Nuclear Polarization in Muonic Atoms

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Based on:
2501.xxxxx appears tomorrow or Wed
2412.05932
2407.09743
2311.00044
2309.16893
2212.02681
2208.03037

With
Ben Ohayon
Bijaya Sahoo
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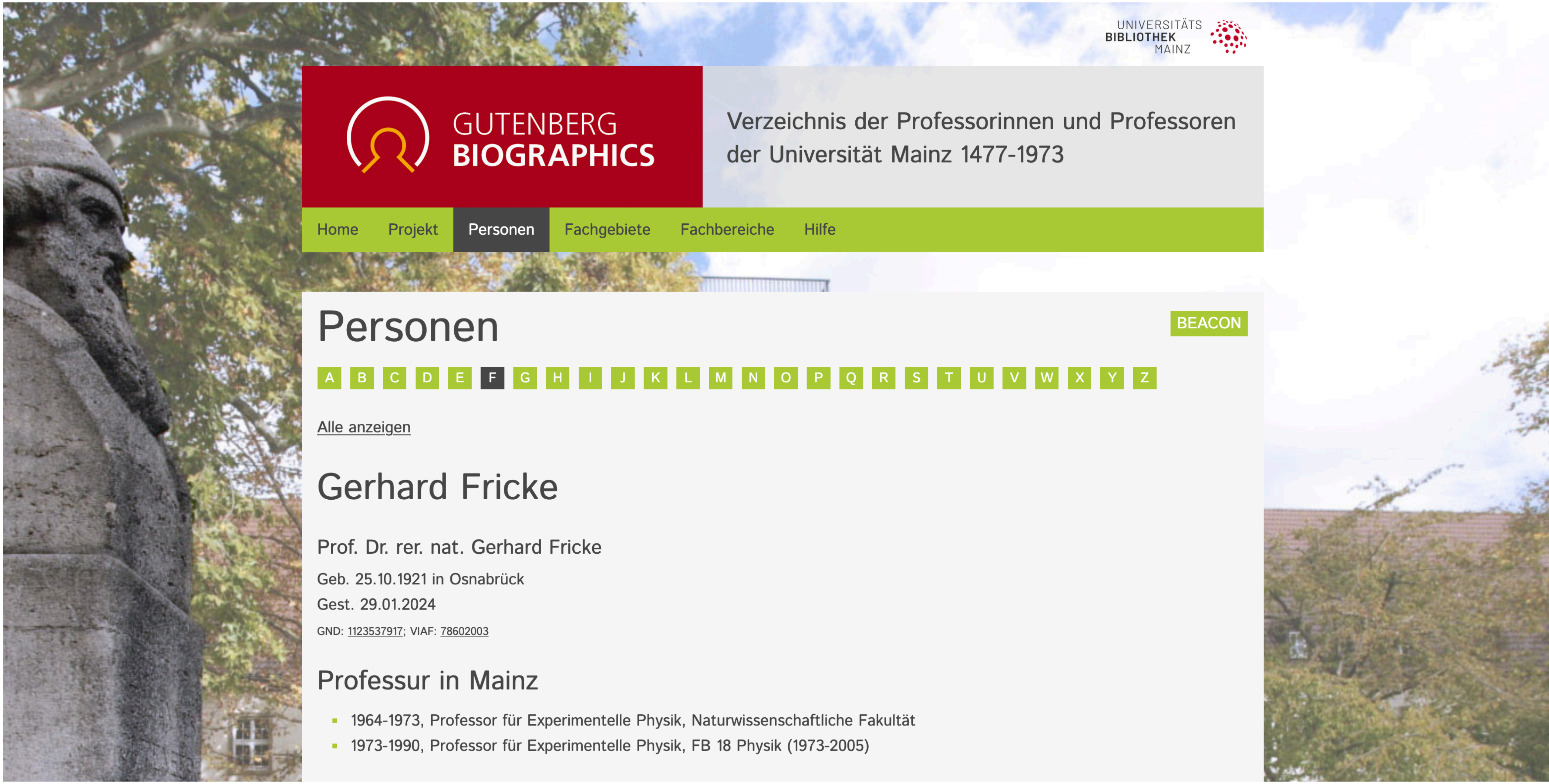
John Behr
Arup Chakraborty
Vaibhav Katyal

Technical Meeting on Compilation and Evaluation of Tables of Nuclear Charge Radii
IAEA Vienna — January 27-30, 2025



UNIVERSITÄTS BIBLIOTHEK MAINZ

GUTENBERG BIOGRAPHICS

Verzeichnis der Professorinnen und Professoren der Universität Mainz 1477-1973

Home Projekt Personen Fachgebiete Fachbereiche Hilfe

Personen

BEACON

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

Alle anzeigen

Gerhard Fricke

Prof. Dr. rer. nat. Gerhard Fricke

Geb. 25.10.1921 in Osnabrück

Gest. 29.01.2024

GND: 1123537917; VIAF: 78602003

Professur in Mainz

- 1964-1973, Professor für Experimentelle Physik, Naturwissenschaftliche Fakultät
- 1973-1990, Professor für Experimentelle Physik, FB 18 Physik (1973-2005)

Prof. Fricke was at the KPH Institute’s Christmas Party in December 2023

Outline

Tests of Cabibbo unitarity with nuclear β decays

Nuclear inputs: theory & Experiment

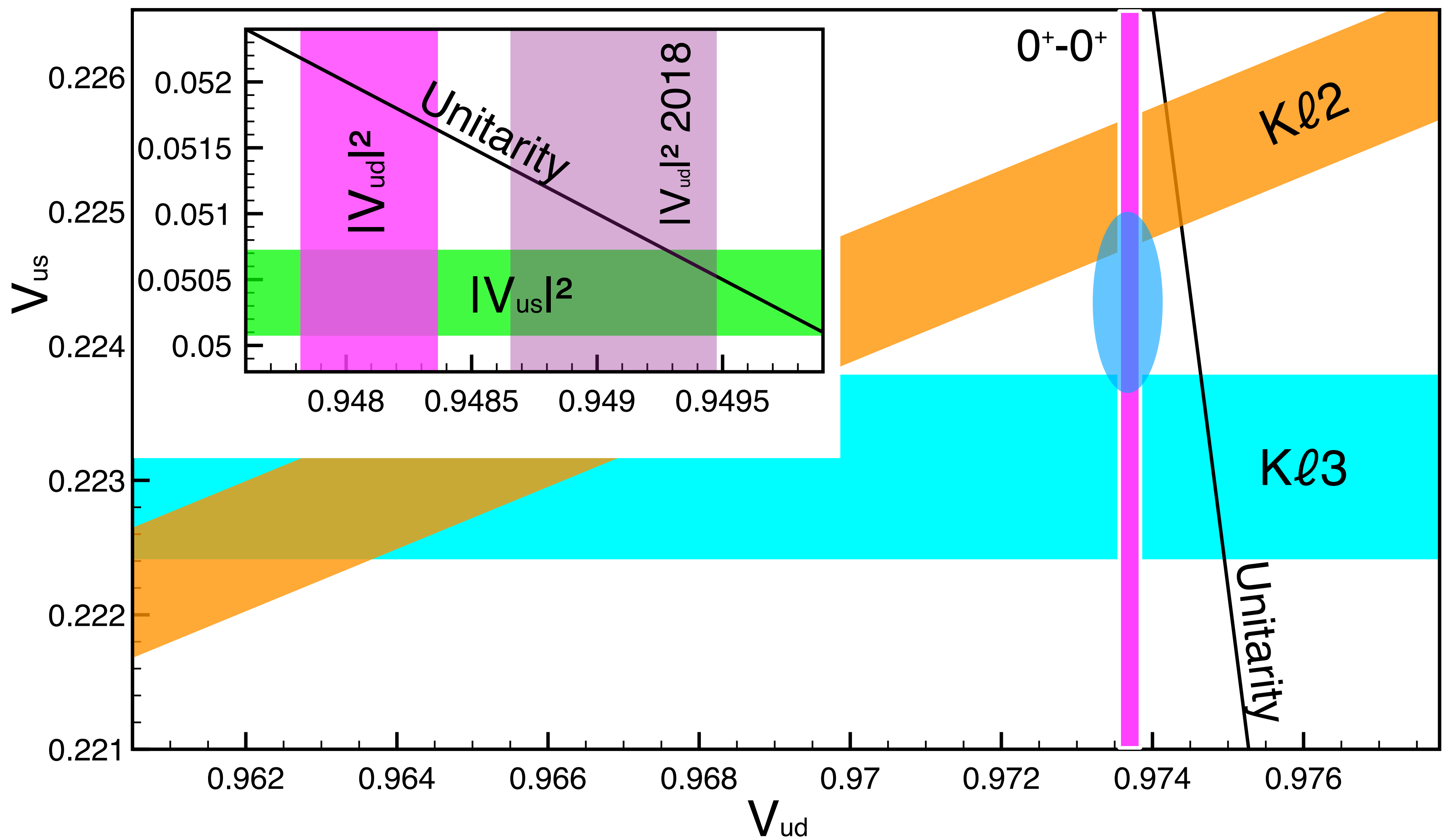
Nuclear polarization in muonic atoms

Status? Update? Outlook?

Precision tests of the Standard Model with β-decays

Top-row CKM unitarity deficit

$$\underset{\sim 0.95}{|V_{ud}|^2} + \underset{\sim 0.05}{|V_{us}|^2} + \underset{\sim 10^{-5}}{|V_{ub}|^2} = 0.9985(6)_{V_{ud}}(4)_{V_{us}}$$



Inconsistencies between measurements of V_{ud} and V_{us} and SM predictions

Most precise V_{ud} from superallowed nuclear decays

Status of V_{ud}

0^+-0^+ nuclear decays: long-standing champion

$$|V_{ud}|^2 = \frac{2984.43s}{\mathscr{F}t(1+\Delta_R^V)}$$

$$|V_{ud}^{0^+-0^+}| = 0.97370\,(1)_{exp,\,nucl}\,(3)_{NS}\,(1)_{RC}[3]_{total}$$

Nuclear uncertainty x 3

Neutron decay: discrepancies in lifetime τ_n and axial charge g_A; competitive!

$$|V_{ud}|^2 = \frac{5024.7\text{ s}}{\tau_n(1+3g_A{}^2)(1+\Delta_R)}$$

Single best measurements only

$$|V_{ud}^{\text{free n}}| = 0.9733\,(2)_{\tau_n}\,(3)_{g_A}\,(1)_{RC}[4]_{total}$$

PDG average

$$|V_{ud}^{\text{free n}}| = 0.9733\,(3)_{\tau_n}\,(8)_{g_A}\,(1)_{RC}[9]_{total}$$

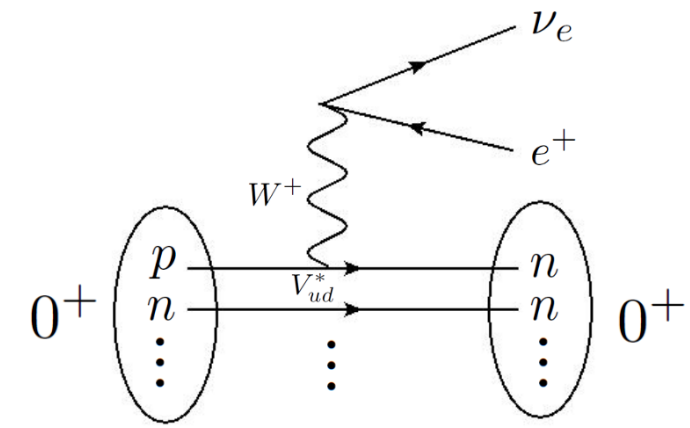
RC not a limiting factor: more precise experiments a-coming

Neutron decay gradually catches up; new experiments a-coming
(expect to match superallowed nuclear decays in the next decade)

Superallowed decays: improvements needed on the theory side

V_{ud} from superallowed nuclear decays

Precise V_{ud} from superallowed decays

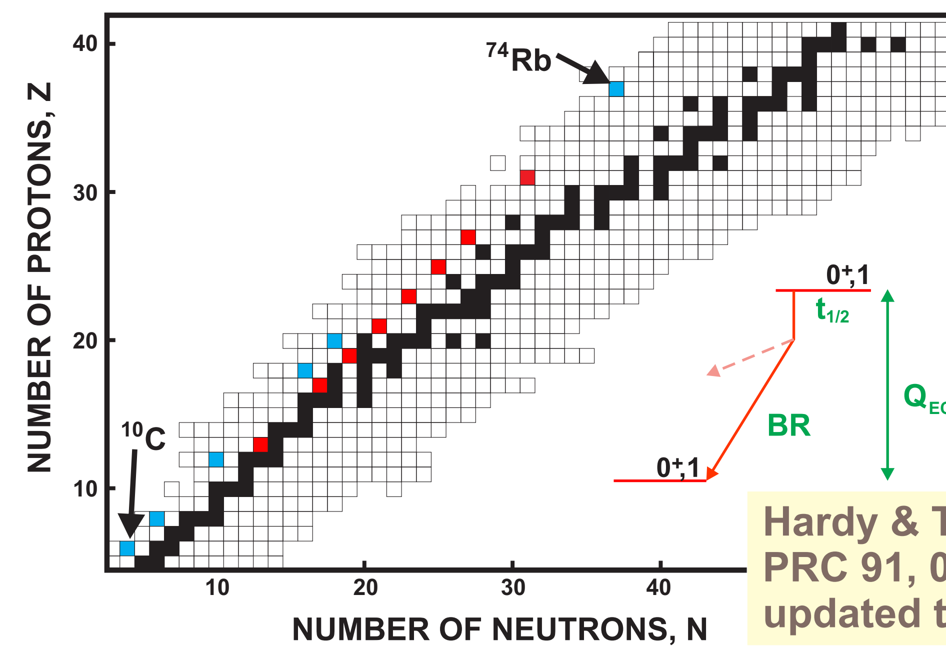


Superallowed 0^+-0^+ nuclear decays:

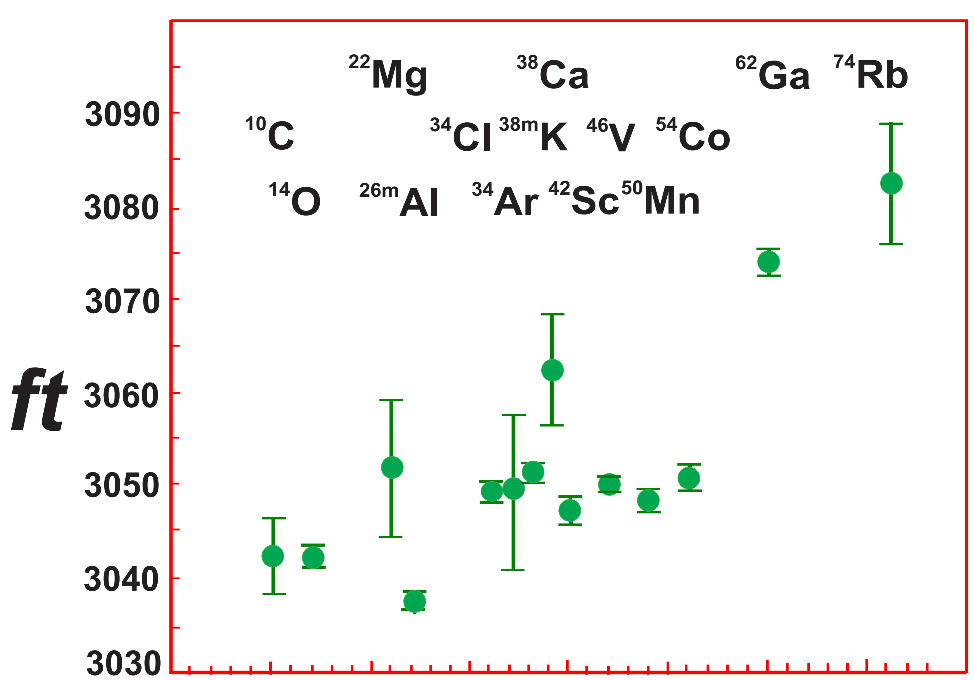
- only conserved vector current
- many decays
- all rates equal modulo phase space

Experiment: **f** - phase space (Q value) and **t** - partial half-life ($t_{1/2}$, branching ratio)

- **8 cases with *ft*-values measured to <0.05% precision; 6 more cases with 0.05-0.3% precision.**
- **~220 individual measurements with compatible precision**



Hardy & Towner
PRC 91, 025501 (2015)
updated to 2017



ft values: same within ~2% but not exactly!
Reason: SU(2) slightly broken

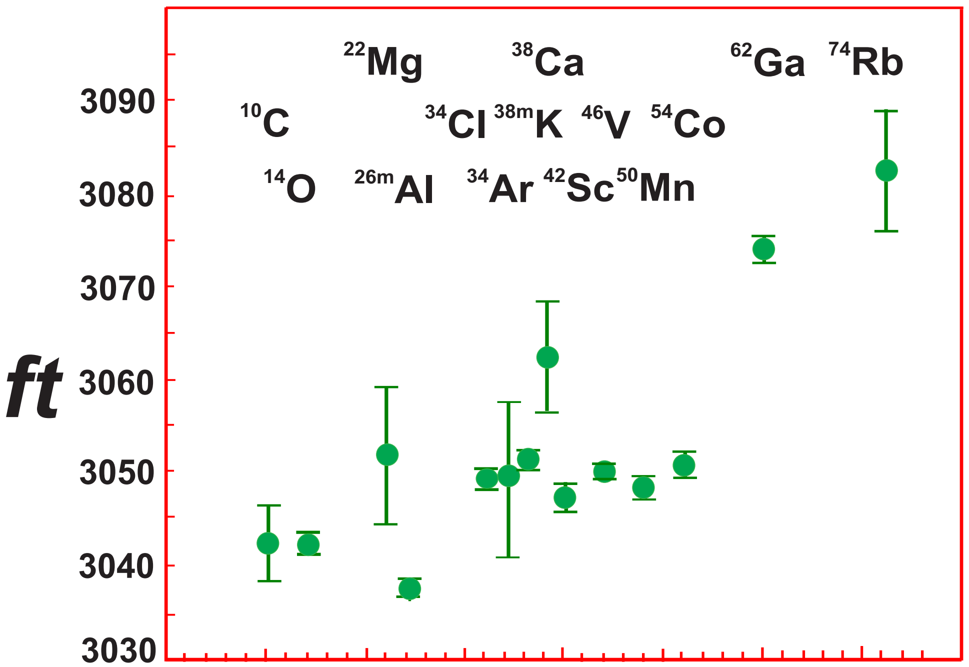
a. RC (e.m. interaction does not conserve isospin)
b. Nuclear WF are not SU(2) symmetric (proton and neutron distribution not the same)

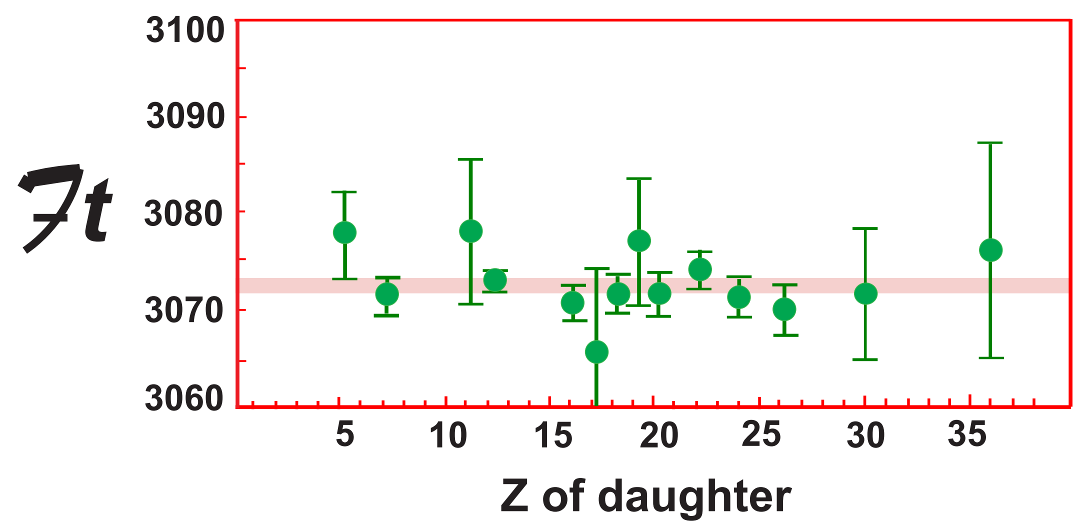
V_{ud} extraction: Universal RC and Universal Ft

To obtain Vud —> absorb all decay-specific corrections into universal **Ft**

$$ft(1+\mathrm{RC}+\mathrm{ISB}) = \mathscr{F}\mathrm{t}(1+\Delta_R^V) = \mathrm{ft}(1+\delta_R')(1-\delta_C+\delta_{NS})(1+\Delta_R^V)$$

~ Measured QED Isospin-breaking Nuclear structure Universal RC





Average of 14 decays Hardy, Towner 1972 - 2020

Pre-2018: $\overline{\mathscr{F}t} = 3072.1 \pm 0.7\, s$

PDG 2022: $\overline{\mathscr{F}t} = 3072 \pm 2\, s$

$$|V_{ud}|^2 = \frac{2984.43 s}{\mathscr{F}t(1+\Delta_R^V)}$$

$$|V_{ud}^{0^+-0^+}| = 0.9737\,(1)_{exp,\,nucl}(3)_{NS}(1)_{RC}[3]_{total}$$

How do Nuclear Radii Enter V_{ud}?

Nuclear Structure Inputs in ft

$$f = m_e^{-5} \int_{m_e}^{E_0} dE_e |\vec{p}_e| E_e (E_0 - E_e)^2 F(E_e) C(E_e) Q(E_e) R(E_e) r(E_e)$$

QED

Fermi Fn: daughter **nuclear charge form factor** $F_{Ch}(q^2)$

Shape factor: **nuclear weak CC transition FF** $F_{CW}(q^2)$

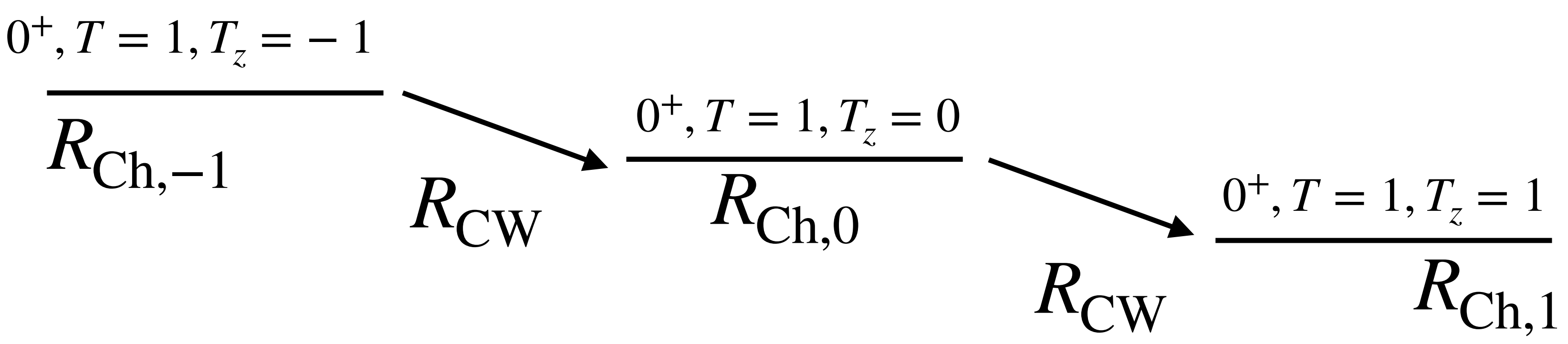
Charge form factors: combination of e-scattering, X-ray/laser/optical atom spectroscopy
Slope of the charge FF at origin: nuclear charge radius
Not all radii are known —> have to be guessed (theory)

Charged-current weak transition form factors: only accessible with the decay itself (tough);
Historically estimated in nuclear shell model with 1B current (Wilkinson; Hardy & Towner; …)
Typical result: very similar to charge FF

New development:
use isospin symmetry and known charge radii to predict the weak transition radius!

Isospin symmetry + Charge Radii in 0^+ isotriplet

CY Seng, 2212.02681

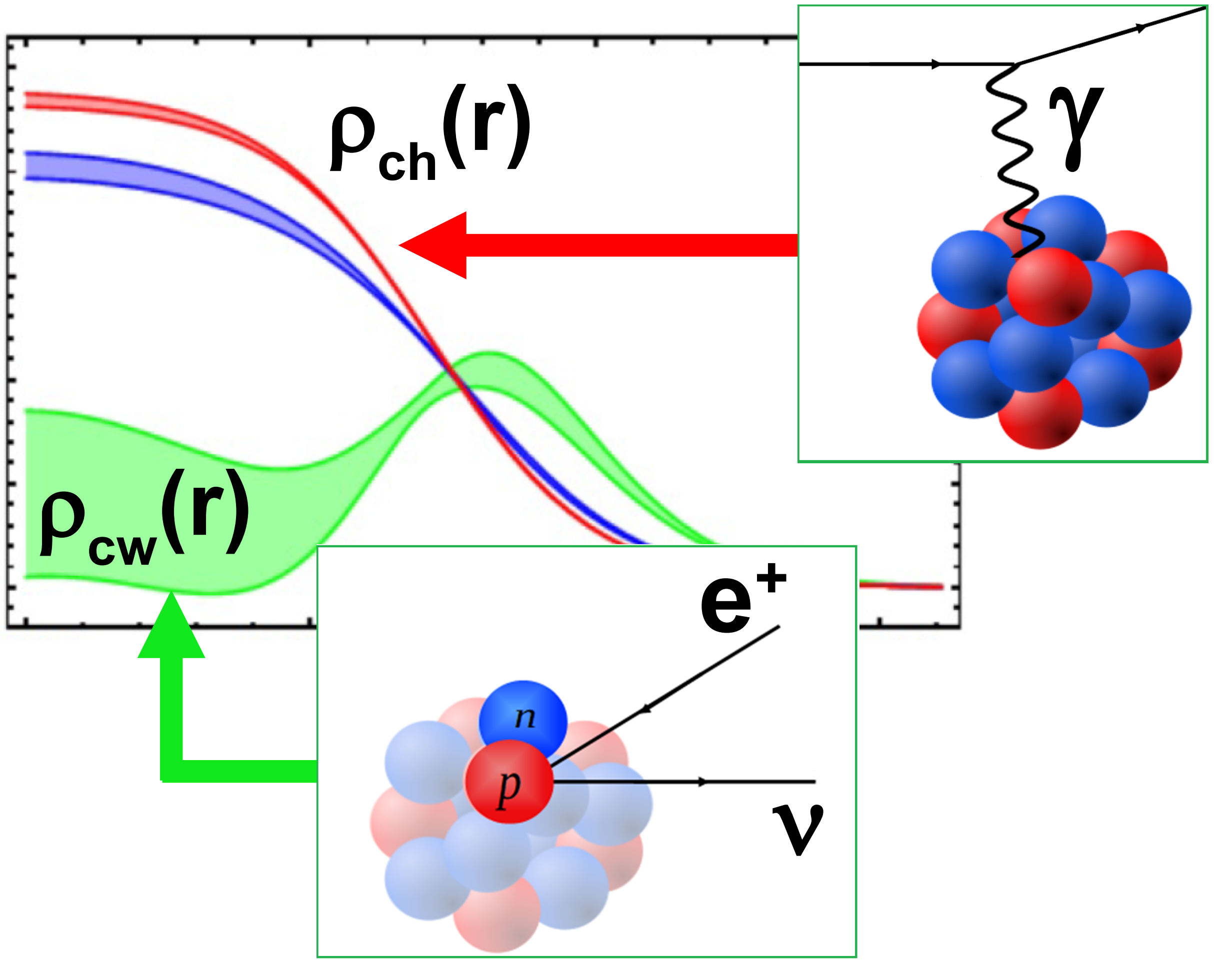


How is R_{CW} related to $R_{Ch,Tz}$?
Charged-Current weak current: pure isovector
Electromagnetic current isovector + isoscalar

Remove isoscalar part:
Relate weak <—> charge radii

$$\begin{aligned} R_{\mathrm{CW}}^2 &= R_{\mathrm{Ch},1}^2 + Z_0(R_{\mathrm{Ch},0}^2 - R_{\mathrm{Ch},1}^2) \\ &= R_{\mathrm{Ch},1}^2 + \frac{Z_{-1}}{2}(R_{\mathrm{Ch},-1}^2 - R_{\mathrm{Ch},1}^2) \end{aligned}$$

Large factors ~Z multiply small differences



Photon probes the entire nuclear charge
Only the outer protons can decay: all neutron states in the core occupied

Isospin symmetry + Charge Radii in $T = 1, O^{+}$ isotriplet

A	$\langle r^2_{\mathrm{ch},-1}\rangle^{1/2}$ (fm)	$\langle r^2_{\mathrm{ch},0}\rangle^{1/2}$ (fm)	$\langle r^2_{\mathrm{ch},1}\rangle^{1/2}$ (fm)	$\langle r^2_{\mathrm{cw}}\rangle^{1/2}$ (fm)
10	${}^{10}_{6}$C	${}^{10}_{5}$B(ex)	${}^{10}_{4}$Be: 2.3550(170)[a]	N/A
14	${}^{14}_{8}$O	${}^{14}_{7}$N(ex)	${}^{14}_{6}$C: 2.5025(87)[a]	N/A
18	${}^{18}_{10}$Ne: 2.9714(76)[a]	${}^{18}_{9}$F(ex)	${}^{18}_{8}$O: 2.7726(56)[a]	3.661(72)
22	${}^{22}_{12}$Mg: 3.0691(89)[b]	${}^{22}_{11}$Na(ex)	${}^{22}_{10}$Ne: 2.9525(40)[a]	3.596(99)
26	${}^{26}_{14}$Si	${}^{26m}_{13}$Al: 3.130(15)[f]	${}^{26}_{12}$Mg: 3.0337(18)[a]	4.11(15)
30	${}^{30}_{16}$S	${}^{30}_{15}$P(ex)	${}^{30}_{14}$Si: 3.1336(40)[a]	N/A
34	${}^{34}_{18}$Ar: 3.3654(40)[a]	${}^{34}_{17}$Cl	${}^{34}_{16}$S: 3.2847(21)[a]	3.954(68)
38	${}^{38}_{20}$Ca: 3.467(1)[c]	${}^{38m}_{19}$K: 3.437(4)[d]	${}^{38}_{18}$Ar: 3.4028(19)[a]	3.999(35)
42	${}^{42}_{22}$Ti	${}^{42}_{21}$Sc: 3.5702(238)[a]	${}^{42}_{20}$Ca: 3.5081(21)[a]	4.64(39)
46	${}^{46}_{24}$Cr	${}^{46}_{23}$V	${}^{46}_{22}$Ti: 3.6070(22)[a]	N/A
50	${}^{50}_{26}$Fe	${}^{50}_{25}$Mn: 3.7120(196)[a]	${}^{50}_{24}$Cr: 3.6588(65)[a]	4.82(39)
54	${}^{54}_{28}$Ni: 3.738(4)[e]	${}^{54}_{27}$Co	${}^{54}_{26}$Fe: 3.6933(19)[a]	4.28(11)
62	${}^{62}_{32}$Ge	${}^{62}_{31}$Ga	${}^{62}_{30}$Zn: 3.9031(69)[b]	N/A
66	${}^{66}_{34}$Se	${}^{66}_{33}$As	${}^{66}_{32}$Ge	N/A
70	${}^{70}_{36}$Kr	${}^{70}_{35}$Br	${}^{70}_{34}$Se	N/A
74	${}^{74}_{38}$Sr	${}^{74}_{37}$Rb: 4.1935(172)[b]	${}^{74}_{36}$Kr: 4.1870(41)[a]	4.42(62)

Seng, 2212.02681
MG, Seng 2311.16755

Weak radii differ significantly from R_{ch}
Shape factor—> Fermi Fn —> ft

Transition	$(ft)_{\mathrm{HT}}$ (s)	$(ft)_{\mathrm{new}}(s)$
^{18}Ne→^{18}F	2912 ± 79	2912 ± 80
^{22}Mg→^{22}Na	3051.1 ± 6.9	3050.4 ± 6.8
^{26}Si→^{26m}Al	3052.2 ± 5.6	3050.7 ± 5.6
^{34}Ar→^{34}Cl	3058.0 ± 2.8	3057.1 ± 2.8
^{38}Ca→^{38m}K	3062.8 ± 6.0	3062.2 ± 5.9
^{42}Ti→^{42}Sc	3090 ± 88	3085 ± 86
^{50}Fe→^{50}Mn	3099 ± 71	3098 ± 72
^{54}Ni→^{54}Co	3062 ± 50	3063 ± 49
^{26m}Al→^{26}Mg	3037.61 ± 0.67	3036.5 ± 1.0
^{34}Cl→^{34}S	$3049.43^{+0.95}_{-0.88}$	3048.0 ± 1.1
^{38m}K→^{38}Ar	3051.45 ± 0.92	3050.5 ± 1.1
^{42}Sc→^{42}Ca	3047.7 ± 1.2	3045.0 ± 2.7
^{50}Mn→^{50}Cr	3048.4 ± 1.2	3046.1 ± 3.6
^{54}Co→^{54}Fe	$3050.8^{+1.4}_{-1.1}$	$3051.3^{+1.7}_{-1.4}$
^{74}Rb→^{74}Kr	3082.8 ± 6.5	3086 ± 11

New ft vs estimates by Hardy and Towner

Relative shift downwards of 0.01-0.1%
Non-negligible given the precision goal 0.01%

More -and more precise- charge radii necessary!
Working closely with exp. (PSI, FRIB, ISOLDE, TRIUMF)

Isospin symmetry + Charge Radii in $T = 1, O^{+}$isotriplet

Above treatment assumes isospin symmetry — but we know that it is slightly broken!
Why isospin symmetry assumption is good enough?

Shape factor and finite size effects are ~small corrections to Fermi function
1-2% ISB effect on top of a RC may be assumed negligible (but needs to be tested)

Test requires that all 3 nuclear radii in the isotriplet are known;
Currently only the case for A=38 system

26	$^{26}_{14}$Si	$^{26m}_{13}$Al: 3.130(15)[f]	$^{26}_{12}$Mg: 3.0337(18)[a]	4.11(15)
30	$^{30}_{16}$S	$^{30}_{15}$P(ex)	$^{30}_{14}$Si: 3.1336(40)[a]	N/A
34	$^{34}_{18}$Ar: 3.3654(40)[a]	$^{34}_{17}$Cl	$^{34}_{16}$S: 3.2847(21)[a]	3.954(68)
38	$^{38}_{20}$Ca: 3.467(1)[c]	$^{38m}_{19}$K: 3.437(4)[d]	$^{38}_{18}$Ar: 3.4028(19)[a]	3.999(35)
42	$^{42}_{22}$Ti	$^{42}_{21}$Sc: 3.5702(238)[a]	$^{42}_{20}$Ca: 3.5081(21)[a]	4.64(39)
46	^{46}Cr	^{46}V	^{46}Ti: 3.6070(22)[a]	N/A

ISB-sensitive combination

$$\Delta M_B^{(1)} \equiv \frac{1}{2}\left(Z_1 R_{p,1}^2 + Z_{-1} R_{p,-1}^2\right) - Z_0 R_{p,0}^2 \; = 0 \quad \text{if isospin symmetry exact}$$

$$\frac{1}{2}\left(20 \times 3.467(1)^2 + 18 \times 3.4028(19)^2\right) - 19 \times 3.437(4)^2 = -0.00(12)(14)(52)$$

Improvement of K-38m radius necessary! (Plans at TRIUMF on IS K-38m, K-37?)

Isospin symmetry breaking in superallowed β-decay

Tree-level Fermi matrix element

$$M_F = \langle f | \tau^+ | i \rangle$$

τ^+ — Isospin operator

$|i\rangle$, $|f\rangle$ — members of T=1 isotriplet

If isospin symmetry were exact, $M_F \to M_0 = \sqrt{2}$

Isospin symmetry is broken in nuclear states (e.g. Coulomb, nucleon mass difference, …)

In presence of isospin symmetry breaking (ISB):

$$|M_F|^2 = |M_0|^2(1 - \delta_C)$$

ISB correction is crucial for V_{ud} extraction

HT: shell model with *phenomenological* Woods-Saxon potential locally adjusted to:

- Masses of the isotriplet T=1, 0^+ (IMME)
- Neutron and proton separation energies
- **Known charge radii**

TABLE X. Corrections δ'_R, δ_{NS}, and δ_C that are applied to experimental ft values to obtain $\mathcal{F}t$ values.

Parent nucleus	δ'_R (%)	δ_{NS} (%)	δ_{C1} (%)	δ_{C2} (%)	δ_C (%)
$T_z = -1$					
^{10}C	1.679	−0.345(35)	0.010(10)	0.165(15)	0.175(18)
^{14}O	1.543	−0.245(50)	0.055(20)	0.275(15)	0.330(25)
^{18}Ne	1.506	−0.290(35)	0.155(30)	0.405(25)	0.560(39)
^{22}Mg	1.466	−0.225(20)	0.010(10)	0.370(20)	0.380(22)
^{26}Si	1.439	−0.215(20)	0.030(10)	0.405(25)	0.435(27)
^{30}S	1.423	−0.185(15)	0.155(20)	0.700(20)	0.855(28)
^{34}Ar	1.412	−0.180(15)	0.030(10)	0.665(55)	0.695(56)
^{38}Ca	1.414	−0.175(15)	0.020(10)	0.745(70)	0.765(71)
^{42}Ti	1.427	−0.235(20)	0.105(20)	0.835(75)	0.940(78)
$T_z = 0$					
^{26m}Al	1.478	0.005(20)	0.030(10)	0.280(15)	0.310(18)
^{34}Cl	1.443	−0.085(15)	0.100(10)	0.550(45)	0.650(46)
^{38m}K	1.440	−0.100(15)	0.105(20)	0.565(50)	0.670(54)
^{42}Sc	1.453	0.035(20)	0.020(10)	0.645(55)	0.665(56)
^{46}V	1.445	−0.035(10)	0.075(30)	0.545(55)	0.620(63)
^{50}Mn	1.444	−0.040(10)	0.035(20)	0.610(50)	0.645(54)
^{54}Co	1.443	−0.035(10)	0.050(30)	0.720(60)	0.770(67)
^{62}Ga	1.459	−0.045(20)	0.275(55)	1.20(20)	1.48(21)
^{66}As	1.468	−0.060(20)	0.195(45)	1.35(40)	1.55(40)
^{70}Br	1.486	−0.085(25)	0.445(40)	1.25(25)	1.70(25)
^{74}Rb	1.499	−0.075(30)	0.115(60)	1.50(26)	1.62(27)

J. Hardy, I. Towner, Phys.Rev. C 91 (2014), 025501

$$\delta_C \sim 0.17\% - 1.6\% \, !$$

Phenomenological constraints on δ_C

δ_C dominated by Coulomb repulsion between protons (hence C)

Coulomb interaction generates both δ_C and ISB combinations of nuclear radii

Miller, Schwenk 0805.0603; 0910.2790; Auerbach 0811.4742; 2101.06199;
Seng, MG 2208.03037; 2304.03800; 2212.02681

Nuclear Hamiltonian: $H = H_0 + V_{\rm ISB} \approx H_0 + V_C$

Coulomb potential for uniformly charged sphere $$V_C \approx -\frac{Ze^2}{4\pi R_C^3}\sum_{i=1}^{A}\left(\frac{1}{2}r_i^2 - \frac{3}{2}R_C^2\right)\left(\frac{1}{2} - \hat{T}_z(i)\right)$$

ISB due to IV monopole, $V_{\rm ISB} \approx \dfrac{Ze^2}{8\pi R^3}\sum_i r_i^2 \hat{T}_z(i) = \dfrac{Ze^2}{8\pi R^3}\hat{M}_0^{(1)}$

Same operator generates nuclear radii $$R_{p/n,\phi} = \sqrt{\frac{1}{X}\langle\phi|\sum_{i=1}^{A} r_i^2\left(\frac{1}{2} \mp \hat{T}_z(i)\right)|\phi\rangle}$$

Impact of precise nuclear radii on Ft and V_{ud}

Recent measurement at IGISOL

Plattner et al, arXiv: 2310.15291
Wilfried's talk (?), Ben's talk (?)

Isotope shift in aluminum 27-26m
$3s^2 3p\ ^2P_{3/2} \longrightarrow 3s^2 4s\ ^2S_{1/2}$ transition

$$\delta\nu^{27,26m} = F\delta\langle r^2\rangle^{27,26m} + M\frac{m_{26m} - m_{27}}{m_{27}(m_{26m} + m_e)}$$

Input from atomic theory: F,M
Reference radius Al-27 from μ atoms

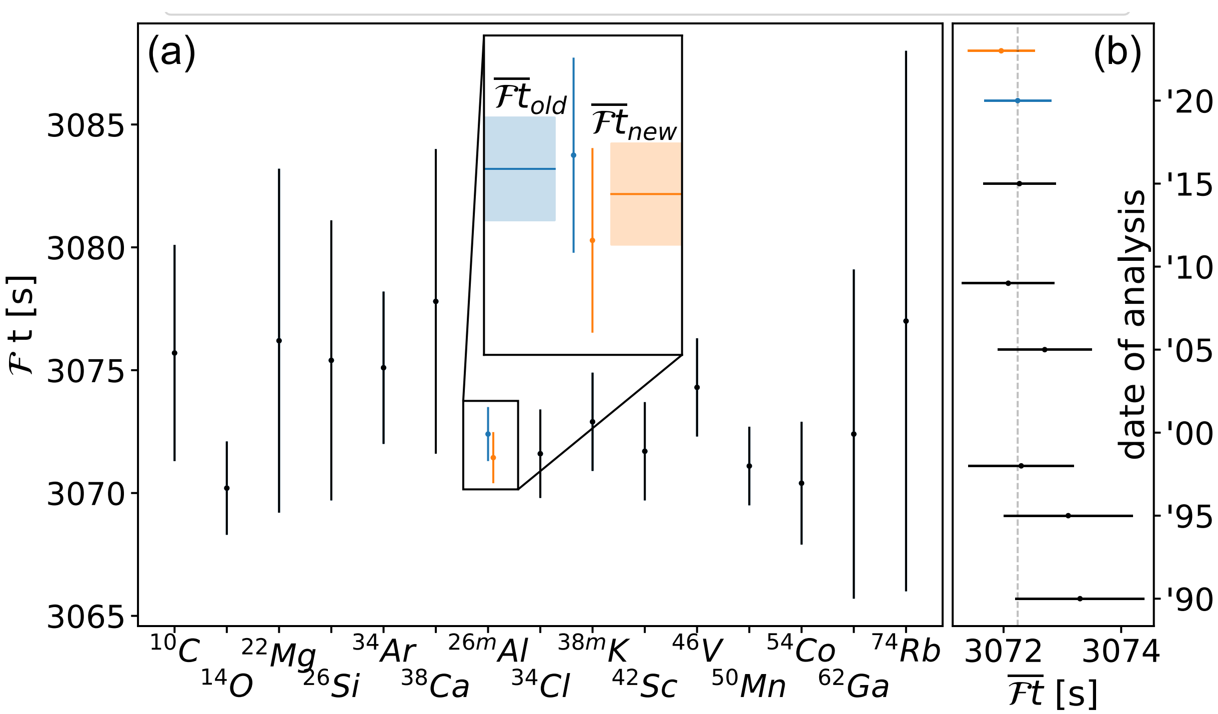
$$R_c(^{26m}\mathrm{Al}) \equiv \langle r^2\rangle^{1/2}_{26m} = \sqrt{R_c(^{27}\mathrm{Al})^2 + \delta\langle r^2\rangle^{27,26m}}$$

$$\delta\nu^{27,26m} = 377.5(3.4)\ \mathrm{MHz} \longrightarrow R_c(^{26m}\mathrm{Al}) = 3.130(15)\ \mathrm{fm}$$

Previously guessed $R_c(^{26m}Al) = 3.040(20)\,\mathrm{fm}$

Quantity	Previous value	This Letter
R_c	3.040(20) fm [27]	3.130(15) fm
δ_{C2}	0.280(15)% [10]	0.310(14)%
$\mathcal{F}t(^{26m}Al)$	3072.4(1.1) s [10]	3071.4(1.0) s
$\overline{\mathcal{F}t}$	3072.24(1.85) s [10]	3071.96(1.85) s
Δ_{CKM}	$152(70) \times 10^{-5}$ [7]	$144(70) \times 10^{-5}$

Al-26m the most precisely measured transition!
Direct impact on Ft and Vud extraction



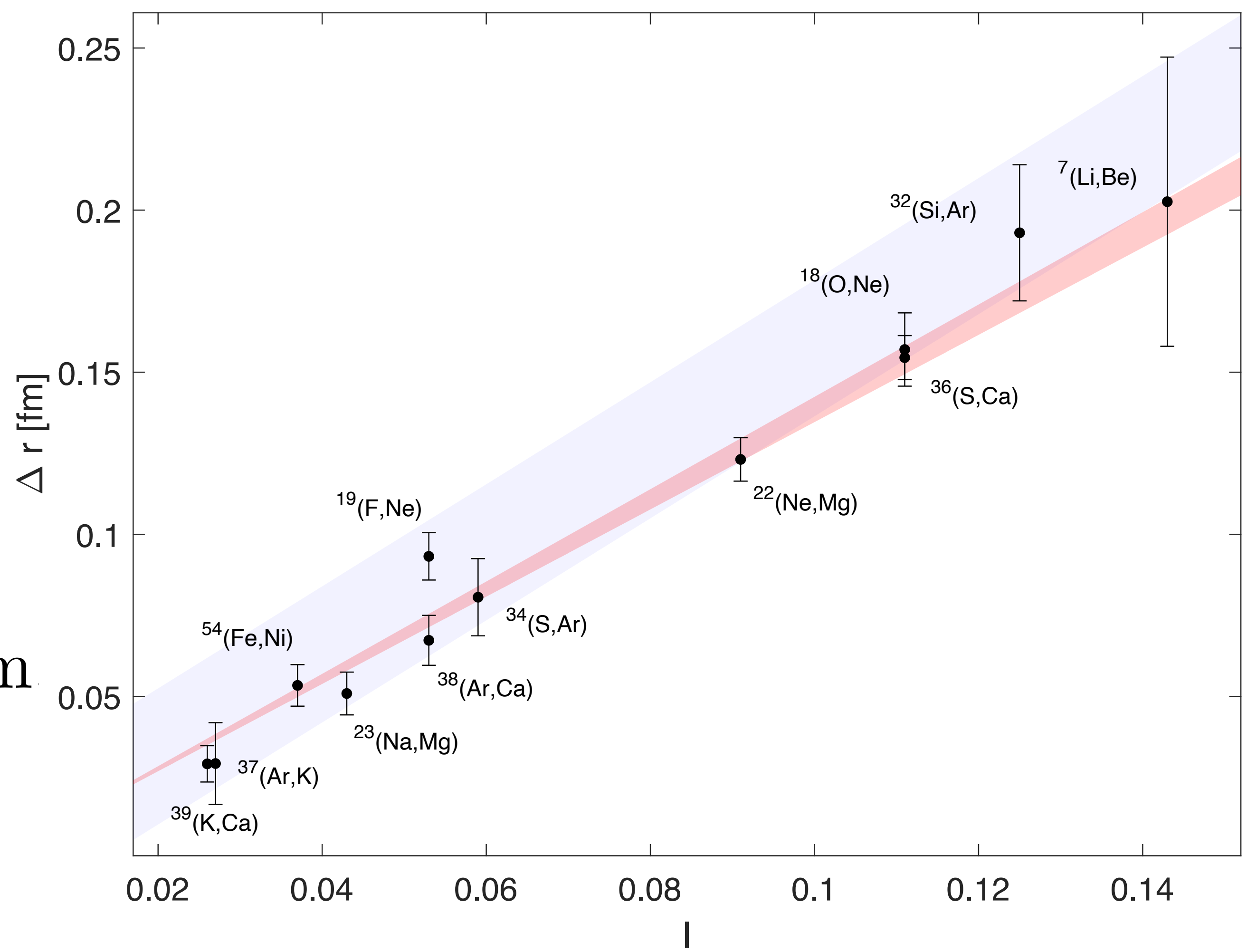
Global fit to Radii of Mirror Nuclei

Ben Ohayon, 2409.08193

Superallowed isotriplets contain mirrors
Use info about radii of other mirror nuclei

$$I = (N - Z)/A$$

$$\Delta_I = r_{N,Z}(I) - r_{Z,N}(I) = 1.382(34) \times I \text{ fm}$$



Agrees well with ab-initio nuclear theory (Novario et al, 2111.12775) but is more precise

Ben's talk

Test of isospin symmetry using mirror fit

Fill in missing entries using fit

$$r_{-1}^2 - r_{+1}^2 = \Delta_I(2r_{+1} + \Delta_I)$$

$$r_{0,\mathrm{SE}}^2 = r_{+1}^2 + \frac{Z_{-1}}{2Z_0}\Delta_I(2r_{+1} + \Delta_I)$$

	r_{-1} fm		$r_{0,\mathrm{SE}}$ fm	$r_{0,\mathrm{exp}}$ fm		r_{+1} fm	$\Delta M_B^{(1)}$ fm^2	r_{CW}^2 fm^2	Ref. [38]
$^{10}_{6}$C	2.638(36)	$^{10}_{5}$B*	2.531(38)		$^{10}_{4}$Be	2.361(36)		9.72(25)	N/A
$^{14}_{8}$O	2.706(11)	$^{14}_{7}$N*	2.623(10)		$^{14}_{6}$C	2.508(09)		10.41(12)	N/A
$^{18}_{10}$Ne	2.934(09)	$^{18}_{9}$F*	2.863(07)		$^{18}_{8}$O	2.777(07)		12.08(12)	13.4(5)
$^{22}_{12}$Mg	3.071(05)	$^{22}_{11}$Na*	3.017(05)		$^{22}_{10}$Ne	2.948(04)		13.24(12)	12.9(7)
$^{26}_{14}$Si	3.137(04)	$^{26m}_{13}$Al	3.088(04)	3.132(08)	$^{26}_{12}$Mg	3.030(03)	−3.5(0.7)	13.77(12)	N/A
$^{30}_{16}$S	3.224(07)	$^{30}_{15}$P*	3.181(06)		$^{30}_{14}$Si	3.132(06)		14.50(13)	N/A
$^{34}_{18}$Ar	3.365(11)	$^{34}_{17}$Cl	3.328(04)		$^{34}_{16}$S	3.284(04)		15.66(13)	15.6(5)
$^{38}_{20}$Ca	3.469(04)	$^{38m}_{19}$K	3.440(07)	3.437(05)	$^{38}_{18}$Ar	3.402(06)	0.6(1.1)	16.58(13)	16.0(3)
$^{42}_{22}$Ti	3.576(05)	$^{42}_{21}$Sc	3.545(04)	3.558(16)	$^{42}_{20}$Ca	3.510(04)	−2.0(2.4)	17.46(13)	21.5(3.6)
$^{46}_{24}$Cr	3.670(05)	$^{46}_{23}$V	3.642(05)		$^{46}_{22}$Ti	3.610(04)		18.29(14)	N/A
$^{50}_{26}$Fe	3.719(04)	$^{50}_{25}$Mn	3.693(04)	3.728(41)	$^{50}_{24}$Cr	3.664(04)	−6.6(7.8)	18.73(14)	23.2(3.8)
$^{54}_{28}$Ni	3.741(05)	$^{54}_{27}$Co	3.715(04)		$^{54}_{26}$Fe	3.688(04)		18.93(14)	18.3(9)
$^{58}_{30}$Zn	3.820(03)	$^{58}_{29}$Cu*	3.797(03)		$^{58}_{28}$Ni	3.773(03)		19.66(14)	N/A
$^{62}_{32}$Ge	3.927(06)	$^{62}_{31}$Ga	3.906(06)		$^{62}_{30}$Zn	3.883(06)		20.65(15)	N/A
$^{74}_{38}$Sr	4.205(12)	$^{74}_{37}$Rb	4.187(12)	4.194(17)	$^{74}_{36}$Kr	4.168(12)	−1.9(6.5)	23.32(19)	19.5(5.5)

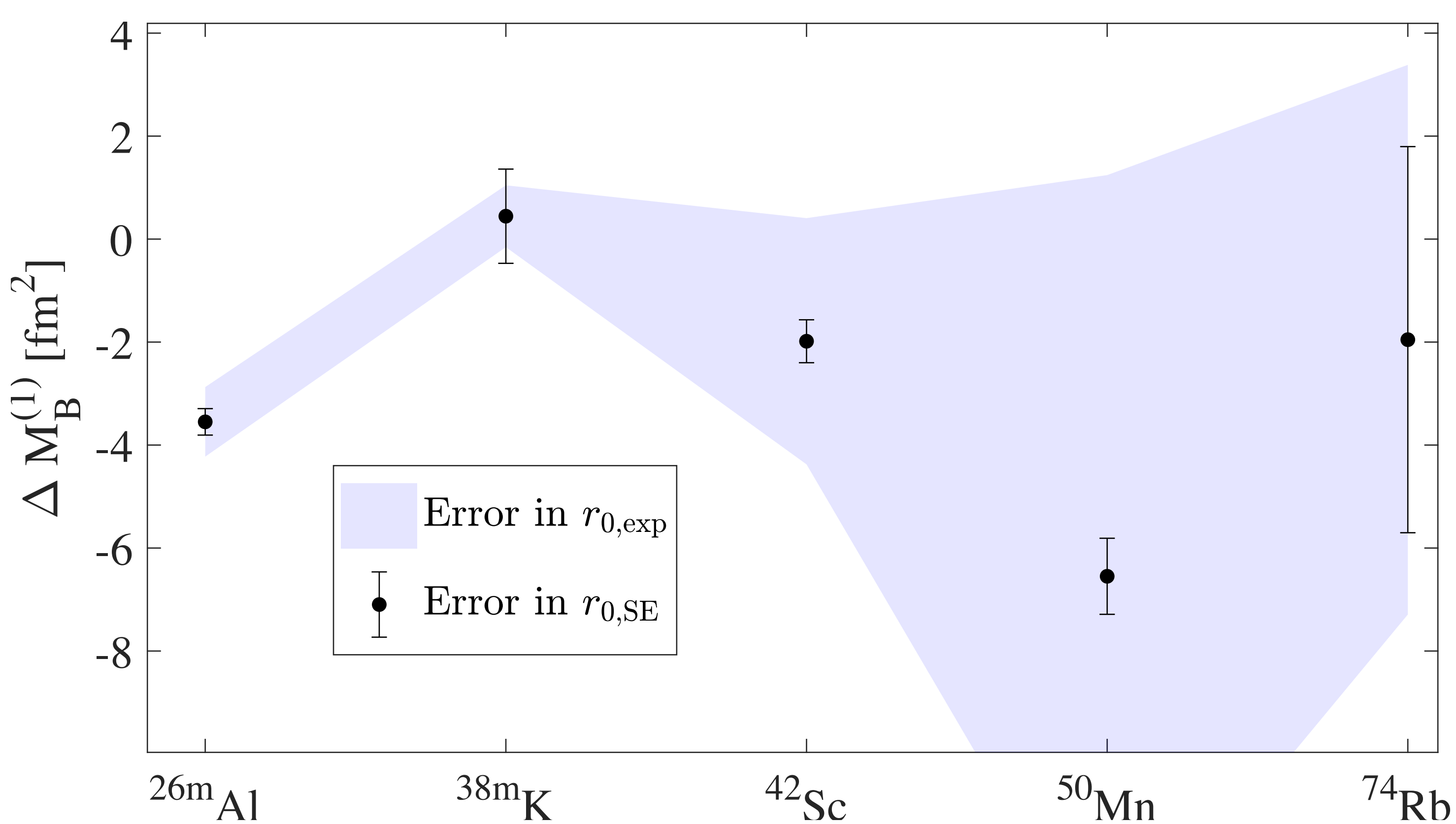
Combine into ISB-sensitive combination

$$\Delta M_B^{(1)} \equiv \frac{1}{2}\left(Z_1 R_{p,1}^2 + Z_{-1} R_{p,-1}^2\right) - Z_0 R_{p,0}^2$$

At present can test 5 isotriplets

A=26 shows significant ISB (??)

Others consistent with 0 within errors



Impact of precise nuclear radii on Ft and V_{ud}

Sensitivity to the charge radii: $\delta_C \approx 0.310(17)\,\% + 0.33\,\% \left[r(^{26m}\mathrm{Al})/\mathrm{fm} - 3.040 \right]$

We find even higher sensitivity of f compared to δ_C (preliminary)

Dedicated paper addressing all ingredients is in preparation (MG, B. Ohayon, B. Sahoo, C-Y Seng)

To summarize

- Nuclear radii are indispensable input for extracting Vud from nuclear beta decays
- Tests of isospin symmetry involve cancellations between radii — precision matters!

Nuclear Polarization

See also Natalia's talk

Where do we get the nuclear radii from?

Everyone takes nuclear radii from tables, e.g. Angeli-Marinova or Fricke-Heilig — Istvan's talk

F&H explicitly specify nuclear polarizability as stemming from Rinker, Speth 1978

However: compare to other works by same people

Disagreement ~ 30-40% — larger than exp. error

$Z-$Element	A	From F & H 2004, 30% error assumed
26−Fe	54	362(109)
	56	403(121)
	57	390(117)
	58	400(120)
27−Co	59	438(131)
28−Ni	58	437(131)
	60	461(138)
	61	426(138)
	62	458(138)
	64	438(138)
29−Cu	63	538(161)
	65	489(147)
30−Zn	64	609(183)
	66	595(179)
	68	581(174)
	70	615(184)

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Systematics of nuclear charge distributions in Fe, Co, Ni, Cu, and Zn deduced from muonic x-ray measurements*

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^{54}Fe	1260.011(45)	0.546
^{56}Fe	1257.054(42)	0.582
^{57}Fe	1255.921(51)	0.600
^{58}Fe	1254.485(49)	0.624
^{59}Co	1341.461(46)	0.588
^{58}Ni	1432.564(44)	0.689
^{60}Ni	1429.369(43)	0.693
^{61}Ni	1428.393(49)	0.632
^{62}Ni	1426.829(43)	0.703
^{64}Ni	1425.229(46)	0.725
^{63}Cu	1514.433(44)	0.739
^{65}Cu	1512.516(45)	0.749
^{64}Zn	1602.718(44)	0.857
^{66}Zn	1600.544(43)	0.909
^{68}Zn	1598.763(44)	0.917
^{70}Zn	1596.898(109)	0.973

Nuclear Charge Radii from μ atoms

Lepton feels pointlike Coulomb potential far outside the nucleus

Finite size effects modify this potential in the vicinity of the nucleus

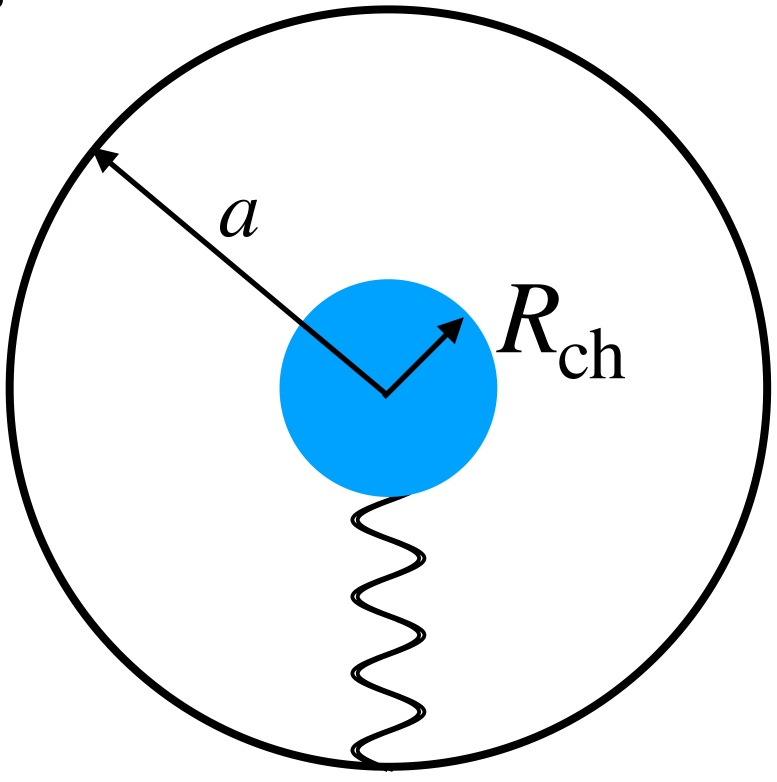
Interplay between atomic and nuclear radii

$$a_{1S}^{eA} = (Z\alpha m_{er})^{-1} \approx 500\,000\,\mathrm{fm}\,Z^{-1}$$

$$\Downarrow$$

$$a_{1S}^{\mu A} = (Z\alpha m_{\mu r})^{-1} \approx 250\,\mathrm{fm}\,Z^{-1}$$

$$R_{\mathrm{ch}} \approx 1.1\,\mathrm{fm} \times A^{1/3}$$



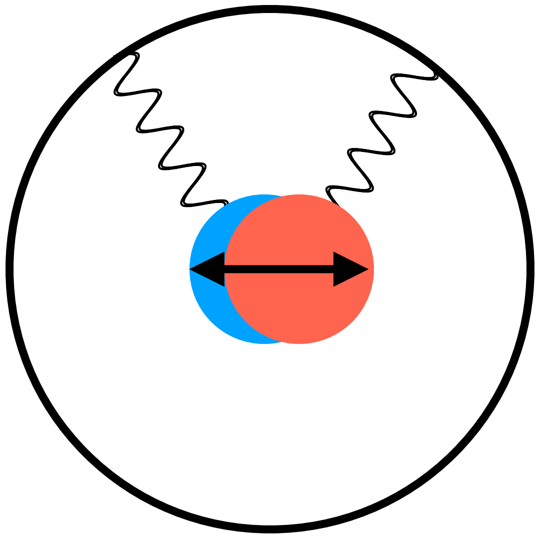
From Z ~ 50 $R_{\mathrm{ch}} \approx a_{1S}^{\mu}$ — very sensitive to nuclear radii

$$\Delta E_{1S} \propto Z\alpha m_r (R_{\mathrm{ch}}/a_{1S}^{\mu})^2$$

For precision: include higher-order corrections (QED + nuclear structure)

QED: numerical solutions of Dirac/Schroedinger radial equations, or analytical $Z\alpha$-expansion

In presence of nuclear polarization



Muon may induce polarization of the nucleus

Structure constant α_{E1} —> electric dipole polarizability

Charges inside nucleus are displaced against each other

α_{E1} has dimension of volume

$$\Delta E_{1S} \propto -Z\alpha m_r \, \alpha_{E1}/(a^{\mu}_{1S})^3$$

Empirical scaling (giant dipole resonance) $\alpha_{E1} \approx 0.00225 A^{5/3}\,\mathrm{fm}^3$

Effectively shifts the extracted radius by

$$\frac{\delta R_{\mathrm{ch}}}{R_{\mathrm{ch}}} \propto \frac{\alpha_{E1}}{2R^2_{\mathrm{ch}}\, a^{\mu}_{1S}} \propto \frac{Z\alpha m_r\, 0.00225\,\mathrm{fm}^3 \times A^{5/3}}{2\times(1.1\,\mathrm{fm}\times A^{1/3})^2} \sim 3.6 ZA \times 10^{-6}$$

Typical precision $\delta R/R \sim 10^{-4}$ —> precision requirement on NP $\quad 10^4\frac{\delta R_{\mathrm{ch}}}{R_{\mathrm{ch}}} \sim 7\frac{Z}{10}\frac{A}{20}$

Accuracy of calculated NP reflects directly in the precision of nuclear radii (not via this formula)

Nuclear polarization - basics

2nd order perturbation theory: $$\Delta E_p = \sum_{N \neq 0} \langle 0' \mid \Delta H_c \mid N \rangle \left[\sum_n \frac{|n\rangle\langle n|}{\epsilon_0 - \epsilon_n - \omega_N} \right] \langle N \mid \Delta H_c \mid 0' \rangle$$

Ericson, Hüfner 1972
Friar 1977

Perturbation: transition induced by Coulomb interaction

$$\Delta H_c(\vec{\mathbf{r}}) = -\alpha \int \frac{d^3 \vec{\mathbf{r}}_N}{|\vec{\mathbf{r}} - \vec{\mathbf{r}}_N|} \hat{\rho}(\vec{\mathbf{r}}_N)$$

First approximation:

nucleus much smaller than atom

nuclear energy splittings much larger than atomic energy

$$\left[\sum_n \frac{|n\rangle\langle n|}{\cancel{\epsilon_0 - \epsilon_n} - \omega_N} \right]$$

Start with leading-order result: $$\Delta E_{n\ell} = \frac{8\alpha^2 m}{i\pi} \phi_{n\ell}(0)\,|^2 \int d^4 q \frac{(q^2 - \nu^2)T_2 - (q^2 + 2\nu^2)T_1}{q^4(q^4 - 4m^2\nu^2)}$$

Bernabeu-Jarlskog 1974
Rosenfelder 1983

Npol-induced potential - δ-function at origin; relativistic treatment of nuclear system

Nuclear polarization - basics

Im parts of forward Compton amplitudes

~ photoabsorption data

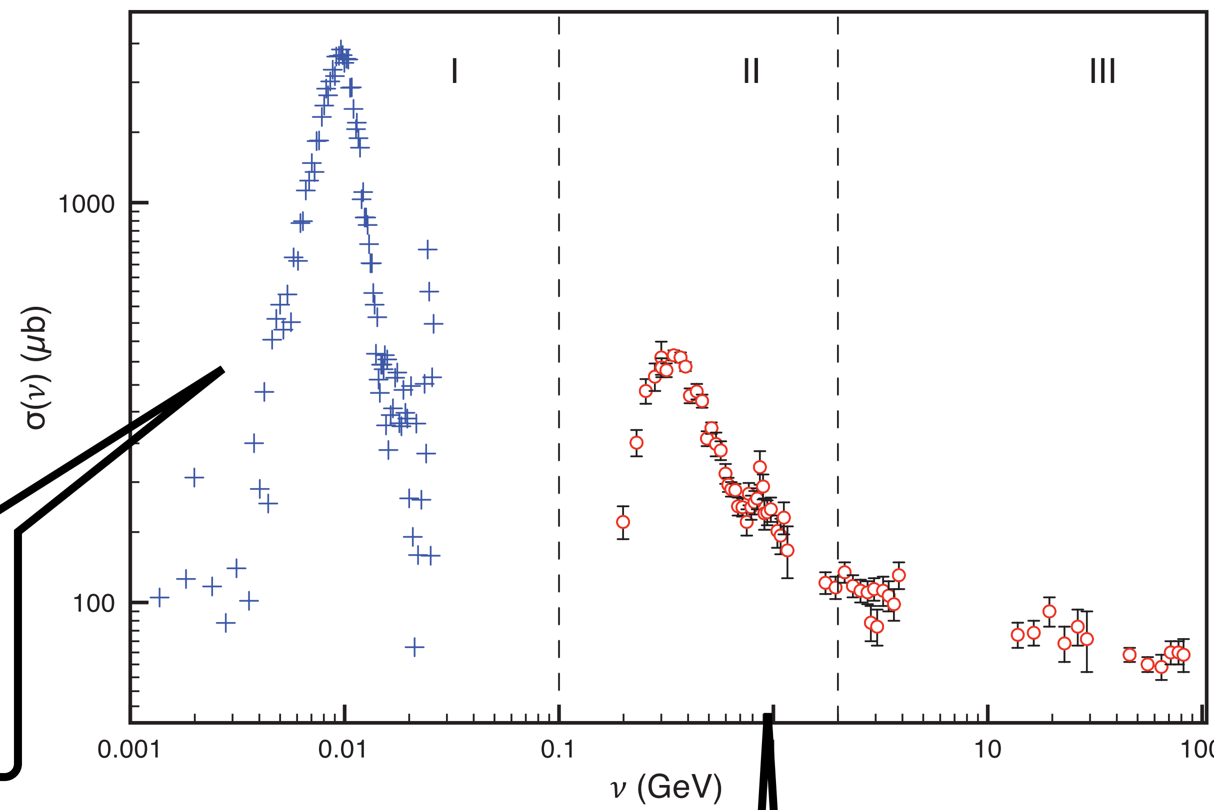
$$\Delta E_{n\ell} = \frac{8\alpha^2 m}{i\pi} \phi_{n\ell}(0)|^2 \int d^4q \frac{(q^2-\nu^2)T_2 - (q^2+2\nu^2)T_1}{q^4(q^4-4m^2\nu^2)}$$

$$\mathrm{Im}\, T_1(\nu, q^2) = \frac{1}{4M} F_1(\nu, q^2) \qquad \mathrm{Im}\, T_2(\nu, q^2) = \frac{1}{4\nu} F_2(\nu, q^2)$$

Real photoabsorption data:

Nuclear range $\nu < 140\,\mathrm{MeV}$

Hadronic range $\nu \geq 140\,\mathrm{MeV}$



Nonrelativistic version:
Migdal sum rule

$$\alpha_{E1} = \frac{1}{2\pi^2}\int_{\mathrm{thr}}^{\nu_{mas}} \frac{d\nu}{\nu^2}\sigma_\gamma(\nu)$$

Baldin sum rule (relativistic)

$$\alpha_E + \beta_M = \frac{1}{2\pi^2}\int_{\mathrm{thr}}^{\infty} \frac{d\nu}{\nu^2}\sigma_\gamma(\nu)$$

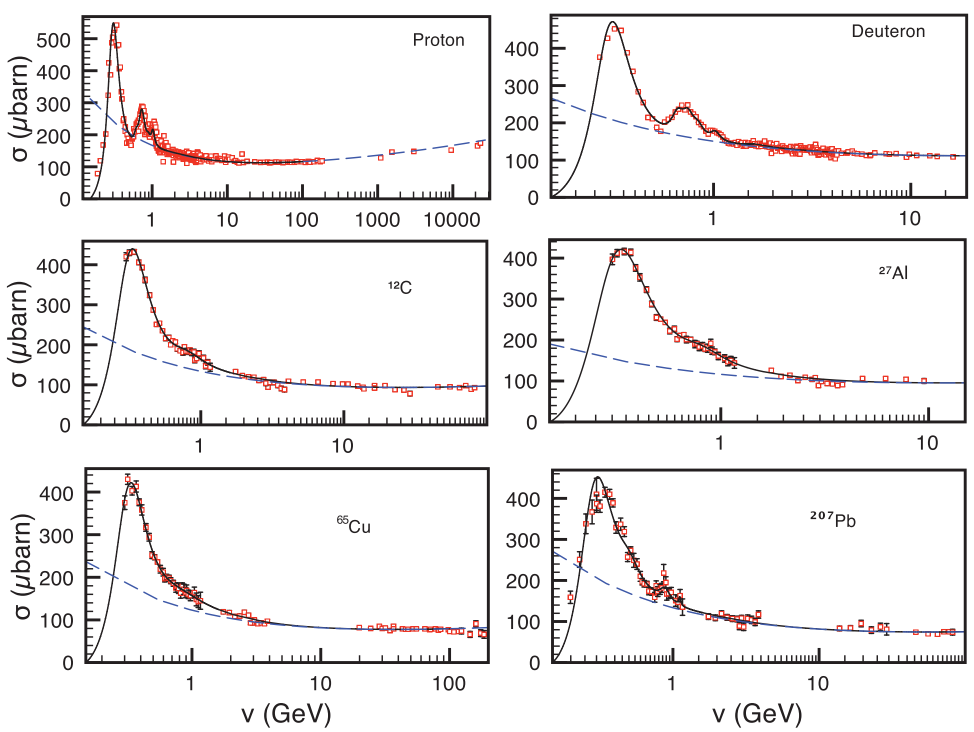
Total photoabsorption data in hadronic range: scales as ~A

Nuclear polarizability scales as $A^{5/3}$

A-scaling of total photoabsorption in hadronic range

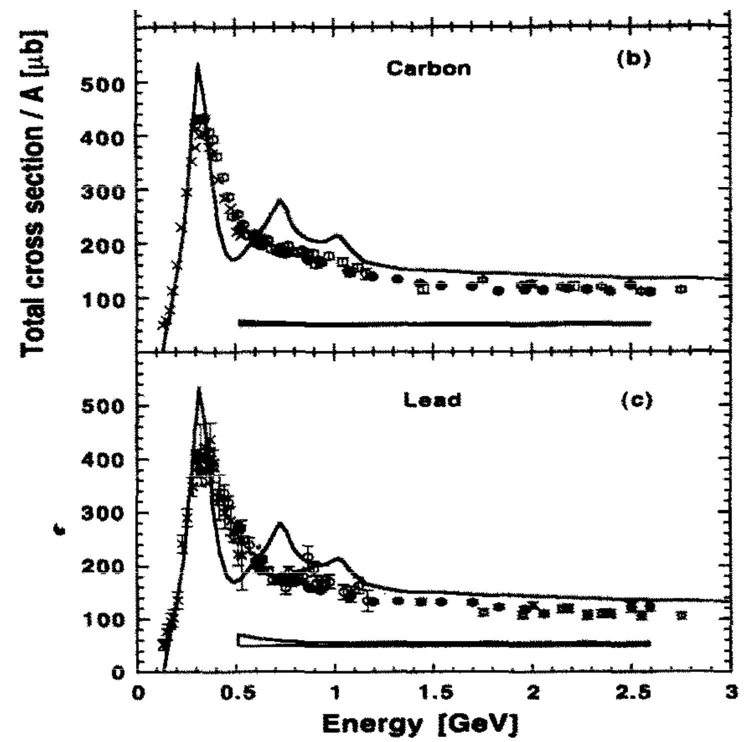
Fit to nuclear photoabsorption — CS per nucleon

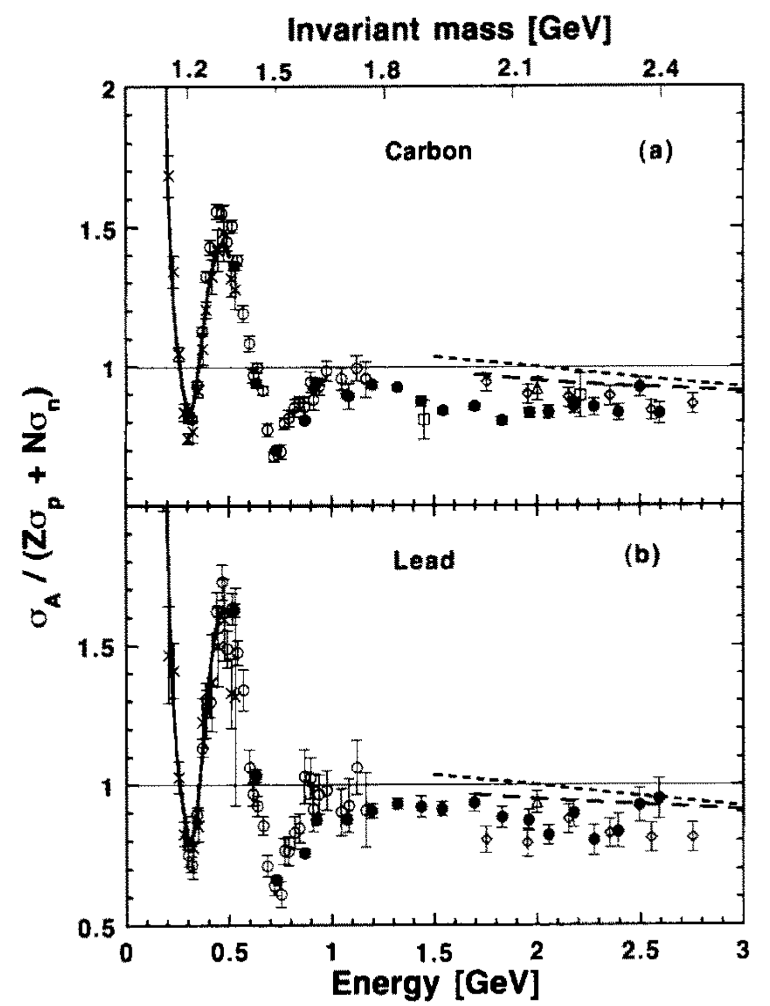
MG et al, 1110.5982



Total hadronic photoabsorption on carbon and lead in the shadowing threshold region

M. Mirazita[a], H. Avakian[a], N. Bianchi[a,*], A. Deppman[a], E. De Sanctis[a], V. Gyurjyan[a], V. Muccifora[a], E. Polli[a], P. Rossi[a], R. Burgwinkel[b], J. Hannappel[b], F. Klein[b], D. Menze[b], W. Schwille[b], F. Wehnes[b]





Oscillating around A_{eff} = A in resonance region;

Shadowing (A_{eff} < A) at high energies

For μ-atoms: $\bar{\nu} = \sigma_{-1}/\sigma_{-2} \sim 500\,\mathrm{MeV}$

A-scaling of total photoabsorption in nuclear range

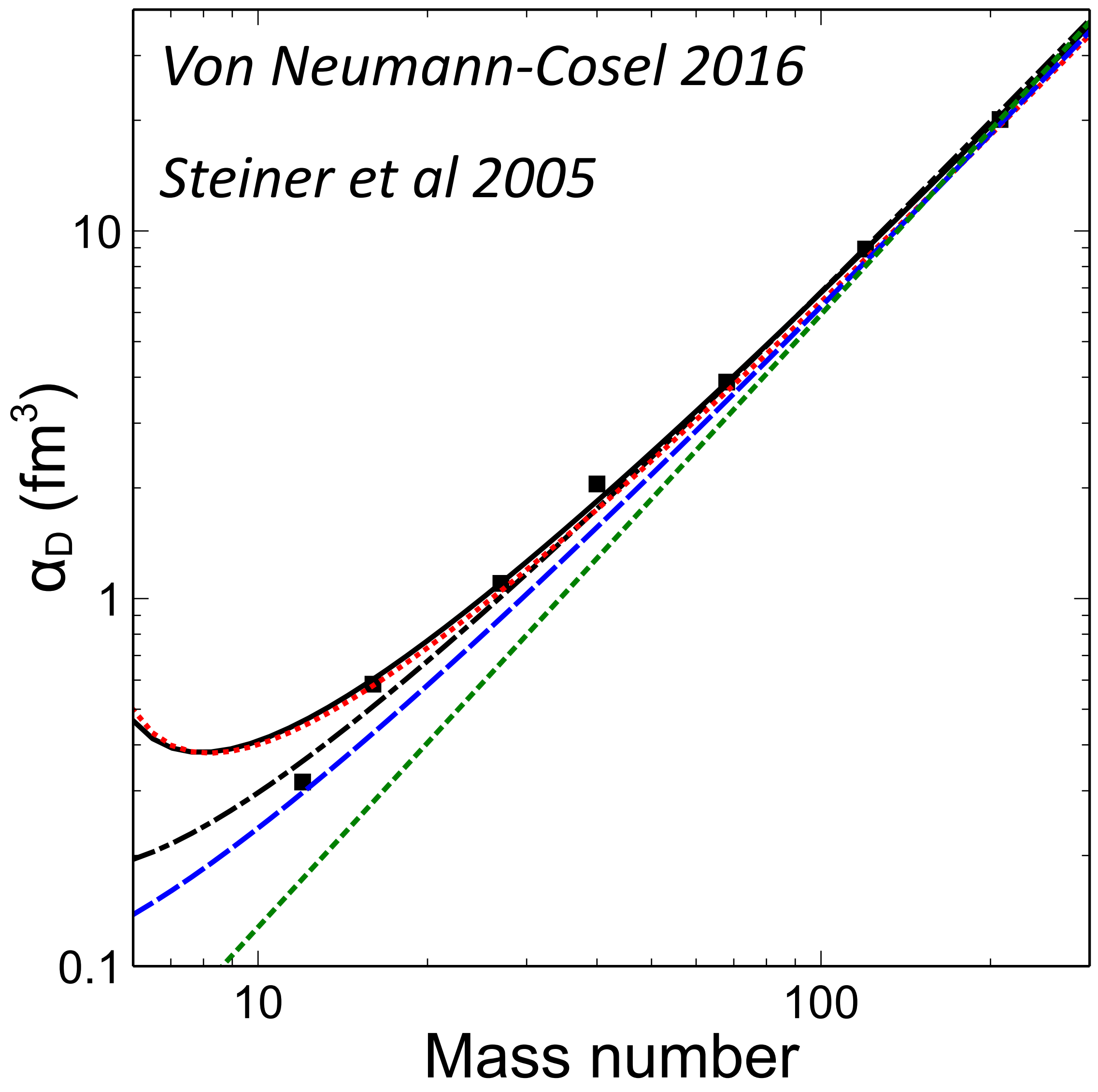
Fit to dipole polarizability from O to Pb

$$\alpha_{E1} = \frac{0.0518\,\mathrm{MeV\ fm}^3 A^2}{S_v(A^{1/3} - \kappa)}$$

$S_v = 27.3(8)$ MeV and $\kappa = 1.69(6)$

Some lighter nuclei: data (Ahrens et al, 1974)

	$\hat{E}$ (MeV)	$\sum_{-2}$ (mb/MeV)	±(%)
Li	100	0.196	1.1
	140	0.197	1.1
	210	0.198	1.1
Be	100	0.192	2.5
	140	0.194	2.5
	210	0.195	2.5
C	100	0.313	1.7
	140	0.316	1.7



Nuclear polarization - leading order

Loop integral is evaluated in two different ways for nuclear and hadronic parts
—> *nuclear* polarization (NP) and *nucleon* polarization (nP)

Hadronic: exact relativistic expression; direct use of real and virtual photoabsorption data

Evaluated on H, He isotopes — use the A-scaling of cross section to extrapolate to arbitrary A

$$\left[\Delta E_{2S}^{\mathrm{hadr}}\right]_{\mu D} = -28(2)\,\mu\mathrm{eV} \longrightarrow \left[\Delta E_{nS}^{\mathrm{nP}}\right]_{\mu A} = -28(2)\,\mu\mathrm{eV}\,\frac{|\phi_{nS}^{\mu A}(0)|^2}{|\phi_{2S}^{\mu D}(0)|^2}\,\frac{A}{2}$$

Carlson, Vanderhaeghen 2011; Carlson, MG, Vanderhaeghen 2013, 2016; …

Nuclear shadowing (A_{eff} < A) concentrated at high energies, ~does not affect Npol

Nuclear: keep dominant longitudinal response

$$\Delta E_{nS}^{NP} = -8\alpha^2|\phi_{nS}(0)|^2 \int_0^\infty \frac{d\mathbf{q}}{\mathbf{q}^2}\int_0^\infty \frac{d\nu S_L(\nu,\mathbf{q})}{\nu + \mathbf{q}^2/2m}$$

$$S_L(\nu,\mathbf{q}) = \mathbf{q}^2\,\frac{\sigma_\gamma(\nu)}{4\pi^2\alpha\nu}F^2(\mathbf{q})$$

$$\alpha_{E1} = \frac{1}{2\pi^2}\int_{\mathrm{thr}}^{\nu_{mas}}\frac{d\nu}{\nu^2}\sigma_\gamma(\nu)$$

Leading-order nuclear polarization

$$\Delta E_{nS}^{\mathrm{NP}} = -2\pi\alpha|\phi_{nS}(0)|^2\alpha_{E1}\sqrt{2m\bar\nu}\,e^{\beta^2(\bar\nu)}\mathrm{Erfc}(\beta(\bar\nu))$$

E.g., Rosenfelder 1983

Nuclear polarization - beyond leading approximation

Approximation scheme: define small parameters

$$\epsilon_1 = Z\alpha m_r R_{\text{ch}} = R_{\text{ch}}/a^{\mu}_{1S}$$

$$\epsilon_2 = (Z\alpha)^2 \frac{m_r}{2\nu_N} = \left| \frac{E_{1S}}{E_{\mu}^{\text{Nucl. Exc.}}} \right|$$

Corrections in ϵ_1: variation of atomic WF over the nucleus volume

$$F_R = \int_0^{\infty} r^2 dr e^{-2Z\alpha m_r r} \rho_{\text{Nuc}}(r)$$

Corrections in ϵ_2: keep Coulomb energy in the Green's function

$$\left[\sum_n \frac{|n\rangle\langle n|}{\epsilon_0 - \epsilon_n - \omega_N} \right]$$

Obtained via radial integral with Coulomb GF and atomic WF

$$K = -\sqrt{\frac{\nu_N}{2m_r}} \int_0^{\infty} dr \int_0^{\infty} dr' \phi_{nS}(r) \frac{g_1(-\nu_N, r, r')}{rr'} \phi_{nS}(r')$$

*New closed-form expressions for Coulomb distortion corrections obtained

Nuclear polarization - beyond leading approximation

Final expression: leading-order + corrections

$$\Delta E_{nS}^{\rm TOT} = \Delta E_{nS}^{\rm NP}\, F_R(\epsilon_1)\, K^{(1)}(\sqrt{\epsilon_2}) + \Delta E_{nS}^{\rm nP}\, F_R(\epsilon_1) K^{(1)}(\sqrt{\epsilon_2^n});$$

$$\epsilon_2^n = (Z\alpha)^2 m_r / 2\nu_n$$

$$\nu_n \approx 500 \text{ MeV}$$

All ingredients have simple parametrization in terms of few input parameters

Easy to use and reproduce! Evaluate and compare to entries in Fricke, Heilig (used to extract radii)

Rinker, Speth 1978:

$$\Delta E_a = \frac{\alpha^2 B^2 k^2 Z}{2M} \langle r^{2k-2} \rangle \left[\frac{Z}{A} \langle E_{\rm N}^{(b)} - E_{\rm N}^{(a)} \rangle_{\tau=0}^{-2} + \frac{N}{A} \langle E_{\rm N}^{(b)} - E_{\rm N}^{(a)} \rangle_{\tau=1}^{-2} \right]$$

Energy-weighted (TRK) sum rule to normalize

Polarizability ~ inverse energy sum rule —> enhanced sensitivity to low-lying states (PDR)

Long-range part of the induced dipole potential $\sim \alpha_{E1}/r^4$ taken between atomic WF

Already noted in Ericson, Hüfner 1972

Uncertainties:

Polarizability 10%

F_R (Gauss vs hard sphere)

Coulomb distortion
(higher orders in ϵ_2)

Good-ish agreement with F&H

For light elements

Should not be taken for granted!

Approaches are different

Nucleon polarization non negligible

From Ca on exceeds exp. precision

Z−Element	A	$-\Delta E_{1S}^{NP}$	$-\Delta E_{1S}^{nP}$	Total NP	Entry in [7]	$\sigma_{\exp}$
4−Be	9	0.44(4)(0)(0)	0.063(6)(0)(0)	0.50(4)	1.0(3)	10
5−B	10	0.99(10)(0)(1)	0.13(1)(0)(0)	1.12(10)	1.0(3)	7
6−C	12	2.1(2)(0)(0)	0.27(3)(0)(0)	2.4(2)	2.5(7)	0.5
7−N	14	3.8(4)(0)(1)	0.48(5)(0)(0)	4.3(4)	3.0(9)	5
8−O	16	7.8(0.8)(0.1)(0.1)	0.79(8)(1)(1)	8.6(8)	5.0(1.5)	4
9−F	19	11.9(1.2)(0.1)(0.2)	1.28(13)(1)(1)	13.2(1.2)	9.0(2.7)	2
10−Ne	20	15.7(1.6)(0.2)(0.3)	1.78(18)(2)(1)	17.5(1.6)	19(6)	5
	21	17.0(1.7)(0.2)(0.4)	1.88(19)(2)(1)	19(2)	18(5)	4
	22	18.0(1.8)(0.2)(0.4)	1.98(20)(2)(1)	20(2)	18(5)	4
11−Na	23	23.3(2.3)(0.3)(0.6)	2.64(26)(4)(1)	26(3)	25(8)	2
12−Mg	24	30.0(3.0)(0.5)(0.8)	3.46(35)(6)(2)	33(3)	38(11)	2
	25	31.3(3.1)(0.5)(0.8)	3.61(36)(6)(2)	35(3)	31(9)	3
	26	32.3(3.2)(0.5)(0.9)	3.75(38)(6)(2)	36(3)	33(10)	3
13−Al	27	42.2(4.2)(0.8)(1.2)	4.80(48)(9)(3)	48(5)	40(12)	2
14−Si	28	51.5(5.2)(1.1)(1.5)	5.99(60)(12)(4)	58(6)	55(16)	5
	29	53.9(5.4)(1.1)(1.6)	6.21(62)(13)(4)	60(6)	53(16)	45
	30	56.1(5.6)(1.2)(1.6)	6.42(64)(13)(4)	63(6)	51(15)	45
15−P	31	67.5(6.8)(1.6)(2.1)	7.86(79)(18)(6)	76(7)	61(18)	11
16−S	32	79.7(8.0)(2.0)(2.6)	9.48(95)(24)(7)	89(9)	83(25)	12
	34	85.6(8.6)(2.2)(2.8)	10.1(1.0)(0.3)(0.1)	97(9)	79(24)	14
	36	91.8(9.2)(2.4)(3.0)	10.6(1.1)(0.3)(0.1)	102(10)	75(23)	13
17−Cl	35	98.5(9.9)(2.9)(3.4)	11.9(1.2)(0.3)(0.1)	110(11)	-	-
	37	106(11)(3)(4)	12.6(1.3)(0.4)(0.1)	119(12)	-	-
18−Ar	36	116(12)(4)(4)	14(1.4)(0.4)(0.1)	130(12)	118(36)	24
	38	124(12)(4)(5)	15(1.5)(0.5)(0.1)	139(14)	107(32)	24
	40	132(13)(4)(5)	16(1.6)(0.5)(0.1)	148(15)	126(38)	25
19−K	39	141(14)(5)(5)	18(1.8)(0.6)(0.2)	159(16)	119(36)	32
	41	150(15)(5)(6)	18(1.8)(0.6)(0.2)	168(17)	132(40)	28
20−Ca	40	160(16)(6)(6)	20(2.0)(0.7)(0.2)	181(18)	142(40)	25
	42	170(17)(6)(7)	21(2.1)(0.8)(0.2)	191(19)	166(50)	29
	43	176(18)(7)(7)	21(2.1)(0.8)(0.2)	198(20)	145(43)	27
	44	180(18)(7)(7)	22(2.2)(0.8)(0.2)	203(21)	175(52)	26
	46	193(19)(7)(8)	23(2.3)(0.8)(0.2)	216(22)	156(47)	107
	48	206(21)(8)(8)	24(2.4)(0.9)(0.2)	230(24)	153(46)	26
21−Sc	45	203(20)(8)(9)	25(2.5)(1.0)(0.2)	230(24)	203(61)	41
22−Ti	46	226(23)(10)(10)	28(2.8)(1.2)(0.3)	256(27)	257(77)	26
	47	230(23)(10)(11)	29(2.9)(1.2)(0.3)	259(27)	252(76)	25
	48	237(24)(10)(11)	29(2.9)(1.3)(0.3)	266(28)	241(72)	26
	49	246(25)(11)(11)	30(3.0)(1.3)(0.3)	276(29)	215(64)	33
	50	253(25)(11)(11)	31(3.1)(1.3)(0.3)	284(30)	216(65)	26
23−V	51	276(28)(13)(13)	35(3.5)(1.6)(0.4)	319(33)	245(73)	26
24−Cr	50	286(29)(14)(14)	37(4)(2)(1)	323(35)	333(100)	27
	52	304(30)(15)(15)	39(4)(2)(1)	343(37)	299(90)	21
	53	310(31)(15)(15)	39(4)(2)(1)	349(38)	302(91)	25
	54	316(32)(16)(15)	40(4)(2)(1)	356(39)	318(96)	31
25−Mn	55	351(35)(19)(17)	44(4)(2)(1)	395(44)	364(109)	34

Agreement deteriorates for larger Z

But keep in mind estimates

included in Shera et al, 1976

^{54}Fe	1260.011(45)	0.546
^{56}Fe	1257.054(42)	0.582
^{57}Fe	1255.921(51)	0.600
^{58}Fe	1254.485(49)	0.624
^{59}Co	1341.461(46)	0.588
^{58}Ni	1432.564(44)	0.689
^{60}Ni	1429.369(43)	0.693
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^{62}Ni	1426.829(43)	0.703
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^{65}Cu	1512.516(45)	0.749
^{64}Zn	1602.718(44)	0.857
^{66}Zn	1600.544(43)	0.909
^{68}Zn	1598.763(44)	0.917
^{70}Zn	1596.898(109)	0.973

If disagree with older calculations

— also extracted radii disagree

How robust is the uncertainty?

Z−Element	A	$-\Delta E_{1S}^{NP}$	$-\Delta E_{1S}^{nP}$	Total NP	Entry in [7]	Goal
26−Fe	54	371(37)(21)(19)	48(5)(3)(1)	419(47)	362(109)	48
	56	384(38)(22)(20)	49(5)(3)(1)	433(49)	403(121)	44
	57	391(39)(22)(20)	50(5)(3)(1)	441(50)	390(117)	56
	58	397(40)(23)(20)	50(5)(3)(1)	447(50)	400(120)	54
27−Co	59	433(43)(26)(23)	56(6)(4)(2)	489(56)	438(131)	50
28−Ni	58	459(46)(29)(25)	59(6)(4)(1)	518(60)	437(131)	46
	60	467(47)(30)(25)	61(6)(4)(1)	528(61)	461(138)	45
	61	476(48)(30)(26)	62(6)(4)(1)	538(63)	426(138)	54
	62	484(48)(31)(26)	62(6)(4)(1)	546(64)	458(138)	45
	64	502(50)(33)(27)	64(6)(4)(1)	566(66)	438(138)	49
29−Cu	63	506(51)(35)(29)	68(7)(5)(1)	574(68)	538(161)	47
	65	530(53)(36)(30)	70(7)(5)(1)	600(71)	489(147)	49
30−Zn	64	545(54)(39)(32)	73(7)(5)(1)	618(75)	609(183)	47
	66	565(56)(41)(33)	75(8)(5)(1)	640(78)	595(179)	45
	68	585(59)(43)(34)	77(8)(6)(1)	662(81)	581(174)	32
	70	606(61)(45)(35)	79(8)(6)(1)	685(84)	615(184)	131
31−Ga	69	616(62)(48)(37)	83(8)(6)(1)	699(87)	567(169)	12
	71	647(65)(50)(38)	86(9)(7)(1)	733(91)	551(165)	12
32−Ge	70	662(66)(54)(40)	89(9)(7)(1)	751(95)	706(212)	16
	72	671(67)(55)(42)	92(9)(8)(1)	763(97)	738(221)	12
	73	683(68)(56)(42)	93(9)(8)(1)	776(99)	700(210)	24
	74	694(69)(57)(43)	94(9)(8)(1)	788(101)	839(242)	17
	76	719(72)(60)(44)	96(10)(8)(1)	815(104)	819(246)	15
33−As	75	737(74)(64)(47)	101(10)(9)(2)	838(109)	761(228)	10
34−Se	76	775(78)(71)(50)	107(11)(10)(2)	882(117)	1036(311)	16
	77	790(79)(72)(51)	109(11)(10)(2)	899(119)	790(237)	16
	78	805(80)(74)(52)	110(11)(10)(2)	915(122)	949(285)	13
	80	835(83)(76)(54)	113(11)(10)(2)	948(126)	872(262)	12
	82	865(87)(79)(56)	116(12)(11)(2)	981(133)	814(244)	19
35−Br	79	850(85)(81)(56)	117(12)(11)(2)	967(131)	933(280)	17
	81	883(88)(84)(58)	120(12)(11)(2)	105(136)	827(248)	20
36−Kr	78	858(86)(86)(57)	121(12)(12)(2)	979(136)	1183(355)	40
	80	892(89)(90)(59)	124(12)(12)(2)	1016(141)	1071(321)	40
	82	927(93)(93)(62)	128(13)(13)(2)	1055(146)	938(281)	40
	83	946(95)(95)(63)	129(13)(13)(2)	1075(149)	936(281)	47
	84	962(96)(96)(64)	131(13)(13)(2)	1093(152)	838(251)	39
	86	997(100)(100)(67)	134(13)(13)(2)	1133(157)	866(260)	34
37−Rb	85	1014(101)(106)(69)	139(14)(14)(2)	1151(163)	853(256)	10
	87	1051(105)(109)(71)	142(14)(15)(2)	1193(169)	807(242)	14
38−Sr	84	1034(103)(112)(71)	145(14)(16)(3)	1179(169)	1136(341)	24
	86	1061(106)(115)(73)	147(15)(16)(3)	1208(174)	929(279)	11
	87	1082(108)(118)(75)	149(15)(16)(3)	1231(178)	843(253)	49
	88	1101(110)(120)(76)	151(15)(16)(3)	1252(181)	937(281)	8
39−Y	89	1165(116)(132)(81)	158(16)(18)(3)	1323(195)	867(260)	9
40−Zr	90	1218(122)(143)(86)	166(17)(20)(3)	1384(208)	975(292)	10
	91	1198(120)(142)(86)	167(17)(20)(3)	1365(206)	957(287)	33
	92	1212(121)(144)(87)	169(17)(20)(3)	1381(209)	984(295)	13
	94	1237(124)(148)(89)	171(17)(20)(3)	1408(214)	946(284)	15
	96	1266(127)(153)(91)	174(17)(21)(3)	1440(220)	966(293)	36
41−Nb	93	1264(126)(156)(92)	177(18)(20)(3)	1441(223)	1127(338)	16

Conclusions
Status & Outlook

Presumably a quote by Wolfgang Pauli:

Nothing is worse than a wrong theory describing data

- Nuclear charge radii: crucial input to SM tests and BSM searches at low energies
- Cabibbo (CKM) unitarity and V_{ud}: nuclear corrections current bottleneck - use R_{ch} as input
- Nuclear charge radii rely on very precise experiments — is theory up to the task?
- Leap in exp. precision: MuX, QUARTET, MUSEUM, RefRad (μ atoms)
- Nuclear polarization crucial to extraction of R_{ch} from atomic transitions
- Are uncertainties of NP firmly under control?
- Personal wish: an open-source nuclear polarization (formula, parametrization, code)
- NP is related to dispersion corrections in e-scattering and to NS correction in β-decay
- Look for a uniform treatment of all of these
- What is the path to these goals?
- Ab-initio methods are hot right now: (potentially) very accurate and systematically improvable — are not easy to understand and are very expensive computationally; viable recipe for nuclear radii tables? — no single ab-initio method covers full nuclear chart
- Generally, μ atoms difficult: nuclear and atomic scales are not well separated! full-blown ab-initio nuclear calculation per se is not enough to guarantee precision