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SELF-ORGANIZED STATES OF ALFVÉN EIGENMODES AND ZONAL MODES VIA CROSS-SCALE INTERACTIONS

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Abstract

In scenarios where a sustained energetic particle source strongly drives toroidal Alfvén eigenmodes (TAE), and phase-space transport is insufficient to saturate TAE, this novel theory of TAE-zonal mode (ZM)-turbulence – self-regulated by cross-scale interactions (including collisionless ZF damping) – merits consideration. Zonal modes are driven by Reynolds and Maxwell stresses, without the onset of modulational instability. TAE evolution in the presence of ZMs conserves energy and closes the system feedback loop. The saturated zonal shears can be sufficient to suppress ambient drift-ITG turbulence, achieving an enhanced core confinement regime. The saturated state is regulated by linear and turbulent zonal flow drag. This regulation leads to bursty TAE spectral oscillations, which overshoot while approaching saturation. Heating by both collisional and collisionless ZM damping deposits alpha particle energy into the thermal plasma, achieving effective alpha channeling. This theory offers a mechanism for EP-induced transport barrier formation, and predicts a novel thermal ion heating mechanism.

1. INTRODUCTION

Energetic particles (EP) are known to excite instabilities such as toroidal Alfvén eigenmodes (TAE) and energetic particle modes (EPM), potentially leading to significant EP transport and so affecting fusion plasma confinement[1]. However, recent experiments yielded results that contradict this expectation, showing that EP can be associated with *improved thermal plasma confinement*, through the formation of internal transport barriers (ITB)[2, 3, 4, 5, 6]. These findings highlight discrepancies between experimental results and previous theoretical predictions for EP physics. In addition, conventional turbulent transport models don't account for the effect of EP on thermal confinement. To address these discrepancies, mechanisms have been proposed, such as fast particle dilution[7, 8], electromagnetic stabilization[9] and zonal mode (ZM) stabilization[10, 11, 12, 13]. ZM can address two key problems: the saturation of EP-driven instabilities via ZM[4, 14] to prevent strong EP transport, and the suppression of turbulence by sheared ZM[15] to form a transport barrier. The TAE/ZM system is an ideal framework for studying these issues. TAE can spontaneously generate ZM via modulational instability, which requires a threshold TAE amplitude[16]. Conversely, the directly driven (forced-driven) process[17, 18, 19, 20, 21, 14] does not require a threshold TAE amplitude. The scenario of ZM drive by waves is a well-known paradigm for pattern and structure formation, such as atmospheric jets, Jovian bands and drift wave-zonal flow (DW-ZF) turbulence in tokamaks. Here, the new element is the presence of *two* wave populations - the TAEs and DWs - which couple via the ZM. Meanwhile, the saturation mechanism for the directly driven process is lacking, especially the proper mechanisms for ZM collisionless damping.

As we will show, with the sustained EP profile and strongly excited TAE, the interactions between TAEs and directly driven ZM lead to an interesting self-organized state of the system, which arises from the feedback loop structure as shown in Fig. 1: ZM is directly driven by TAE via Reynolds and Maxwell stresses, and damped by collisional and collisionless drag processes. This, in turn, influences the evolution of the TAE via wave-ZM interaction. ZM damping regulates the TAE saturation level and the oscillations of TAE and ZM as they approach saturation. The geodesic acoustic transference (GAT) is a relevant collisionless damping mechanism, which requires sufficient turbulent mixing to be effective. GAT heats ions nonlinearly, ultimately functioning as an alpha channeling process.

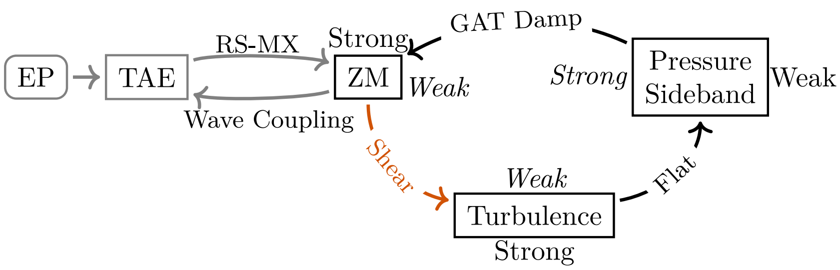

FIG. 1. A feedback diagram for TAE and ZM with energy deposition from EP to thermals. TAEs directly drive the evolution of the ZM through RS-MX stresses, while ZM appears in the TAE evolution equation in the form of a wave coupling term, ensuring energy conservation and influencing the saturation process of TAE. The GAT process as a collisionless damping mechanism for ZM, in which thermal plasma damps ZM and so is heated. The saturated ZM can generate $E \times B$ shearing to suppress DW turbulence. The outer feedback (shown in regular font) and the inner feedback (shown in italic font) on the right side represent a possible interaction between the TAE-ZF and DW-ZF systems, when the shearing effect of the TAE-driven ZM on turbulence is included in the model.

EP transport is connected to thermal turbulence by cross-scale interactions. The shearing effect from the saturated ZM can be sufficient to suppress DW turbulence, leading to a transport barrier for thermals.

2. PREDATOR-PREY MODEL FOR TAE AND ZM

To investigate the feedback interactions between TAE and ZM, we follow the notations in Refs.[18, 19], use $\delta\phi$ and $\delta A_\parallel$ for scalar potential and parallel vector potential respectively, and define $\delta L \equiv \delta\phi - (v_\parallel \delta A_\parallel / c)$, $\delta\psi \equiv \delta A_\parallel \omega/(c k_\parallel)$. Let $\delta\phi = \delta\phi_Z + \delta\phi_T$, where $\delta\phi_Z$ is the zonal component and $\delta\phi_T \equiv \delta\phi_0 + \delta\phi_{0*}$, with $\delta\phi_0$ and $\delta\phi_{0*}$ having the opposite real frequencies. Therefore, a collision of two counter-propagating eigenmodes is considered here. Using the ballooning decomposition in (r, θ, ϕ), we have

$$\delta\phi_0 = \hat{A}_0 e^{i\int \hat{k}_{r,0} \mathrm{d}r + i(n\phi - m_0\theta - \omega_0 t)} \sum_j e^{-ij\theta} \Phi_0(x - j)$$
$$\delta\phi_Z = \hat{A}_Z e^{i\int \hat{k}_Z \mathrm{d}r - i\omega_Z t} \sum_m \Phi_Z$$

where Φ_0 above accounts for the fine scale structure of TAE, $\hat{A}_0$ is the envelope amplitude, and $\hat{k}_{r,0}$ stands for the radial envelope wave number. From the gyrokinetic nonlinear vorticity equation[22, 18, 19, 1], one can obtain the nonlinear evolution of TAE $\delta\phi_0$ as equation (1):

$$-k_\parallel^2 \delta\psi_0 + \frac{\omega_0^2}{V_A^2}\delta\phi_0 = \frac{4\pi\omega_0 e}{c^2 k_{\perp,0}^2}\langle J_{k_0}\omega_d \delta H_{k_0}\rangle_{\mathcal{E}} - i\frac{c}{B_0}\frac{k_Z k_{\theta,0}}{k_{\perp,0}^2}(k_Z^2 - k_{\perp,0}^2)\frac{\omega_0}{V_A^2}\delta\phi_0(\delta\phi_Z - \delta\psi_Z) \tag{1}$$

where $\mathcal{E} \equiv v^2/2$, $\mathbf{k}\delta\phi = [k_\parallel \hat{b} + k_\theta \hat{\theta} + (\hat{k}_r - inq'\partial_x \ln\Phi)\hat{r}]\delta\phi$, $V_A^{-2} \equiv 4\pi n_0 e^2 \rho_i^2/(c^2 T_i)$, $J_k = J_0(k_\perp \rho_L)$ is the Bessel function, EP distribution function is $F = F_0 + \delta H$. The ZM contributions are the second term on the R.H.S. The breaking of the ideal MHD condition is expressed as:

$$\delta\phi_0 - \delta\psi_0 = i\frac{c}{B_0}\frac{k_\theta k_Z}{\omega_0}(\delta\psi_Z - \delta\phi_Z)\delta\phi_0 \tag{2}$$

In equation (1), the integral $\langle\cdots\rangle_{\mathcal{E}} \equiv \int \sqrt{\mathcal{E}}\mathrm{d}\mathcal{E}$ of the fast particle distribution δH appears. This is obtained from the nonlinear gyrokinetic equation:

$$\left(-i\omega + v_\parallel \partial_l + i\omega_d + \Omega_Z\right)\delta H_k = -i\frac{e_s}{m}Q_0 F_0 J_k \delta L_k - \frac{c}{B_0}\Lambda_k J_{k'}\delta L_{k'}\delta H_{k''} \tag{3}$$

where $\Lambda_k \equiv \sum_{k=k'+k''} \hat{b}\cdot\mathbf{k}''\times\mathbf{k}'$. Then the "linear" response to the TAE mode $\delta\phi_0$ is written as[23, 24]:

$$\delta H_0^L = -\frac{e}{m}Q_0 F_0 e^{i\lambda_{d0}} J_{k_0}\delta L_0 \sum_l \frac{(-1)^l J_l(\hat{\lambda}_{d0}) e^{il(\theta-\theta_0)}}{\omega_0 - k_{\parallel,0}v_\parallel - l\omega_t - i\Omega_Z} \tag{4}$$

where $\omega_d = \hat{v}_{d,\mathcal{E}}(k_r \sin\theta + k_\theta \cos\theta)$, $\hat{v}_{d,\mathcal{E}} = (v_\perp^2 + 2v_\parallel^2)/(2\Omega_i R_0)$, $Q_0 F_0 \equiv (\omega_0 \partial_{\mathcal{E}} - \omega_*)F_0$, $\omega_* F_0 = \mathbf{k}\cdot\mathbf{b}\times\nabla F_0/\Omega_i$, $L_{n0E}^{-1} \equiv -\mathrm{d}\ln n_{0E}/\mathrm{d}r$, Ω_i is the thermal ion cyclotron frequency, $\Omega_Z \equiv c k_{\theta,0} k_Z J_Z \delta L_Z / B_0 \simeq k_{\theta,0} V_{E\times B}$ is the nonlinear phase shift from ZM, $\lambda_{d0} = \hat{\lambda}_{d0}\sin(\theta - \theta_0)$, $\hat{\lambda}_{d0} = k_{\perp,0}\hat{\rho}_{d,\mathcal{E}}$, $\hat{\rho}_{d,\mathcal{E}} = qR_0\hat{v}_{d,\mathcal{E}}/v_\parallel$, $\tan\theta_0 = k_r/k_\theta$, $J_l(\hat{\lambda}_{d0})$ is the l-th order Bessel function of $\hat{\lambda}_{d0}$. For large parallel velocity, there is $\omega_t = v_\parallel/qR_0$.

Multiplying equation (1) by $\delta\phi_{0*}$, utilizing equations (2) and (4), and assuming a slowing down distribution function $F_{0E} = n_{0E}\mathcal{E}^{-3/2}/\ln(\mathcal{E}_f/\mathcal{E}_c)$ with $\mathcal{E}_c \leq \mathcal{E} \leq \mathcal{E}_f$ and $\mathcal{E}_f = v_f^2/2$ ($\mathcal{E}_f$ is the maximum fast particle energy, $\mathcal{E}_c$ is the critical energy of ions slowing down by the electrons), we have the evolution equation for $|\delta\phi_0|^2$ or, equivalently, a wave kinetic equation:

$$\left(1 - \frac{k_{\parallel,0}^2 V_A^2}{\omega_0^2}\right)\partial_t|\delta\phi_0|^2 = \left(1 - \frac{k_{\parallel,0}V_A}{\omega_0}\right)\frac{\omega_A\pi}{2\sigma}q^3 k_{\theta,0}\rho_A\beta_E f_r\frac{R_0}{L_{n0E}}|\delta\phi_0|^2 - \left(1 - \frac{k_Z^2}{k_{\perp,0}^2} - \frac{k_{\parallel,0}^2 V_A^2}{\omega_0^2}\right)\frac{2c}{B_0}k_{\theta,0}k_Z|\delta\phi_0|^2\delta\phi_Z \tag{5}$$

Here we have only considered $l = \pm 1$ transit resonances, $\omega_A \equiv V_A/(qR_0)$, $\rho_A \equiv V_A/\Omega_i$, $\beta_E \equiv 8\pi n_{0E}m_i\mathcal{E}_f/B_0^2$, $\sigma \equiv \ln(\mathcal{E}_f/\mathcal{E}_c)\mathcal{E}_f/V_A^2$, f_r is a factor related to the fraction of resonant particles[23]. Equation (5) consists of the usual linear growth from the wave-particle resonance (R.H.S. term $\propto |\delta\phi_0|^2$)[23, 24], and damping from the nonlinear interaction between TAE and ZM (R.H.S. term $\propto |\delta\phi_0|^2\delta\phi_Z$). Therefore, equation (5) can be cast into the form of $\partial_t|\delta\phi_0|^2 = \gamma_L|\delta\phi_0|^2 - \gamma_d|\delta\phi_0|^2\delta\phi_Z$, as equation (10) discussed later.

The evolution equation for ZM is obtained from the zonal components of gyrokinetic vorticity equation as below[22, 18, 19, 1]:

$$\frac{e^2}{T_i}\left\langle(1 - J_Z^2)F_{0,i}\right\rangle_{\mathcal{E}}\partial_t\delta\phi_Z = -\textstyle\sum_s\langle e_s J_Z i\omega_d\delta H_0\rangle_{\mathcal{E},Z} + (\text{RS-MX})_Z \tag{6}$$

On the R.H.S., the first term is the curvature coupling term (CCT), where the bracket denotes an integral in velocity space and then zonal-averaging. Because $\langle(\cdots)\delta H_0^L\rangle_{\mathcal{E},Z} = 0$ with the linear EP response equation (4), so the possible contribution of CCT to ZM only come from the nonlinear response of the distribution $\langle(\cdots)\delta H_0^{NL}\rangle_{\mathcal{E},Z} \neq 0$[18]. However, as a higher order response, the CCT contribution should be small. The second term accounts for Reynolds and Maxwell stresses (RS-MX), which are primarily thermal contributions and can be included following Refs.[25, 16, 18, 19]. Considering $k_\perp\rho_i \ll 1$ and $k_\perp\hat{\rho}_{d,\mathcal{E}} \ll 1$, we have the evolution of zonal potential as:

$$\partial_t\delta\phi_Z = \frac{ck_{\theta,0}\hat{F}}{\hat{\chi}_{iZ}B_0}\left[\frac{n_{0E}\hat{\rho}_{d,f}^2}{n_0\rho_i^2}\hat{G} + \left(1 - \frac{k_{\parallel,0}^2V_A^2}{\omega_0^2}\right)\right]|\delta\phi_0|^2 \tag{7}$$

where $\hat{\chi}_{iZ} \equiv \chi_{iZ}/(k_Z^2\rho_i^2) \simeq 1.6q^2/\sqrt{\varepsilon}$ is the neoclassical polarizability of ZM[26], $\hat{\rho}_{d,f} \equiv qv_f/\Omega_i$. The two parts in the bracket on the R.H.S. of equation (7) come from CCT and RS-MX, respectively. We corrected the results from Ref.[18], and found that CCT is equivalent to a turbulent stress like RS-MX. They share similar structures:

(a) Both are proportional to the TAE amplitude $|\delta\phi_0|^2$ and $\hat{F} \equiv i(k_{r,0} - k_{r,0*})$, indicating that the collision of two TAEs and the asymmetry of mode structure are necessary for ZM generation.
(b) Both exhibit polarization effects, either from gyro-motion or EP drift.
(c) Both require the breaking of ideal MHD condition $1 - (k_{\parallel,0}^2V_A^2/\omega_0^2) \sim \varepsilon \neq 0$ (for CCT from $\hat{G}$).

Because $\hat{G} \propto \varepsilon f_r|\Omega_{*E}/\omega_0|$ and not every portion in EP density is at resonant ($f_r < 1$), we notice that:

$$\frac{\text{CCT}}{\text{RS-MX}} \sim \frac{f_r n_{0E}}{n_0}\frac{\hat{\rho}_{d,f}^2}{\rho_i^2}\left|\frac{\Omega_{*E}}{\omega_0}\right| \sim f_r q^2\frac{P_E}{P_i}\left|\frac{\Omega_{*E}}{\omega_0}\right| \ll 1 \tag{8}$$

for the strongly driven scenario $f_\beta \equiv P_E/P_i \sim 1$, where $P_E = n_{0E}T_E$, $P_i = n_0T_i$, $T_E \equiv m_i\mathcal{E}_f$, $\Omega_{*E} \equiv k_\theta L_{n_{0E}}^{-1}cT_i/eB$. Consequently, RS-MX predominantly drives the generation of ZM. Another issue in retaining CCT [18, 19] as a ZF drive source is the necessity of a corresponding ***sink*** to ensure the energy conservation. However, there is no such "sink" explicitly for CCT in other evolution equations to balance the energy transfer to ZM. Given that the contribution of CCT is secondary and that energy conservation is essential, we omit the CCT in equation (7) and later cast it in the form of equation (11).

Using a quasi-linear approach, we can estimate the effects of TAE on the EP distribution[27, 28]. Then the evolution equation of the EP profile, accounting for external EP sources such as neutral beam heating or ion cyclotron resonance heating, is formally written as below:

$$\partial_t n_{0E} = -D_{\text{res}}\hat{F}L_{n_{0E}}^{-1}n_{0E} + \text{Source} + \cdots \tag{9}$$

where the TAE transport coefficient is $D_{\rm res} \simeq \frac{\pi}{2\sigma}\frac{k_Z^2\hat{\rho}_{d,f}^2 V_A f_r}{k_{\parallel,0}}\left(\frac{\delta B_r}{B_0}\right)^2$.

Now we can simplify the evolution of TAE and ZM, neglecting CCT, and rewriting equations (5) and (7) as equations (10)-(11):

$$\partial_t|\delta\phi_0|^2 = \gamma_1|\delta\phi_0|^2 - \gamma_d|\delta\phi_0|^2\delta\phi_Z \tag{10}$$

$$\hat{\chi}_{iZ}\partial_t\delta\phi_Z = \gamma_2|\delta\phi_0|^2 - \nu\delta\phi_Z \tag{11}$$

where,

$$\gamma_1 \equiv \frac{\pi\omega_A}{2\sigma}q^3 k_{\theta,0}\rho_A\beta_E f_r\frac{R_0}{L_{n0E}}, \quad \gamma_d \equiv \frac{2c}{B_0}k_{\theta,0}k_Z, \quad \gamma_2 \equiv \frac{c}{B_0}k_{\theta,0}\hat{F}\varepsilon$$

$\nu = (\nu_{ii} + \nu_G)$ is the damping of ZM. ν_{ii} is the ion-ion collisional damping[29, 26] and ν_G stands for possible collisionless damping. Here, ν is caused by interactions in thermal plasma, therefore, it will not affect the calculation of EP-driven TAEs. Equations (9)-(11) constitute a system describing the dynamics of EP, TAE and ZM. Note that for $\varepsilon\hat{F}/\hat{\chi}_{iZ} = k_Z$, the RS-MX terms cancel perfectly upon addition of the evolution equation for TAE and ZM amplitude! This reflects energy conservation. This system is similar to the well-known "Predator-Prey" system[17, 29, 15], as shown in Fig. 1. Here we are focusing on the cases where EP profile is sustained by external sources, continuum damping is negligible and wave-particle trapping is unable to saturate the TAE ($\gamma_1/\omega_0 > 10^{-2}$, see equation (4.176) in Ref.[1]). In essence, *this system is not near marginal*. Hence, we further simplify the system by hereafter ignoring the evolution equation (9) for EP. This simplified system manifests a closed feedback loop, featuring a robust EP source, strongly excited TAEs, and ZM generation through RS-MX. Subsequently, the ZM is damped and feeds back to the evolution of TAE through the nonlinear wave-ZM interaction, leading to the saturation in both TAE and ZM. Assuming γ_1, γ_2, γ_d and ν are constants, the saturation levels are straightforwardly obtained as below, showing that the saturation of TAE is controlled by the damping of ZM.

$$(\delta\phi_Z)_S = \gamma_1/\gamma_d \tag{12}$$

$$(|\delta\phi_0|^2)_S = \gamma_1\nu/(\gamma_d\gamma_2) \tag{13}$$

3. COLLISIONLESS ZM DAMPING

When only collisional damping is present and as $\nu = \nu_{ii} \to 0$, the TAE saturation level decreases, accompanied by intense oscillations in both TAE and ZM ($f_{\rm osc} \sim \sqrt{\nu\gamma_1}/\omega_A$ when $\nu \lesssim 4\gamma_1$). This can appear as an intermittent (bursty) signal of TAE prior to saturation, and differs fundamentally from the quasi-periodic chirping caused by profile relaxation[30, 31, 11]. Furthermore, this property emphasizes the crucial role of collisionless ZM damping. Considering the significant oscillation of ZM when $\nu_{ii} \to 0$, which can drive thermal ion and electron oscillations in space, we introduce a simple mechanism for collisionless ZM damping, namely the geodesic acoustic transference[32, 33, 34]. According to Refs.[32, 33, 34], ZF couples with a pressure sideband through the geodesic curvature coupling:

$$\partial_t\langle\tilde{u}^y\rangle = \cdots - \omega_B\langle p_s\sin\vartheta\rangle \tag{14}$$

where $\omega_B \equiv 2L_\perp/R$, $L_\perp = (L_T, L_n)$ is the perpendicular scale of thermal profile, $\langle\tilde{u}^y\rangle$ is ZF, ϑ is the poloidal angle, p_s is the pressure sideband ($m = 1, n = 0$). The evolution of electron pressure sideband can be expressed as[34]:

$$\partial_t\langle p_{e,s}\sin\vartheta\rangle = -\tau_{\rm turb}^{-1}\langle p_{e,s}\sin\vartheta\rangle + \frac{\omega_B}{2}\langle\tilde{u}^y\rangle \tag{15}$$

Here, magnetic flutter effects and ion flow sideband are neglected, defining $\tau_{\rm turb}^{-1} \equiv D_{\rm turb}/L_\perp^2$. A quasilinear turbulent coefficient $D_{\rm turb} \simeq {\rm Re}\sum_{\tilde{\mathbf{k}}} i\frac{c^2}{B_0^2}|\phi_{\tilde{\mathbf{k}}}|^2/(\omega - \tilde{\mathbf{k}}_\perp\cdot\mathbf{v}_E)$ is assumed, with $\tilde{\mathbf{k}}_\perp$ representing the wave number of the thermal turbulence and $\mathbf{v}_E$ denotes $E\times B$ advection. The energy in the pressure sideband couples to thermal particles, mostly via turbulent mixing and dissipation of parallel current[34]. The resonance of the thermal particle turbulence with zonal $E\times B$ flows gives the irreversibility which underpins the turbulent mixing. In the collisionless regime, resistivity should be weak, making turbulent mixing the dominant process.

Dedimensionalizing equations (10), (11) and (15) yields equation (16)-(18), respectively. Geodesic acoustic coupling appears as the third term on the R.H.S. of equation (17).

$$\partial_\tau x = \hat{\gamma}_1 x - \hat{\gamma}_d xy \tag{16}$$

$$\partial_\tau y = \hat{\gamma}_2 x - \hat{\nu}_{ii}y - \hat{\gamma}_{G1}z \tag{17}$$

$$\partial_\tau z = \hat{\gamma}_{G2}y - (\omega_A\tau_{\rm turb})^{-1}z \tag{18}$$

Here $\tau \equiv \omega_A t$, $x \equiv |e\delta\phi_0/T_i|^2$, $y \equiv e\delta\phi_Z/T_i$, $z \equiv (1+\tau_i)\langle \hat{p}_{e,s} \sin\vartheta\rangle$, $\tau_i \equiv T_i/T_e$, $\hat{p}_{e,s}$ is the electron pressure sideband normalized to equilibrium pressure. Coefficients are dedimensionlized:

$$\hat{\gamma}_1 \equiv \frac{\gamma_1}{\omega_A}, \quad \hat{\gamma}_d \equiv \frac{2D_B k_{\theta,0} k_Z}{\omega_A}, \quad \hat{\gamma}_2 = \frac{\hat{\gamma}_d}{2}, \quad \hat{\nu}_{ii} \equiv \frac{\nu_{ii}}{\hat{\chi}_{iZ}\omega_A},$$
$$\hat{\gamma}_{G1} \equiv \frac{C_S^2 q R_0 \omega_B}{ik_Z D_B \hat{\chi}_{iZ}\omega_A L_\perp^2}, \quad \hat{\gamma}_{G2} \equiv (1+\tau_i)\frac{ik_Z D_B}{V_A}\frac{\omega_B}{2}$$

where $D_B \equiv cT_i/(eB_0)$ and $\varepsilon\hat{F}/\hat{\chi}_{iZ} = k_Z$ is applied. It's easy to find the geodesic acoustic oscillation from $\hat{\gamma}_{G1}\hat{\gamma}_{G2} = \omega_{\text{GAM}}^2/(\hat{\chi}_{iZ}\omega_A^2)$, where $\omega_{\text{GAM}}^2 \equiv 2(1+\tau_i)C_S^2/R^2$ is the geodesic acoustic mode (GAM) frequency, $C_S^2 \equiv T_e/M_i$. The GAT can provide extra damping of the ZM, which is obtained from the static sideband $z_{\text{static}} = \hat{\gamma}_{G2}\omega_A\tau_{\text{turb}} y$ and $\hat{\gamma}_{G1} z_{\text{static}}$ as:

$$\hat{\nu}_G = \frac{1}{\hat{\chi}_{iZ}} \frac{\omega_{\text{GAM}}^2}{\omega_A \tau_{\text{turb}}^{-1}} \sim O(10^{-2}) - O(10^{-1}) \tag{19}$$

Note that turbulent mixing is necessary for collisionless ZM damping here. When compared to $\hat{\nu}_{ii} \leq O(10^{-4})$, collisionless damping from GAT will be dominant.

4. SATURATION OF TAE AND ZM

The saturation of TAE. Specifically, using $k_Z^2\rho_i^2 \lesssim k_\perp^2\rho_i^2 \sim \varepsilon^2$ and $k_\parallel = 1/(2qR_0)$, we obtain the saturation level of TAE from equation (13) as:

$$\left(\frac{\delta B_r}{B_0}\right)_S^2 \sim \frac{\pi q^2}{4\sigma\hat{\chi}_{iZ}\varepsilon^2} \frac{\beta_E f_r k_{\theta,0}}{L_{n0E}} \frac{\nu\rho_i^2}{\Omega_i} \tag{20}$$

This is directly proportional to the damping of ZM, akin to the case in the DW-ZF system[29, 15]. Equation (19) demonstrates that turbulence can regulate the saturation of TAE through the ZM damping. In addition, if EP transport is governed by TAE, since $\hat{\nu}_G \propto \tau_{\text{turb}}$, then *as turbulence increases (indicated by the decrease in τ_{turb}), the EP transport coefficient will decrease*. However, as we will discuss latter, the shearing feedback of ZM on turbulence will modify this straightforward conclusion.

Introducing $\tau_Z^{-1} \equiv D_B k_{\theta,0} k_Z$, we notice that when $3\hat{\gamma}_d\hat{\gamma}_2 \sim (\omega_A\tau_{\text{turb}})^{-2}$ or $\tau_Z \sim \tau_{\text{turb}}$, the system will oscillate at the frequency of ω_{GAM} rather than f_{osc}, which is similar to the results in Ref.[20]. This condition indicates that the energy drained by ZM from the TAE equals to the energy dissipated by the pressure sideband. As a result, the energy transfer will be controlled by GAT, leading to oscillations at ω_{GAM}. To generate such oscillations, we take the Bohm coefficient $D_{\text{turb}} = D_B/16$ and $(\rho_i/L_\perp)^2 \sim 0.02$ in the subsequent analyses, ensuring $\tau_Z \sim \tau_{\text{turb}}$ and $1/(\omega_A\tau_{\text{turb}}) \ll 1$. For typical parameters in tokamaks like DIII-D with $q = 1.5$, $\sigma \sim 2.5$, $\beta_E = 1\%$, $f_r = 0.25$, $k_{\theta,0} = nq/a \sim 10$, $L_{n0E} = 0.5a$, we have $\hat{\gamma}_1 \sim 0.12$, $\hat{\gamma}_d \sim 0.1$, $\hat{\gamma}_2 \sim 0.05$, $\hat{\nu}_G \sim 0.1$, $\hat{\gamma}_{G1} \sim -3.9i$, $\hat{\gamma}_{G2} \sim 0.0012i$, $1/(\omega_A\tau_{\text{turb}}) \sim 0.045$. The saturated TAE amplitude is $(\delta B_r/B_0)_S^2 \sim 10^{-7}$. Comparing this to the threshold for spontaneous generation $\rho_i^2/(4\varepsilon(qR_0)^2)$ [16], yields a ratio of $\text{TAE}_{\text{sat/th}} \sim O(10^{-1}) - O(1)$. This suggests that spontaneous generation is possible after the directly driven process saturates. A similar process is found for ITG turbulence simulation[35, 36, 37]. The estimated EP transport coefficient using equation (9) is around $D_{\text{res}} \sim O(10)\text{m}^2/\text{s}$, which exceeds the neoclassical value but is limited, and so does not cause severe EP transport. This also facilitates the maintenance of a sustained EP profile through modulated heating, to align with the assumption of a fixed linear TAE growth rate.

The saturation of ZM. The saturation level of zonal potential can be estimated from equation (12) as:

$$(\delta\phi_Z)_S \sim \frac{\pi}{2\sigma}\frac{T_i}{e}\frac{f_\beta f_r q^2}{k_Z L_{n0E}} \tag{21}$$

Then the ZF shear $\omega_{E\times B} \equiv ck_Z^2\delta\phi_Z/B_0$ is:

$$\omega_{E\times B} = \frac{\pi}{2\sigma}q^2 D_B f_\beta f_r \frac{k_Z}{L_{n_{0E}}} \sim \frac{\pi}{2\sigma}q^2\varepsilon f_\beta f_r \frac{C_S}{L_{n_{0E}}} \tag{22}$$

The radial scale of ZM k_Z is crucial in determining the saturation levels. Factors such as the fraction of EP pressure f_β, the fraction of resonant particles f_r and the EP gradient $L_{n_{0E}}^{-1}$ also influence the strength of ZF[12]. For the typical parameters mentioned previously, the ZF shear reaches around $0.3C_S/a$ in regions where TAEs are excited, a magnitude comparable to the ITG linear growth rate with dilution effect $\gamma_{\rm ITG} \sim 0.1C_S/a$ [38, 39]. Therefore, the suppression of ITG turbulent transport can be anticipated. The threshold condition for shear suppression, $\omega_{E\times B} > \gamma_{\rm ITG}$, translates into a criterion for the TAE linear growth rate $\gamma_{\rm TAE} = \gamma_1/2 > \gamma_{\rm ITG}k_{\theta,0}/k_Z$. This criterion further implies a threshold for the strength of the EP source (e.g., from NBI heating). There may appear to be a conflict concerning strong ZF shearing and necessary turbulence for ZM damping. However, since ZM damping occurs via a pressure sideband, in a strongly (weakly) turbulent state, the pressure sideband weakens (strengthens), resulting in strong (weak) ZM and shearing, and consequently facilitating reduction (enhancement) of turbulence. Moreover, when $\tau_{\rm Z} \sim \tau_{\rm turb}$, the GAM plays a role in dissipating EP energy in this system. In general, zonal shears do affect DW and consequently regulate TAE, a process not explored in this paper. This interaction could result in a dynamical $\hat{\nu}_G$ and thereby influence EP transport. An example of the possible feedback involving both the TAE-ZF and DW-ZF systems is shown in Fig. 1.

The $E \times B$ velocity caused by ZF can be estimated as:

$$V_{E\times B} \sim \frac{\pi}{2\sigma} q^2 f_\beta f_r \frac{C_S \rho_i}{L_{n_{0E}}} \tag{23}$$

The heating power transferred from EP to thermals, estimated from equation (11) as $\nu_G V_{E\times B}^2$ in steady state, can be distributed to both ions and electrons ($\tau_i \sim 1$) or predominately into electrons ($\tau_i \ll 1$). The ZF phase shift (Ω_Z) in equation (4) is smaller than the transit frequency (ω_t) by roughly two orders of magnitude. Thus, Ω_Z affects near marginal transit resonance or trapped particle dynamics[14]. Modulation of wave-particle resonance by ZF could lead mesoscopic structure such as the $E \times B$ staircase[40, 41].

5. CONCLUSIONS

This work introduces a novel Predator-Prey type theory for the collective dynamics of EP, TAE, turbulence and ZM, which highlights the crucial role of collisionless ZM damping and predicts a novel alpha channeling mechanism. This work presents a theoretical explanation for ITB formation driven by EPs through directly driven ZM, and a new explanation of bursty TAE phenomena. These results may contribute to the development of a novel operational scenario in future fusion devices like ITER. We assumed a sustained EP drive to eliminate EP profile relaxation, which is a reasonable strategy for isolating the key new physics – the saturation of AE via collisionless ZM damping. When the system is near-marginal, the interplay between the ZF shift, wave-particle trapping[42, 43, 44] and stochastic scattering[45, 46] is also interesting. This model is relevant to scenarios without significant phase-space transport. Without ZM, our model would lead to an unbounded TAE growth. The most direct correction is to consider the growth rate feedback from profile flattening or by phase-space relaxation in future. In addition, turbulence may also have effects on decorrelating EP phase-space clumps and holes[47]. Our model can serve as a basis for understanding these issues in the future. Finally, this work explores novel cross-scale interactions in fusion plasmas, introducing a compelling mechanism for transferring EP energy to thermal ions via ZM, while facilitating the formation of a TAE-driven ITB to enhance thermal confinement.

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