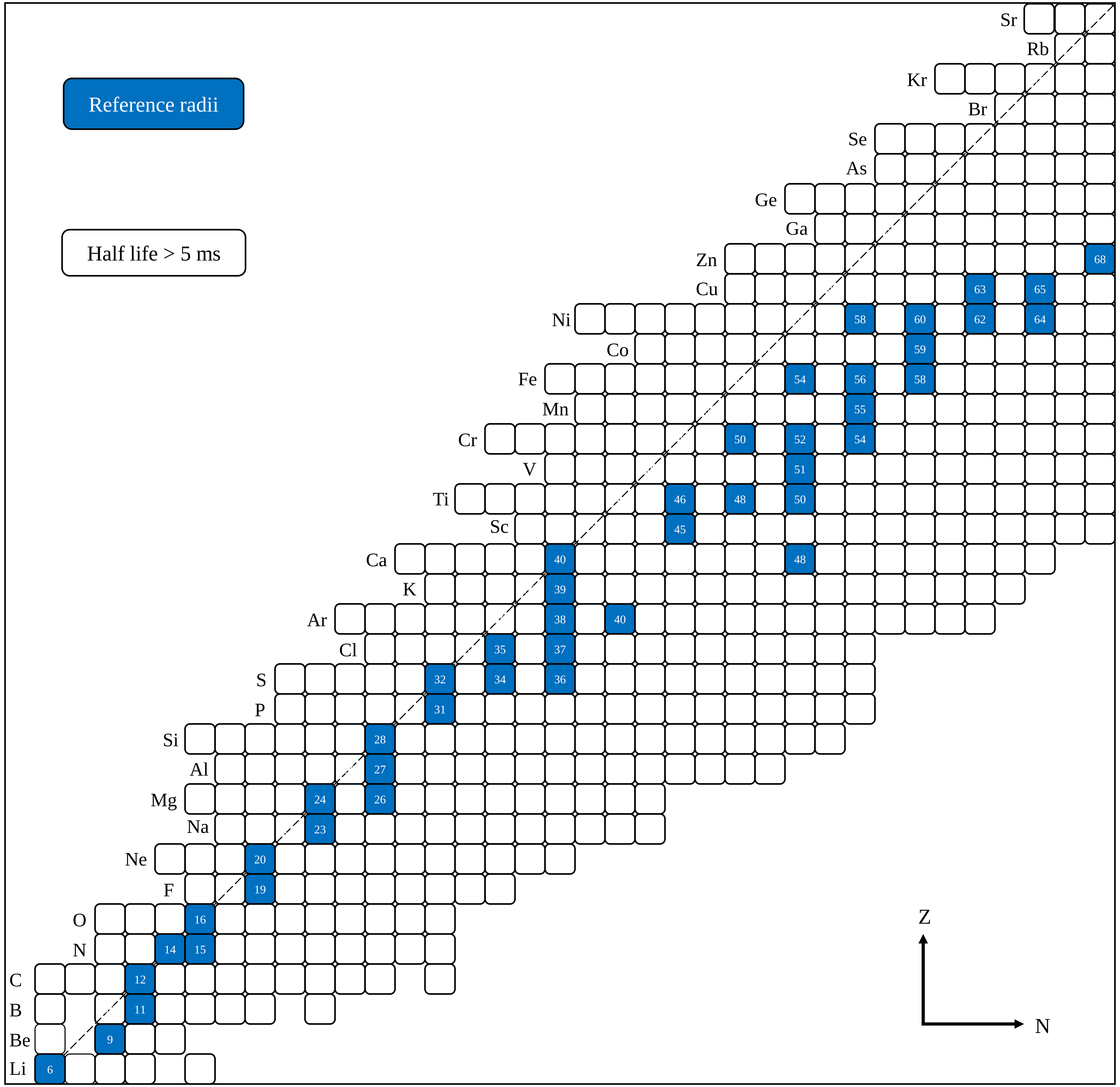


Reference radii and where to find them



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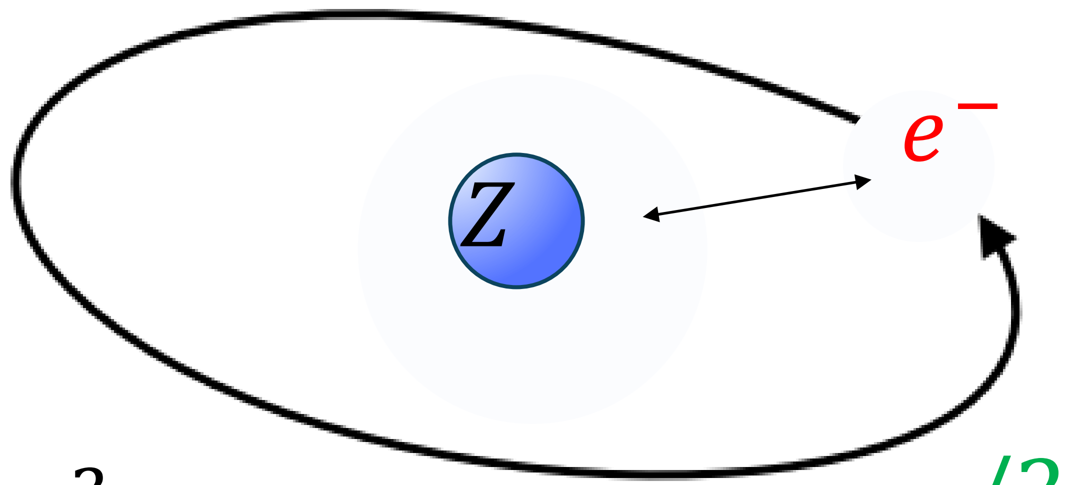
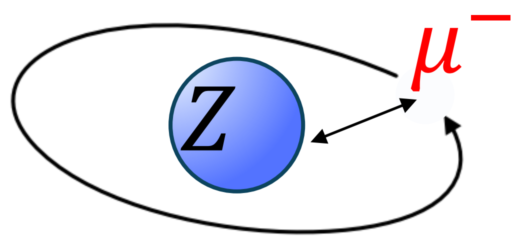
Muonic Atoms 101:

Ordinary atoms



Muonic atoms



Characteristic **length**

(Bohr radius: $a_0 = \frac{\hbar}{m_e c \alpha} \sim 0.5\text{Å}$):

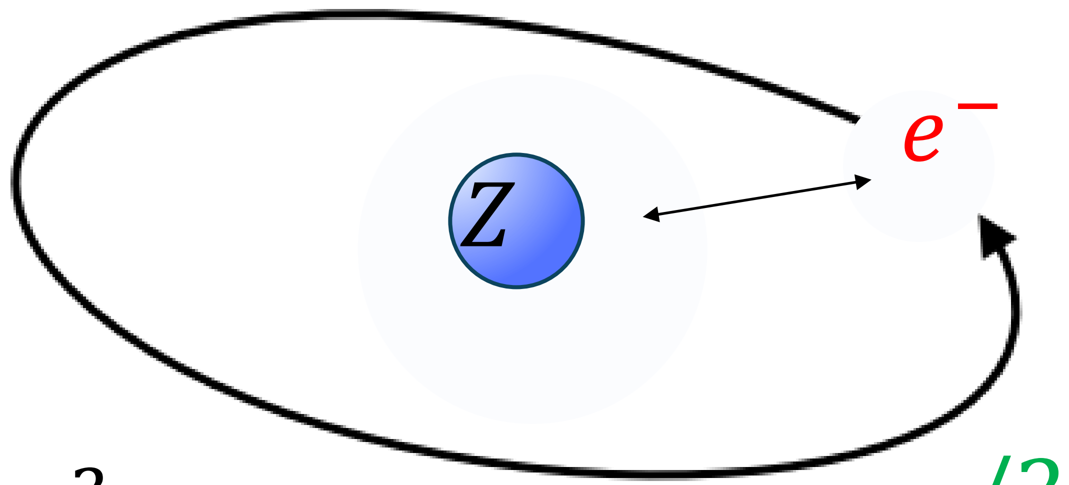
$$a_n = \frac{n^2 a_0}{Z} \xrightarrow{/200} \frac{n^2 a_0}{Z} \frac{m_e}{m_\mu}$$

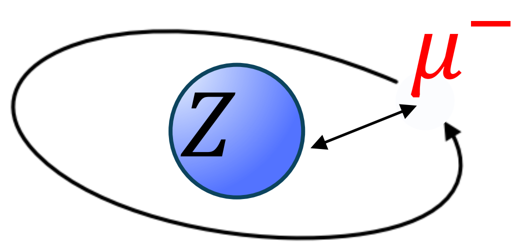
Shorter distances

bohayon@technion.ac.il

Muonic Atoms 101:

Ordinary atoms | Muonic atoms





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Shorter distances

Characteristic **Energy**

(Rydberg: $R_\infty = \frac{\alpha}{2a_0} \sim 13.6\ eV$):

$$E_n = -\frac{Z\alpha}{2a_n} = -\frac{R_\infty Z^2}{n^2} \xrightarrow{\times 200} E_n = -\frac{R_\infty Z^2}{n^2}\frac{m_\mu}{m_e}$$

$MW \rightarrow Laser$

$Laser \rightarrow x - ray$

Higher energies

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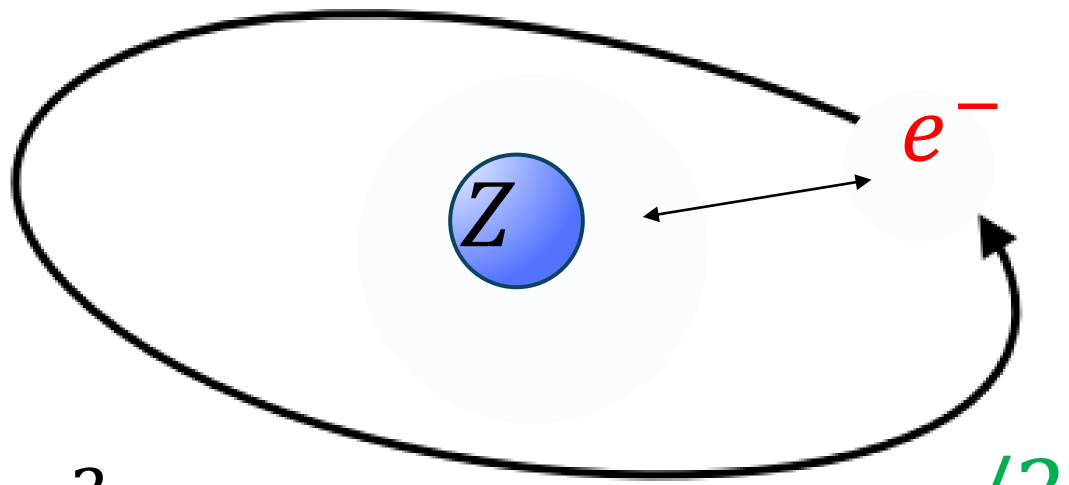
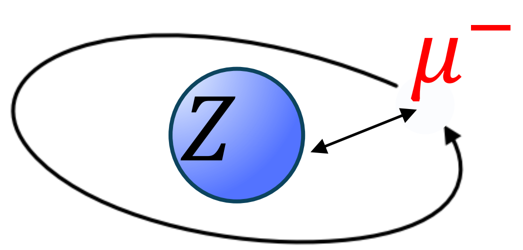
Muonic Atoms 101:

	Ordinary atoms		Muonic atoms	
				
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Characteristic **Energy** (Rydberg: $R_\infty = \frac{\alpha}{2a_0} \sim 13.6\ eV$):	$E_n = -\frac{Z\alpha}{2a_n} = -\frac{R_\infty Z^2}{n^2}$	$\xrightarrow{\times 200}$ *MW → Laser* *Laser → x − ray*	$E_n = -\frac{R_\infty Z^2}{n^2} \frac{m_\mu}{m_e}$	Higher energies
Finite Nuclear Size effect:	$\Delta E_{FNS} \sim \frac{4}{3} \frac{R_\infty Z^4}{n^3} \left(\frac{r_c}{a_0}\right)^2 \delta_{l0}$	$\xrightarrow{\times (200)^3}$	$\frac{4}{3} \frac{R_\infty Z^4}{n^3} \left(\frac{r_c}{a_0}\right)^2 \left(\frac{m_\mu}{m_e}\right)^3 \delta_{l0}$	
For Hydrogen 1s-2p:	$\sim 4\ \text{neV}\ (1\ \text{MHz, ppb})$		$\sim 30\ \text{meV, } 10\ \text{ppm}$	

bohayon@technion.ac.il

Measuring nuclear radii with muonic atoms:

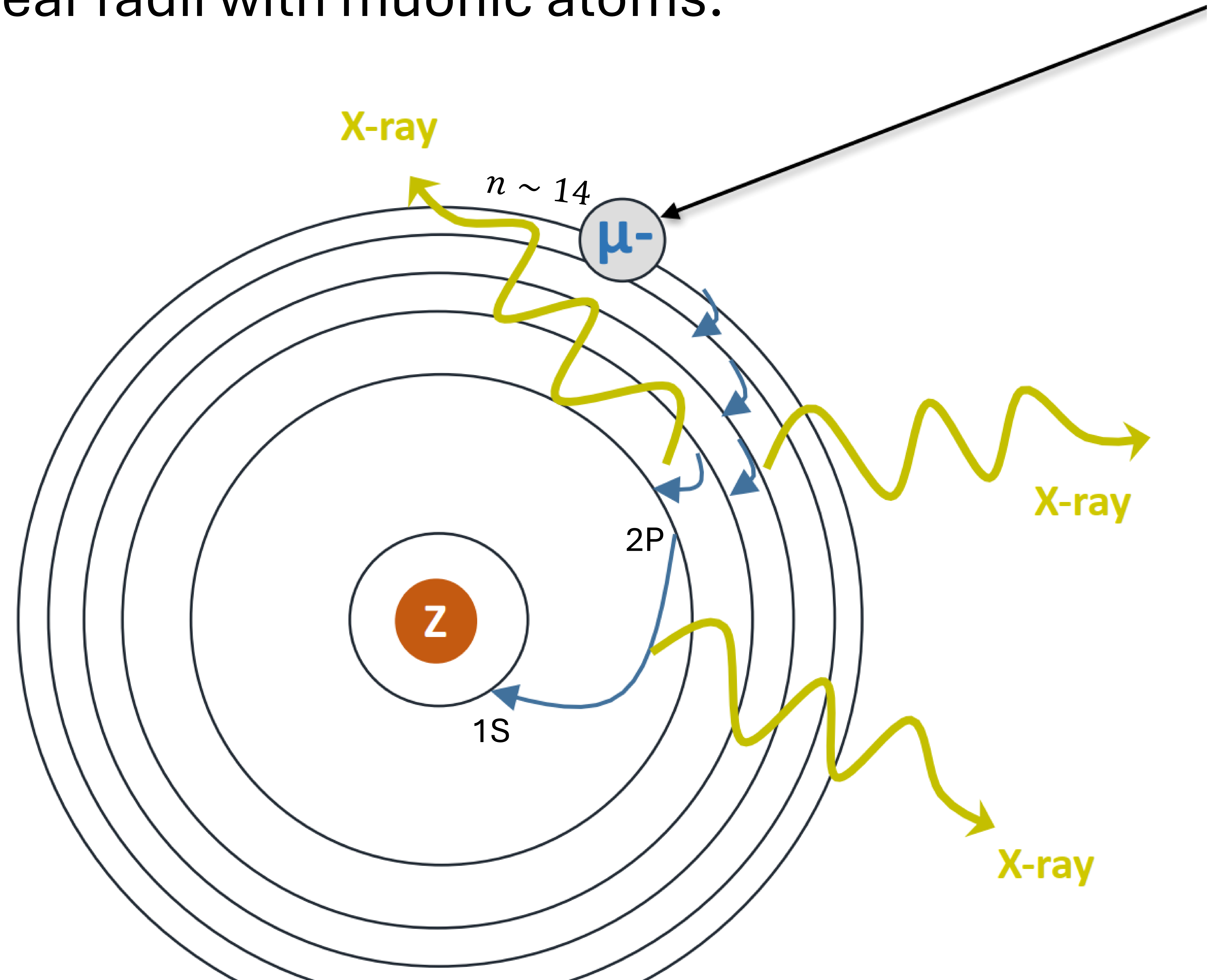
1. Captured around N=14

2. All electrons are emitted

3. Cascade to ground level

4. Muon decay $\sim 2\mu s$

5. $\mathrm{E}_{2\mathrm{P}-1\mathrm{S}} = E_{QED} + \Delta E_{FNS} + \cdots$



Ecosystem of charge radii determinations

Radius of unstable nucleus $r_x^2 = r_{ref}^2 + \delta r_{a,x}^2$

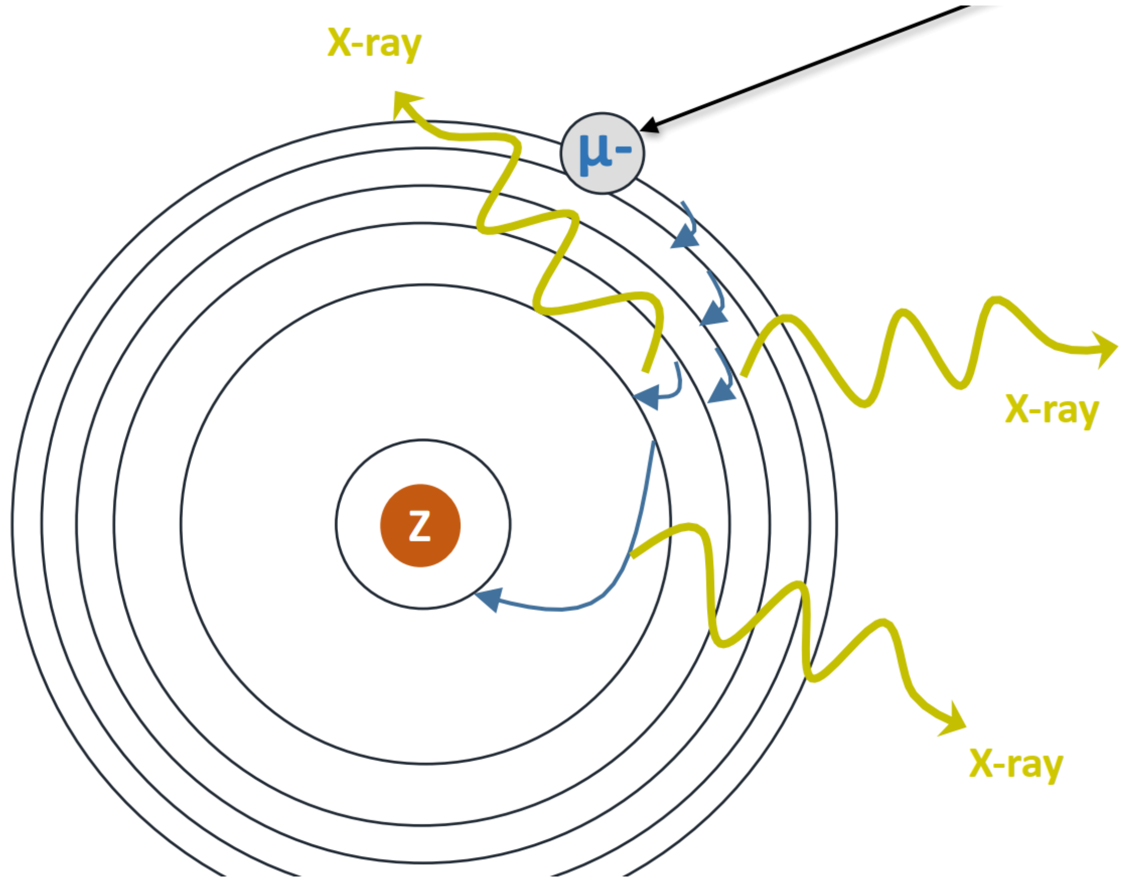
Reference radius (mostly) from **Muonic atoms**

Review

Differential radii (mostly) from radioactive **electronic atoms**

Experiments with exotic atoms

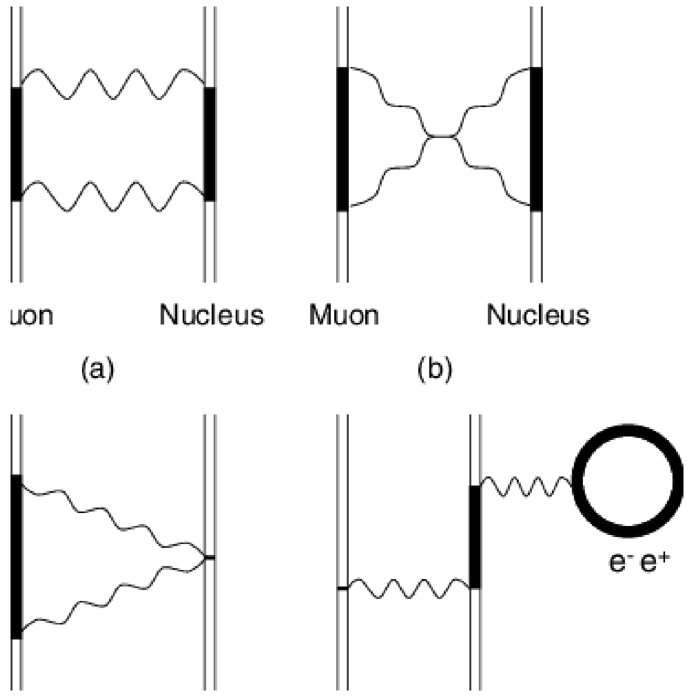
Muonic-atom spectroscopy



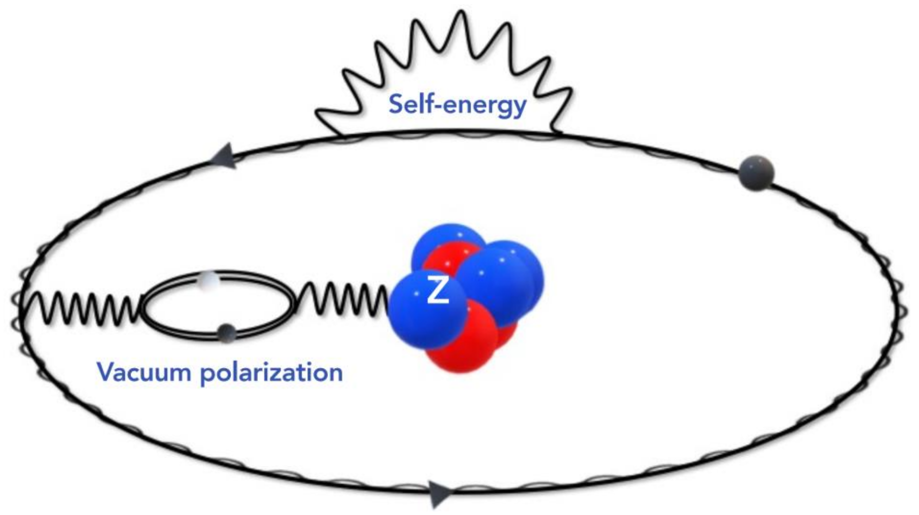
Recent Updates

Theory calculations

Nuclear:



Non-perturbative QED

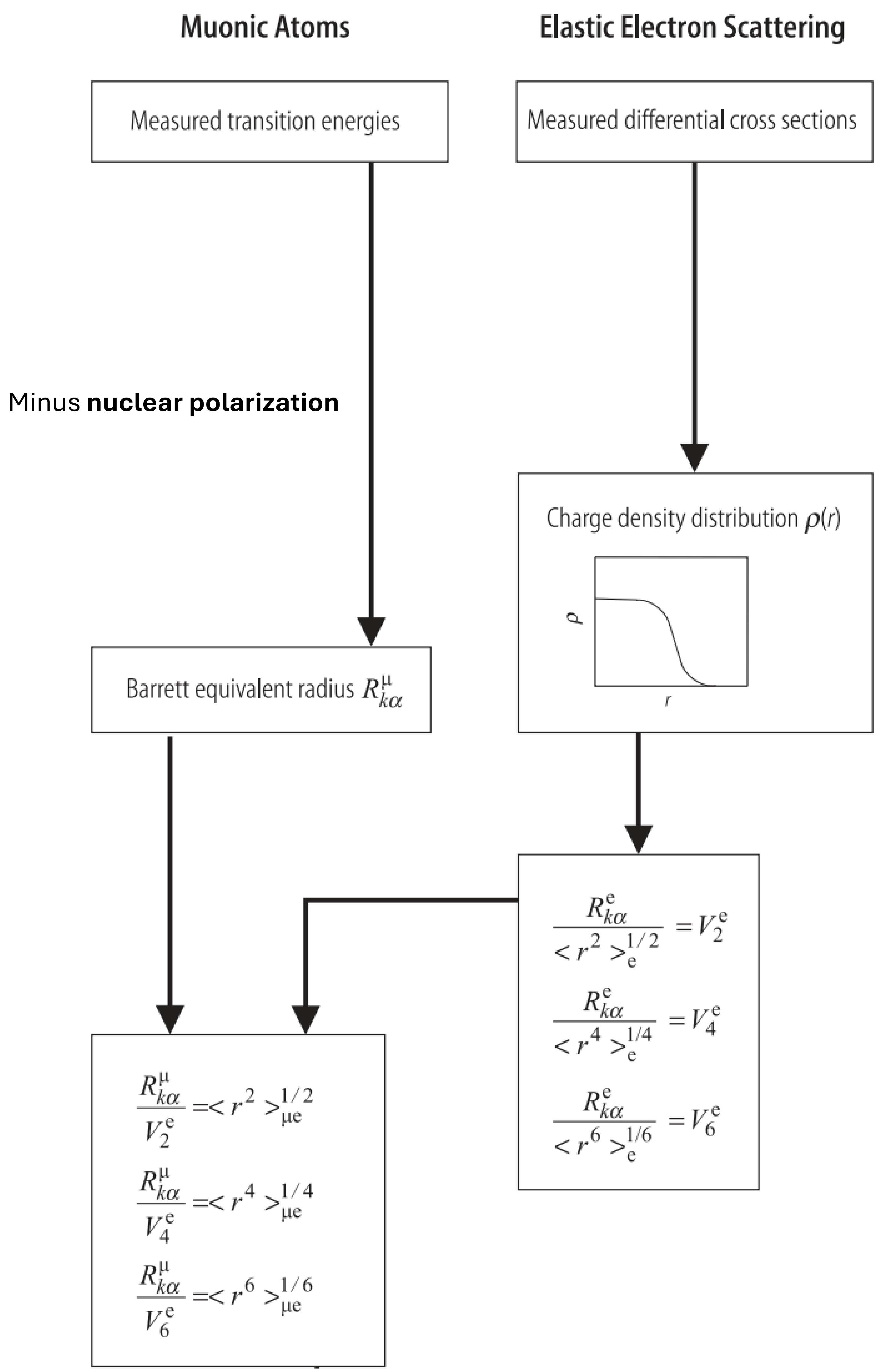


"Old school" combined analysis of muonic atoms and electron scattering:

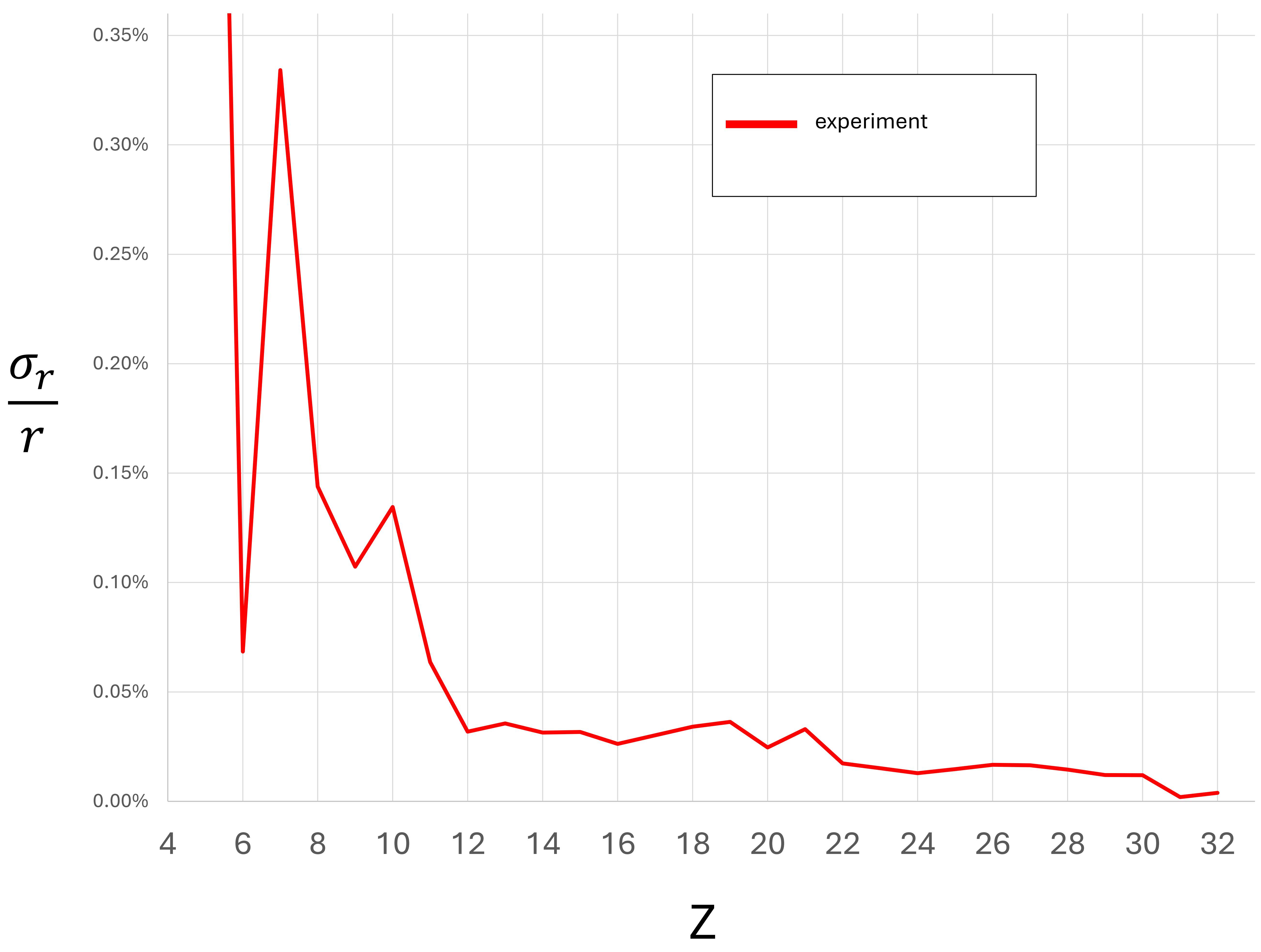
- What is the limitation of the "Barret recipe?"

Three sources of uncertainty:

1. Experiment (muonic atom energies)
2. Theory (nuclear polarization)
3. **Charge distribution** (scattering)



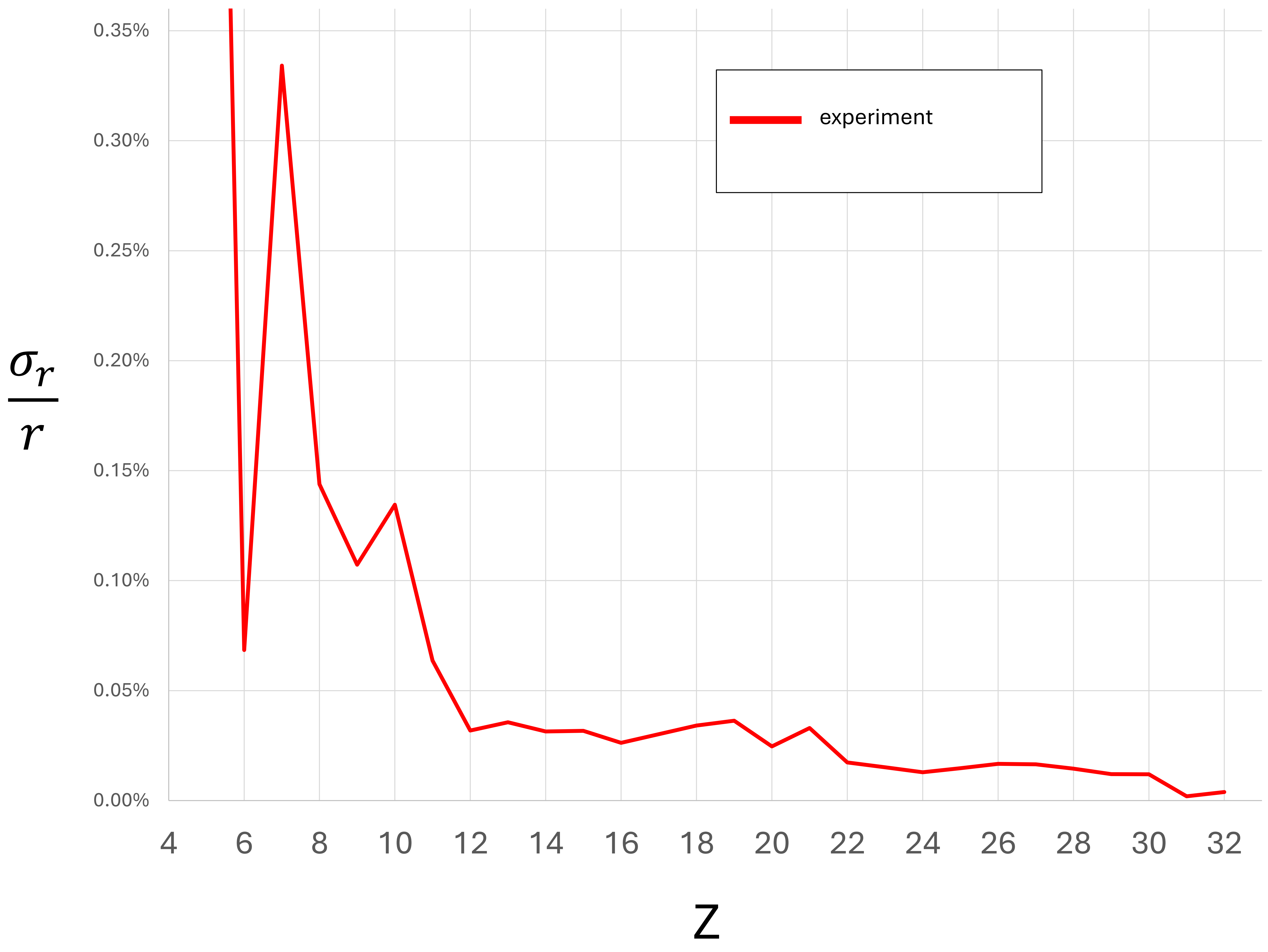
Sources of Uncertainty to Ref. Radii:



bohayon@technion.ac.il

Sources of Uncertainty to Ref. Radii:

- Notice: Fricke & Heilig only include statistical unc. -> consult original papers



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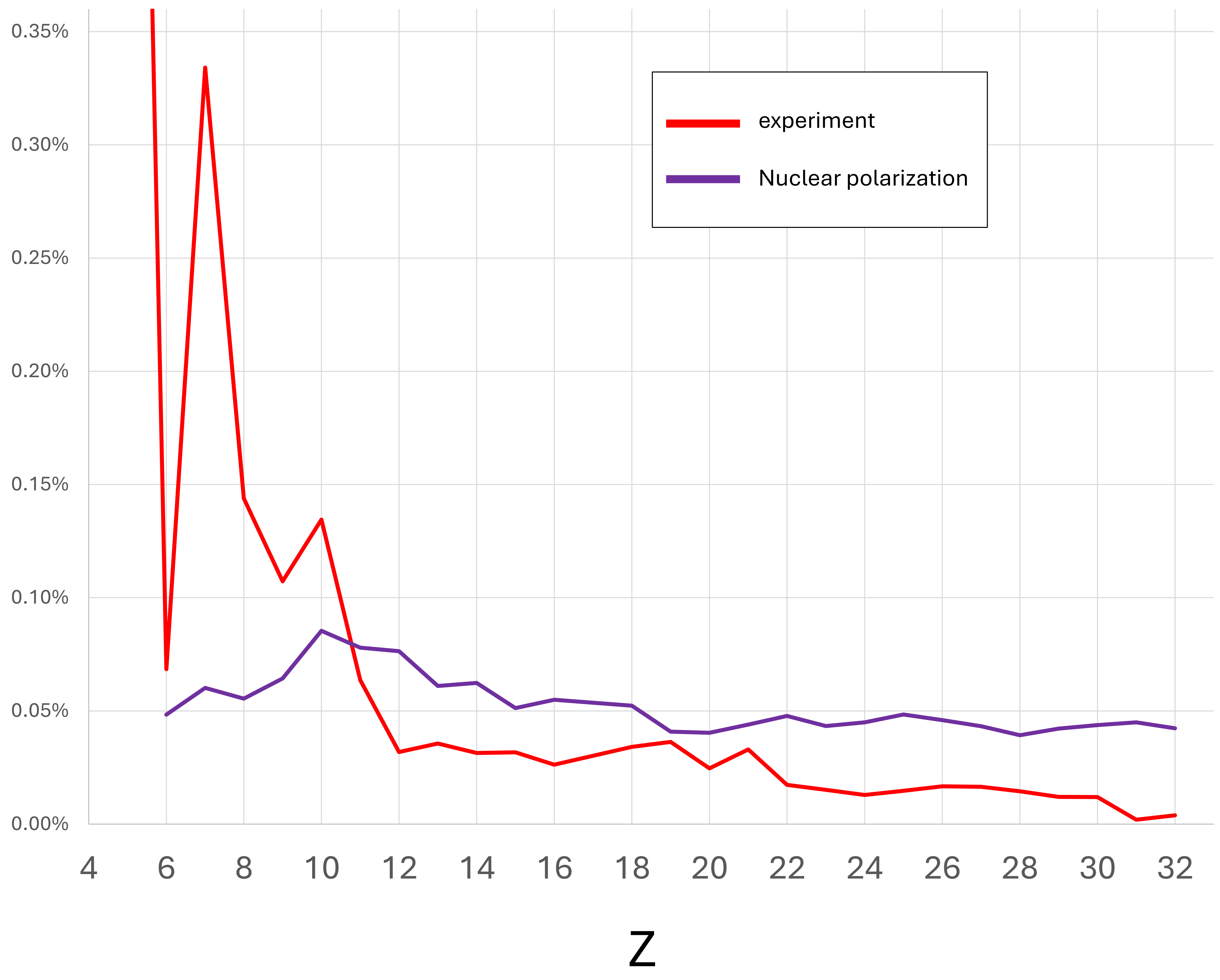
Sources of Uncertainty to Ref. Radii:

- Notice: nuclear polarization from Rinker & Speth 1970s, assumed 30% unc. – **MG & NO talks**!



$$\frac{\sigma_r}{r}$$

How about electron scattering?

- Experimental uncertainty (normalization) goes away for ratios

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APH N.S., Heavy Ion Physics 15/1–2 (2002) 87–102

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Barrett Moments and *rms* Charge Radii

I. Angeli[a]

Institute of Experimental Physics, University of Debrecen
H-4010 Debrecen, P.O. Box 81, Hungary

Received 21 August 2001

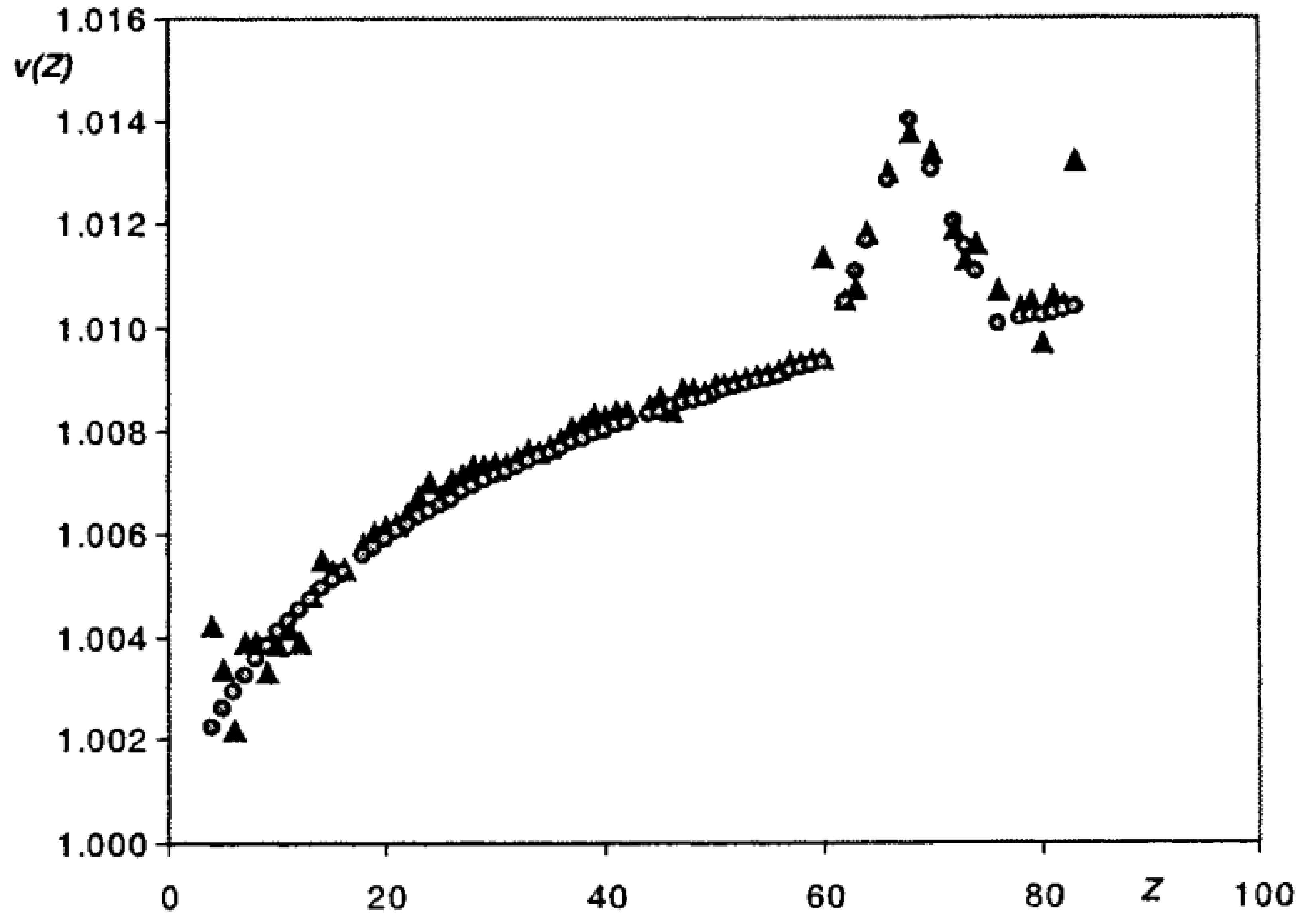

Fig. 1. Z dependence of the ratio v, Eq. (3). Triangles: v values derived from $R_{k,\alpha}$ and $< r^2 >^{1/2}$ data; circles: calculated by the empirical formula (5).

Uncertainty in ratios of moments?

- Experimental uncertainty (normalization) goes away for ratios
- Residual model-dependency from finite momentum transfer
- How much does the second-best scattering measurement deviates from the best one?

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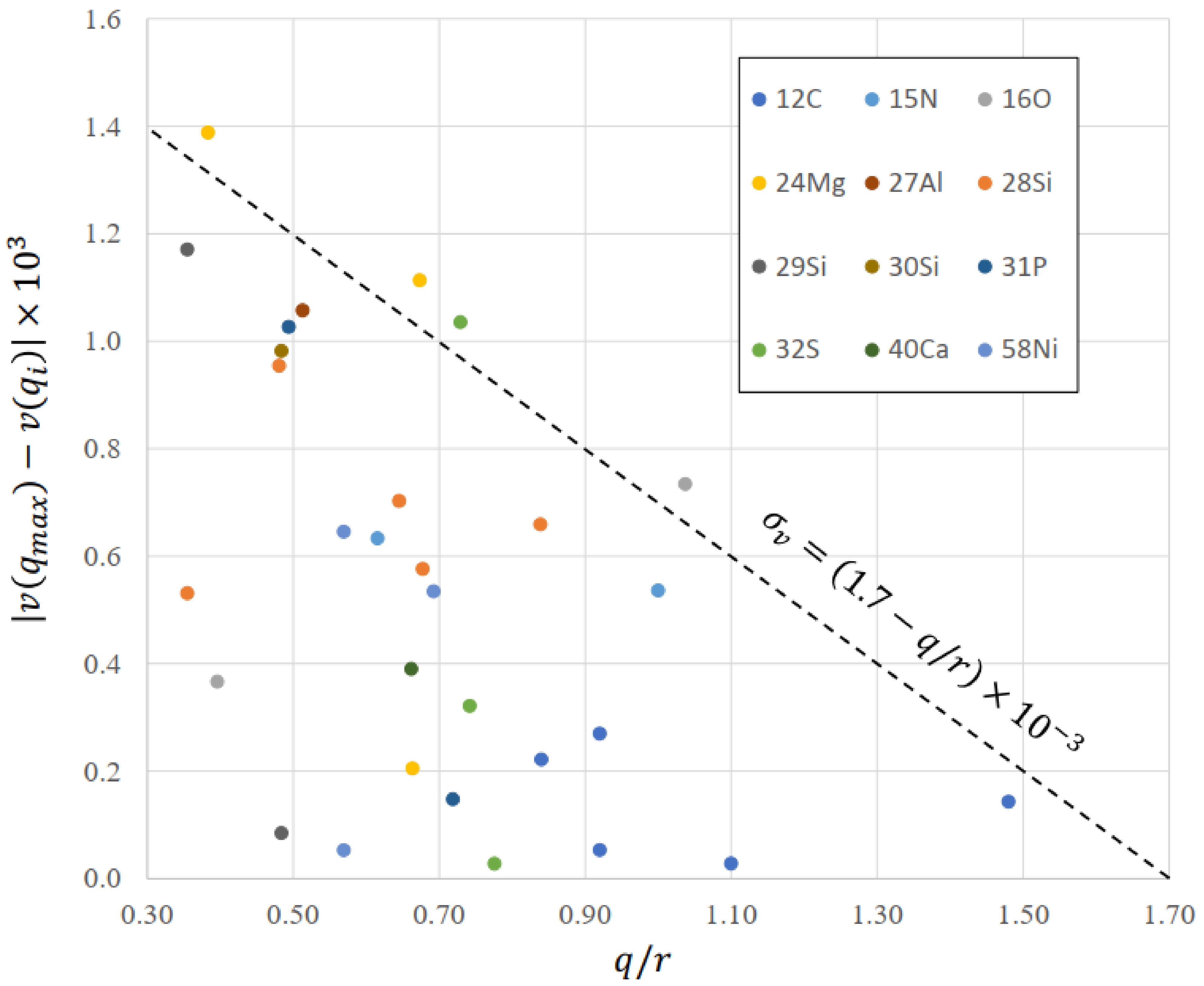
Quantify our intuition:

Best experiment has broad momentum transfer compared to nuclear size.



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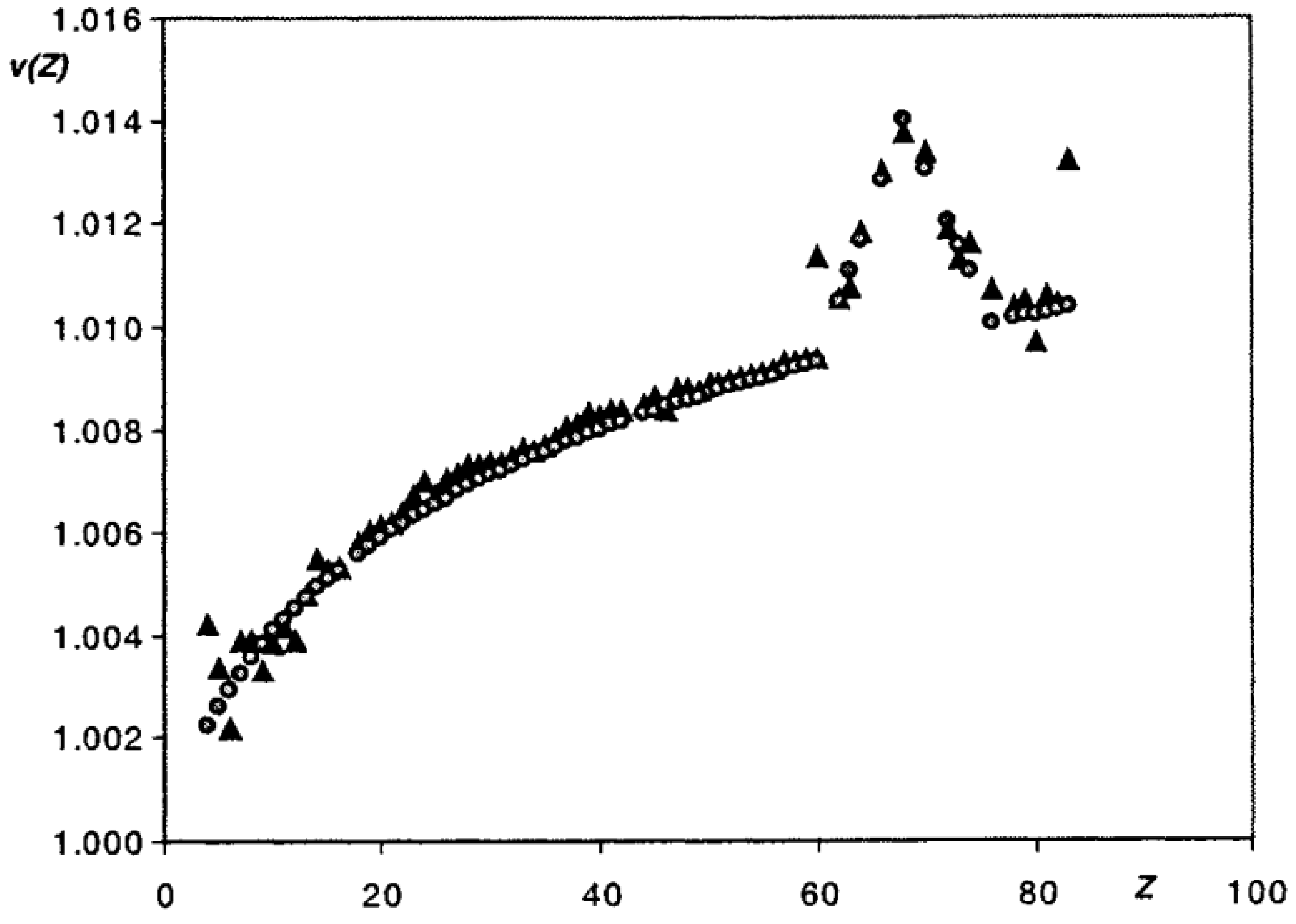

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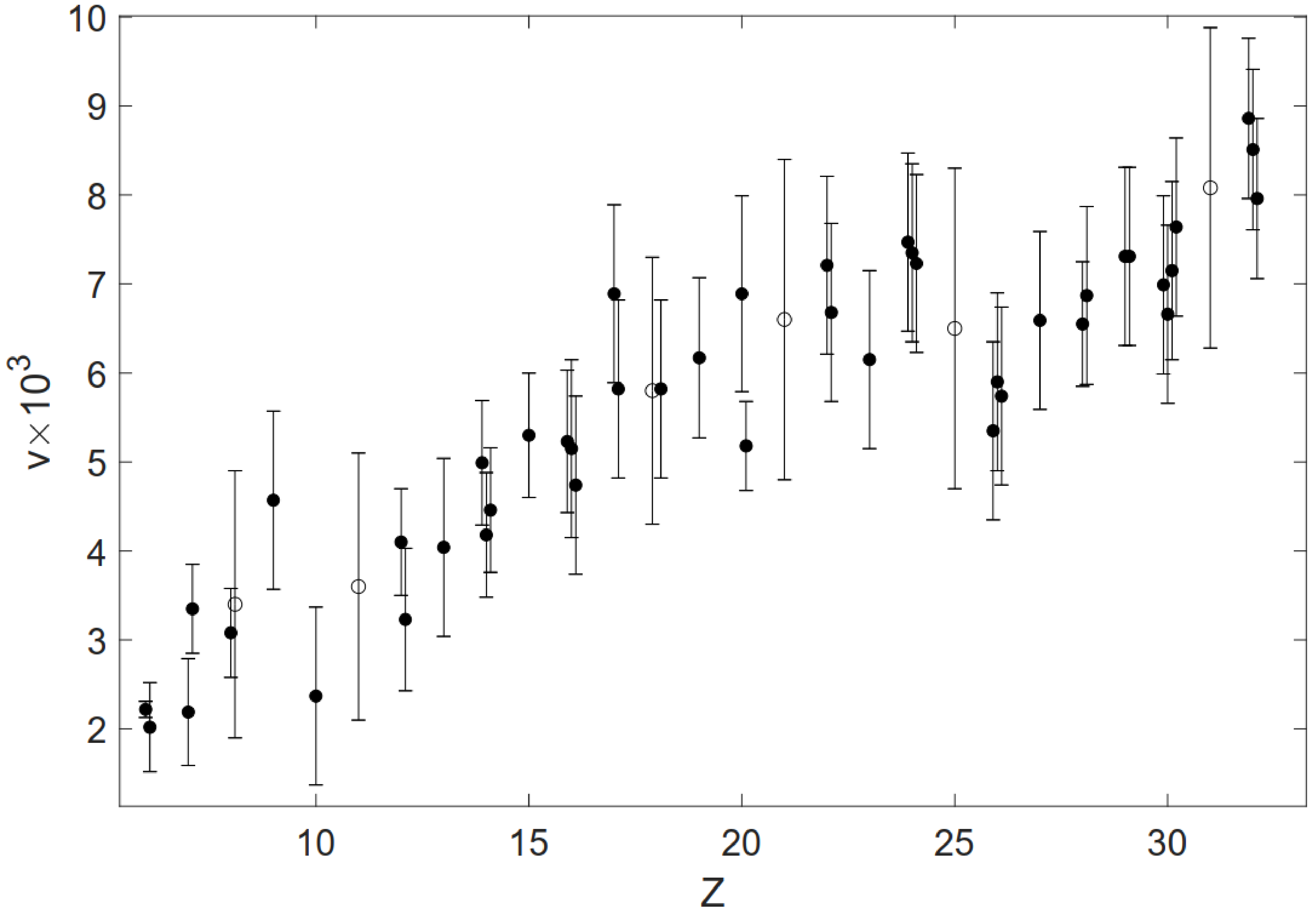
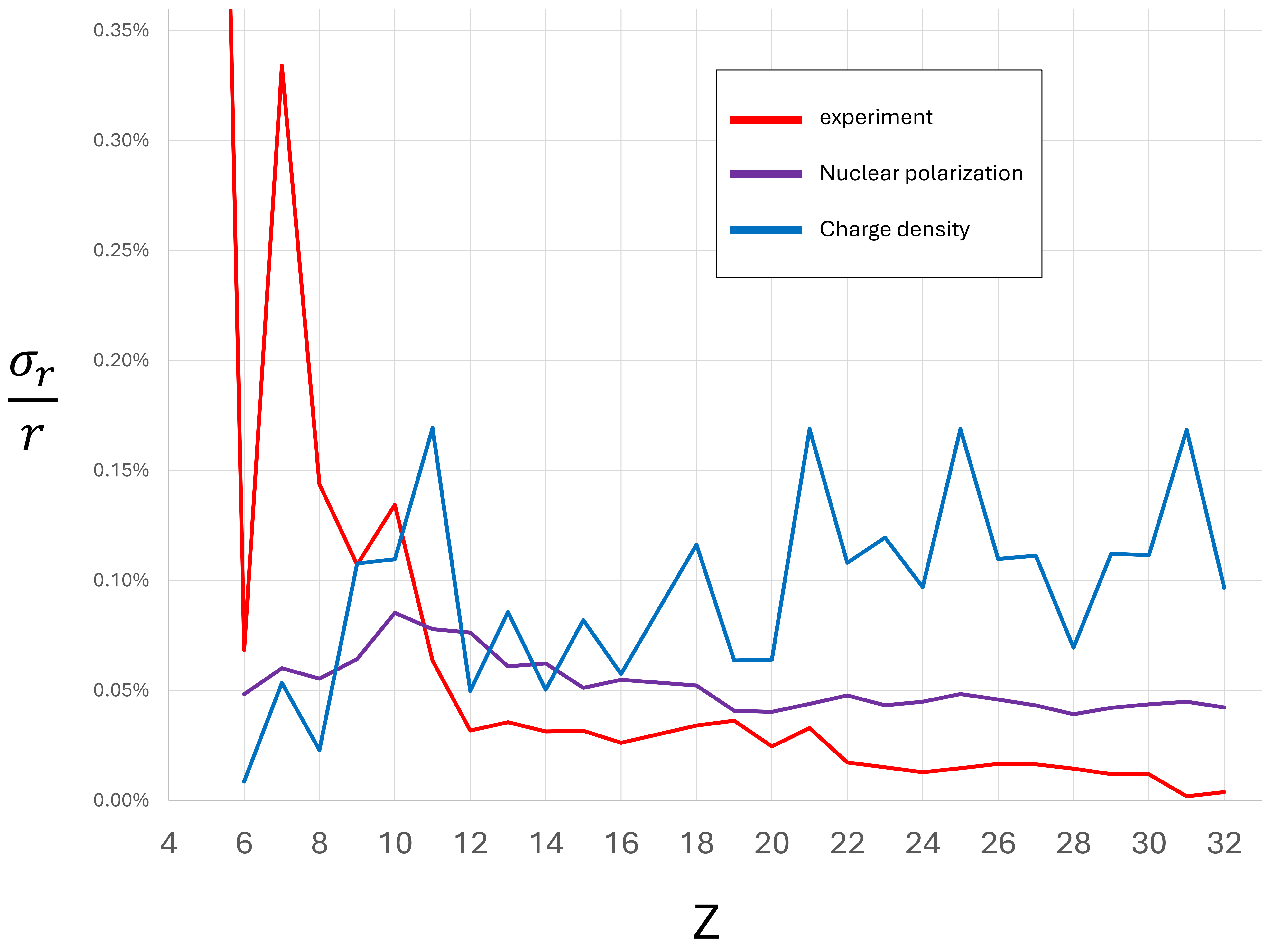


Sources of uncertainty :

Radii of light nuclei from muonic atom x-ray spec.

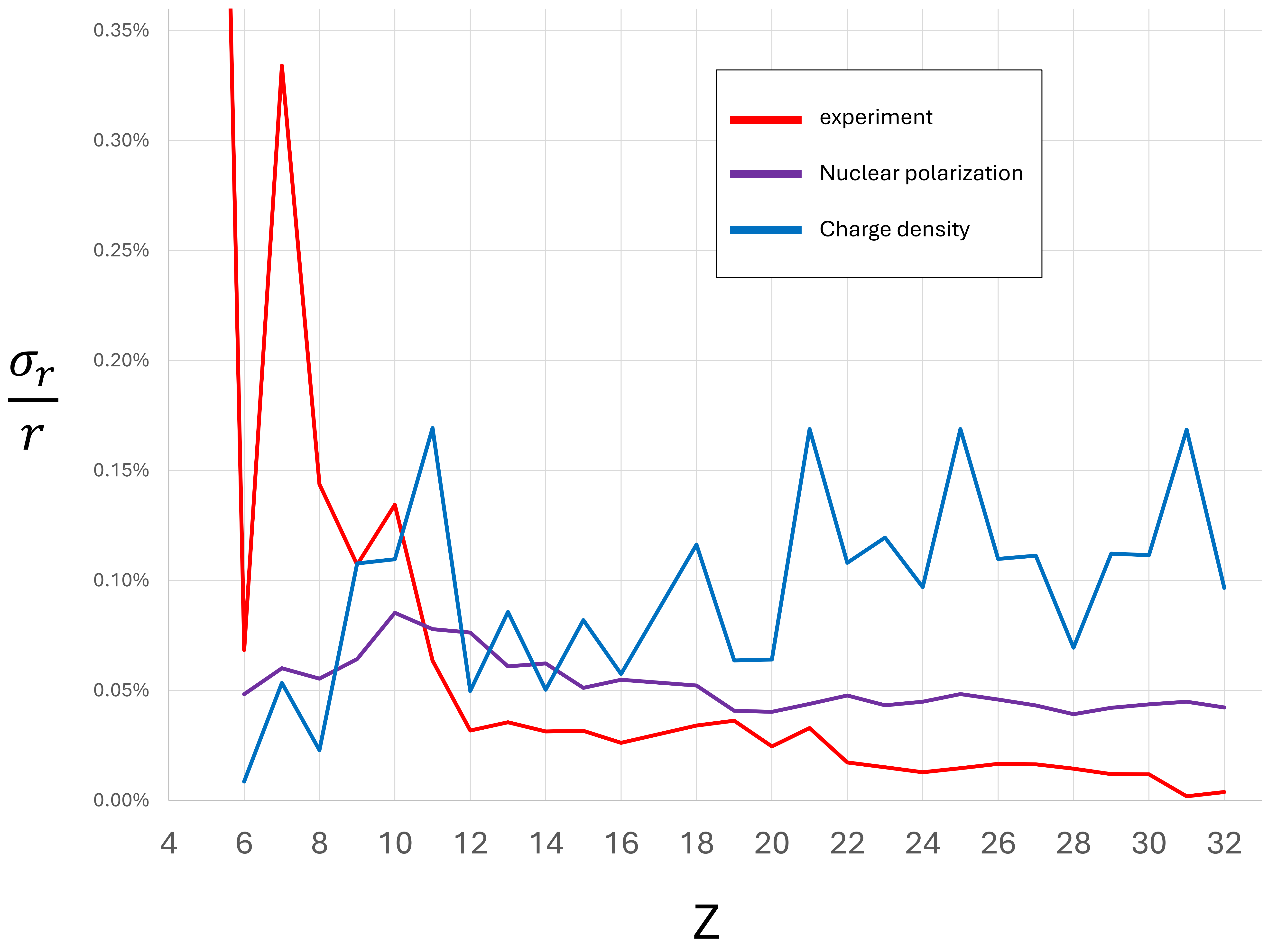


bohayon@technion.ac.il

Sources of uncertainty :

Radii of light nuclei from muonic atom x-ray spec.

- Within Barret-recipe: scattering model dependency matters !



bohayon@technion.ac.il

Transparent tabulation (inviting your input!) arXiv:2409.08193

Table 2

Reference radii used in this work. Unless stated otherwise in the note, they are determined via Eq. 1 and 2 with the $2P_{3/2} - 1S$ Barret radii given in [4] and the v factors from tab. 1. Uncertainties are denoted by σ and correspond to statistics and energy calibration (exp), nuclear polarization (NP), and charge distribution (CD) as resulting from the ν factors of Tab. 1

el.	Z	A	r_{ch}	σ_{exp}	σ_{NP}	σ_{CD}	σ_{tot}	Note
Li	3	6	2.589	0.039			0.039	A
Be	4	9	2.519	0.012		0.030	0.032	B
B	5	11	2.411	0.021			0.021	C
C	6	12	2.483	0.002	0.001	0.000	0.002	D
N	7	14	2.556	0.009	0.002	0.001	0.009	
	7	15	2.612	0.009			0.009	E
O	8	16	2.701	0.004	0.001	0.001	0.004	F
F	9	19	2.902	0.003	0.002	0.003	0.005	†
Ne	10	20	3.001	0.004	0.003	0.003	0.006	†
Na	11	23	2.992	0.002	0.002	0.005	0.006	†
Mg	12	24	3.056	0.001	0.002	0.002	0.003	†
	12	26	3.030	0.001	0.002	0.002	0.003	†
Al	13	27	3.061	0.001	0.002	0.003	0.003	†G
Si	14	28	3.123	0.001	0.002	0.002	0.003	†
P	15	31	3.190	0.001	0.002	0.002	0.003	
S	16	32	3.262	0.001	0.002	0.003	0.003	
	16	34	3.284	0.001	0.002	0.003	0.004	
	16	36	3.298	0.001	0.001	0.003	0.004	
Cl	17	35	3.388	0.015			0.015	H
Cl	17	37	3.384	0.015			0.015	H
Ar	18	38	3.402	0.002	0.003	0.005	0.006	
	18	40	3.427	0.001	0.002	0.003	0.004	
K	19	39	3.435	0.001	0.001	0.003	0.004	
Ca	20	40	3.481	0.001	0.001	0.004	0.004	
	20	48	3.475	0.001	0.001	0.002	0.002	
Sc	21	45	3.548	0.001	0.002	0.006	0.007	

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Example: Ne	10	20	3.001	0.004	0.003	0.003	0.006	AM: 3.006(2) fm
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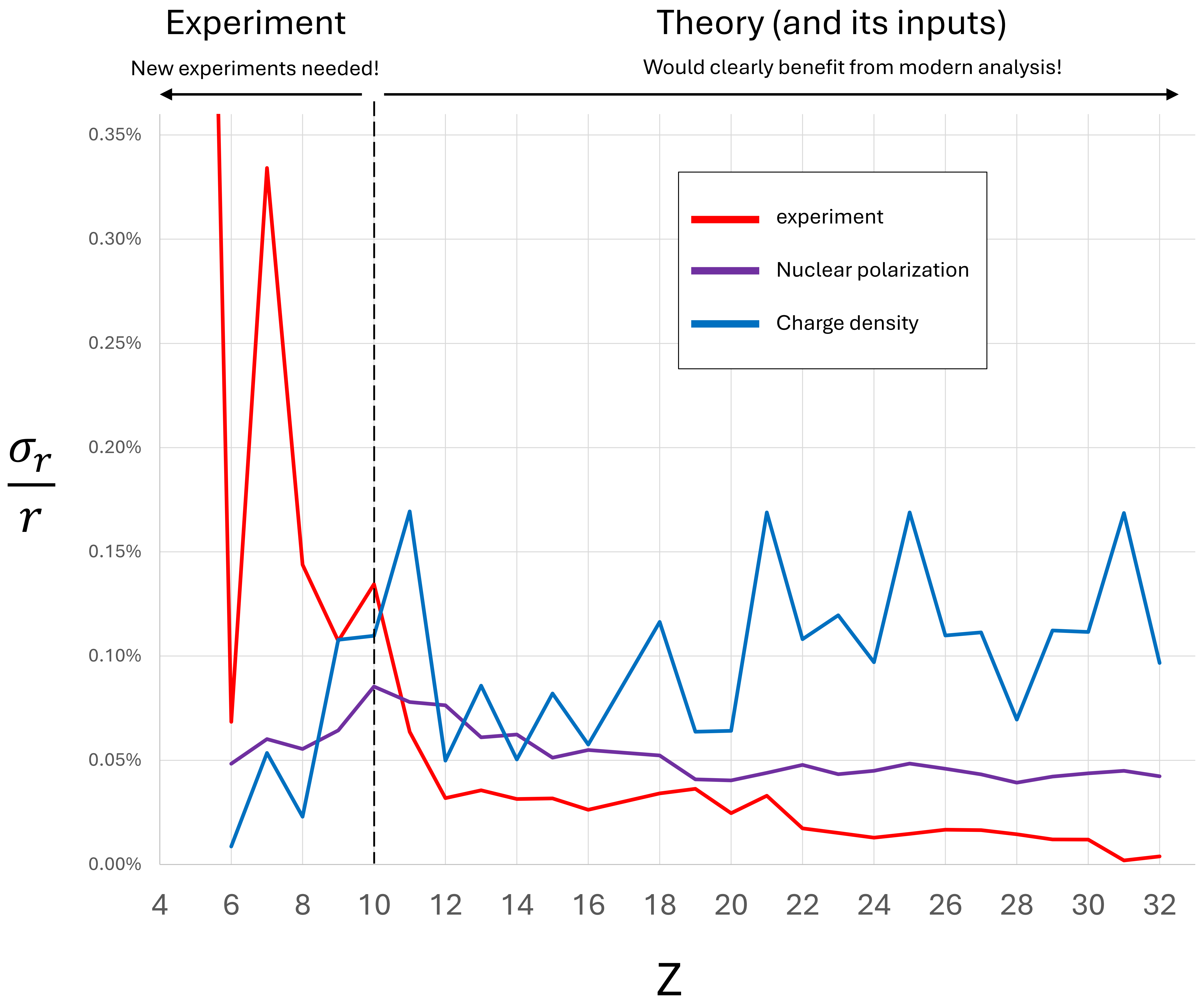
Sources of uncertainty :

Radii of light nuclei from muonic atom x-ray spec.

What is the spike in exp. uncertainty < Z=11?



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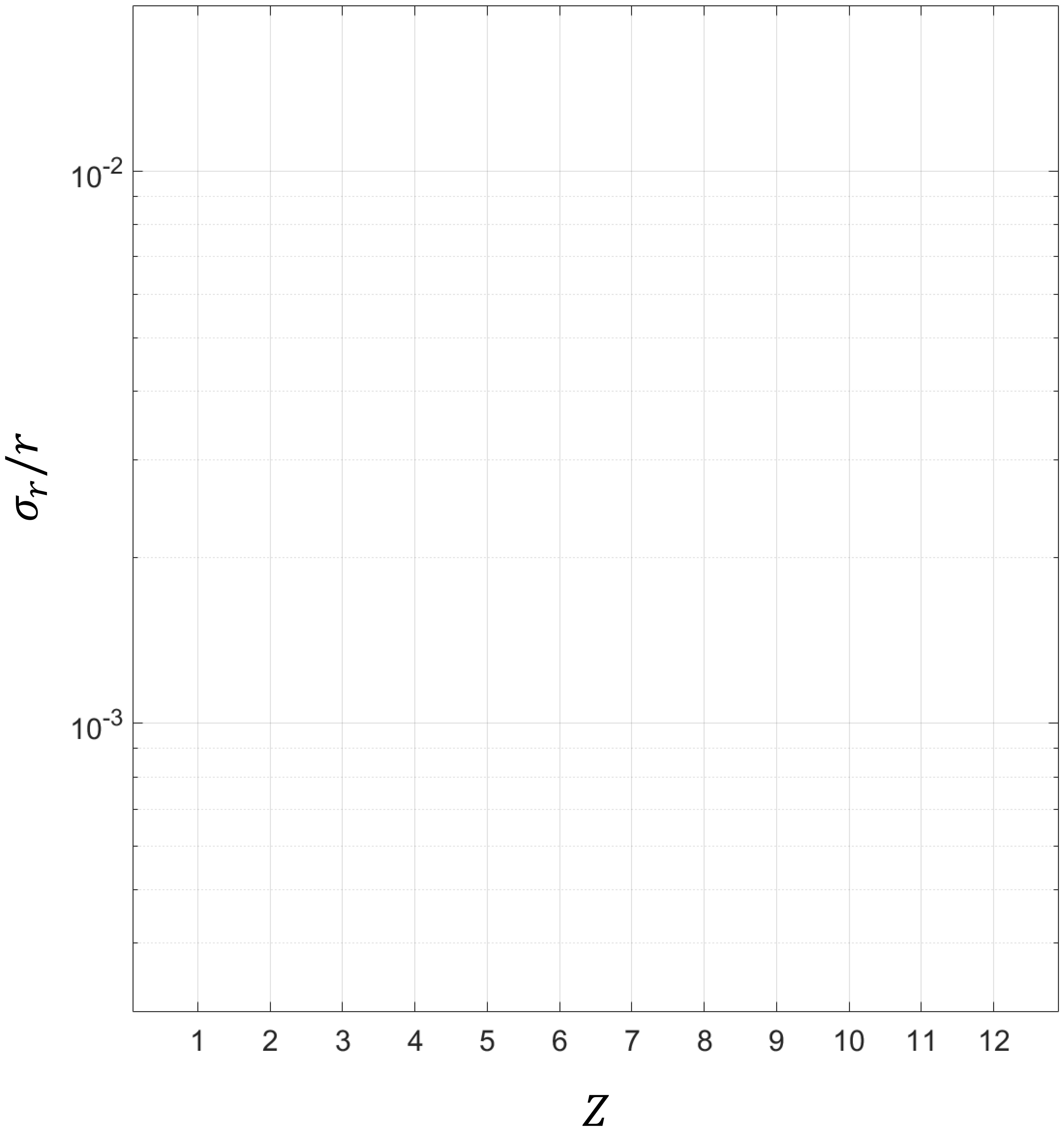
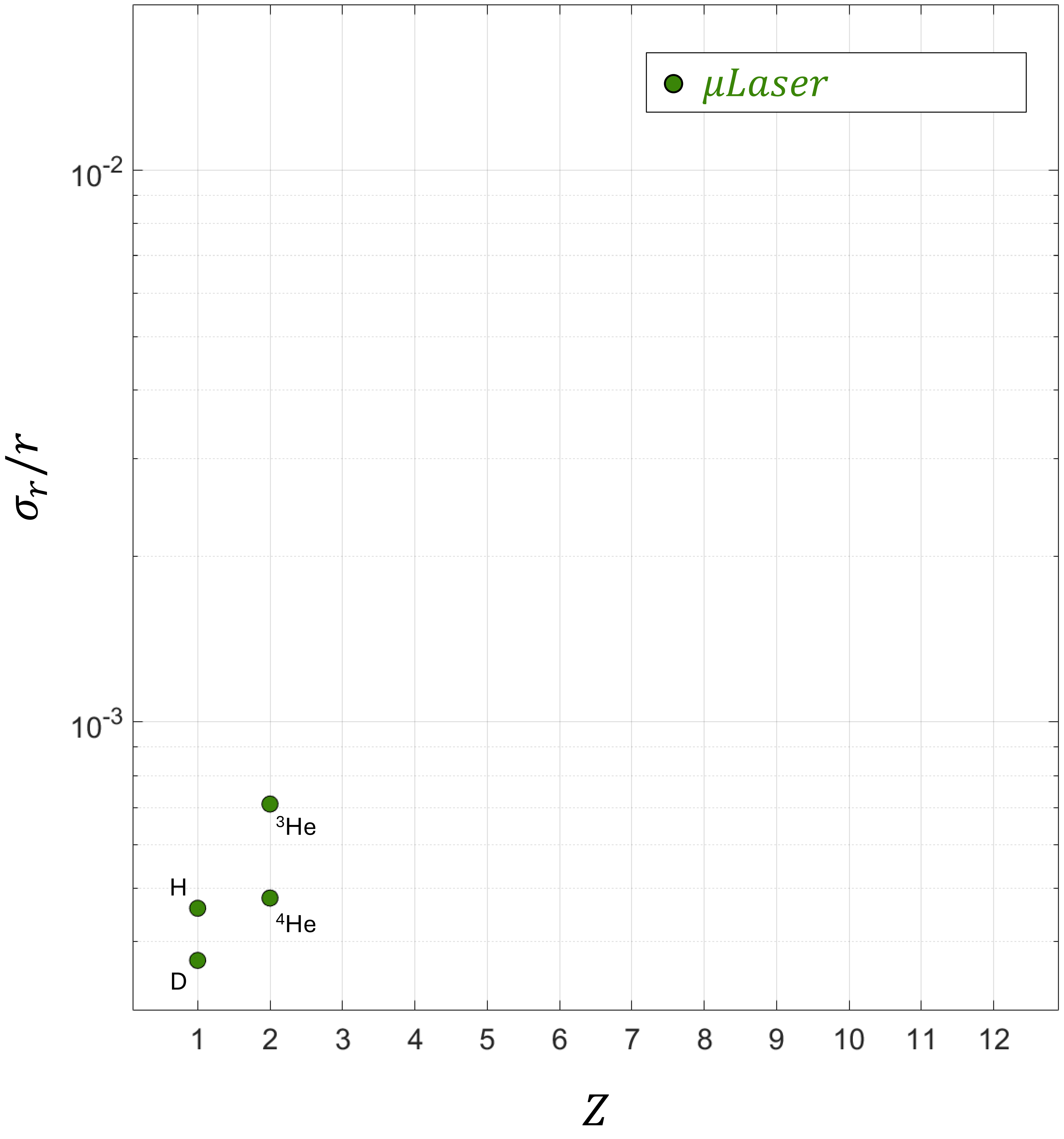
The radius gap



The radius gap

- **For $Z < 3$:**
  Laser spectroscopy of muonic atoms, limited by nuclear theory



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- **For $Z < 3$:**
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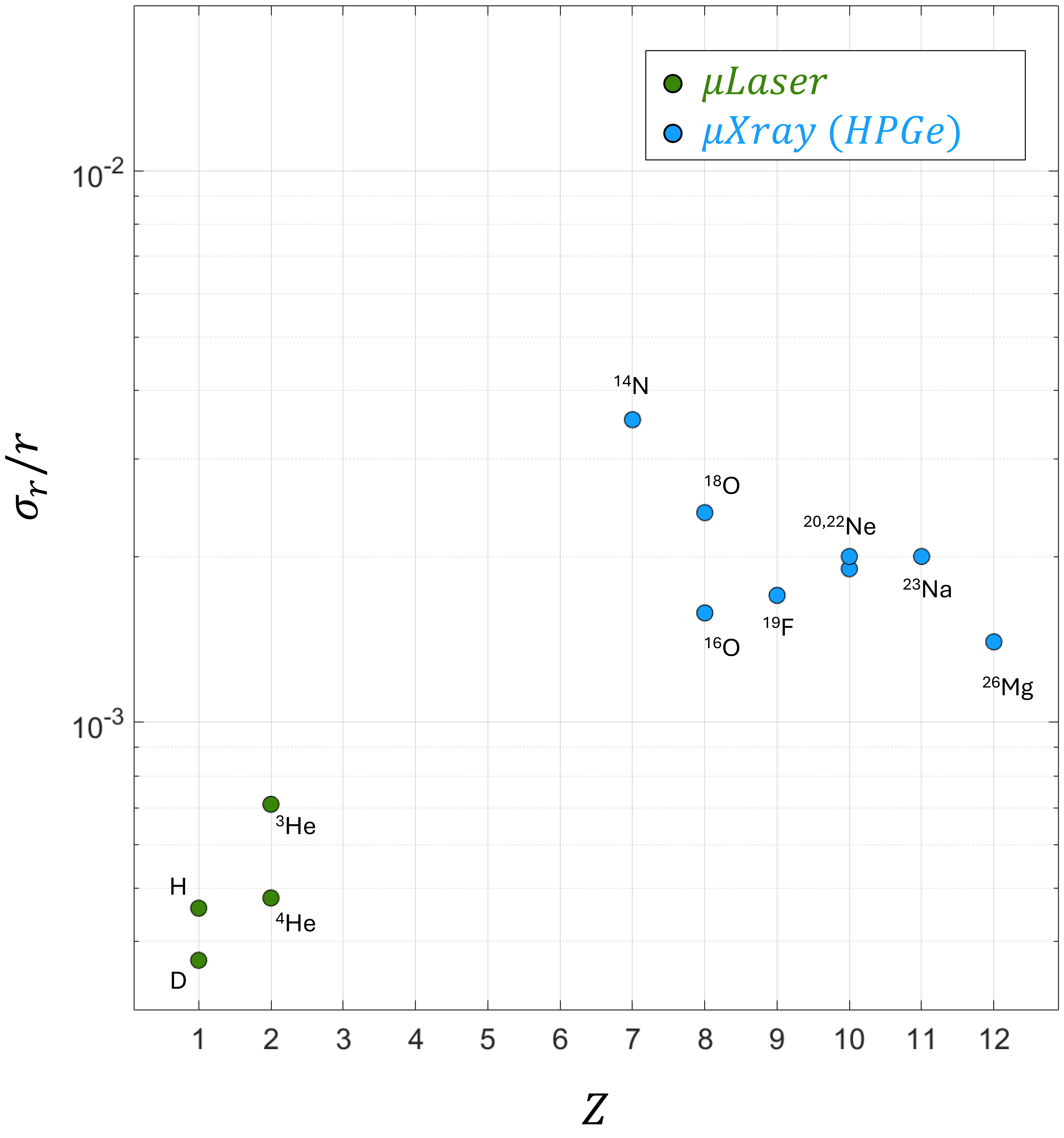
- **For $\mathbf{Z > 6}$:**
  Measured x-rays from muonic atoms using solid-state detectors.



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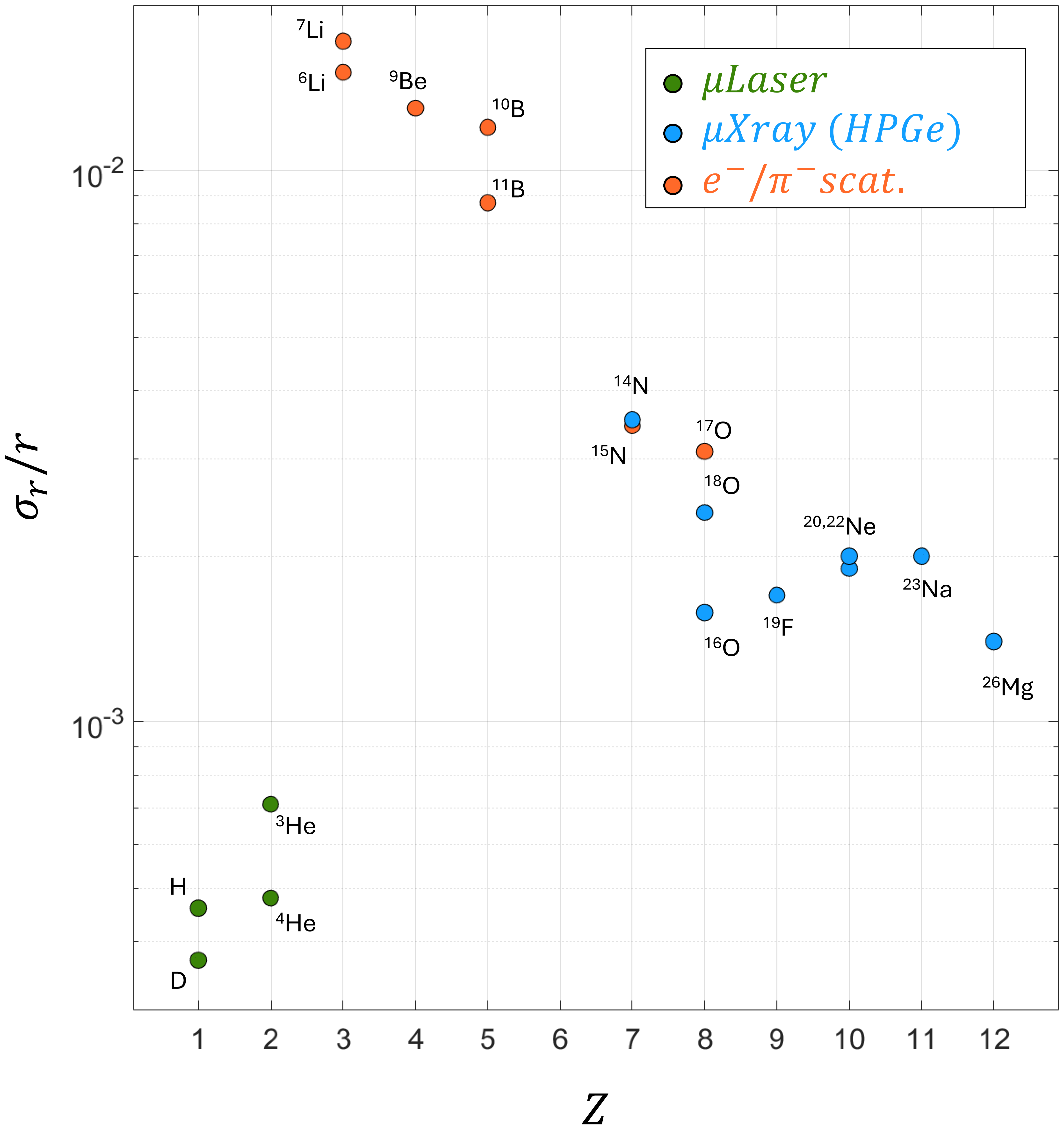
- **For $\mathbf{Z = 3 - 5}$, and others:**
  Electron scattering



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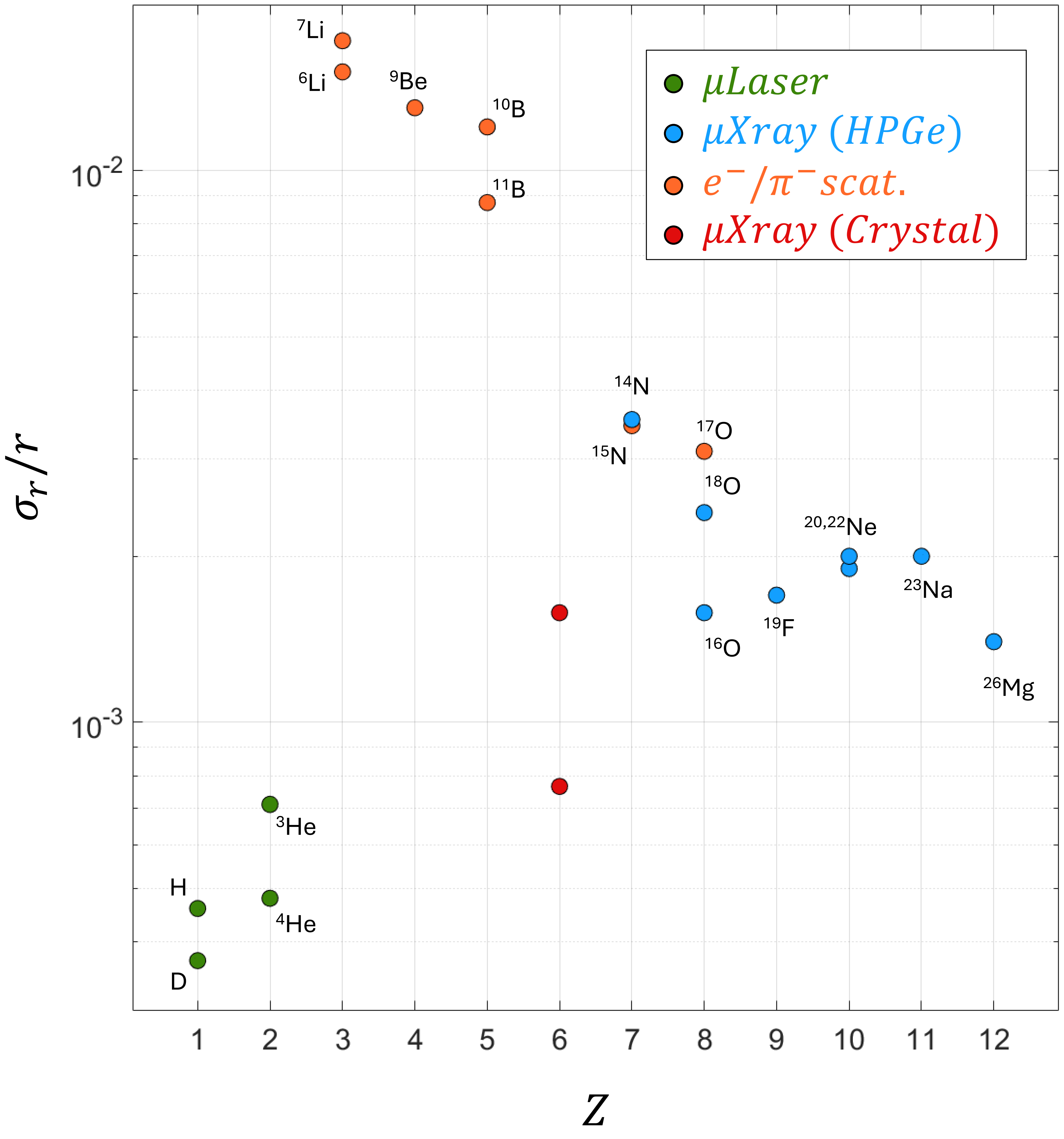
- **For $\mathbf{Z = 6}$**
  Measured with crystal spectrometer. Not widely applicable



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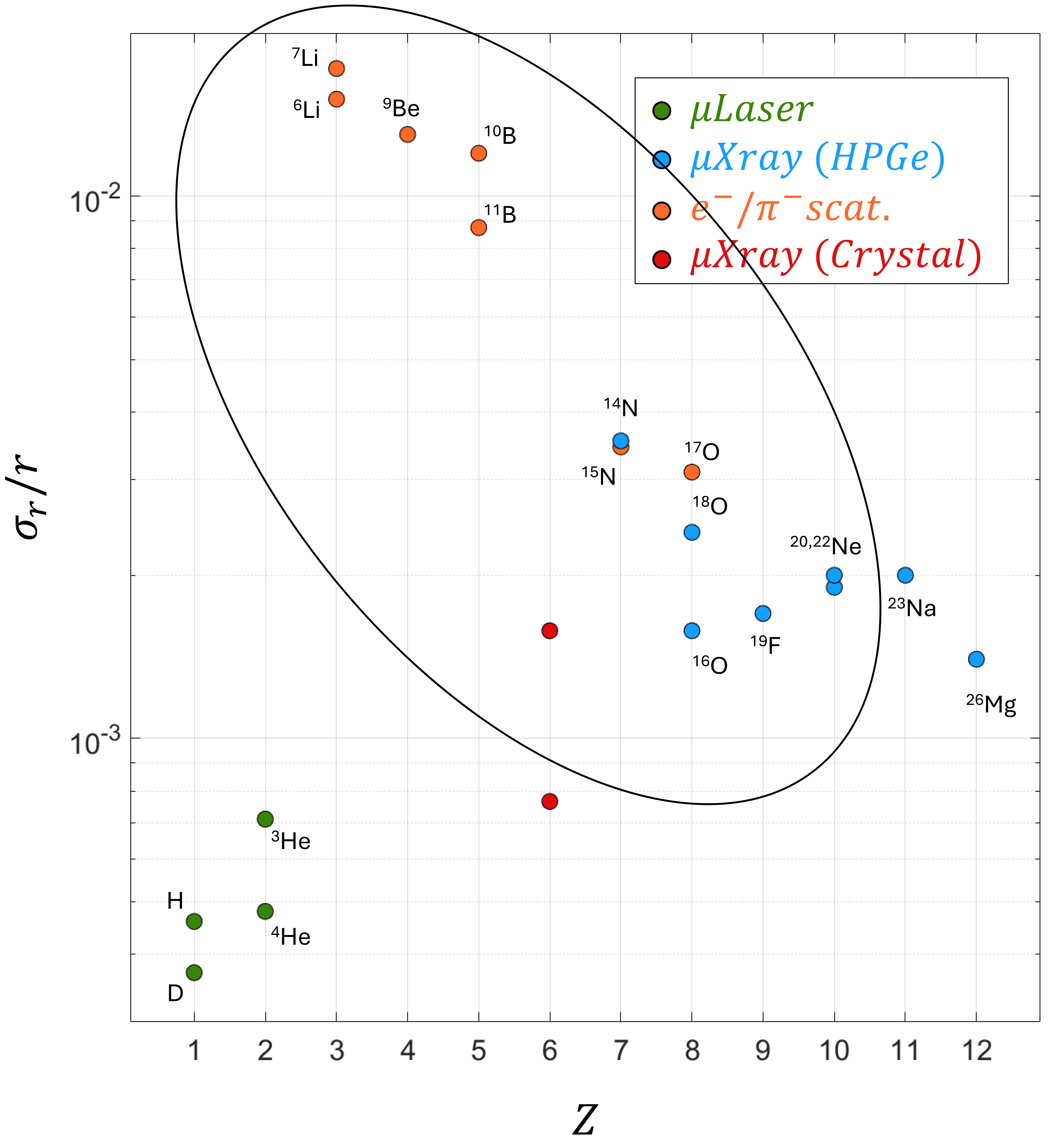
- **For $\mathbf{Z = 6}$**
  Measured with crystal spectrometer. Not widely applicable



bohayon@technion.ac.il

The radius gap

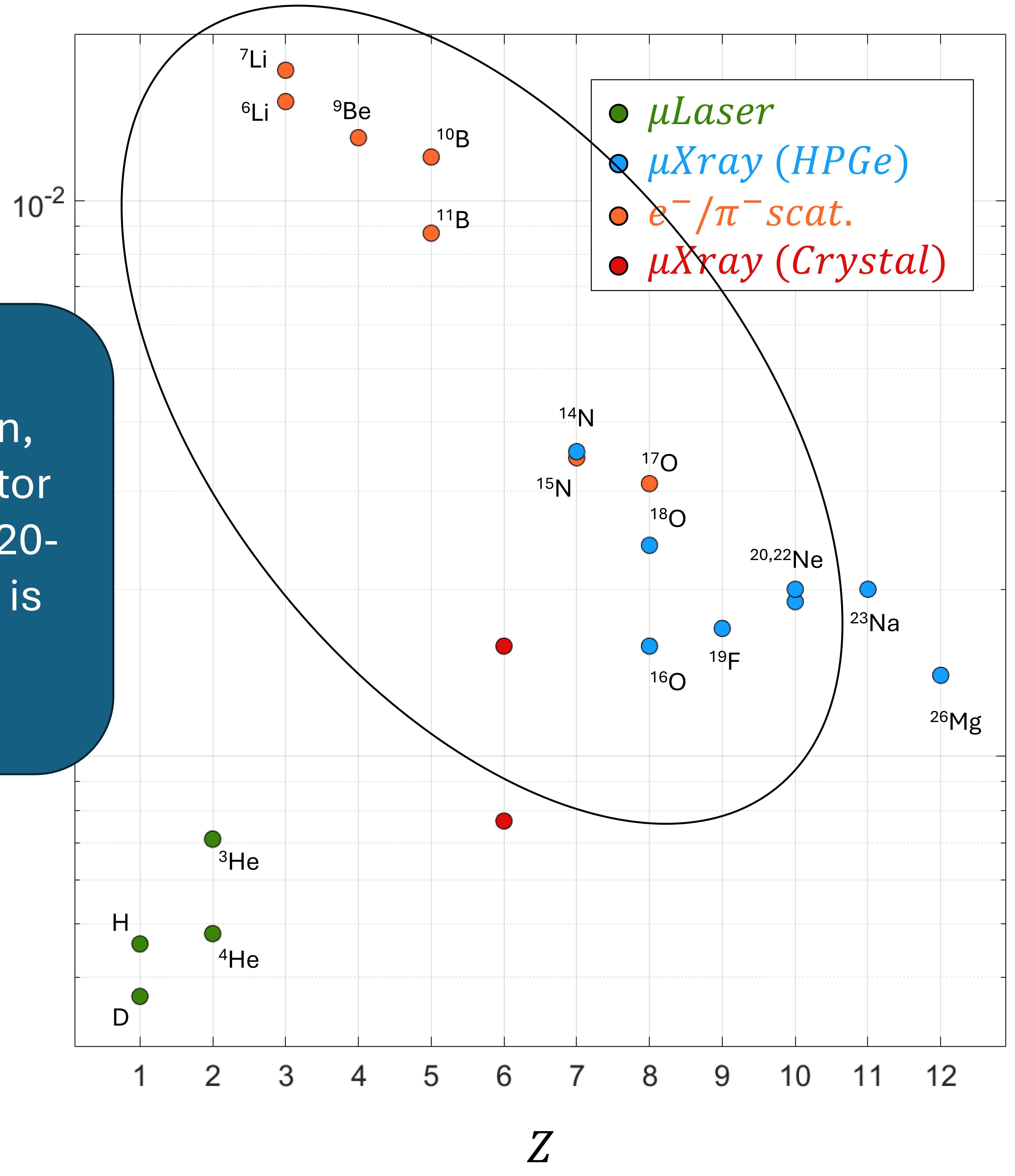
- **For $Z < 3$:**
  Laser spectroscopy of muonic atoms, limited by nuclear theory

- **For $\mathbf{Z > 6}$:**
  Measured x-rays from muonic atoms using solid-state

- **For $\mathbf{Z = 3 - 5}$, and others:**
  Electron scattering

- **For $\mathbf{Z = 6}$**
  Measured with crystal spectrometer. Not widely applicable

High-resolution, efficient, detector for low-energy (20-200 keV) x-rays is needed



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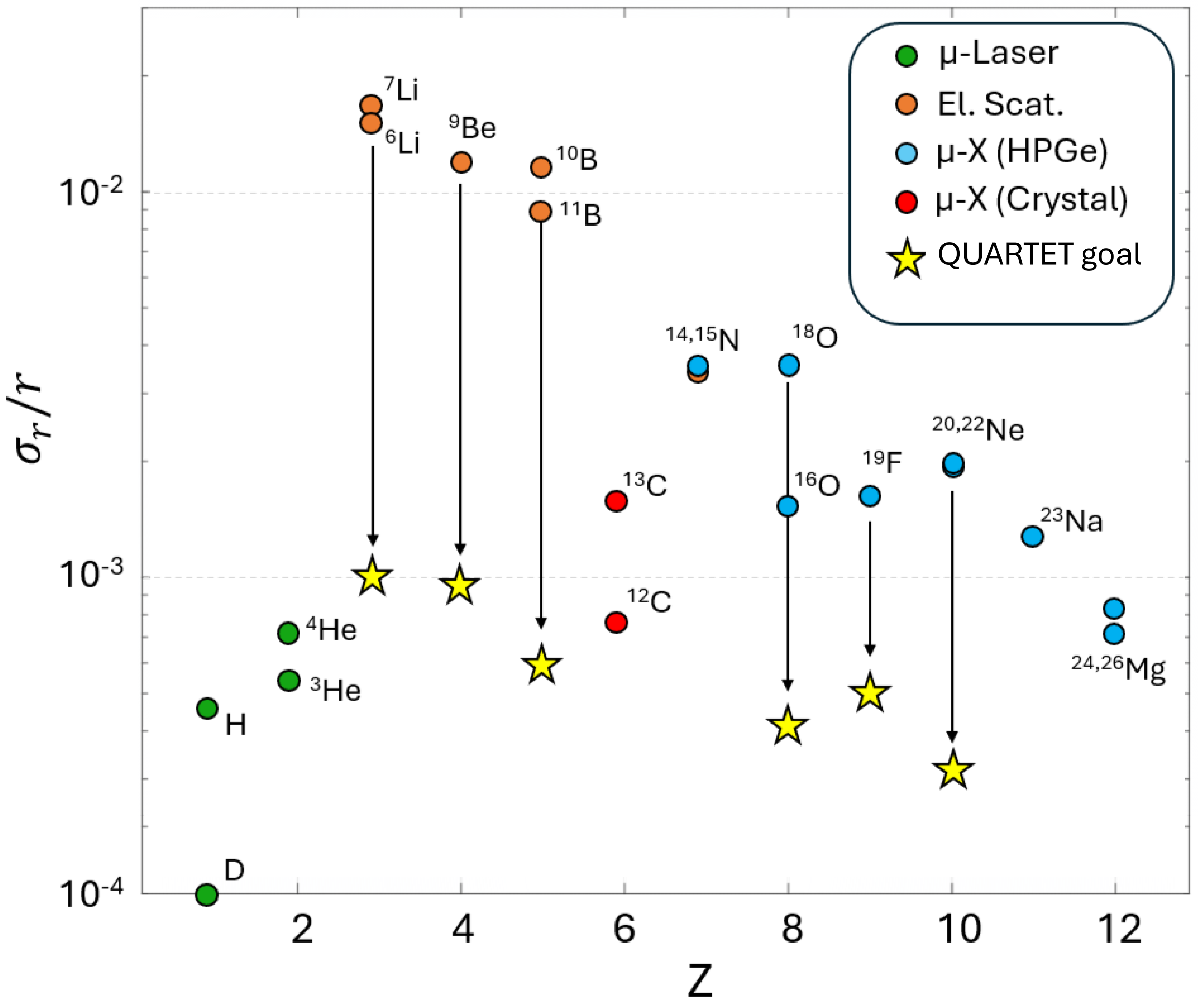
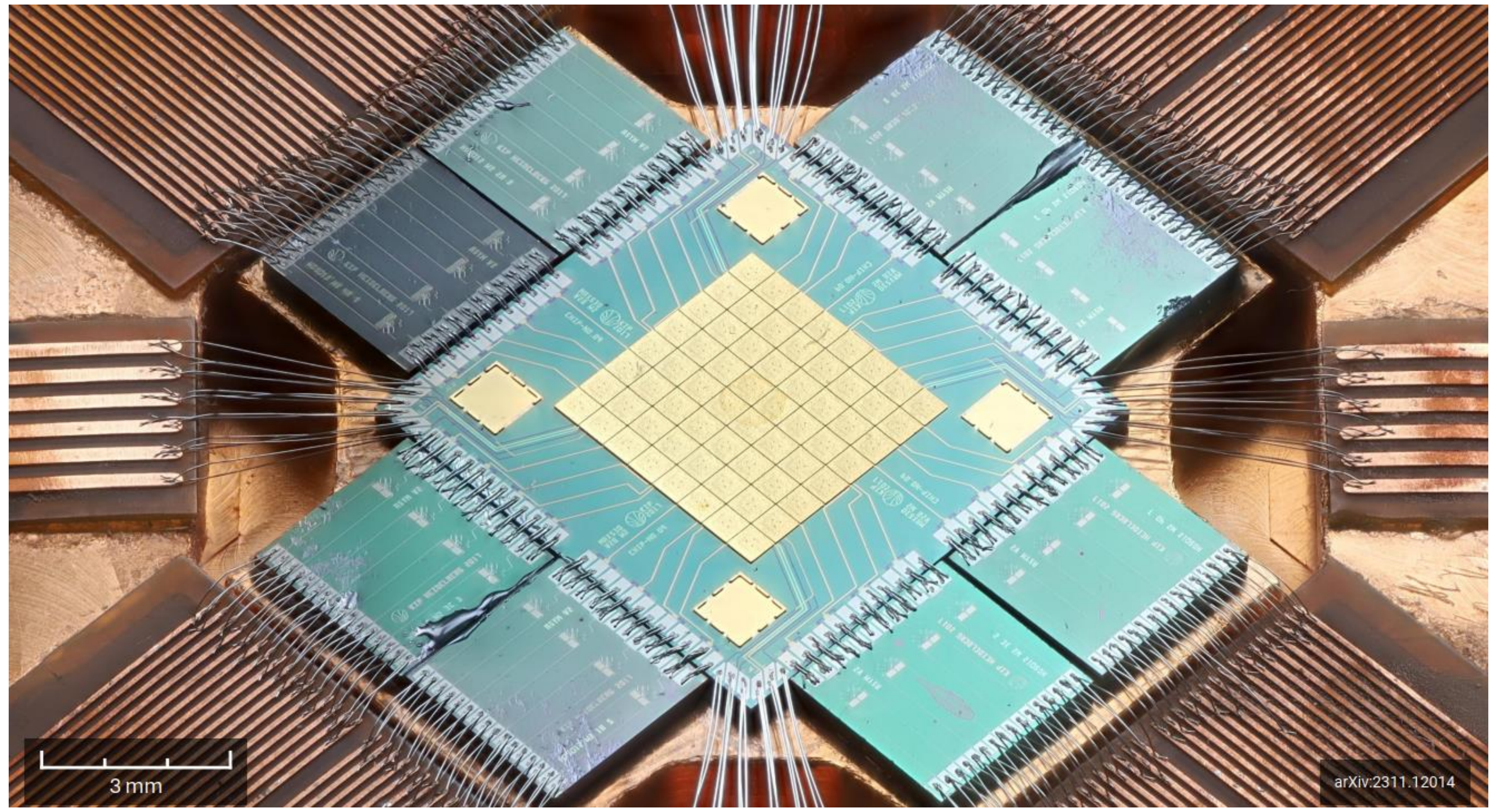
Experimental opportunities with muonic atoms:



For more information about QUARTET
See here, and next talk.

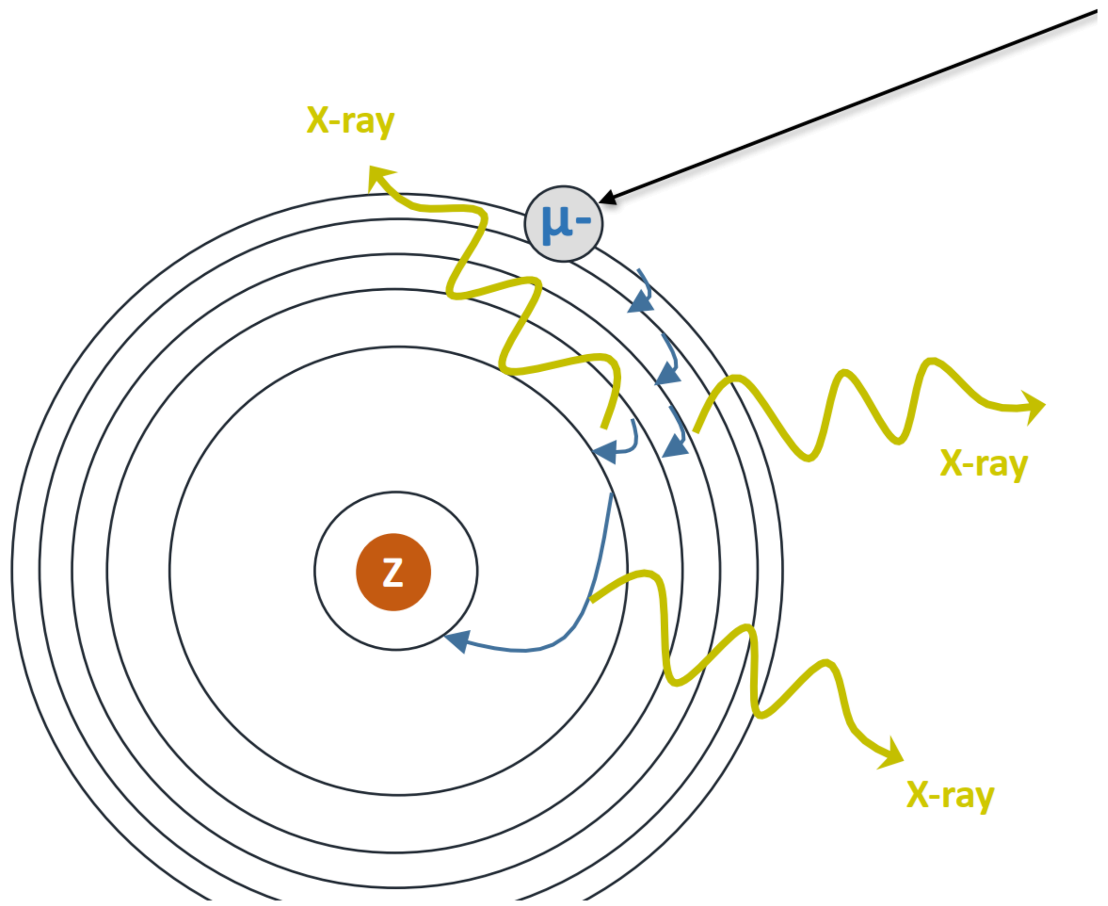
Enter microcalorimeters

Cryogenic microcalorimeters



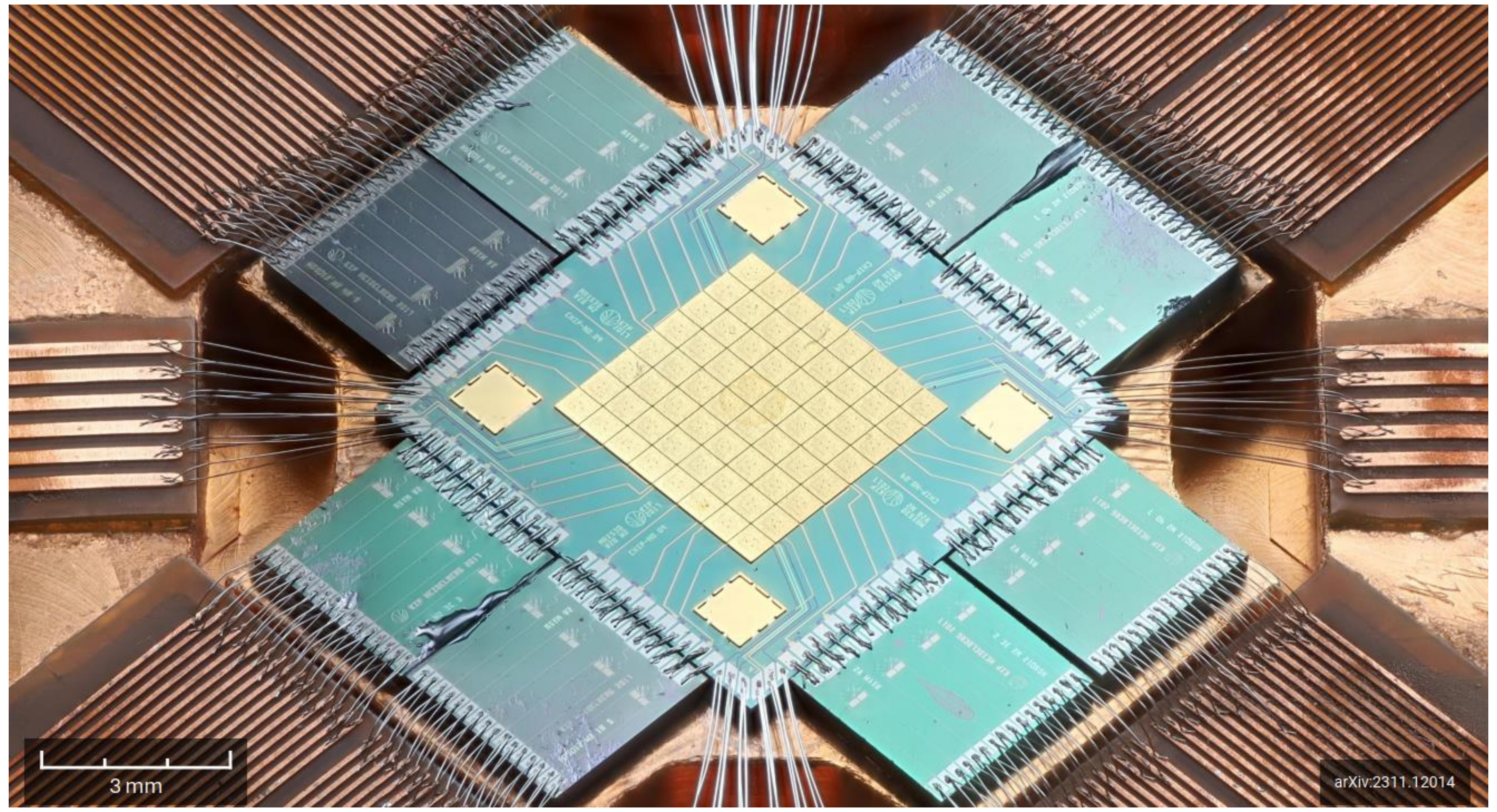
- High quantum efficiency
- Broadband (important for calibration)
- **Superb resolution $\left(\frac{E}{\Gamma_E} > 10^3\right)$**
- Fast rise time

Muonic atoms



$$\frac{\delta E_{FNS}}{E_0} \sim Z^2 \left(\frac{r_c}{a_0}\right)^2 \left(\frac{m_\mu}{m_e}\right)^2 \sim 10^{-4}\, Z^2$$

Cryogenic microcalorimeters



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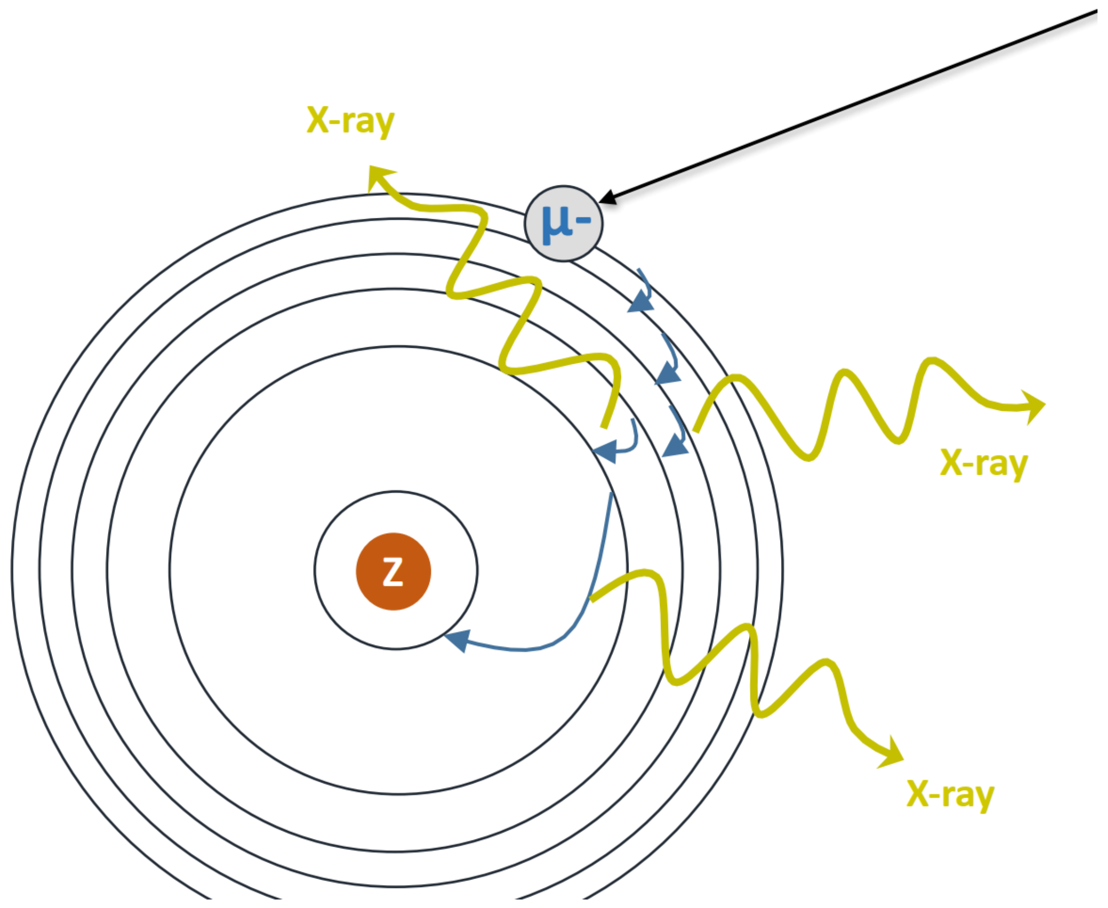

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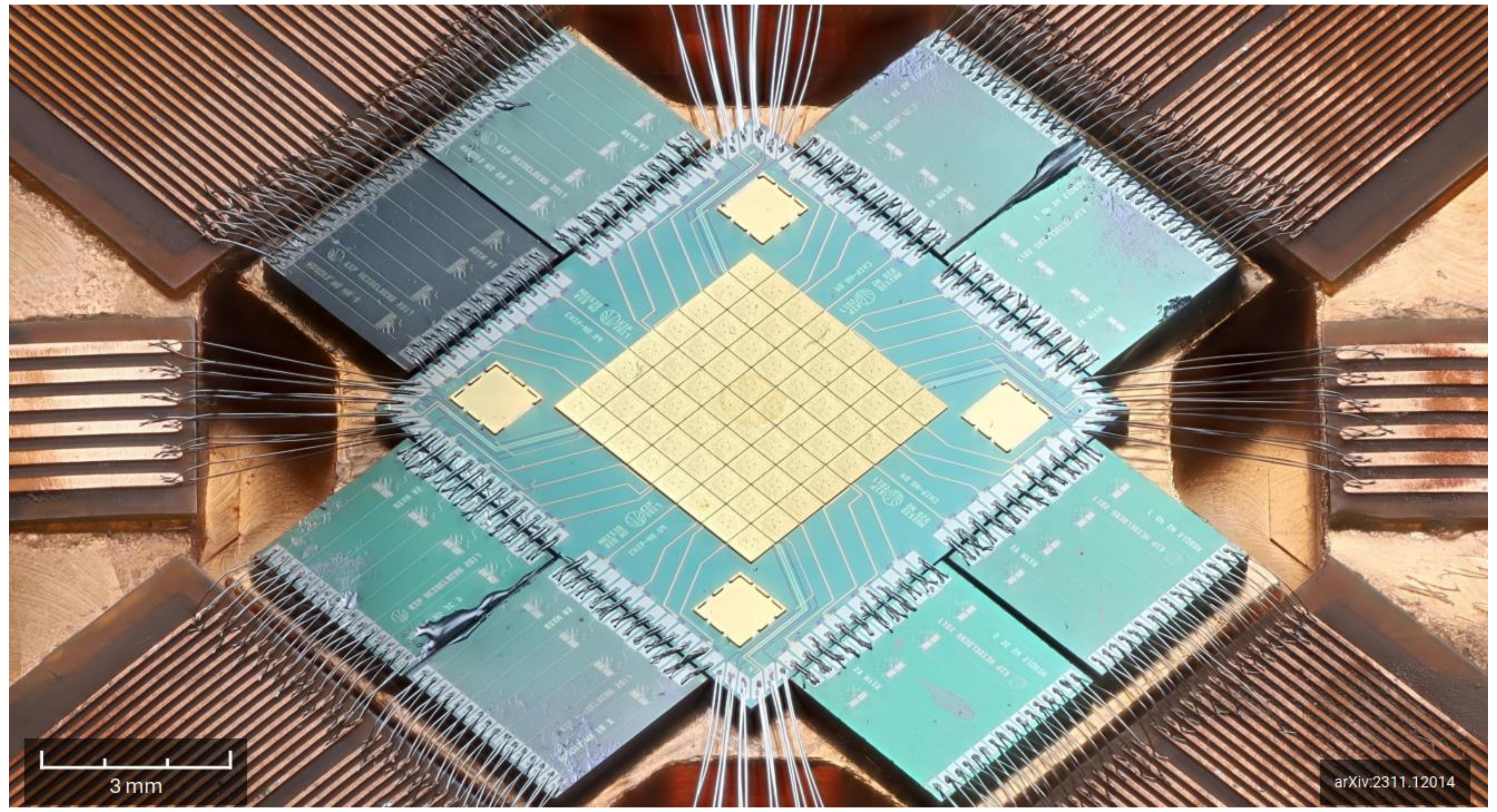

- High quantum efficiency
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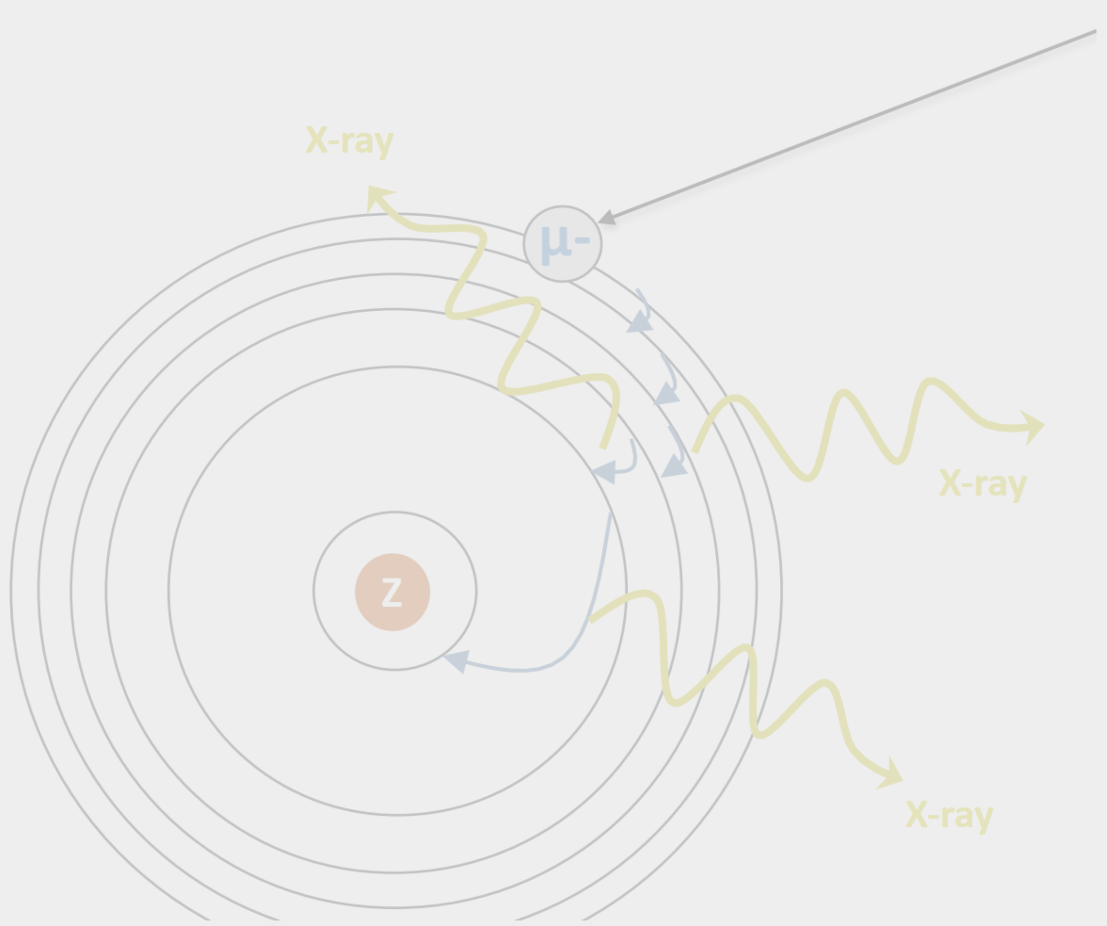
- **Superb resolution** $\left(\frac{E}{\Gamma_E} > 10^3\right)$
- Fast rise time

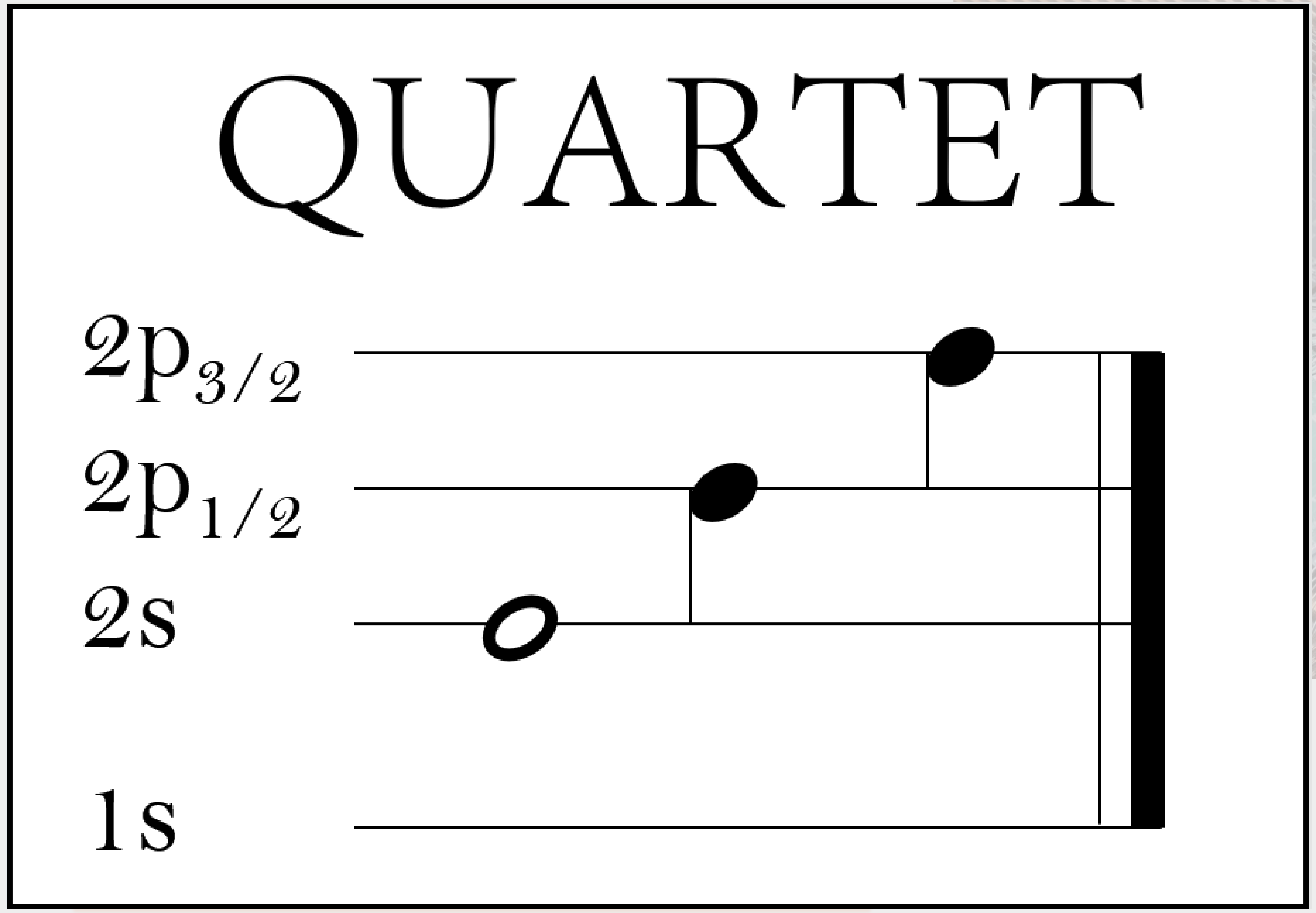
Enter microcalorimeters

Muonic atoms



$$\frac{\delta E_{FNS}}{E_0} \sim Z^2 \left(\frac{r_c}{a_0}\right)^2 \left(\frac{m_\mu}{m_e}\right)^2 \sim 10^{-4}\, Z^2$$

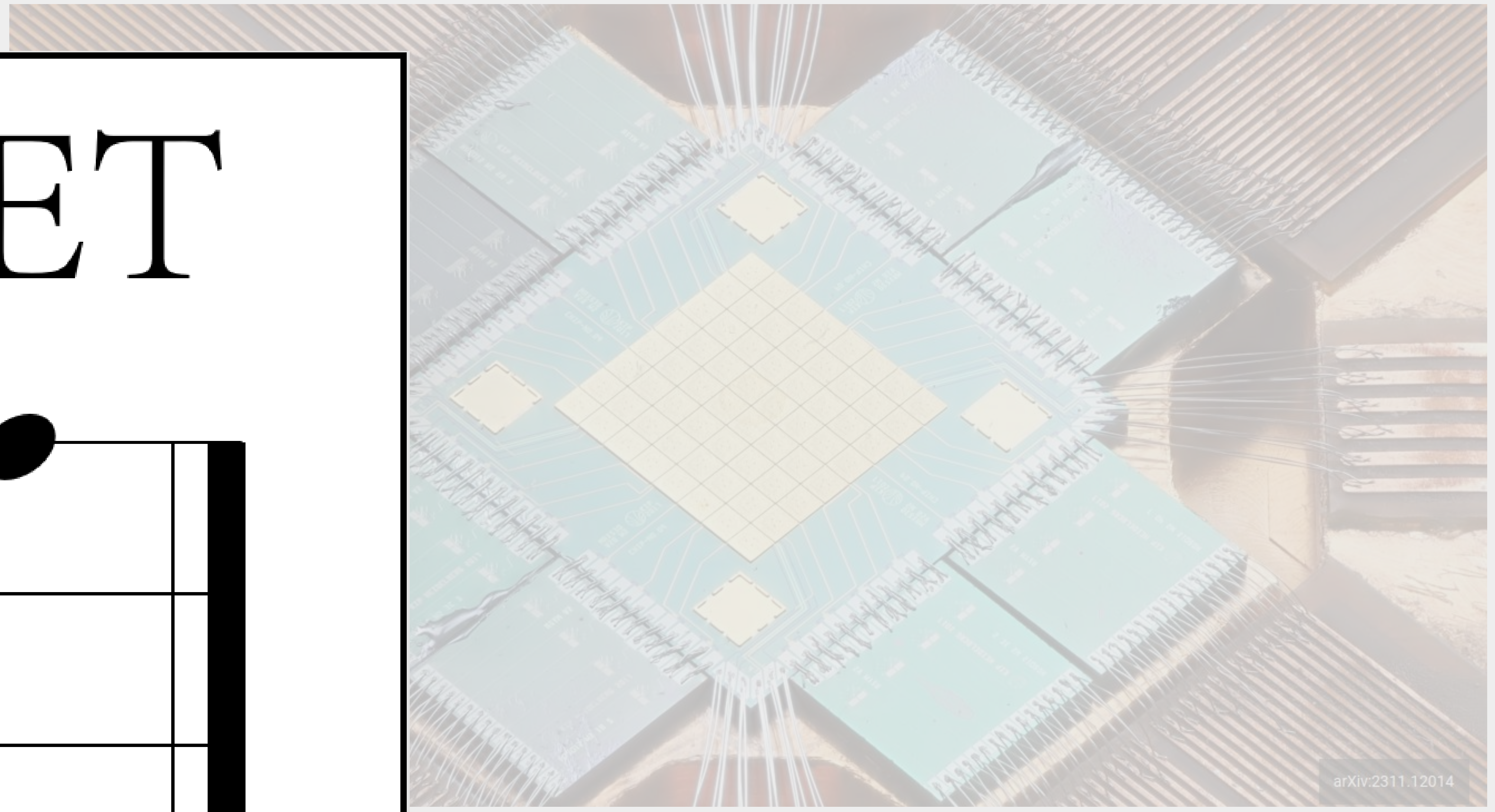
Quantum Interactions with Exotic Atoms



More info:

arXiv:2311.12014
arXiv:2310.03846

Cryogenic microcalorimeters

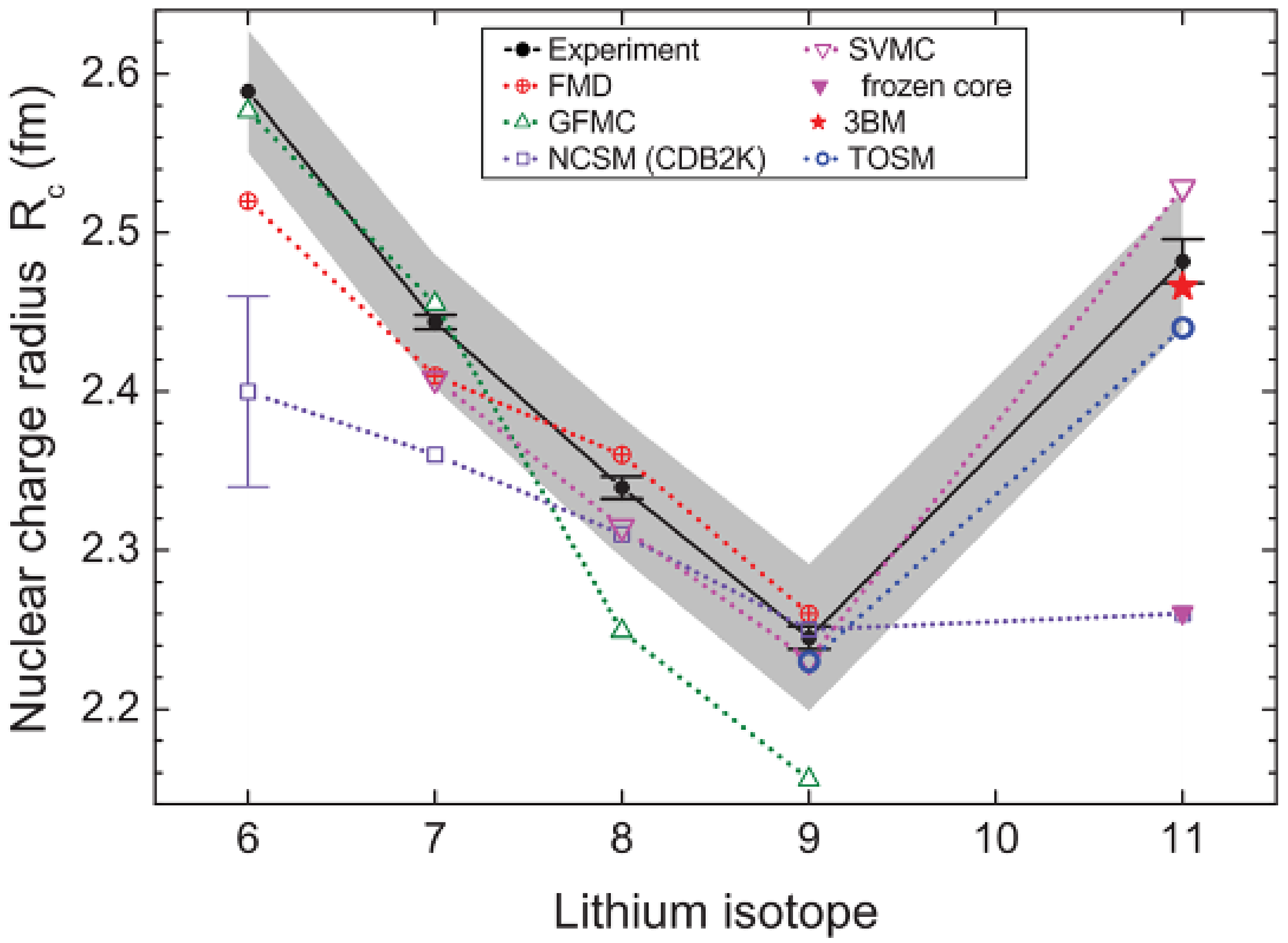


- [H]igh quantum efficiency
- Broadband (important for calibration)
- **Superb resolution** $\left(\frac{E}{\Gamma_E} > 10^3\right)$
- Fast rise time (important for background suppression)

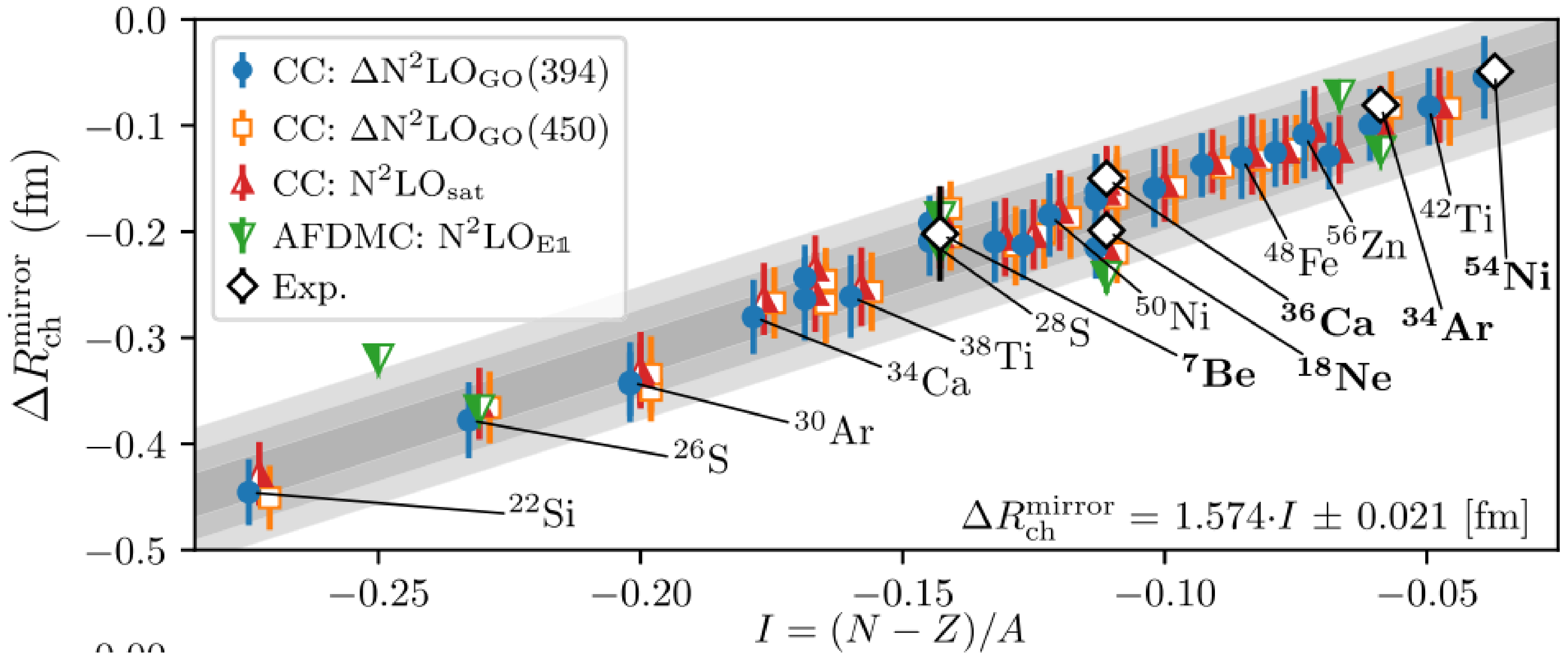
What are light radii good for?

- **Absolute radius:** Li/Be/B → calibrate entire chains, test nuclear chiral EFT, inc. ^{7}Li-^{7}Be and (future) ^{8}Li-^{8}B mirrors
- **Isotope shifts:** compare electronic and muonic atoms to search for new lepton-neutron interactions
- Upcoming optical determinations of absolute radii for helium-like Li to C (Wuhan, Mainz). Important cross check and strong test for new physics beyond isotope shifts.
- **Mirror nuclei:** $\Delta r = r(N,Z) - r(Z,N)$,probe of neutron skins and the nuclear equation of state. **Scarce measurements**, especially in **light nuclei** with large isospin asymmetry: $I = (N-Z)/A$.
- Novel measurements of g-factors in H-like ions **limited by muonic isotope shifts for new physics searches.**

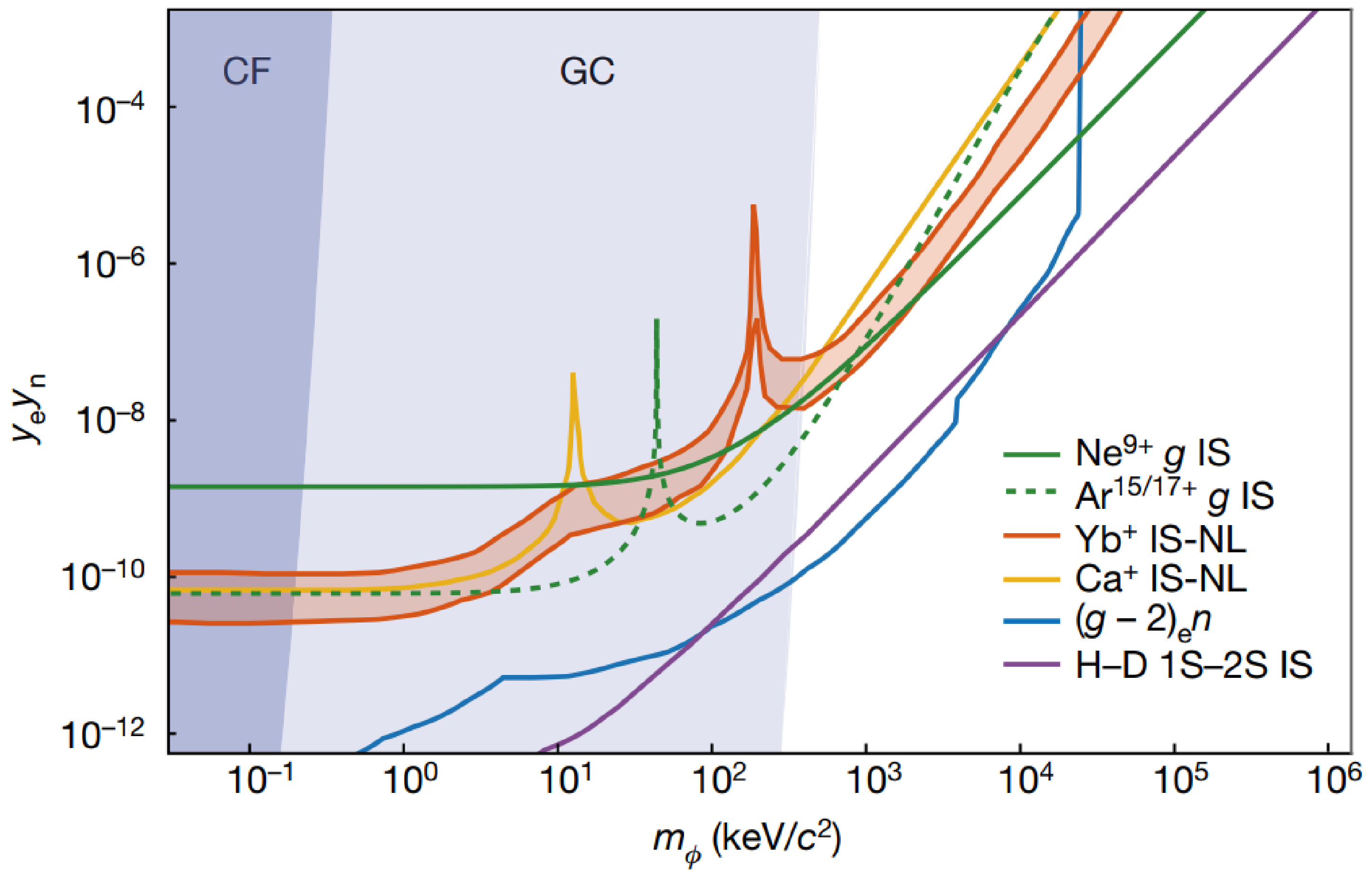
PRC 84, 024307 (2011)



S. J. Novario, et. al., PRL 130, 032501 (2023)



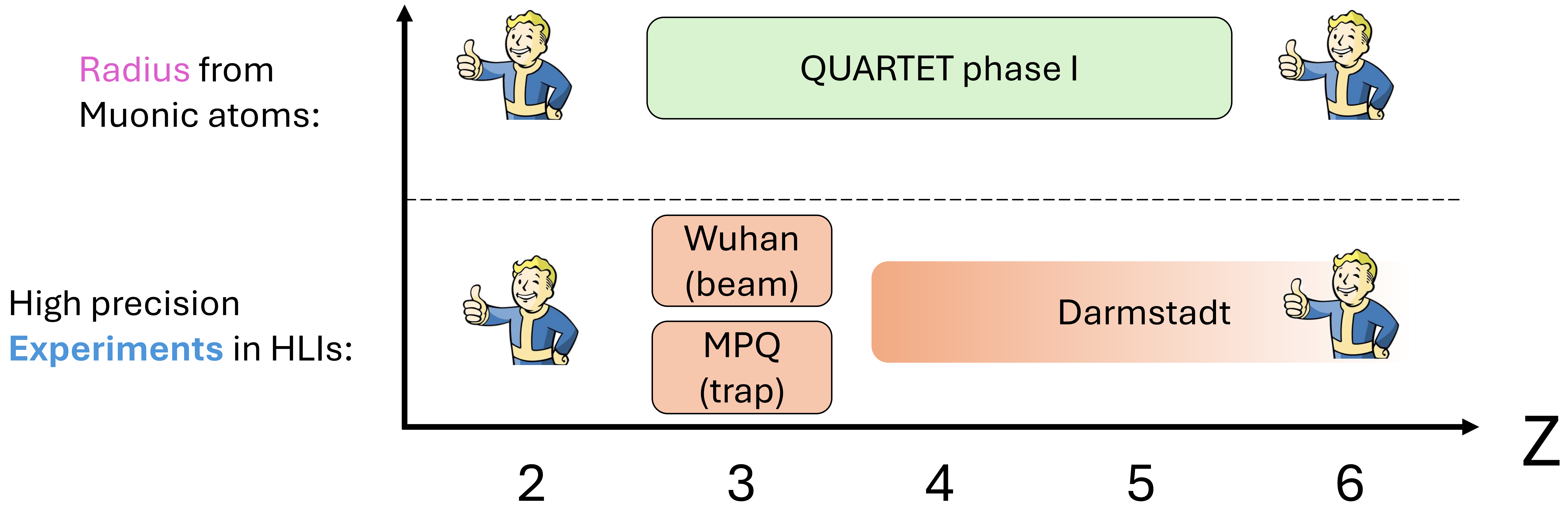
T. Sailer et. al., Nature 606 (2022)



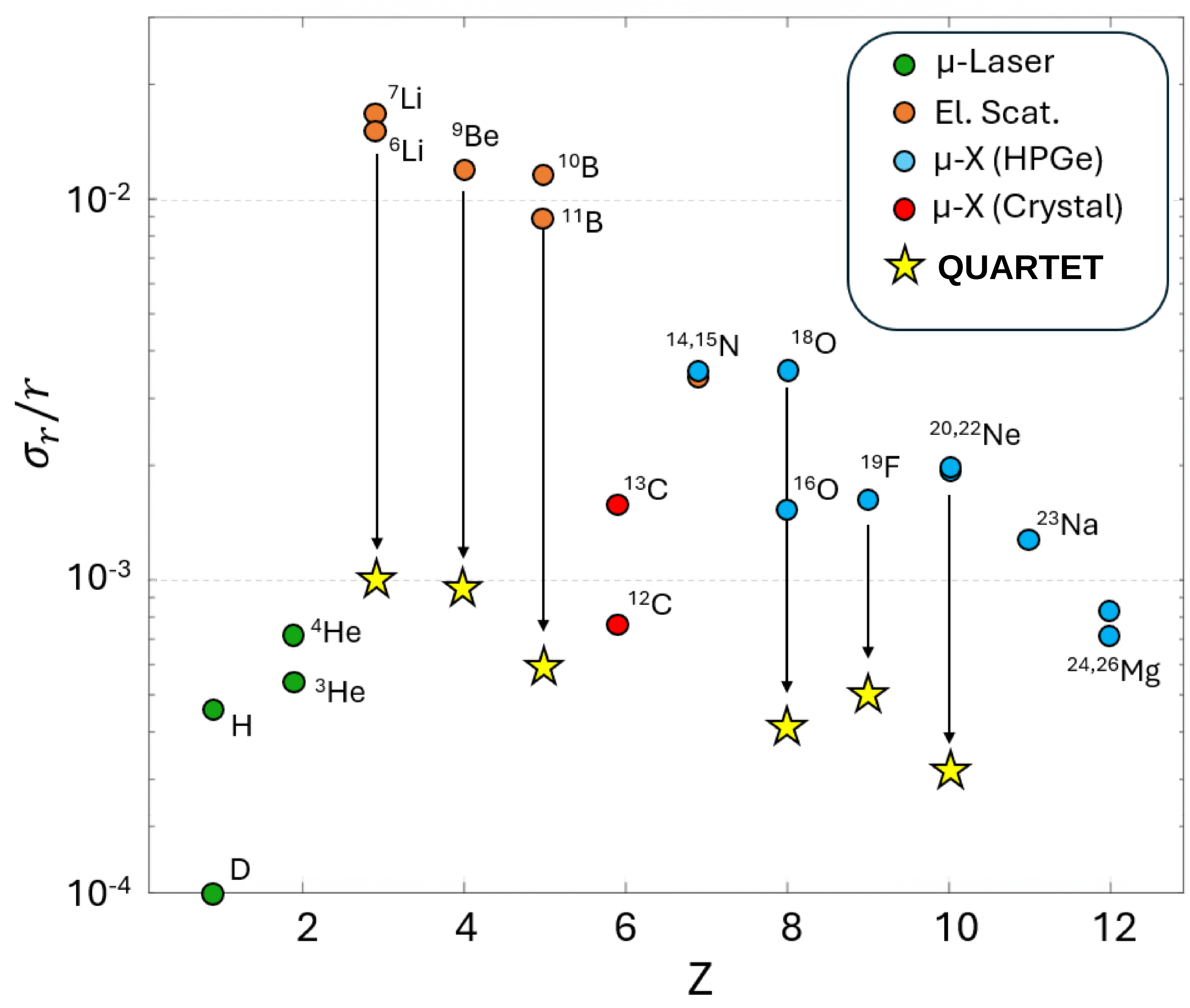
All limited by reference radii

Output II: Z dependence of (unknown) high-order QED in Helium-like atoms

$$E_{HO} \equiv \boldsymbol{E_{exp}} - (E_2 + E_4 + E_5 + E_6 + \boldsymbol{\delta E_{FNS}})$$



What can better radii of light nuclei do for the mirror fit?



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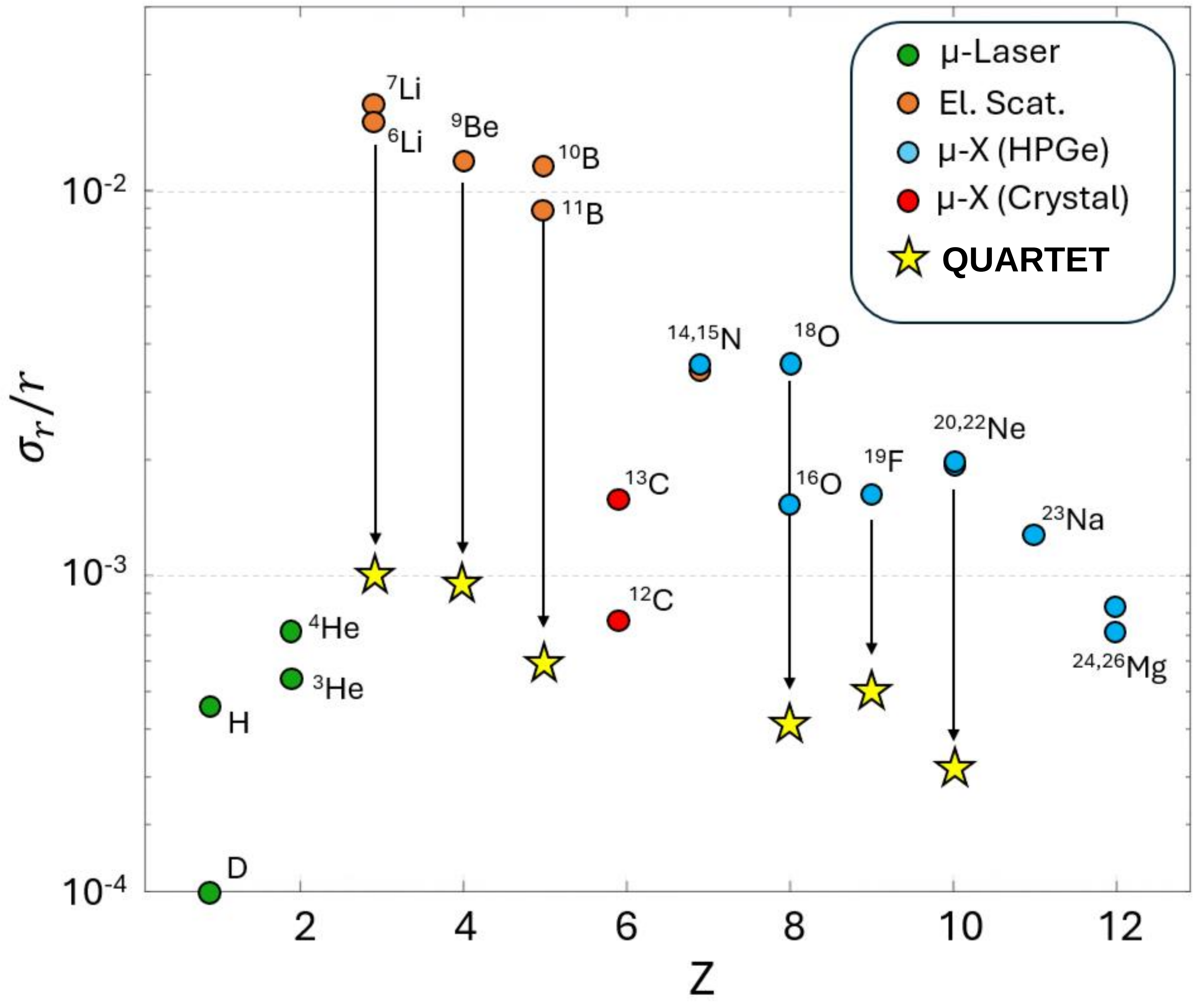

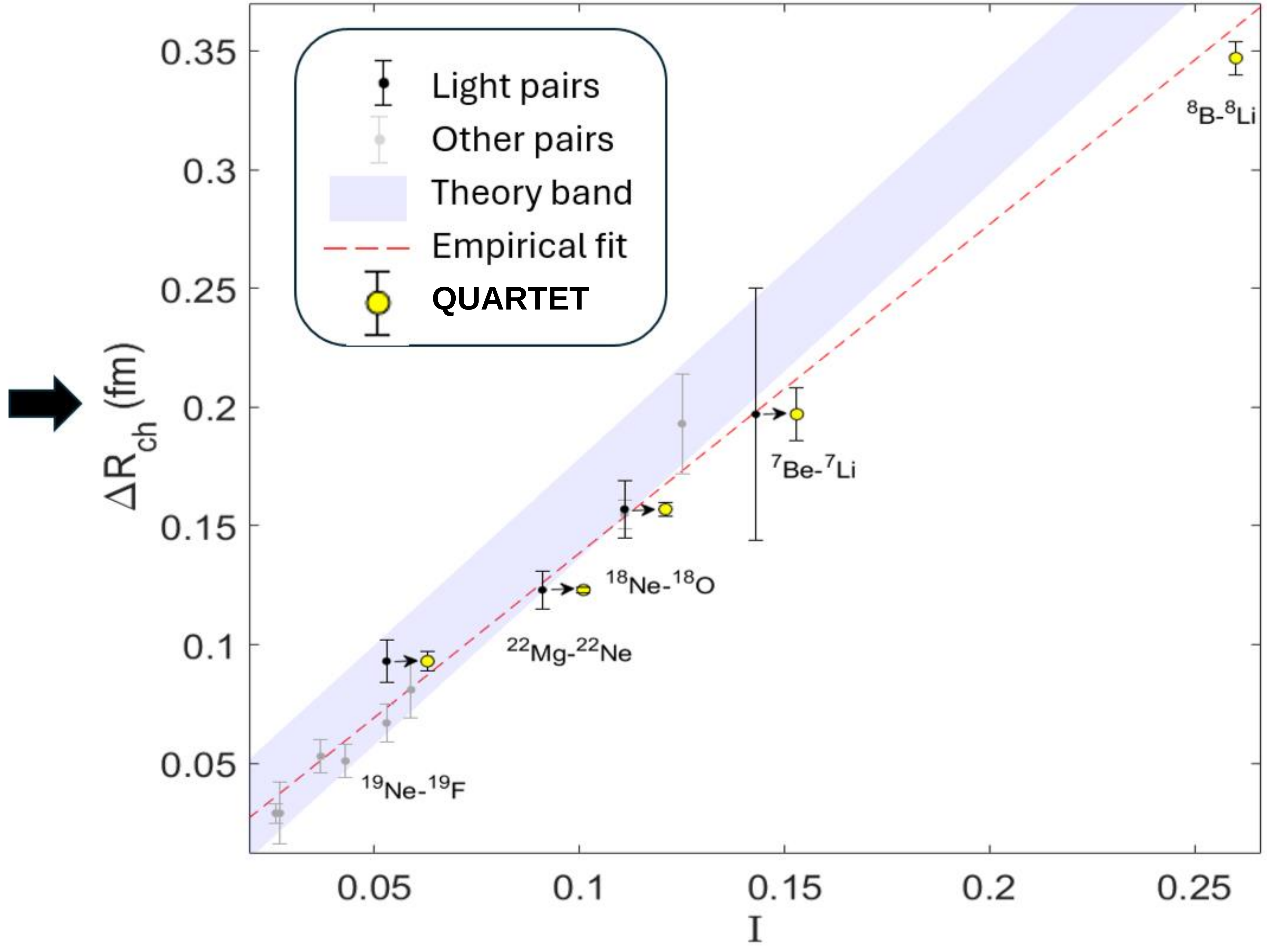

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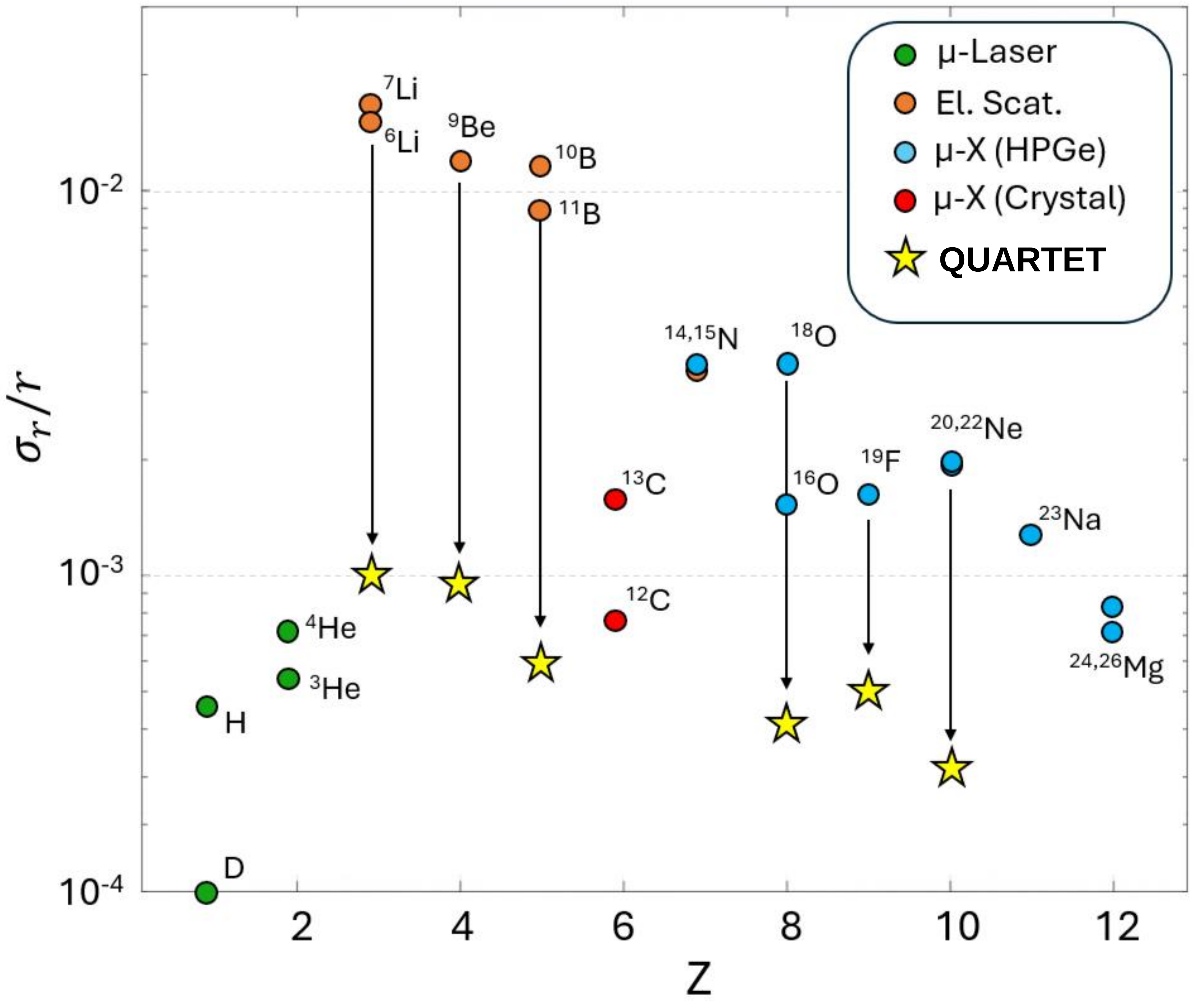
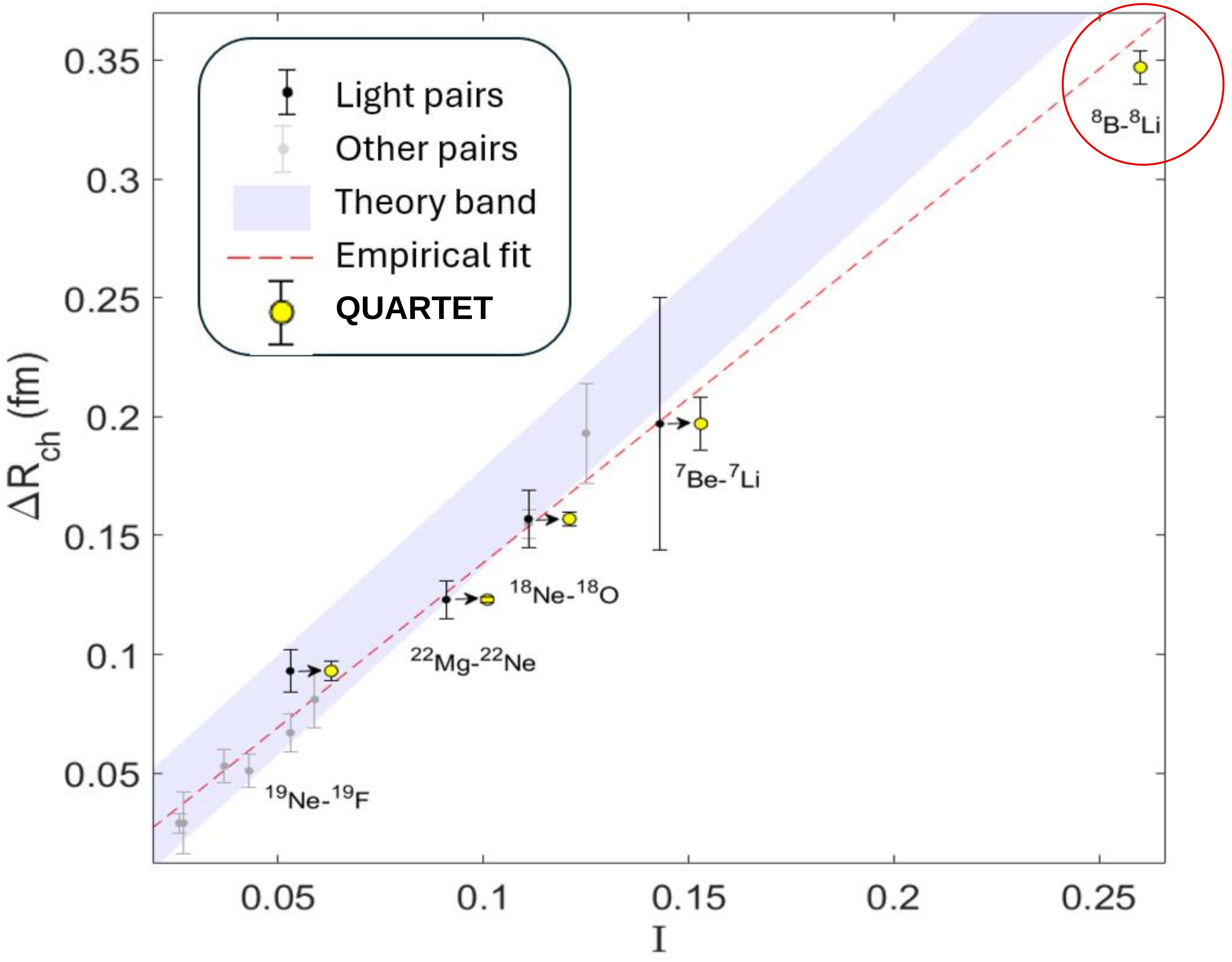
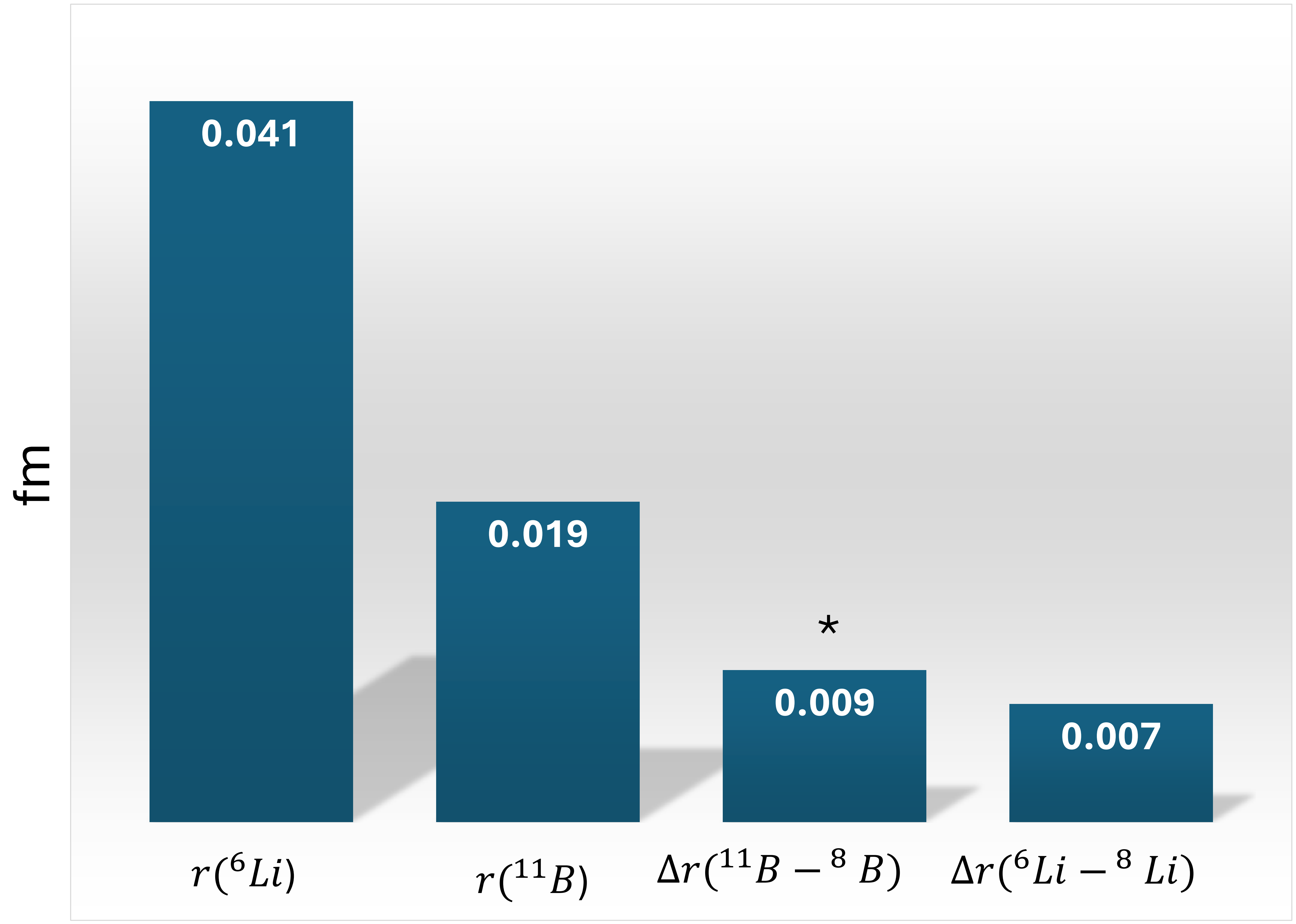
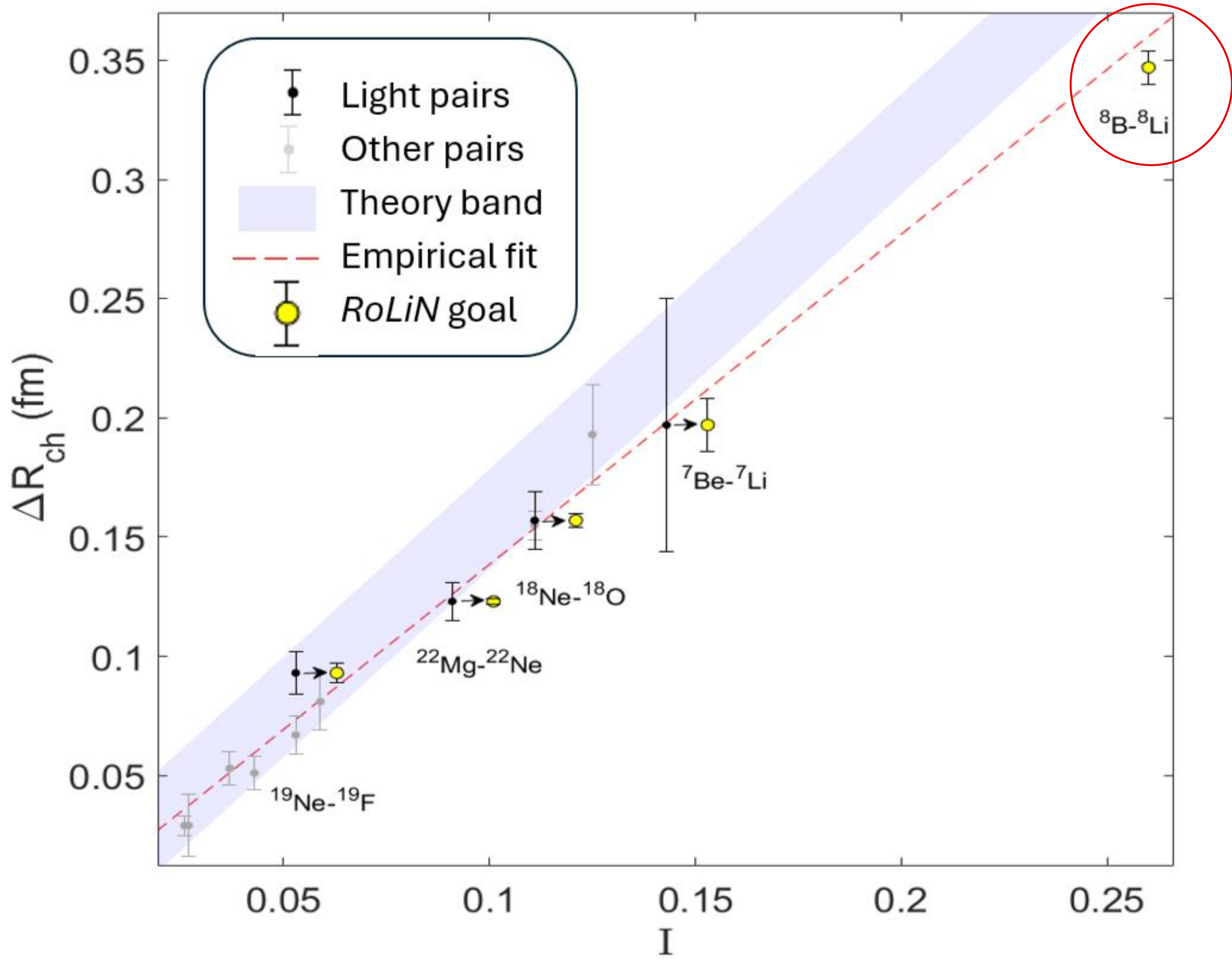




Golden case: A=8

Uncertinaty contributions to $\Delta_{ch}^{mirr}(A=8)$ *

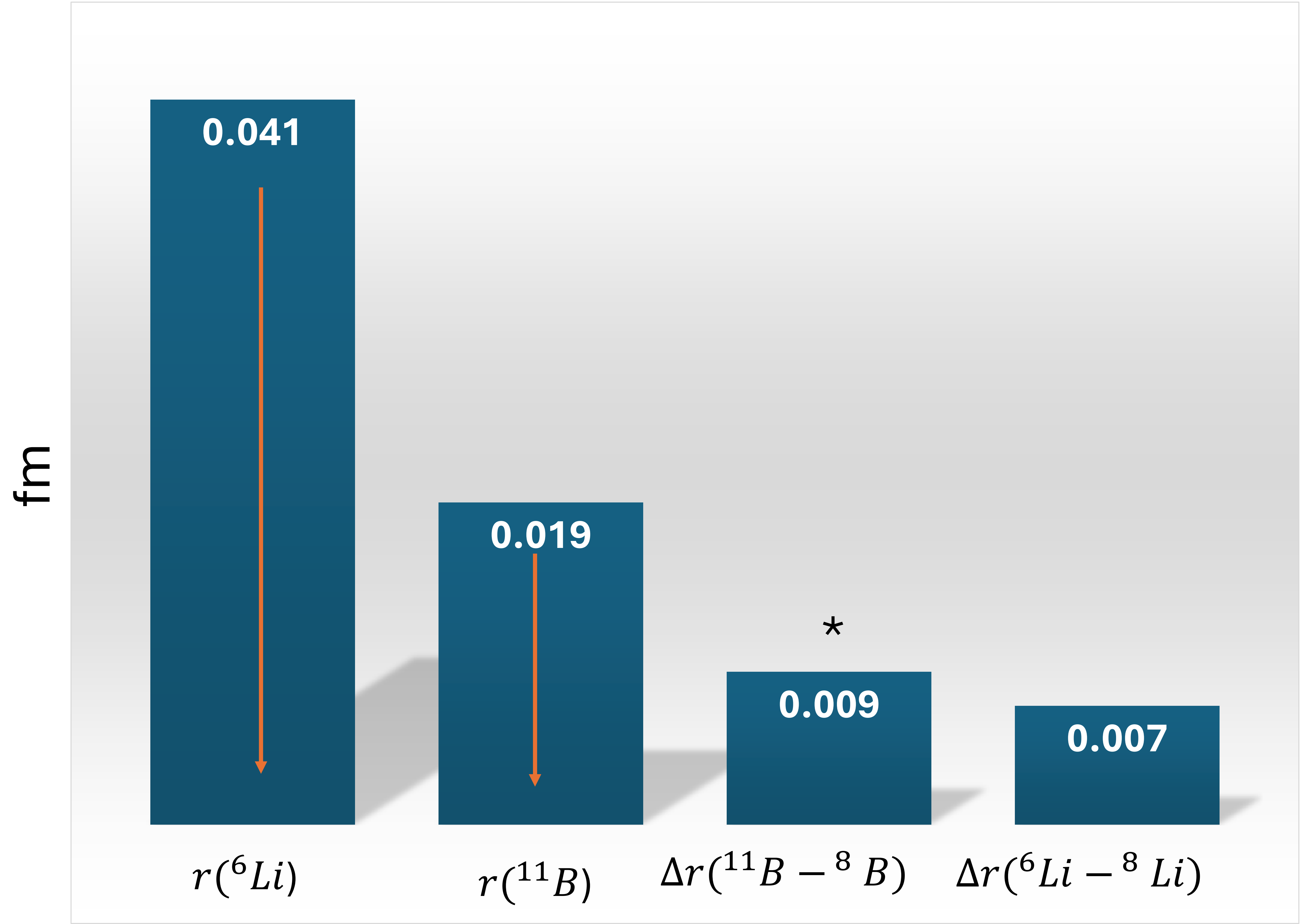




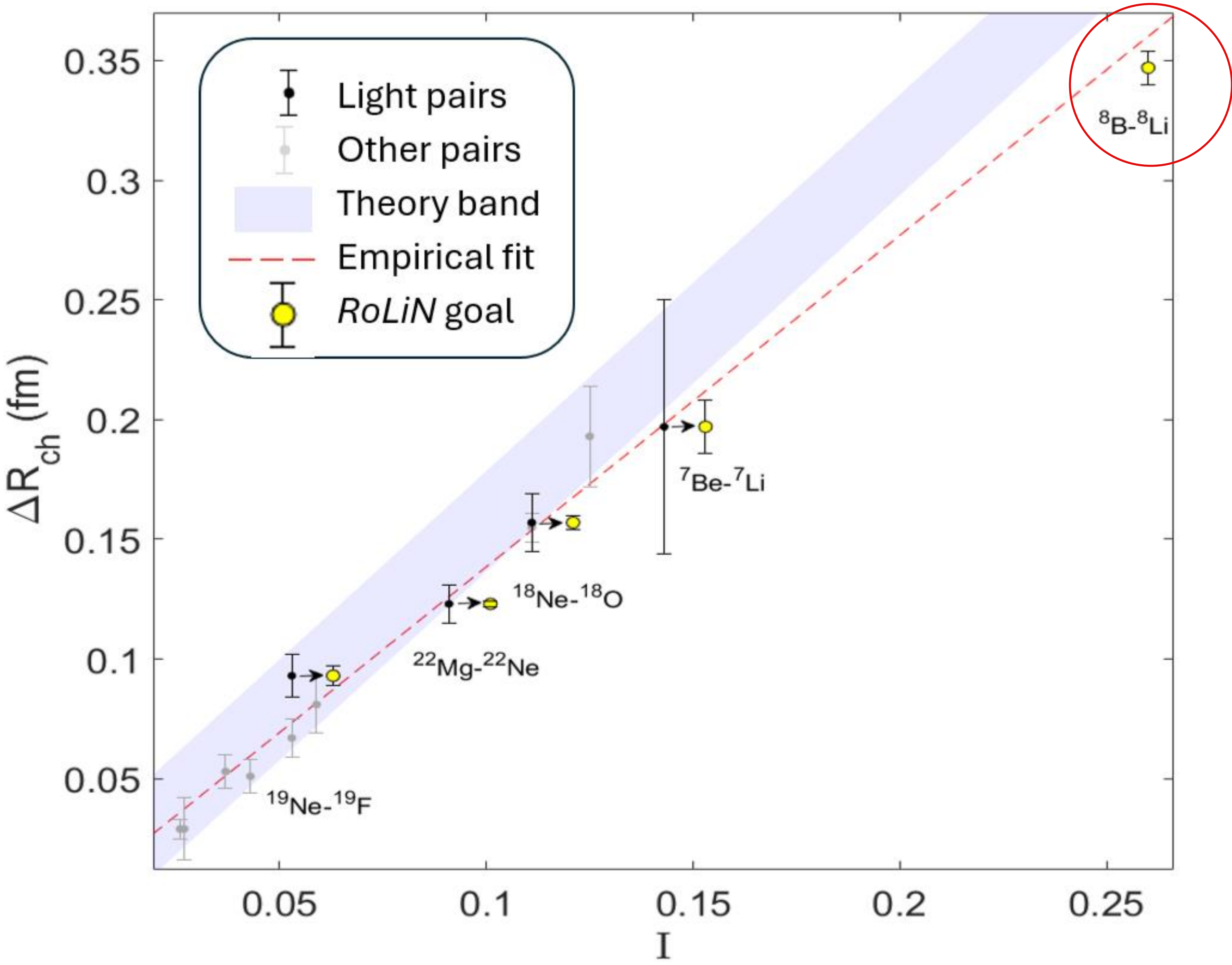
* Private com. Wilfried Nörtershäuser

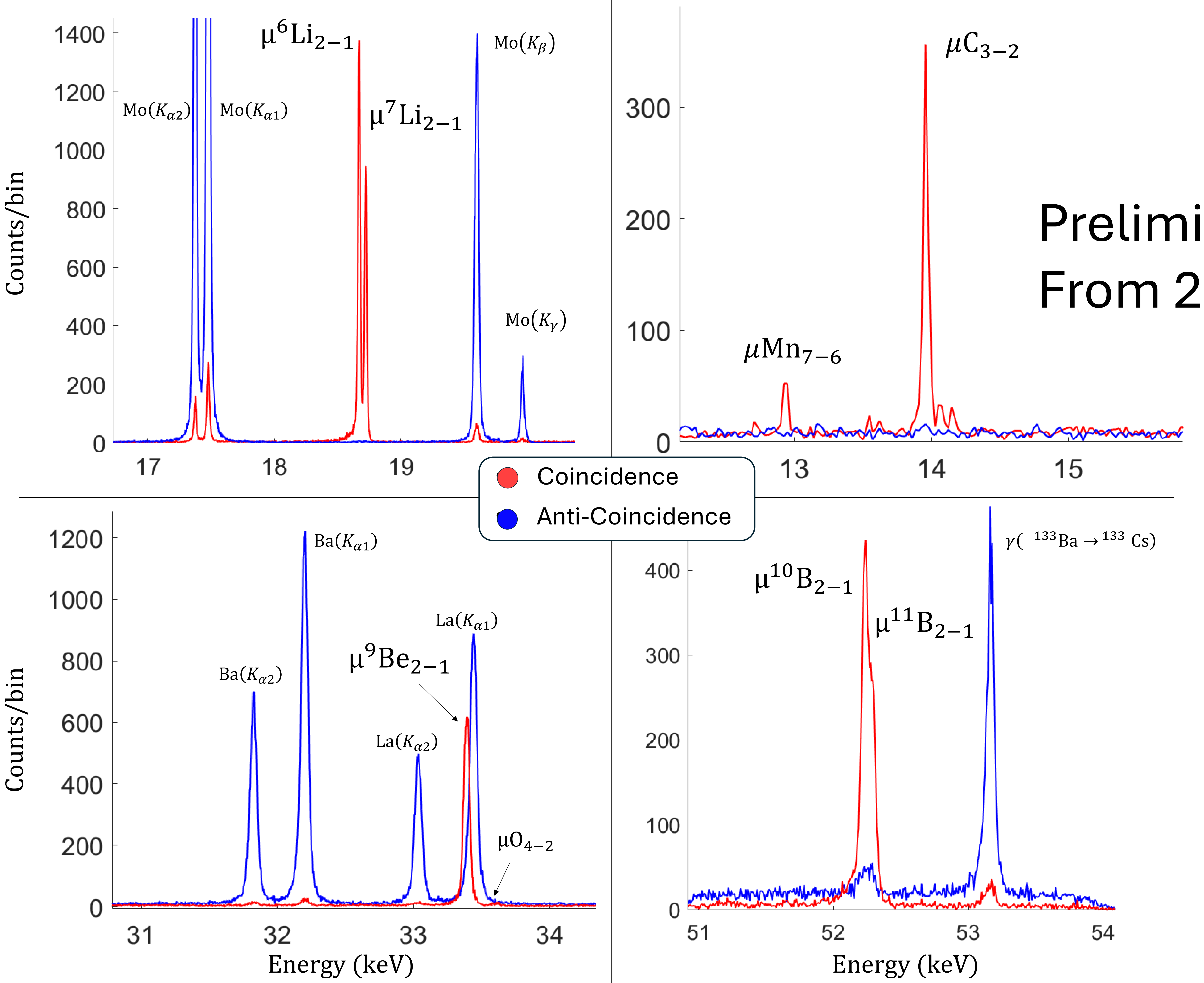
Golden case: A=8

Uncertinaty contributions to $\Delta_{ch}^{mirr}(A=8)$ *

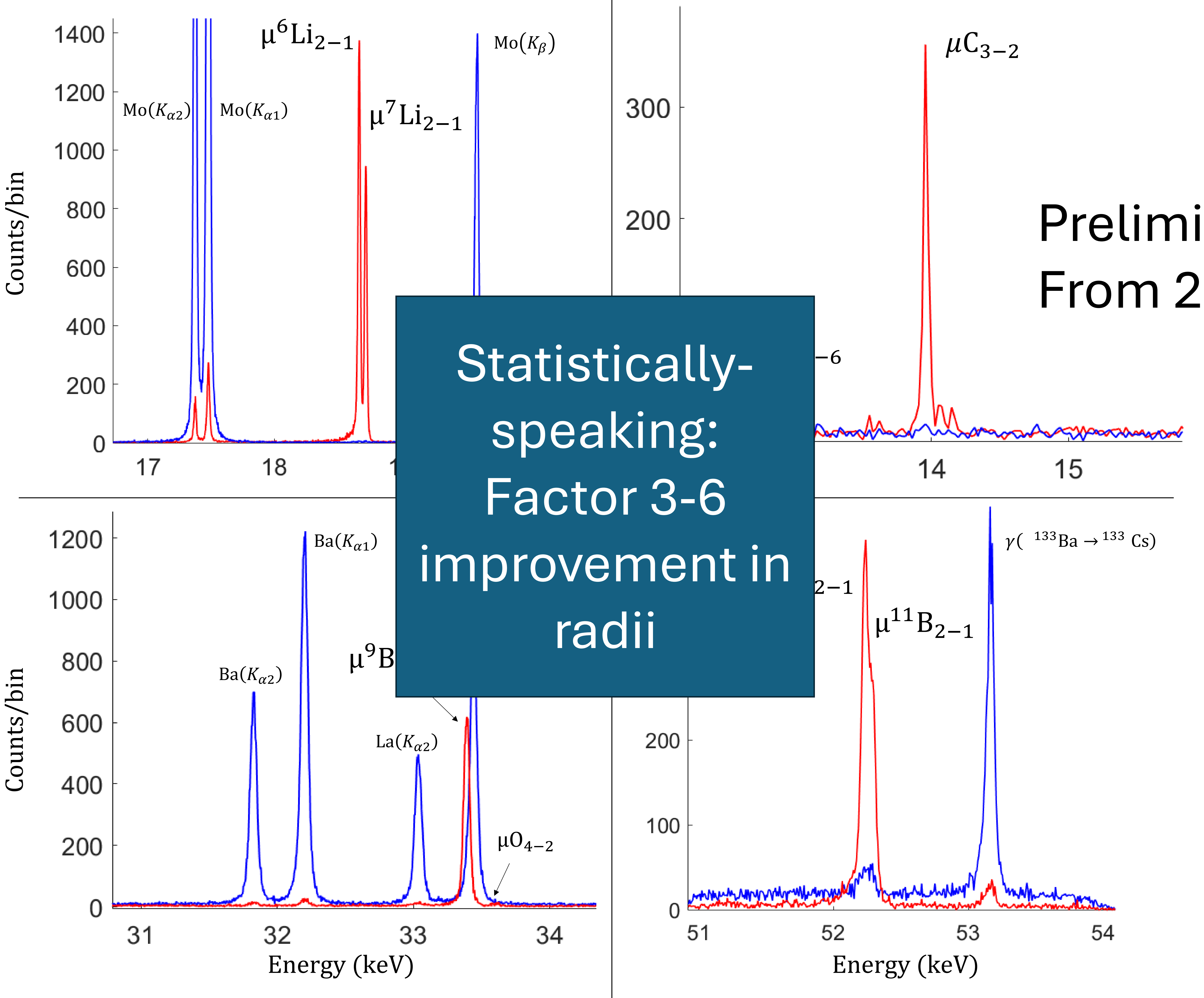


* Private com. Wilfried Nörtershäuser





Preliminary results From 2024

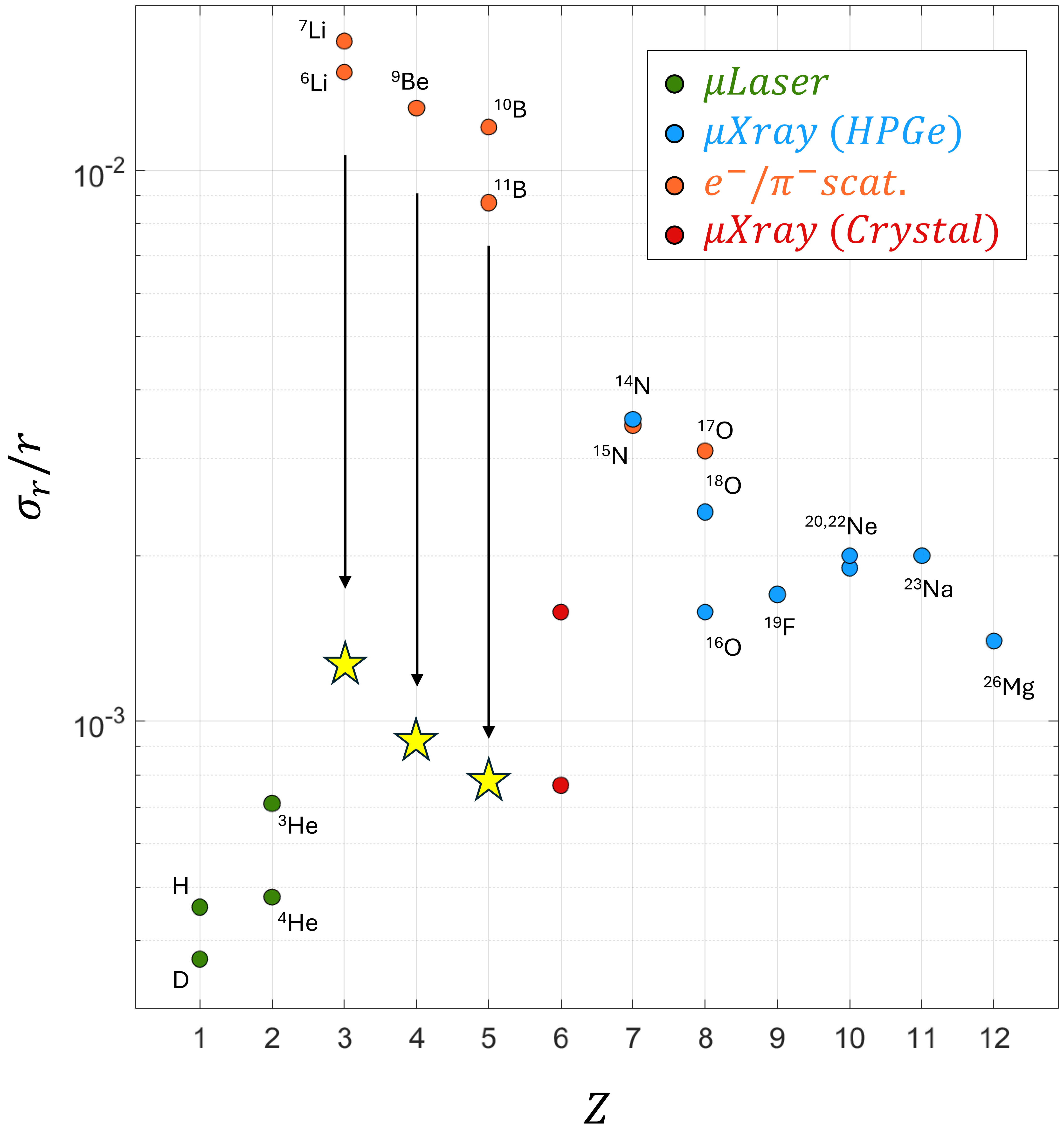


Statistically-speaking: Factor 3-6 improvement in radii

Preliminary results
From 2024

QUARTET goals

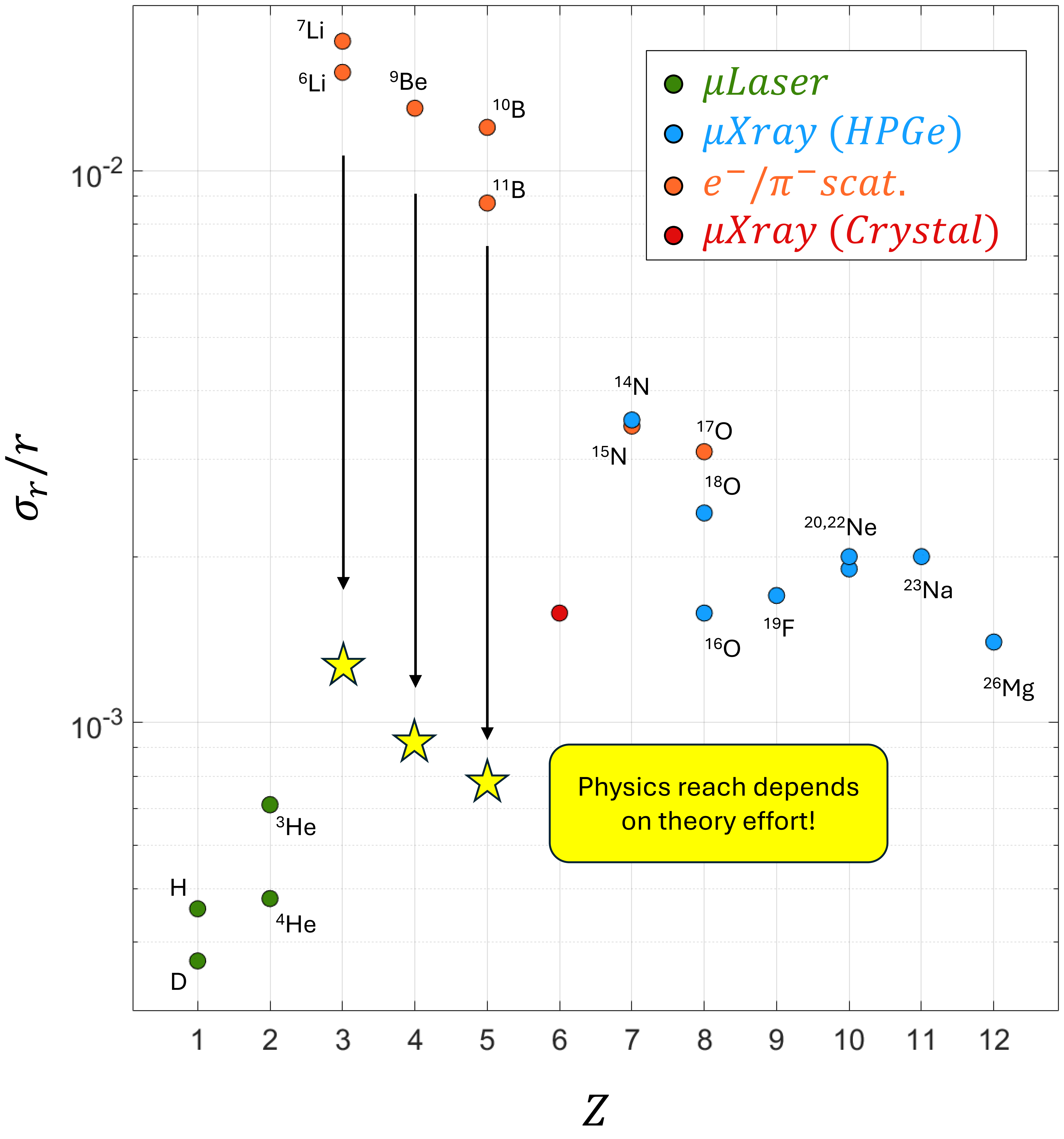
- Phase I: order of magnitude improvement in radii of Li, Be, B



QUARTET goals

- Phase I: order of magnitude improvement in radii of Li, Be, B

- Phase II: Heavier systems

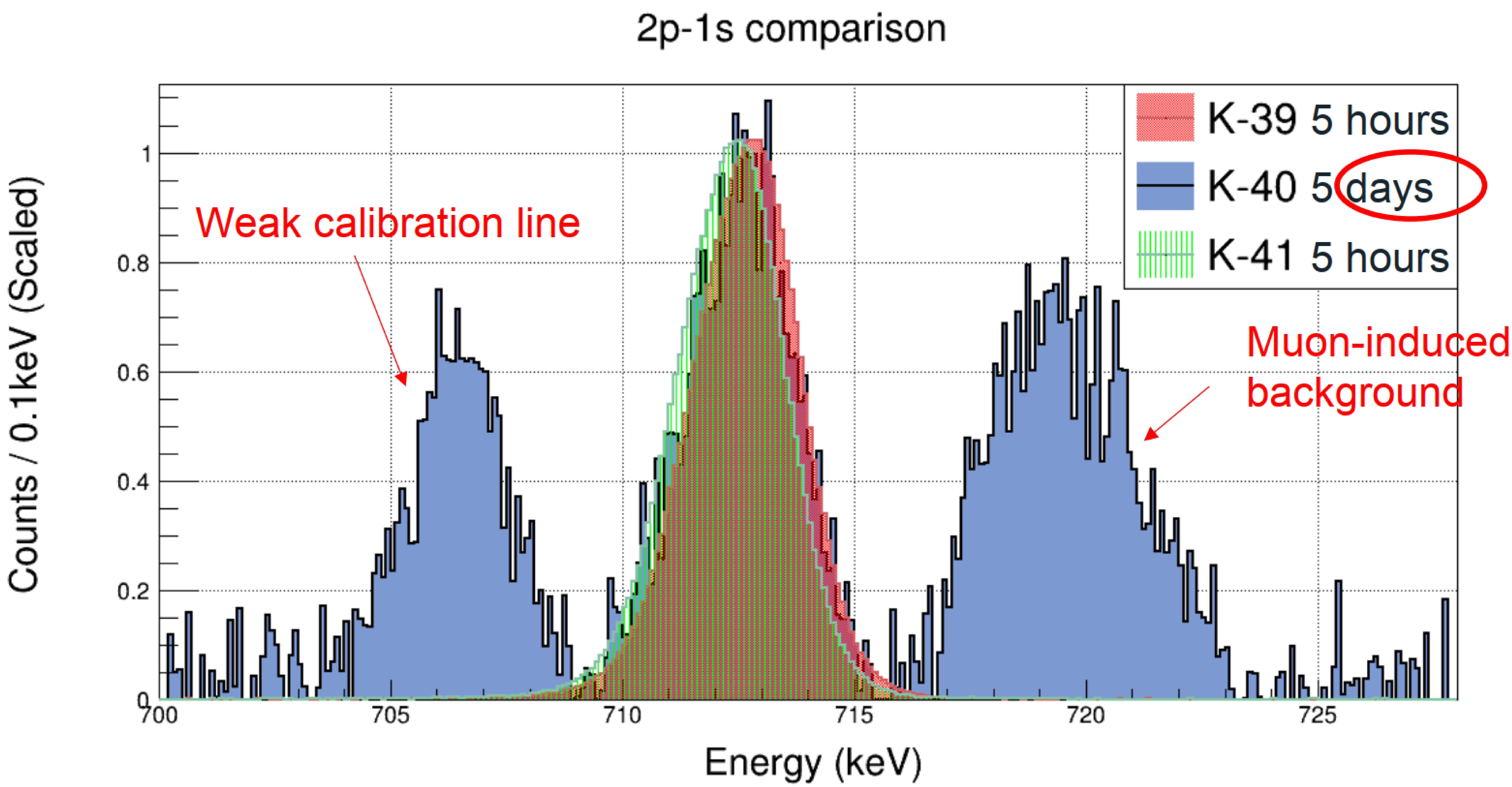


Are there opportunities with heavier systems?

Mostly with microgram targets (HL>years)

- Mux (led by Andreas Knecht)
- Ref-Radii (led by Thomas Cocolios)

Potassium muonic isotope shift



Chlorine measurement (preliminary)

- Muonic 2p-1s energy:

  ^{nat}Cl: 578.56(30) keV

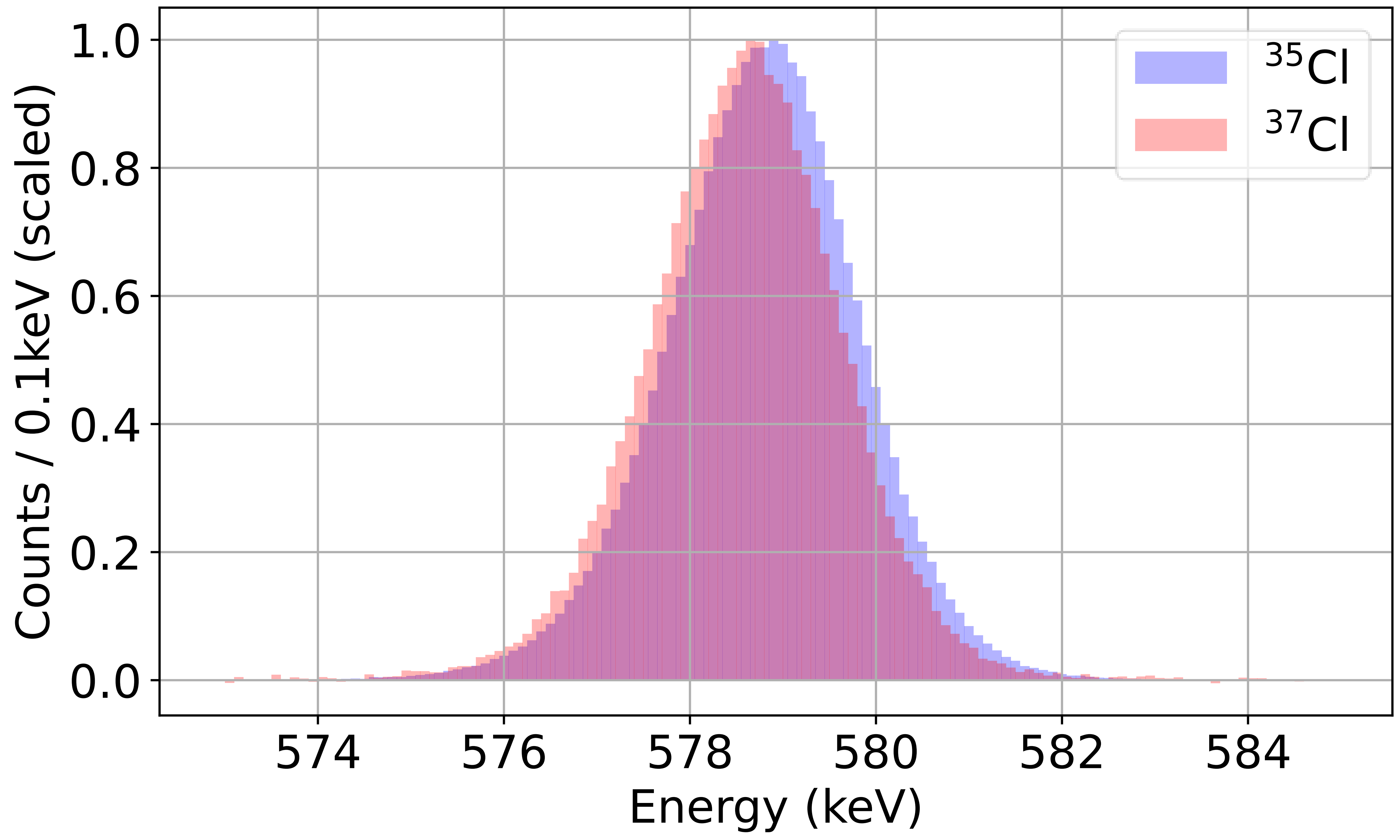
- Expected improvement on 2p-1s transition energy:

  300 eV → < 30 eV

- Expected improvement on radii:

  0.45% → ~0.10-0.15 % (including systematics)

Literature $\delta < r^2 >^{35,37} = 0.03(16)$



Currently worst radius in the region by more than a factor 3

Ecosystem of charge radii determinations

Radius of unstable nucleus $r_x^2 = r_{ref}^2 + \delta r_{a,x}^2$

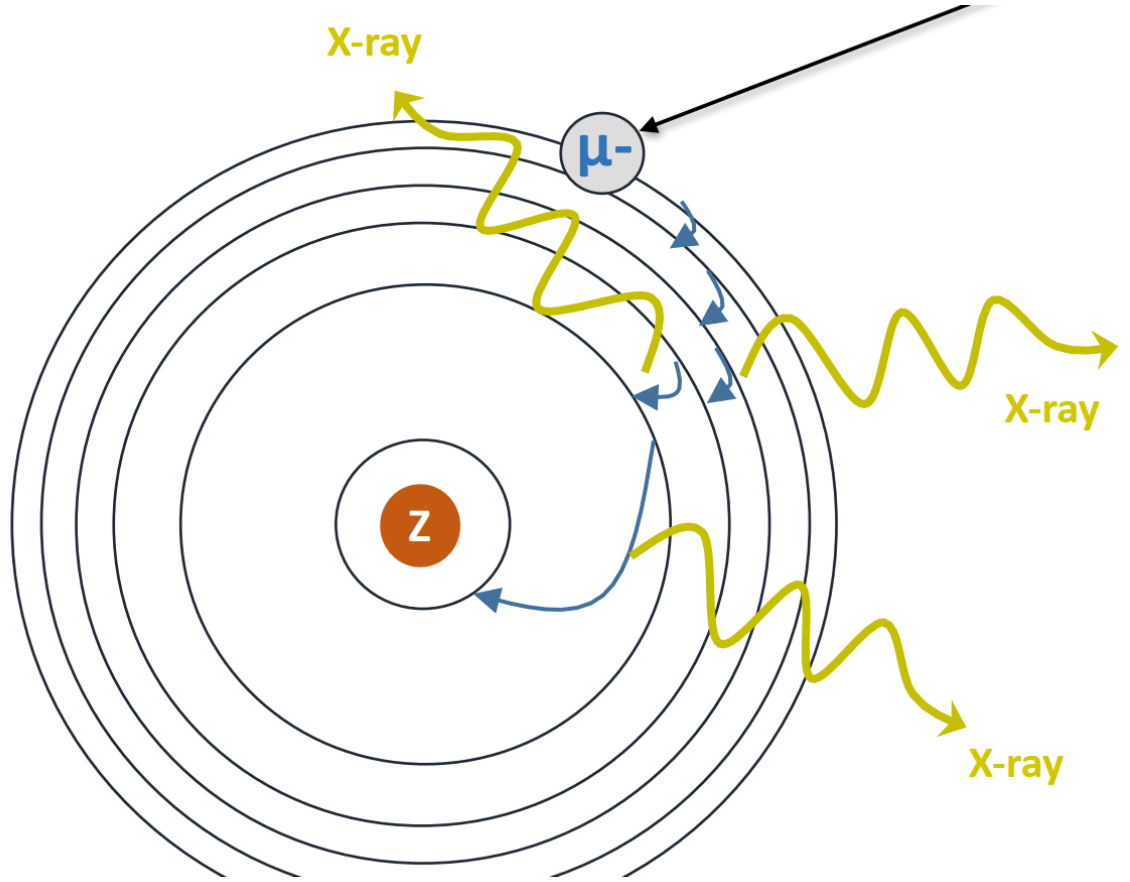
Reference radius (mostly) from **Muonic atoms**

Review

Differential radii (mostly) from radioactive **electronic atoms**

Experiments with exotic atoms

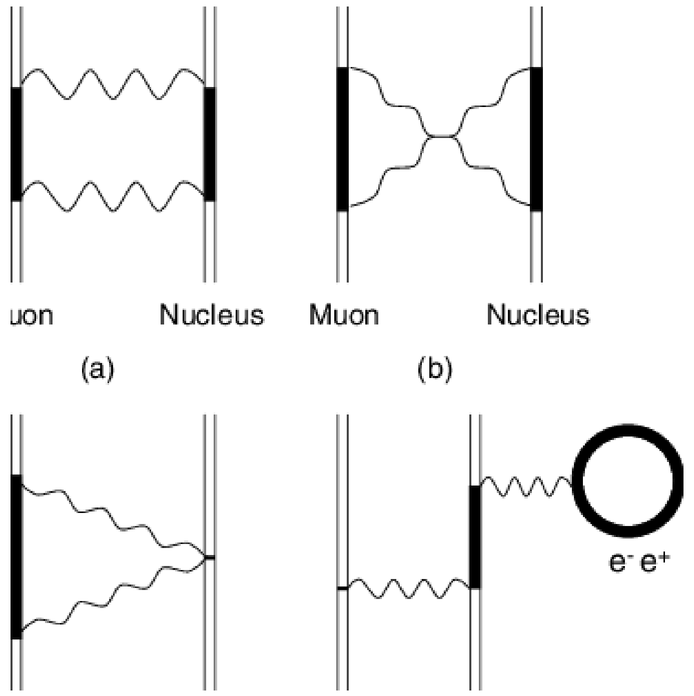
Muonic-atom spectroscopy



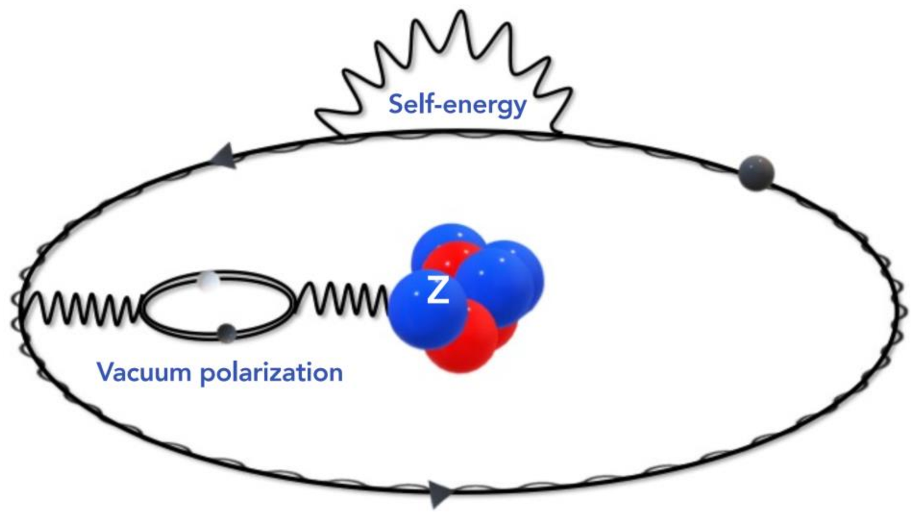
Recent Updates

Theory calculations

Nuclear:



Non-perturbative QED

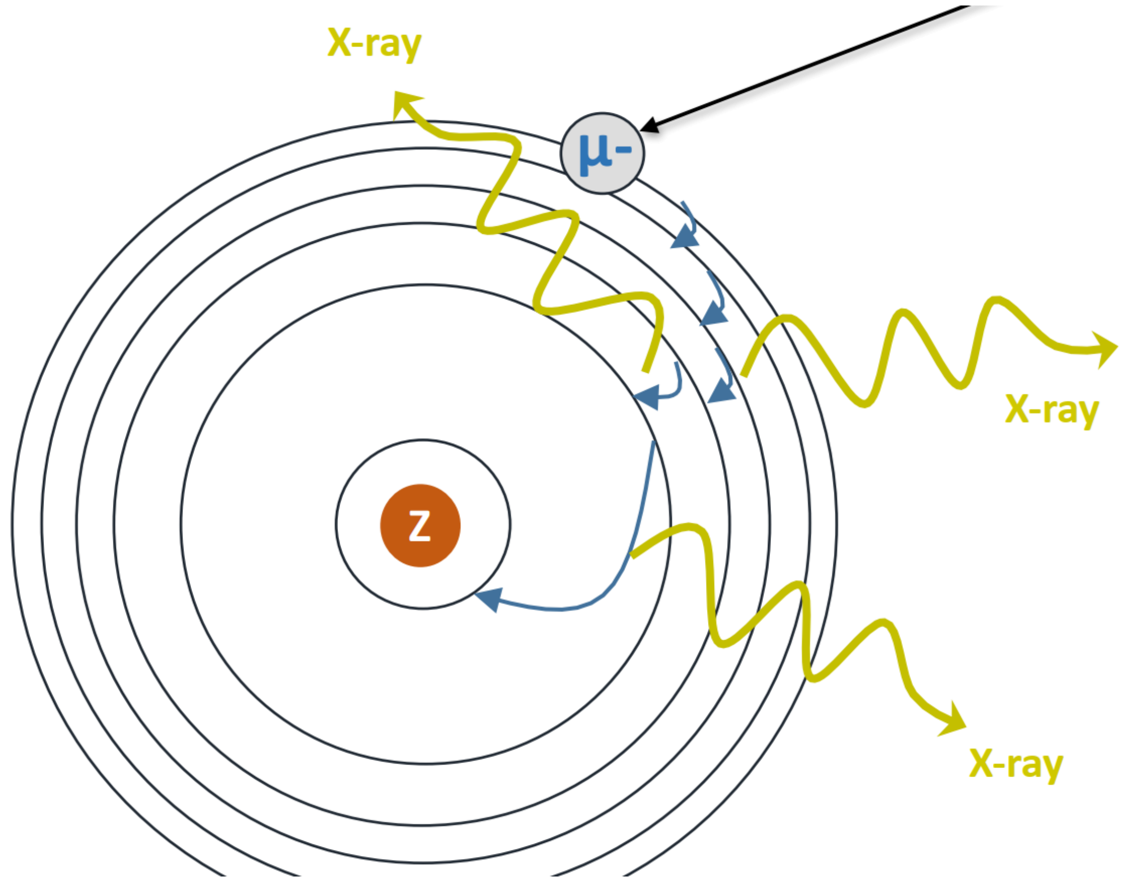


Ecosystem of charge radii determinations

Radius of unstable nucleus $r_x^2 = r_{ref}^2 + \delta r_{a,x}^2$

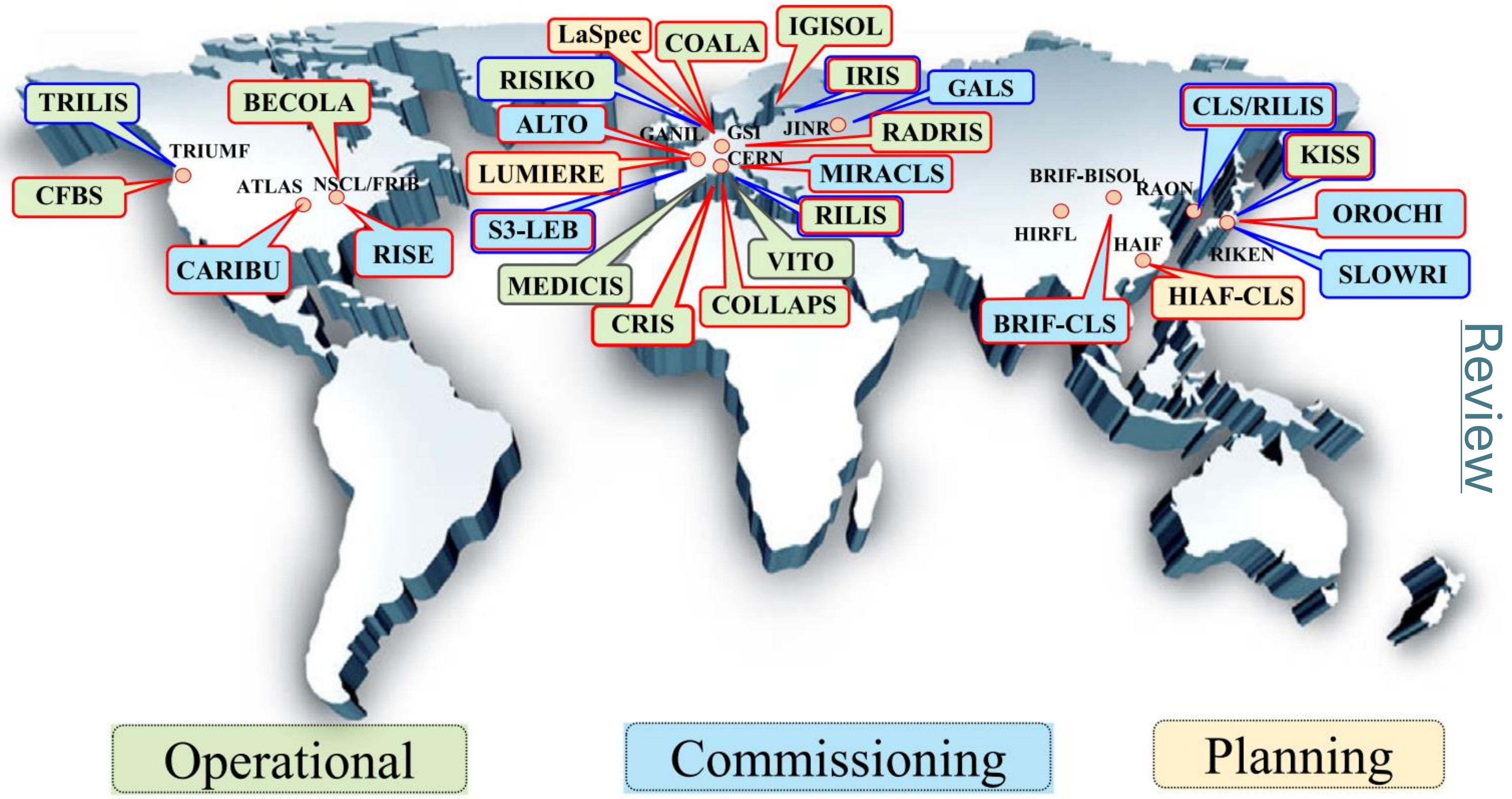
	Reference radius (mostly) from **Muonic atoms** (Review)	Differential radii (mostly) from radioactive **electronic atoms**
Experiments with exotic atoms	Muonic-atom spectroscopy (Recent Updates)	Collinear laser spectroscopy (Review)
Theory calculations	Nuclear: / Non-perturbative QED	Atomic many body calculations (Review)

Muonic-atom spectroscopy



Recent Updates

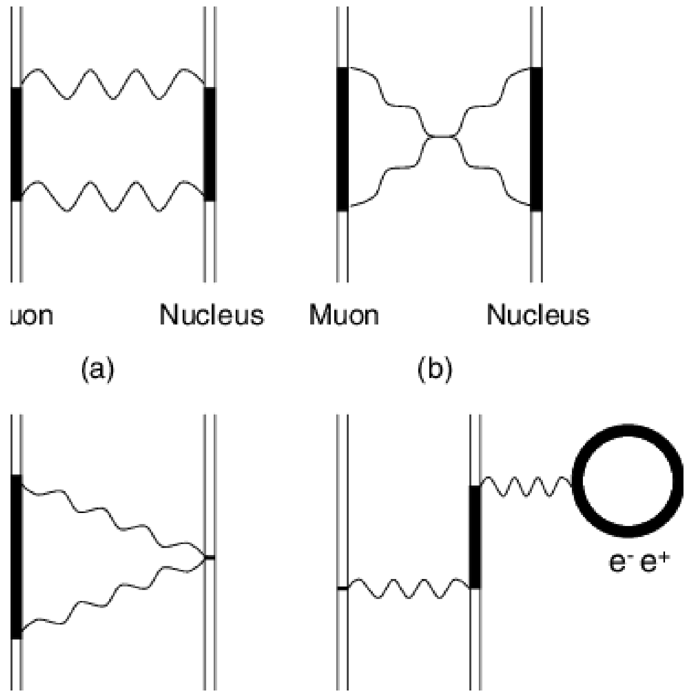
Collinear laser spectroscopy



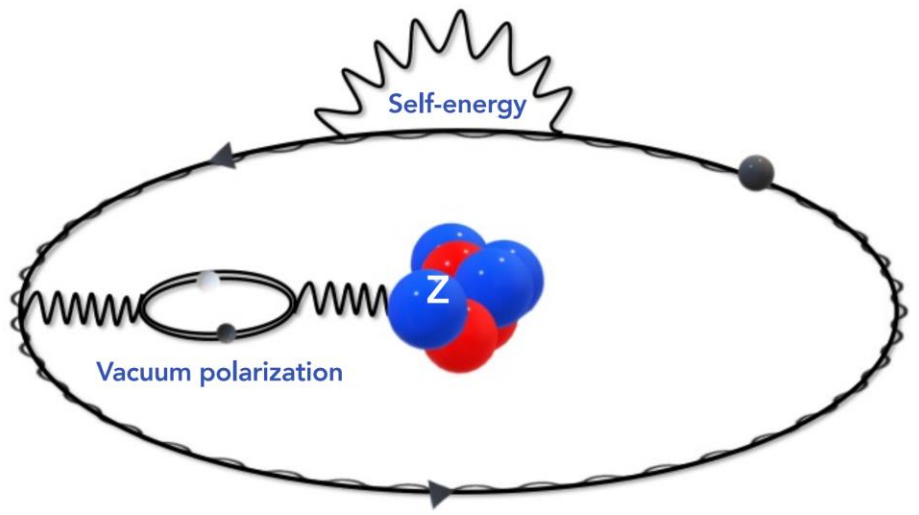
Review

Theory calculations

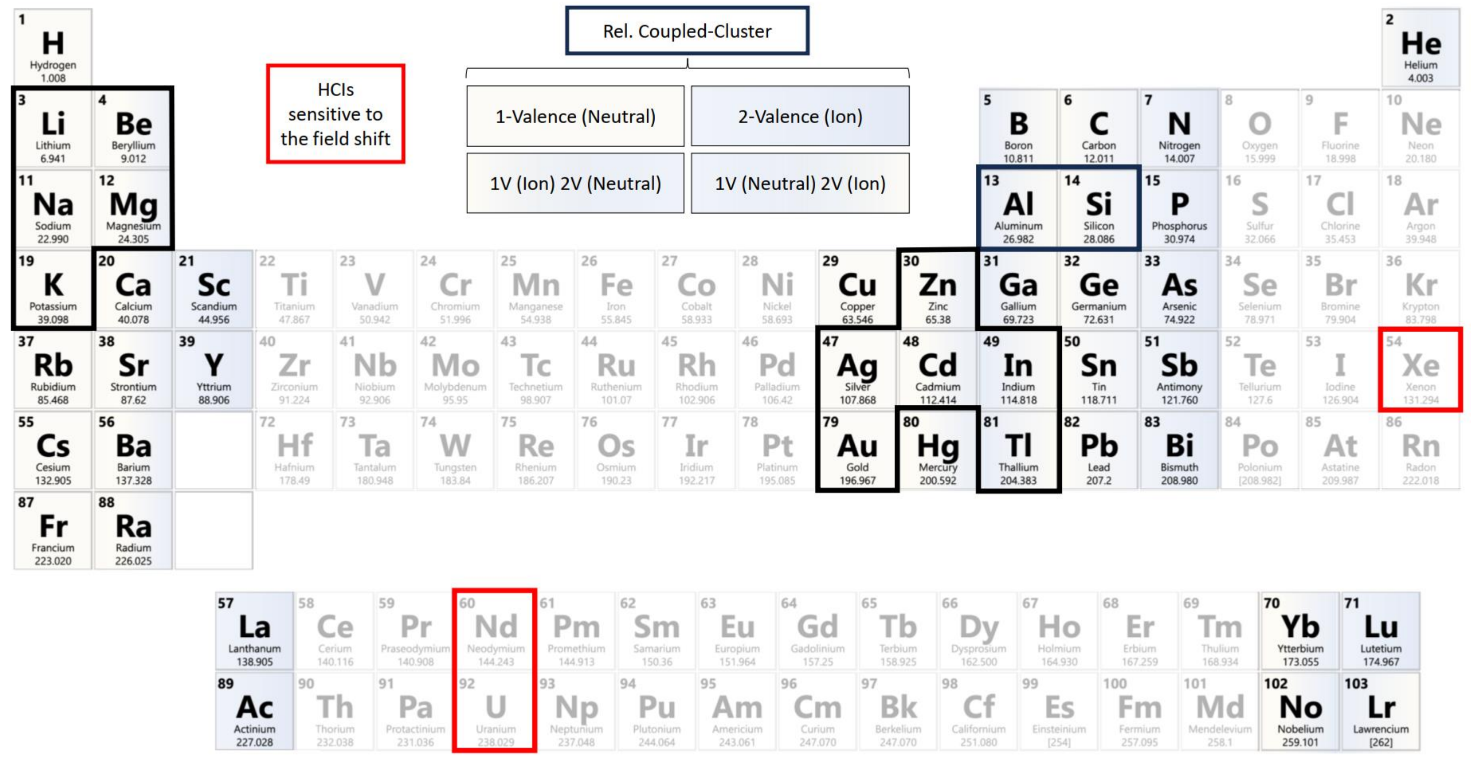
Nuclear:



Non-perturbative QED



Atomic many body calculations



Review

Where do charge radii come from?

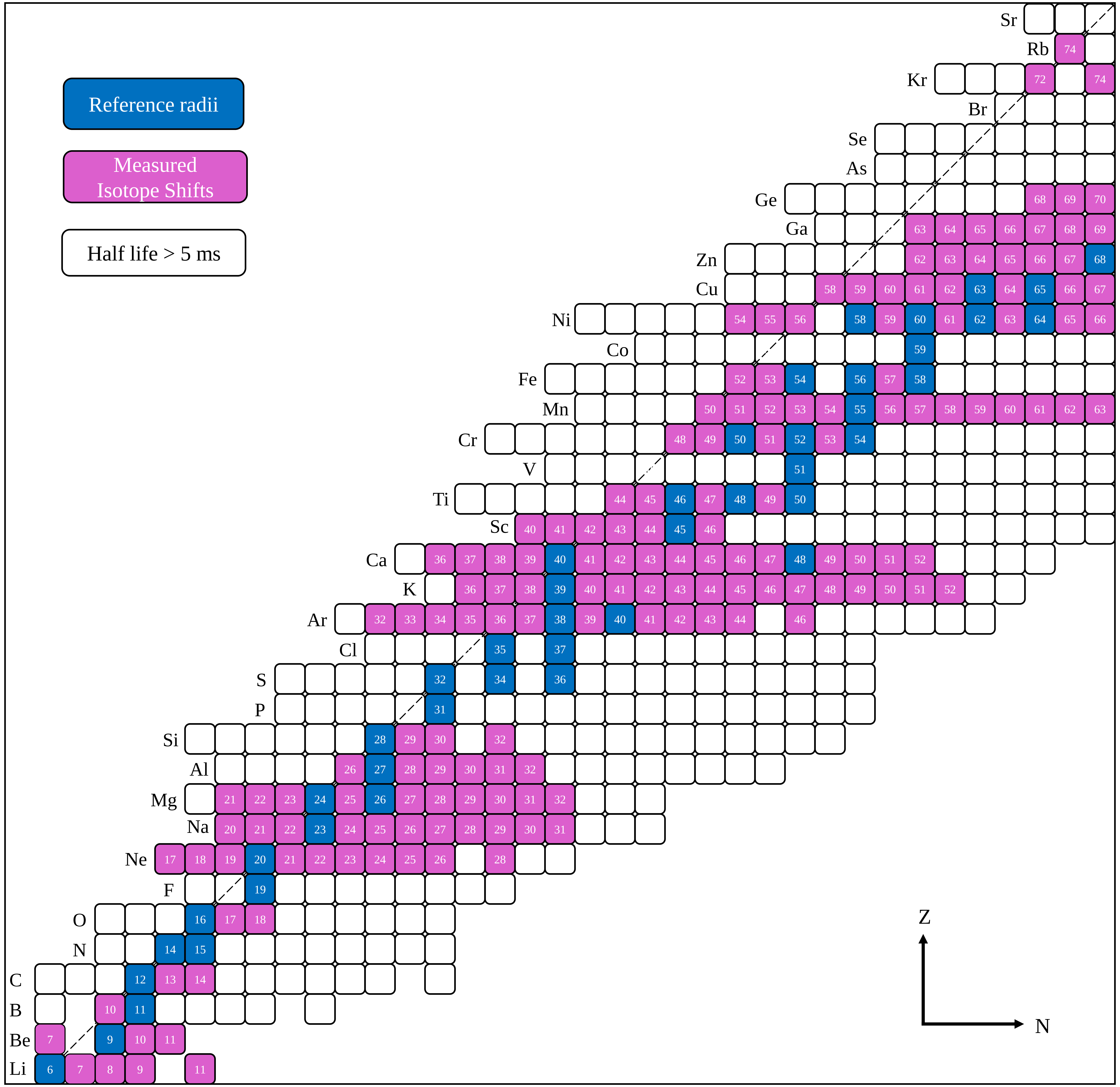
Extraction of MS radius difference from measurements

$$\delta\nu_{A,A'} \approx \left(\frac{1}{M_{A'}} - \frac{1}{M_A}\right) \boldsymbol{K} + \boldsymbol{F}\,\delta r^2_{A,A'}$$

Atomic factors, either calculated or extracted from reference radii (King Plot).

Reference radii connect MS differences with absolutes

$$r^2_{A'} = r^2_A + \delta r^2_{A,A'}$$

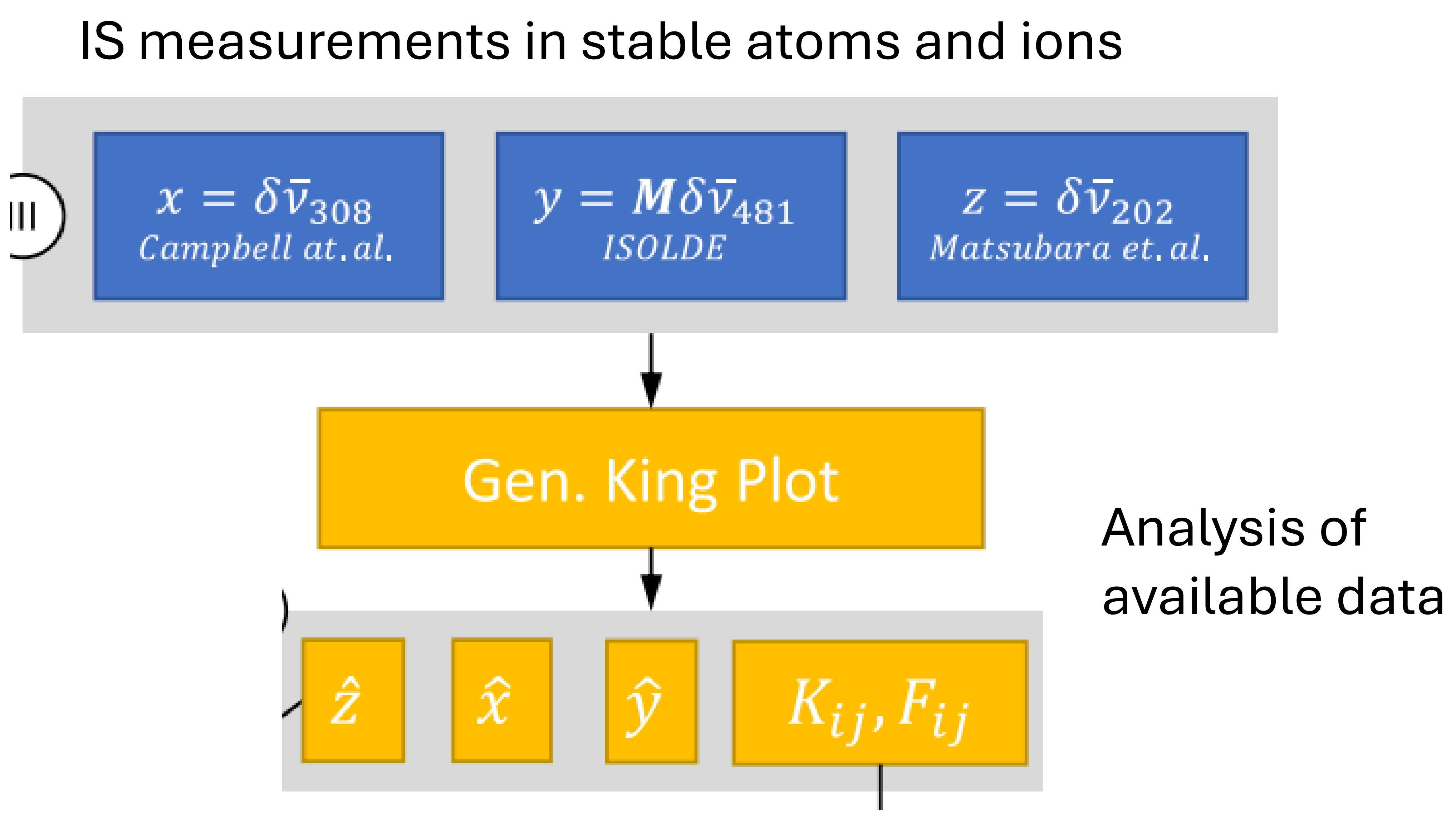


The need for a top view

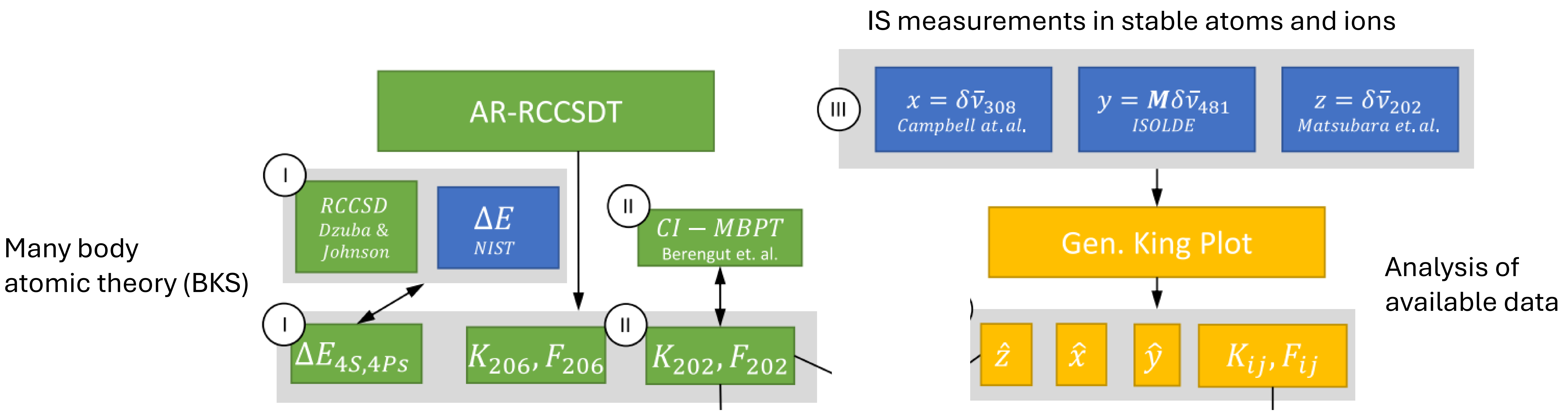
IS measurements in stable atoms and ions

III	$x = \delta\bar{v}_{308}$ *Campbell at. al.*	$y = \boldsymbol{M}\delta\bar{v}_{481}$ *ISOLDE*	$z = \delta\bar{v}_{202}$ *Matsubara et. al.*

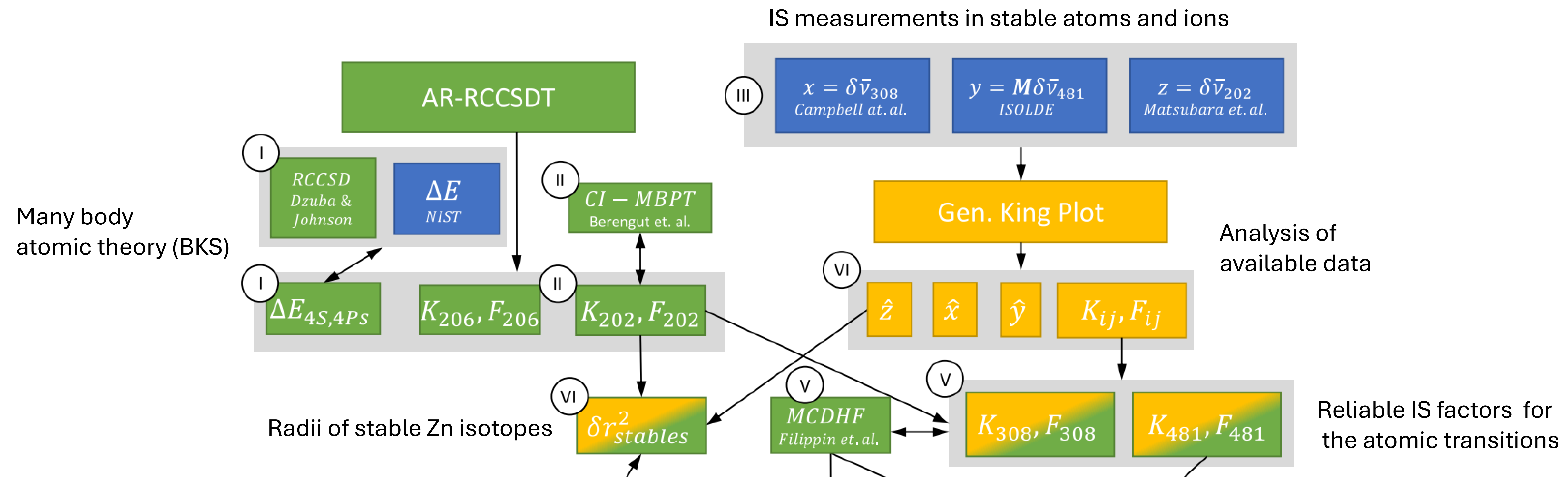
The need for a top view



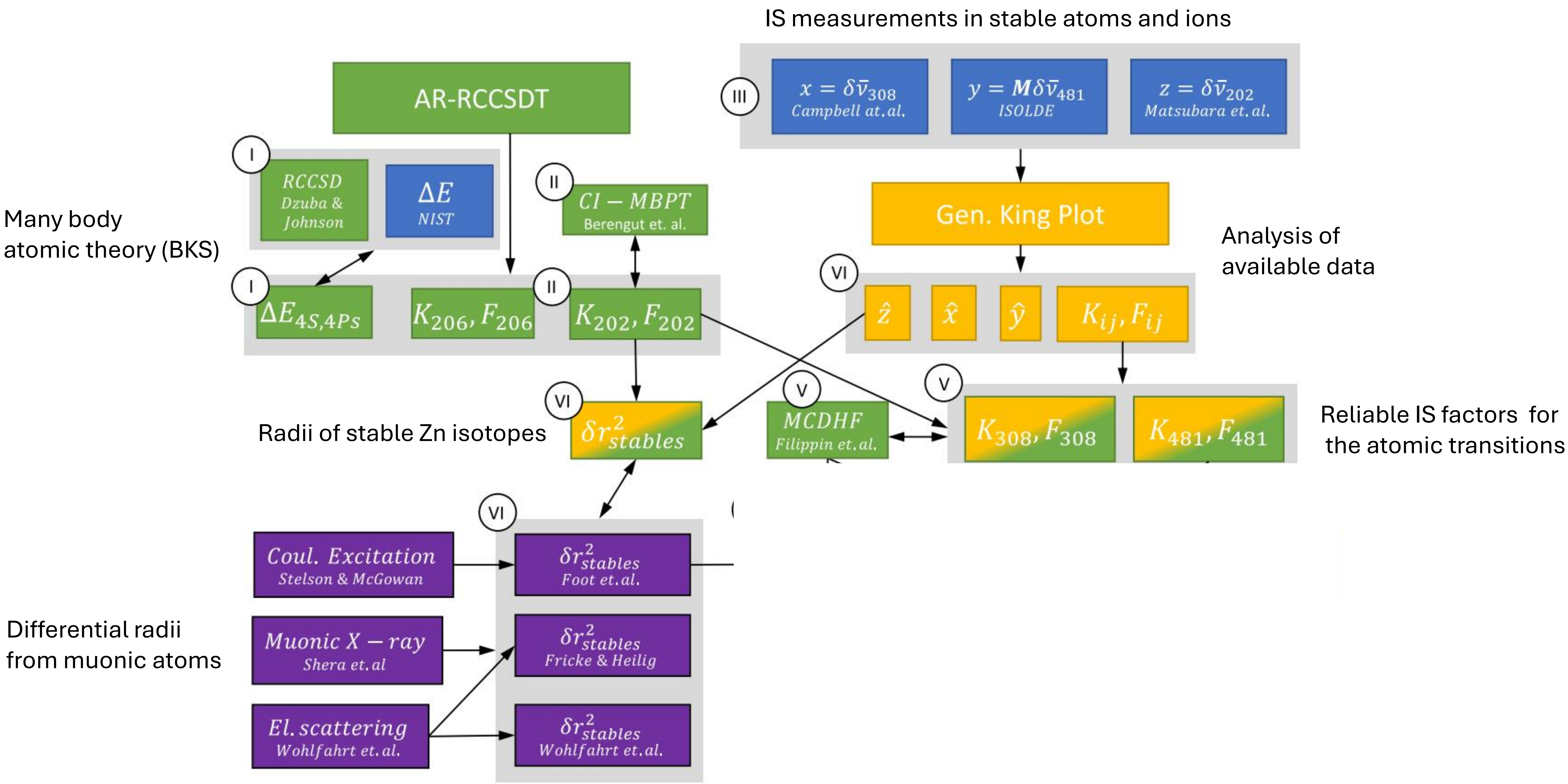
The need for a top view



The need for a top view



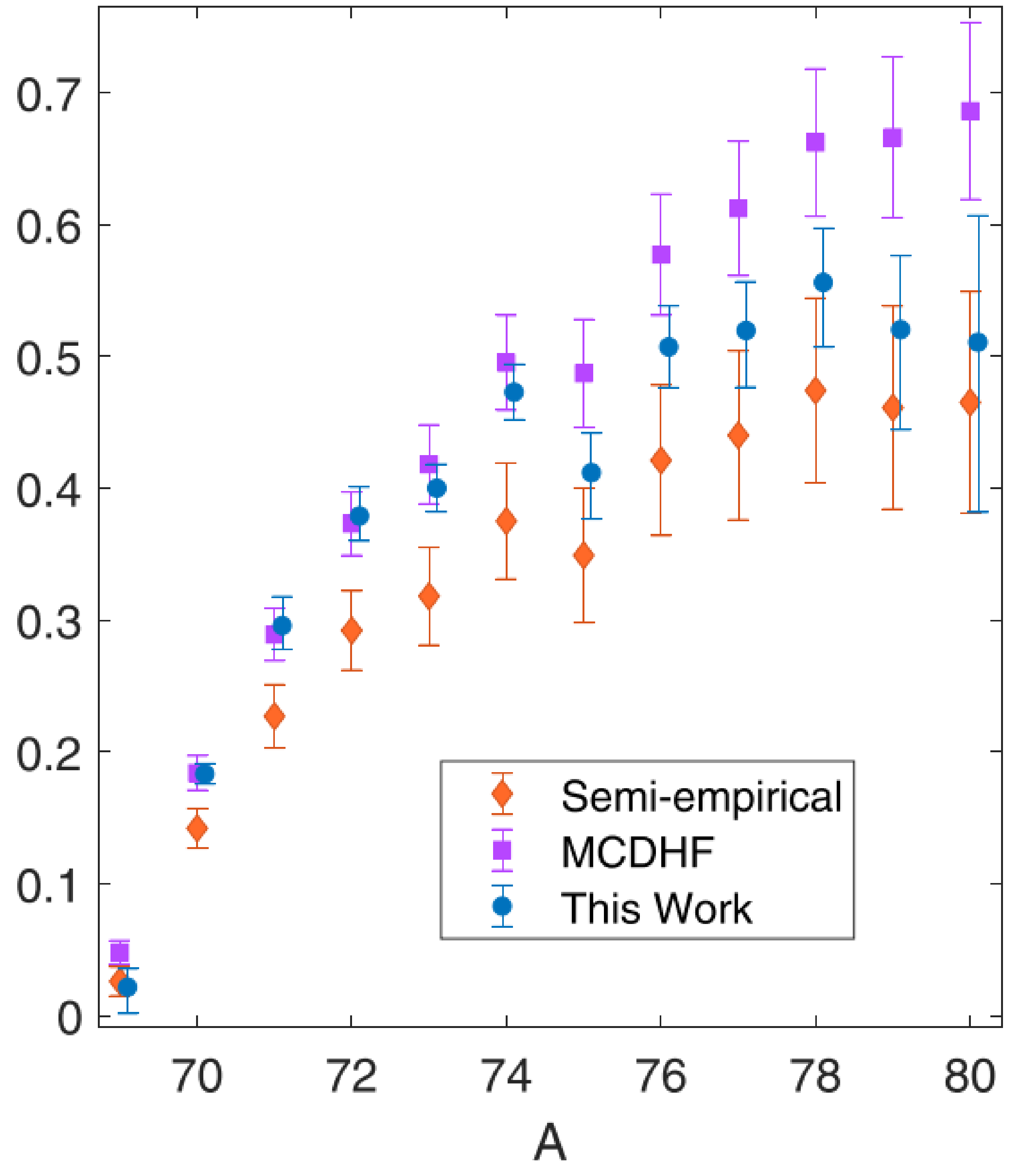
The need for a top view



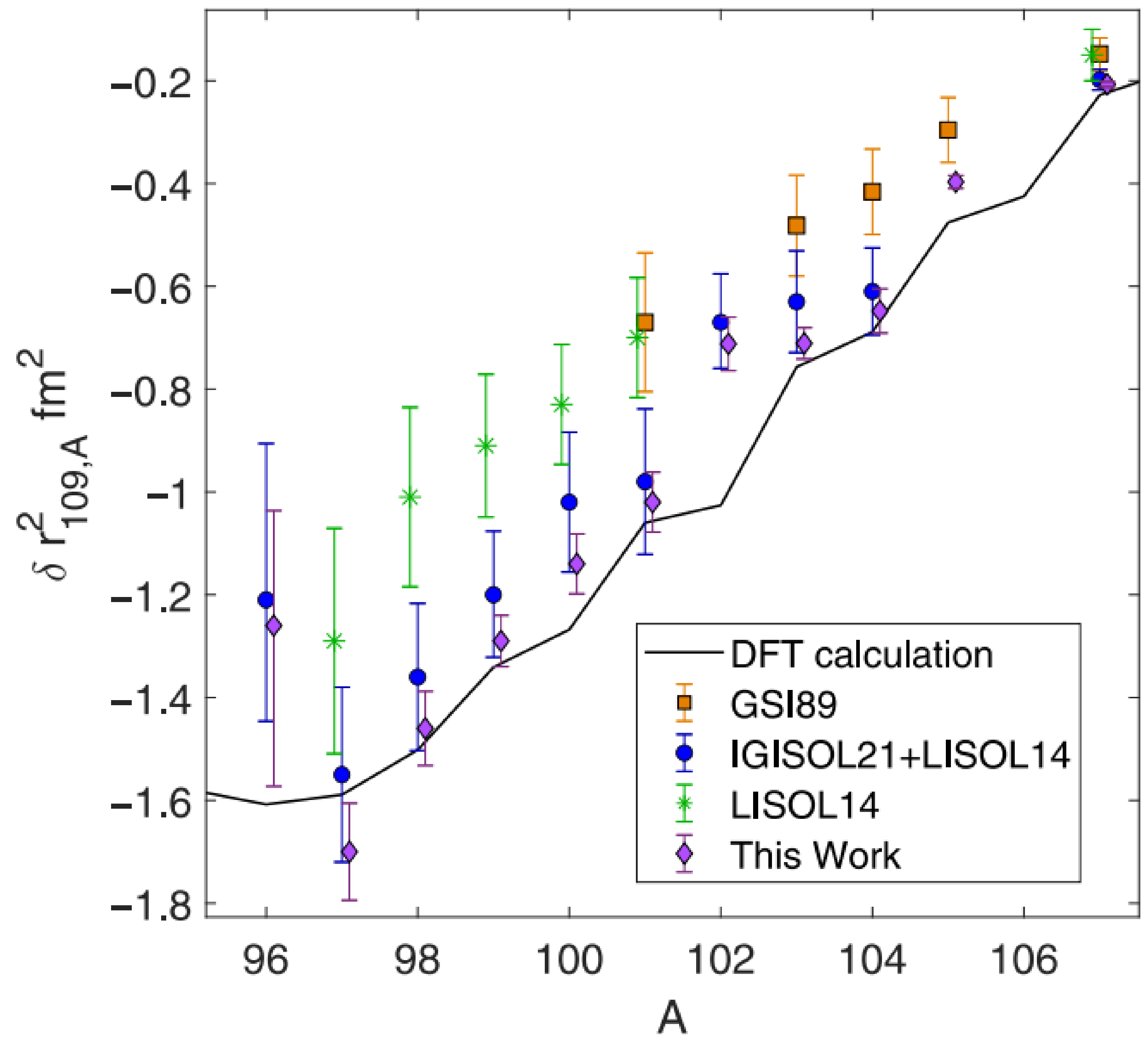
The need for a top view

Differential radii analysis project

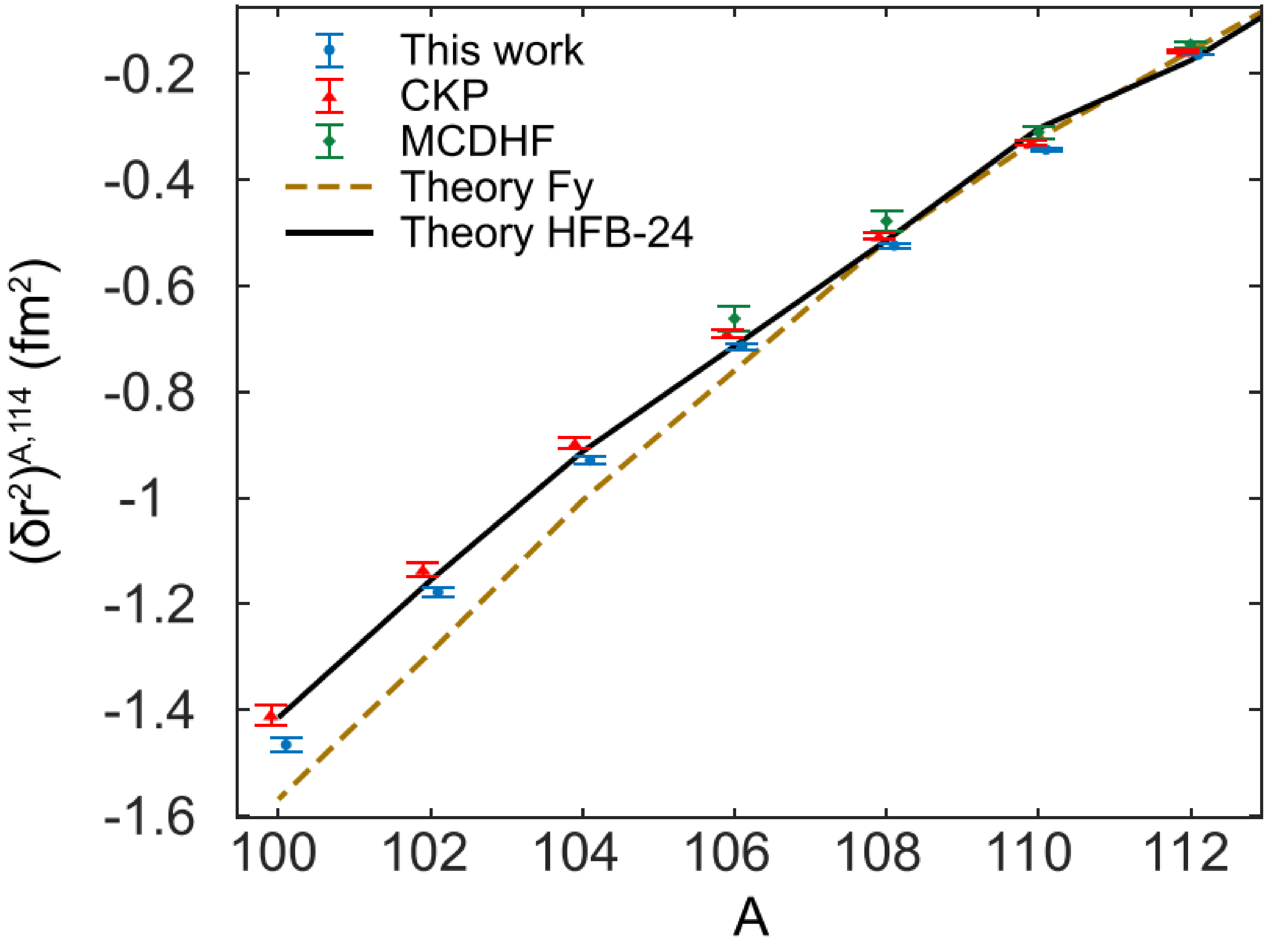
Zn (PRR 5 043142)



Ag (PRR 6 033040)



Cd (NJoP 24 123040)



Also: Ne (PRA 042503), Na+Mg (PRC L031305), K (2412.05932), Al (in prep.)

Take home:

- Simultaneous analysis of muonic and electronic atoms (same nuclear model, etc.)
- Statement that current radii uncertainties are not reliable (give us breathing room).
- Emphasize differential radii of electronic measurements:
  g-factor, helium-like, …
- Importance of alpha_d measurements, especially in the pigmy region
- Cost of the “Only improvements are welcome” approach

Point of discussion on NPOL:

(infinitely heavy nucleus, no QED loops – See NO's talk)

	One photon exchange	Two photon exchange	Three photon exchange
Elastic:	$< r^2 >$	$< r_Z^3 >$ Nuclear shape (muonic atoms)	$< r^2 >^2$ $< r^4 >$ Log terms Electronic atoms
Inelastic:		$NPOL1$	$NPOL2$

Perturbative (ab initio) approach:
(infinitely heavy nucleus, no QED loops)

	One photon exchange	Two photon exchange	Three photon exchange
Elastic:	$< r^2 >$	$< r_Z^3 >$ Nuclear shape (muonic atoms)	$< r^2 >^2$ $< r^4 >$ Log terms Electronic atoms
	Extract		
Inelastic:		$NPOL1$	$NPOL2$

Calculate ab initio

Perturbative (ab initio) approach:
(infinitely heavy nucleus, no QED loops)

	One photon exchange	Two photon exchange	Three photon exchange
Elastic:	$< r^2 >$	$< r_Z^3 >$ Nuclear shape (muonic atoms)	$< r^2 >^2$ $< r^4 >$ Log terms Electronic atoms
	Extract	Partial cancelation	Partial cancelation?
Inelastic:		$NPOL1$	$NPOL2$

Calculate ab initio (NPOL1, NPOL2)

Perturbative (ab initio) approach:
(infinitely heavy nucleus, no QED loops)

$$\Delta E_{2S-2P} = \delta_{\mathrm{QED}} + \frac{m_r^4 (Z\alpha)^4}{12} \langle r_c^2 \rangle + \delta_{\mathrm{TPE}}$$

Elastic

Theory: Lamb shift in muonic D

$\Delta E^{\mu D}_{\mathrm{Lamb}}$ = 228.7740 (3) $\mathrm{meV}_{\mathrm{QED}}$ + 1.7503 (**200**) $\mathrm{meV}_{\mathrm{TPE}}$ – 6.1074 meV/fm^2 * $\mathbf{R_d}^2$

$$\Delta E^{\mathrm{exp}}_{\mathrm{LS}} = 202.8785(31)_{\mathrm{stat}}(14)_{\mathrm{syst}}\,\mathrm{meV}$$

Inelasti

Nuclear structure two (and three!)-photon contributions to the Lamb shift in muonic deuterium.

Calculate ab initio

Cancellation only in perturbative approach (S. Bacca)?

Theoretical derivation of TPE

MG?

$$\delta_{\rm TPE} = \delta^A_{\rm Zem} + \delta^n_{\rm Zem} + \underbrace{\delta^A_{\rm pol} + \delta^n_{\rm pol}}_{\text{MG?}}$$

$$\delta^A_{\rm pol} = \delta^{(0)}_{D1} + \delta^{(1)}_{R3} + \delta^{(1)}_{Z3} + \delta^{(2)}_{R^2} + \delta^{(2)}_{Q} + \delta^{(2)}_{D1D3} + \delta^{(0)}_{C} + \delta^{(0)}_{L} + \delta^{(0)}_{T} + \delta^{(0)}_{M} + \delta^{(1)}_{R1} + \delta^{(1)}_{Z1} + \delta^{(2)}_{NS}$$

$$\delta^A_{\rm Zem} = -\delta^{(1)}_{Z3} - \delta^{(1)}_{Z1}$$

Friar an Payne ('97)

All-order (data-driven) approach:
(infinitely heavy nucleus, no QED loops)

	One photon exchange	Two photon exchange	Three photon exchange
Elastic:	$< r^2 >$	$< r_Z^3 >$ Nuclear shape (muonic atoms)	$< r^2 >^2$ $< r^4 >$ Log terms Electronic atoms
	Extract		
Inelastic:		$NPOL1$	$NPOL2$

Dirac-Coulomb with Input from electron scattering

All-order (data-driven) approach:
(infinitely heavy nucleus, no QED loops)

	One photon exchange	Two photon exchange	Three photon exchange	
Elastic:	$< r^2 >$	$< r_Z^3 >$ Nuclear shape (muonic atoms)	$< r^2 >^2$ $< r^4 >$ Log terms Electronic atoms	Dirac-Coulomb with Input from electron scattering
Inelastic:	Extract	$NPOL1$	$NPOL2$	Nuclear polarization?


ATTENTION TO THE ALARM!
In case of FALSE alarm call immediately 2600 (security center)
Thanks for listening!