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3D hybrid fluid-kinetic simulations of large scale plasma instabilities in runaway electron beams

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CONFERENCE PRE-PRINT

**3D HYBRID FLUID-KINETIC SIMULATIONS OF RESISTIVE INSTABILITIES IN
RUNAWAY ELECTRON BEAMS**

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Abstract

Runaway electrons (REs), generated during plasma disruptions in tokamaks, pose significant challenges due to the risk of causing damage to the first wall of the device. Electrons may be accelerated by the large toroidal electric field arising during a major disruption and cause substantial damage when reaching material walls. Via collisions, an avalanche can set in such that eventually a large fraction of the pre-disruption plasma current can be converted into relativistic runaway electron current. This work aims at contributing to the understanding of MHD mode activity in RE beams, as it plays an important role for instance in the development of benign RE beam termination scenarios that aim to avoid wall damage. In the JOREK code, RE physics can be studied in several ways, e.g., a RE fluid treatment, which is self-consistently coupled to the background plasmas. This model does not include the energy/pitch-angle distribution of the particles or the accurate kinetic orbits, limiting the accuracy with respect to transport predictions. The paper introduces a novel hybrid fluid–kinetic model that achieves high fidelity, treating REs kinetically using a relativistic guiding-center approach, while describing the background plasma by ansatz-based reduced MHD equations. At first, comprehensive benchmark studies are conducted regarding the 2D equilibrium force balance with $J_{total} = J_{RE}$, and the linear stability of 3D tearing mode, verifying the accuracy of the model against analytical predictions and other numerical methods, e.g., the full-orbit approach in JOREK and the fluid model in M3D-C1. These benchmark studies build a solid foundation for applying our model to more complex nonlinear scenarios. Previous analytical studies suggest that in the case of small Δ' , the saturation width of the magnetic island driven by REs is roughly 1.5 times larger than in the otherwise identical Ohmic current scenario. Our simulations are quantitatively in line with this prediction. Besides, the drift orbit shifts of highly energetic REs might change the picture and this paper is trying to address the effect of these orbit shifts onto the linear and non-linear dynamics. The preliminary results prove a strong toroidal coupling effect due to RE shifts.

1. INTRODUCTION

Major disruptions of tokamak fusion plasmas pose a significant risk to the integrity of fusion devices due to large electromagnetic forces, thermal heat loads, and in particular large runaway electron (RE) induced heat loads given the potential generation of large quantities of REs [1]. To mitigate the risks associated to disruption consequences, several techniques have been applied in tokamak experiments, e.g., Massive Gas Injection and Shattered Pellet

Injection. In view of such mitigation scenarios, 3D plasma instabilities and their interaction with REs play a key role. And also the so-called benign termination scenarios [2], which may offer drastically reduced heat loads are highly dependent on the nature of plasma instabilities that cause the sudden loss of RE confinement. Experiments and simulations have demonstrated that REs can influence plasma instabilities [3] and excite kinetic instabilities. Now in JOREK [4], numerical approaches are implemented, ranging from fluid models [5], which treat REs as a separate population, to hybrid kinetic–MHD models with high fidelity [6], combining particle-in-cell (PiC) methods with MHD to explore the influence of kinetic effects of REs. In this work, a drift/gyro kinetic treatment has now been coupled to the MHD equations in JOREK. This article will initially verify the correctness of the model by comparisons regarding the axisymmetric equilibrium of a RE beam and the linear growth rate of tearing modes in presence of REs to analytical and numerical literature results in the fluid limit. Building up on this foundation, it will then turn towards a study of linear and non-linear dynamics of tearing modes in a RE beam, taking into account the kinetic effects governing the interaction.

The rest of the article is structured as follows. In Section 2, the hybrid fluid-kinetic model is introduced for which verification results are successively shown in section 3, where a 2D benchmark was carried out regarding the major-radial force balance of a tokamak equilibrium with $\mathbf{J}_{total} = \mathbf{J}_{RE}$ in subsection 3.1. Moreover, in subsection 3.2, for validation regarding the 3D physics, linear TM simulations in a RE beam were conducted, and the results were examined against analytical theory and other numerical approaches. Building up upon this work, section 4 presents studies regarding the non-linear TM dynamics in a RE beam. Subsection 4.1 pays particular attention to the saturated amplitude. Subsection 4.2 gives some preliminary results to assess the effect of RE shifts mentioned in subsection 3.1. Finally, section 5 concludes the article by summarizing the work and providing a concise outlook to future work.

2. HYBRID FLUID-KINETIC MODEL

In this section, a drift-kinetic version of the hybrid fluid-kinetic model is introduced. In this framework, JOREK evolves thermal particles through the corresponding fluid equations, while relativistic REs are treated kinetically using a full-f relativistic PiC technique. The relativistic guiding center model implemented in the JOREK follows the formulation described in [7]:

$$\begin{aligned}\dot{\mathbf{X}} &= \frac{p_{\parallel}^*}{\gamma_r m_0} \frac{\mathbf{B}^*}{B_{\parallel}} + \frac{\mathbf{b}}{B_{\parallel}} \times \left(p_{\parallel} \frac{\partial \mathbf{b}}{\partial t} - q\mathbf{E} + \frac{\mu}{\gamma_r} \nabla B \right), \\ \dot{p}_{\parallel} &= \frac{\mathbf{B}^*}{B_{\parallel}} \cdot \left(q\mathbf{E} - p_{\parallel} \frac{\partial \mathbf{b}}{\partial t} - \frac{\mu}{\gamma_r} \nabla B \right),\end{aligned}$$

where

$$\begin{aligned}B_{\parallel}^* &= \mathbf{B}^* \cdot \mathbf{b}, \\ \gamma_r &= \sqrt{1 + \frac{p_{\parallel}^2}{(m_0 c)^2} + \frac{2\mu B}{m_0 c^2}},\end{aligned}\tag{1}$$

and $\mathbf{X}$, $p_{\parallel}$, $\mu = \frac{p_{\perp}^2}{2m_0 B}$ represent the guiding center location, the momentum parallel to the magnetic field, and the magnetic moment of the particle, respectively. $\mathbf{E}$ and $\mathbf{B}$ denote the electric and magnetic fields, and the effective magnetic field is given as $\mathbf{B}^* = p_{\parallel} \nabla \times \mathbf{b} + q\mathbf{B}$, q is the charge, m_0 is the rest mass of the particle, and γ_r is the relativistic factor. The speed of light in vacuum is given by c .

In the guiding-center formalism, the RE current consists of two components: the parallel current $\mathbf{J}_{r,\parallel}$ associated with $p_{\parallel}$ in Eq. 1 and the perpendicular current $\mathbf{J}_{r,\perp}$, which arises from guiding-center drifts and the magnetization current:

$$\mathbf{J}_{r,\perp} = \mathbf{J}_r^{drift} + \mathbf{J}_r^{mag}.\tag{2}$$

After some treatment, the total perpendicular current can be defined as,

$$\mathbf{J}_{r,\perp} = \sigma_r \frac{\mathbf{E} \times \mathbf{B}}{B^2} + (\mathcal{P}_{r,\parallel} - \mathcal{P}_{r,\perp}) \frac{\mathbf{B} \times \kappa}{B^2} + \frac{1}{B} (\mathbf{b} \times \nabla \mathcal{P}_{r,\perp}),\tag{3}$$

where the perpendicular and parallel pressure of REs are defined as:

$$\begin{aligned}\mathcal{P}_{r,\perp} &= \frac{1}{2} \int d^3 v \gamma_r m_0 v_{\perp}^2 f, \\ \mathcal{P}_{r,\parallel} &= \int d^3 v \gamma_r m_0 v_{\parallel}^2 f.\end{aligned}\tag{4}$$

Under the assumption that the plasma consists of thermal particles and REs, and thus $\mathbf{J} = \mathbf{J}_i + \mathbf{J}_e + \mathbf{J}_r$, where i and e correspond to thermal ions and electrons, respectively. By integrating directly over the three-dimensional velocity vector and coupling the current density into the MHD momentum equation, we will have the current coupling scheme:

$$\rho \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\sigma_r \mathbf{E} + (\mathbf{J} - \mathbf{J}_r) \times \mathbf{B} - \nabla p - \nabla \cdot \mathbf{\Pi}, \quad (5)$$

Inserting the expressions in eq 3 into 5, we will have the pressure coupling scheme:

$$\rho \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\sigma_r \mathbf{E}_{\parallel} + \mathbf{J} \times \mathbf{B} - \nabla p - \nabla \cdot \mathbf{\Pi} - (\mathcal{P}_{r,\parallel} - \mathcal{P}_{r,\perp}) \kappa - \nabla \mathcal{P}_{r,\perp}. \quad (6)$$

Obviously, in the guiding center theory, these two schemes are equivalent.

Finally, we will also have the Ohm's law:

$$\mathbf{E} = -\mathbf{u} \times \mathbf{B} + \eta(\mathbf{J} - \mathbf{J}_r). \quad (7)$$

3. VERIFICATIONS

3.1. 2D Tokamak Equilibrium with Runaway Electrons

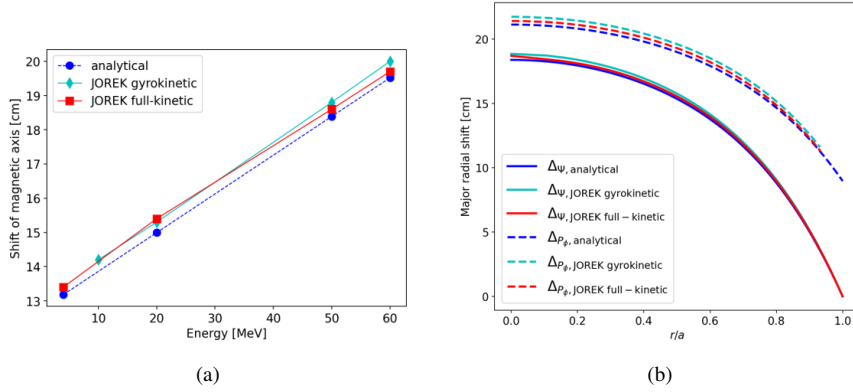


FIG. 1. Comparison with analytical results and full-kinetic treatment (a) shift of magnetic axis varying with RE energy; (b) minor radial shift profile of 50 MeV REs.

As part of the validation for the development of the hybrid fluid-kinetic model in this work, we first conduct a benchmark test using an axisymmetric geometry. After a thermal quench, the cold plasma initially tends toward a force-free equilibrium ($\mathbf{J} \times \mathbf{B} \approx 0$). When a substantial fraction of the current is carried by REs, however, RE pressure modifies the equilibrium, producing a Grad-Shafranov like major radial shift of the flux surfaces. Furthermore, the RE drift orbits are again shifted with respect to these flux surfaces due to the curvature drift. Moreover, the shift may exceed the radial extent of MHD modes or current layers, potentially altering the linear growth and non-linear saturation of the instability. The shift effect on MHD instability will be discussed in subsection 4.2.

In this section, we benchmark our results against the analytical solutions of Ref.[8] and the full-kinetic coupled model in Ref [6].

Figure 1 (a) illustrates the dependency of the Grad-Shafranov like magnetic axis shift on the RE energy, owing to the change in force balance, while (b) presents the radial dependency of the shift of the flux surfaces and RE drift orbit centers for 50 MeV REs. Both figures compare the results to analytical predictions and to results obtained using the full-orbit kinetic JOREK simulations, which agree very well with the results obtained by our model.

3.2. 3D linear Tearing Mode with Runaway Electrons

In the following, a benchmark for 3D MHD instabilities and their interaction with REs will be provided in this subsection. Early studies on the interaction between TMs and REs [10] laid the foundation for this topic. A new γ expression is derived in [9], which describes the RE effect on TM instability:

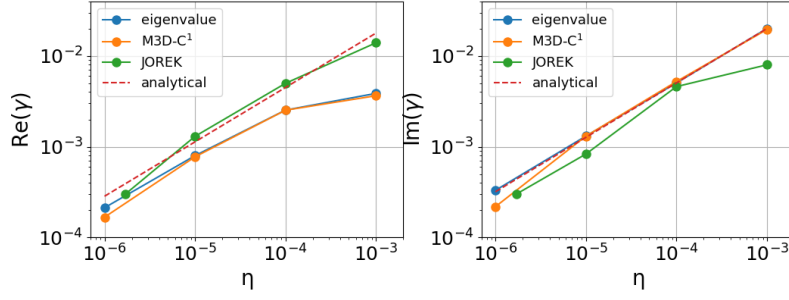


FIG. 2. (a) Real and (b) imaginary parts of γ for the (2,1) TM. Red dashed lines represent analytically calculated values. Blue circles indicate results from the eigenvalue code. Orange circles correspond to results from the M3D-C1 simulations. Green circles denote results from the JOREK simulations. The authors gratefully acknowledge C. Liu for providing the data related to the analytical solutions, M3D-C1, and eigenvalue results from [9].

$$\frac{\gamma^{5/4}}{\eta^{3/4} k_c^{1/2}} \frac{2\pi\Gamma(3/4)}{\Gamma(1/4)} = \Delta' - i\pi \frac{mJ'_{RE0}}{|k_c| r_s} \quad (8)$$

The magnetic flux ψ structure will not be significantly altered by REs, therefore in presence of REs, the standard Furth-Killeen-Rosenbluth growth rate is obtained [11], while a frequency component is excited due to the gradient of the runaway current - a feature absent in ohmic plasmas. The real and imaginary parts of γ from four different approaches – an eigenvalue code, M3D-C1 fluid simulations, the analytical theory, and JOREK kinetic simulations – are compared in Figure 2 as a function of the normalized resistivity of the background plasma, demonstrating consistency between our JOREK fluid-kinetic model and other methods.

4. APPLICATIONS

4.1. Non-linear Tearing Mode Dynamics in a Runaway Beam

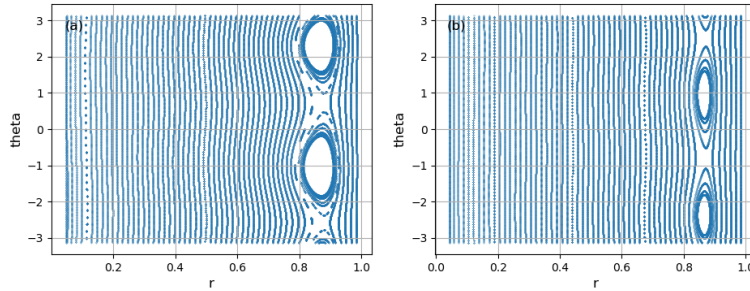


FIG. 3. Poincaré plots of saturated islands formed (a) in the presence of REs and (b) w/o REs, for the case with $\Delta' \approx 0.2$.

In the RE-driven case, the runaway current remains “frozen into” the magnetic field throughout island evolution, whereas in the resistive case, the ohmic current is governed by the flux-surface average of Ohm’s law, leading to different saturated states. In a slab geometry, the expressions predicting the saturated island widths are given in [10]:

$$w_{RE} = -\frac{j_0(0)}{j_0''(0)} \frac{\Delta'}{0.272}, \text{ and } w_{MHD} = -\frac{j_0(0)}{j_0''(0)} \frac{\Delta'}{0.411}. \quad (9)$$

The predictions are made under the condition that the island is small, i.e., $w, \Delta' \ll 1$. The islands in a REs case are approximately 1.5 times larger than in the ohmic case under these assumptions.

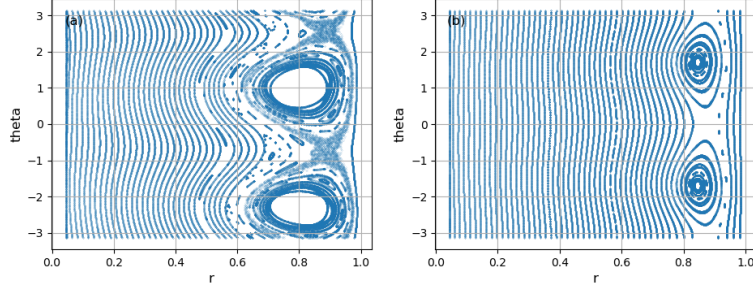


FIG. 4. Poincaré plots of saturated islands formed (a) in the presence of REs and (b) w/o REs, for the case with $\Delta' \approx 2.0$.

In our work, we consider two cases with different equilibria in which the TM has different values of Δ' . Poincaré plots of the saturated islands with and without REs are shown in Figures 3 and 4. The former corresponds to $\Delta' \approx 0.2$, and the latter to $\Delta' \approx 2.0$. In Figure 3, the width ratio is approximately 1.7, close to the analytically predicted value, while for the more unstable case with $\Delta' \approx 2.0$, the ratio is larger than 2. In latter case, larger islands emerge, leading to enhanced mode coupling and a more complex magnetic topology with initial signs of magnetic field stochastization around the island separatrix. In such cases, the evolution becomes complex and more difficult to predict analytically.

4.2. Dependence of Tearing Mode on Runaway Electron Energy

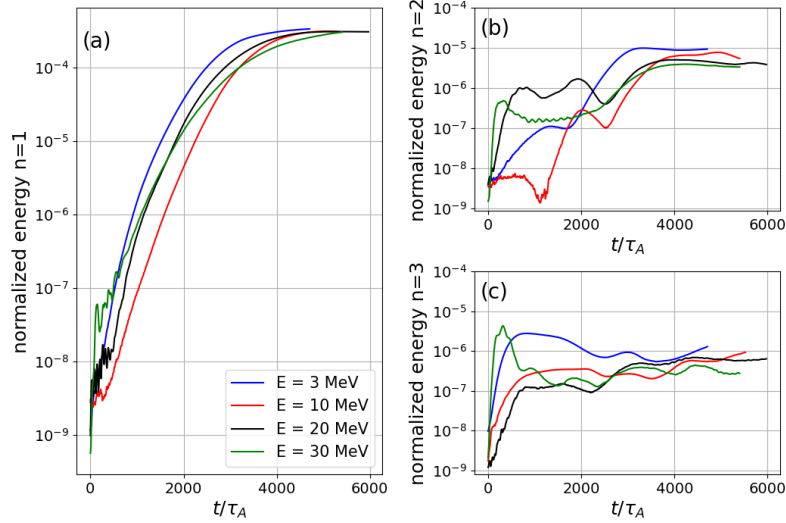


FIG. 5. Time evolution of normalized energy (a) $n=1$ (b) $n=2$ and (c) $n=3$ in the presence of REs. Colors correspond to different RE energies: 3 MeV (blue), 10 MeV (red), 20 MeV (black) and 30 MeV (green).

As shown in section 3.1, RE energy will have a notable effect on MHD equilibrium, and the shift between flux surfaces and RE orbits can exceed the resistive layer width of classical TM instabilities, modifying their stability properties. In this section, a RE energy scan is conducted, and preliminary results are given. In Figure 5, the time evolution of normalized energy (a) $n=1$ (b) $n=2$ and (c) $n=3$ is shown and colors correspond to different RE energies: 3 MeV (blue), 10 MeV (red), 20 MeV (black) and 30 MeV (green). In this simulation, the (2,1) islands are dominant and the shift is several orders of magnitude smaller than the (2,1) island width. Consequently, the $n=1$ mode is barely affected, both linear characteristics and nonlinear saturation. Nevertheless, as shown in Figure 5 (b) and (c), the $n=2$ and $n=3$ modes are significantly affected, depending on the RE energy. A coupling mechanism induced by the RE shift can be identified in Figure 6. In Figure 6, radial profiles of ψ for the (5,3) and (6,3) modes are shown. And obviously, a overlap between two modes is getting wider as RE energy increasing. As illustrated

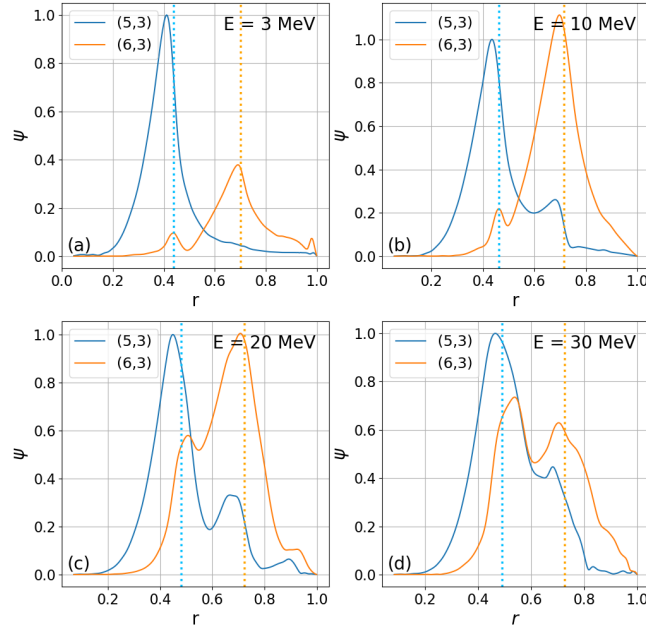


FIG. 6. Radial profiles of ψ for the (5,3) and (6,3) modes, in the presence of REs with different energies: (a) 3 MeV, (b) 10 MeV, (c) 20 MeV, (d) 30 MeV. In each subfigure, the (5,3) is shown in blue, with its corresponding rational surface indicated by a dashed blue line, while the (6,3) mode is shown in orange, with its rational surface indicated by a dashed orange line.

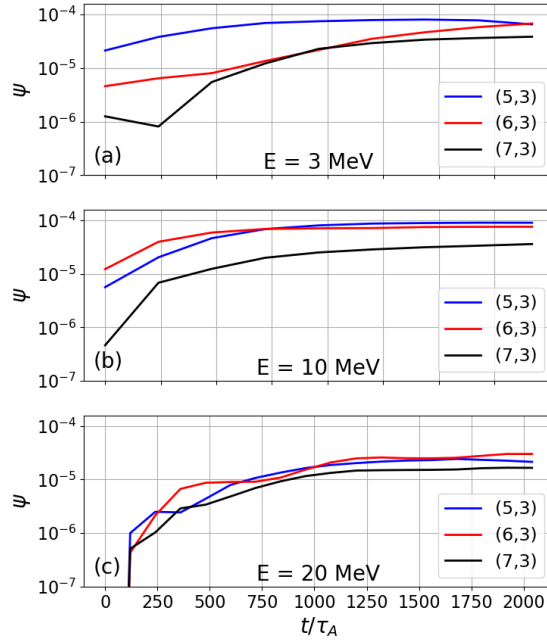


FIG. 7. Time evolution of (5,3) (blue), (6,3) (red), and (7,3) (black) ψ in the presence of REs with different energies: (a) 3 MeV, (b) 10 MeV, (c) 20 MeV.

in Figure 7, several m -harmonic components with the same n number cluster together, further demonstrating that RE energy enhances mode coupling. As depicted in Figure 7, the several m harmonic components with one n number are getting bonding together in 30 MeV RE beam, indicating that RE energy enhances mode coupling. At present, no qualitative and quantitative conclusions can be drawn, as this part of the work is still ongoing. In a toroidal geometry, the gradient pressure will enhance the G-S shift, which in turn strengthens the toroidal coupling [12]. In the presence of REs, a G-S-like shift likewise occurs, and at the same time, RE drift orbits are shifted with respect to these flux surfaces due to the curvature drift. Both shifts may contribute to strengthening toroidal coupling, but it remains to be analyzed which one is dominant.

5. CONCLUSION

In this work, we have developed and validated a hybrid fluid-kinetic model within the JOREK framework to investigate the interaction between REs and large scale MHD instabilities. By exploiting a drift-kinetic approach for the full- f PiC model used to describe the REs, the model is able to efficiently calculate the co-evolution of slowly growing MHD instabilities in self-consistent interaction with the relativistic kinetic RE dynamics. By benchmarking our model against analytical theory and existing numerical results, we have demonstrated its capability to capture the essential physics of RE effect on MHD instabilities. This includes a 2D verification regarding the major-radial force balance and RE drift orbits, as well as a verification of the linear TM growth rates in the presence of a RE beam. In the main studies of this work, we investigated non-linear TM dynamics in a RE beam. In the non-linear regime, REs significantly impact the evolution and saturation of magnetic islands. We observe that the presence of REs increases the saturated island width compared to the classical ohmic case, consistent with theoretical predictions. Furthermore, preliminary results assessing the effect of RE shifts on TM instability are presented, revealing that RE shifts enhance toroidal coupling. Further qualitative and quantitative results are still under investigation.

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