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VERIFICATION OF ENERGETIC AND ANGULAR DISTRIBUTIONS OF NUCLEAR FUSION PRODUCTS IN PLASMAS

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Abstract

Yields and spectra of neutrons belong to central subjects of physics of controlled fusion neutron sources and prospective energy reactors. Flux densities of primary neutrons from plasma are the key parameters for the design of a fusion device from the viewpoint of practical applications such as tritium breeding, production of fissile nuclides, minor actinide burning, neutron physics research and others, as well as from the viewpoint of the selection of structural and functional materials for the reactor systems and estimations of their operating regimes and durability. Therefore, a proper attention needs to be paid to verification of modelling methods used to estimate fluxes and spectra of primary fusion neutrons from plasmas. New analytical results have been obtained for integral angular distributions and integral energetic distributions of nuclear fusion products in plasmas with general anisotropic distributions of fuel nuclei velocities.

1. INTRODUCTION

In the work [1] calculations of such integral characteristics of a fusion neutron source as the energy resolved counting rate $\frac{dg}{dE_n}$ [MeV^{-1} s^{-1}] of a collimated neutron detector and the number of primary neutrons with a certain energy $\frac{d\Gamma}{dE_n}$ [MeV^{-1} cm^{-2} s^{-1}] per unit area per second at a certain point of the first wall of a reactor were first described on the basis of the work [2].

An important component of the ongoing works on the physics basis of controlled nuclear fusion systems, including nuclear fusion neutron sources, is the development and use of verification methods for all mathematical models involved. Rigorous analytical results concerning the calculated quantities and the underlying mathematical relationships can serve as reliable verification methods.

One of the main procedures for verifying the energetic and angular distributions of nuclear fusion products is the calculation of the normalization of these distributions. Integration of the double differential reaction rate coefficient over the entire range of product particle kinetic energies and over the total solid angle of emission should yield a result that coincides with the total reaction rate coefficient, which can be calculated independently.

This paper describes new analytical results representing an independent formula for the total angular distribution of fusion product particles in plasma, integrated over all kinetic energies, as well as an independent formula for the total energetic distribution of fusion product particles in plasma, integrated over all emission angles. In addition to the four verification methods considered earlier in [2], the new analytical results are essential for verification purposes as they allow independent calculations of integral distributions of fusion products. It should be noted that the obtained formulae can be applied not only for verification purposes in mathematical modelling but also for experimental verification of angular anisotropy and neutron spectra.

The essence of the achieved verification capability is that the total angular distribution of fusion products, calculated using the obtained independent formula, should coincide with the result of numerical integration of the calculated double differential reaction rate coefficient over the entire range of kinetic energies of fusion products. Analogously, the total energetic distribution of fusion products, e.g. the integral neutron spectrum, calculated using the obtained independent formula, should coincide with the result of numerical integration of the calculated double differential reaction rate coefficient over the entire range of emission angles of fusion products.

Binary nuclear fusion reactions between two fuel particles with indices “1” and “2” and the formation of two products with indices “3” and “4” are considered.

2. TOTAL ANGULAR DISTRIBUTION

The derivation of the formula for the total angular distribution of the local source of nuclear fusion products in plasma can be illustrated using Fig. 1. First of all, Fig. 1 shows that according to the cosine law, the following identities hold

$$\mathrm{v}_3^2 = \mathrm{V}^2 + \mathscr{L}^2(\upsilon) - 2\mathrm{V}\mathscr{L}(\upsilon)\cos(\pi - \xi), \tag{1}$$

$$\cos\Theta = \frac{\mathrm{v}_3^2 + \mathrm{V}^2 - \mathscr{L}^2(\upsilon)}{2\mathrm{v}_3\mathrm{V}}, \tag{2}$$

where v_3 denotes the absolute value of the velocity of the fusion product "3", V denotes the absolute value of the velocity of the center of mass of the interacting fuel nuclei "1" and "2", $\mathscr{L}(\upsilon)$ denotes the absolute value of the velocity of the fusion product "3" in the center of mass frame, that is

$$\mathscr{L}(\upsilon) = |\mathbf{u}_3| = |\mathbf{v}_3 - \mathbf{V}|. \tag{3}$$

The fact that $\mathscr{L}(\upsilon)$ with high accuracy is a function of the absolute value of the relative velocity υ of the interacting fuel nuclei "1" and "2", was demonstrated in the work [2]. Namely,

$$\mathscr{L}(\upsilon) = \sqrt{\frac{2m_4}{Mm_3}\left(\frac{\mu\upsilon^2}{2} + q_f\right)}, \tag{4}$$

with m_3 and m_4 being the masses of fusion products "3" and "4", M and μ being, respectively, the summary mass and the reduced mass of the fuel nuclei "1" and "2", and $q_f = (M - m_3 - m_4)c^2$ being the fusion energy.

Plugging (1) into (2) gives

$$\cos\Theta = \frac{\mathrm{V} + \mathscr{L}(\upsilon)\cos\xi}{\sqrt{\mathrm{V}^2 + \mathscr{L}^2(\upsilon) + 2\mathrm{V}\mathscr{L}(\upsilon)\cos\xi}}, \tag{5}$$

$$\sin\Theta = \sqrt{1 - \cos^2\Theta}. \tag{6}$$

In other words, the polar angle in the center of mass frame ξ, the absolute value of the relative velocity υ and the absolute value of the velocity of the center of mass V determine the value of the polar angle Θ of the velocity of the center of mass.

The choice of the positive sign in formula (6) is dictated by the fact that the value Θ needs to be within the range between 0 and π, being the angle between the center of mass velocity vector $\mathbf{V}$ and the velocity vector $\mathbf{v}_3$ of the fusion product with particle type index "3", as shown in Fig. 1. The vector $\mathbf{b}$ corresponds to the local direction of the magnetic field. The angle ϑ_3 represents the polar angle of emission of the fusion product particle "3" in the laboratory frame.

Let us next use the formula for the total reaction rate coefficient between fuel nuclei "1" and "2"

$$\mathscr{R}_{12} = \int \sigma(\upsilon)\upsilon f_1(\mathbf{v}_1)\, f_2(\mathbf{v}_2)\, d^3\mathbf{v}_1 d^3\mathbf{v}_2, \tag{7}$$

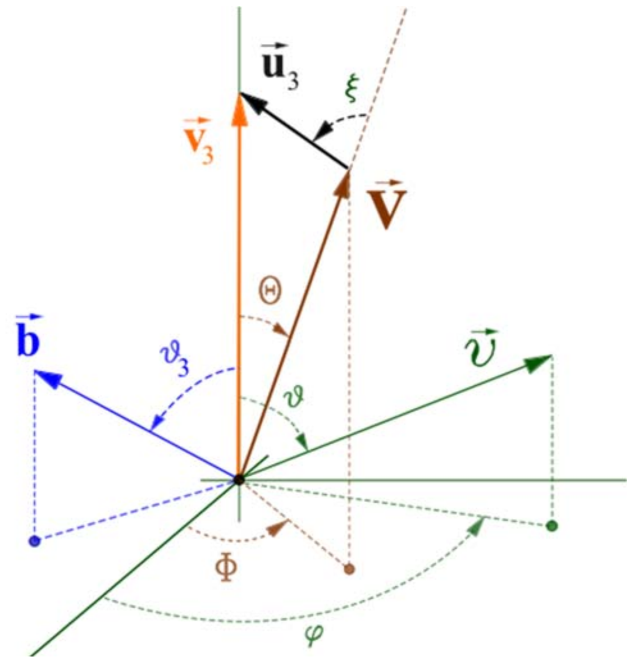


FIG. 1. Kinematics of the nuclear fusion reaction in plasma and illustration of the integration variables in velocity space to derive the formula for the total angular distribution of product particles.

where the value

$$\sigma(\upsilon) = 2\pi \int_0^{\pi} \frac{d\sigma(\upsilon,\xi)}{d\Omega_{C.M.}} \sin\xi d\xi \tag{8}$$

is the total cross section of the fusion reaction, written as the integral of the differential cross section over the angles in spherical polar coordinates in the center of mass frame. Calculations of the differential and total cross sections of the deuterium-deuterium reaction ^{2}H(^{2}H,n)^{3}He, the deuterium-tritium reaction ^{3}H(^{2}H,n)^{4}He and others are described in [3] and [4]. Functions $f_1(\mathbf{v}_1)$ and $f_2(\mathbf{v}_2)$ designate unity normalized distributions of velocities of fuel nuclei "1" and "2", respectively.

Next, let us substitute expression (8) into formula (7) and perform a change of integration variables in formula (7) from the components of the velocity vectors in the laboratory frame $\mathbf{v}_1$ and $\mathbf{v}_2$ to the components of the velocity vector of the center of mass $\mathbf{V}$ and the relative velocity vector υ, and then introduce spherical polar coordinates in velocity space with the polar axis along the velocity vector of the fusion product under consideration, i.e. particle "3", as shown in Fig. 1, obtaining the expression

$$\mathcal{R}_{12} = \int_0^{\infty} \upsilon^3 d\upsilon \int_0^{\pi} 2\pi \frac{d\sigma(\upsilon,\xi)}{d\Omega_{C.M.}} \sin\xi d\xi \int_0^{\pi} \sin\vartheta d\vartheta \int_0^{2\pi} d\varphi \int_0^{\infty} \mathrm{V}^2 d\mathrm{V} \int_0^{\pi} \sin\Theta d\Theta \int_0^{2\pi} d\Phi \;\; f_1(\mathrm{v}_1,\vartheta_1)\, f_2(\mathrm{v}_2,\vartheta_2), \tag{9}$$

where the absolute values of the velocities of the fuel nuclei are expressed via the new integration variables as

$$\mathrm{v}_1 = \sqrt{\mathrm{V}^2 + \frac{\mu^2}{m_1^2}\upsilon^2 + \frac{2\mu}{m_1}\mathrm{V}\upsilon\cos\alpha}\,, \tag{10}$$

$$\mathrm{v}_2 = \sqrt{\mathrm{V}^2 + \frac{\mu^2}{m_2^2}\upsilon^2 - \frac{2\mu}{m_2}\mathrm{V}\upsilon\cos\alpha}\,, \tag{11}$$

with α designating the angle between the vector of the center of mass velocity $\mathbf{V}$ and the vector of the relative velocity υ, i.e.

$$\cos\alpha = \sin\vartheta \sin\Theta \cos(\varphi - \Phi) + \cos\vartheta \cos\Theta . \tag{12}$$

The cosines of the polar angles of velocities of the fuel nuclei are expressed as

$$\cos\vartheta_1 = \frac{\mathrm{V}\cos\Theta_b + \frac{\mu}{m_1}\upsilon\cos\vartheta_b}{\mathrm{v}_1}, \tag{13}$$

$$\cos\vartheta_2 = \frac{\mathrm{V}\cos\Theta_b - \frac{\mu}{m_2}\upsilon\cos\vartheta_b}{\mathrm{v}_2}, \tag{14}$$

where

$$\cos\Theta_b = \sin\vartheta_3 \sin\Theta \cos(\Phi - \varphi_b) + \cos\vartheta_3 \cos\Theta , \tag{15}$$

$$\cos\vartheta_b = \sin\vartheta_3 \sin\vartheta \cos(\varphi - \varphi_b) + \cos\vartheta_3 \cos\vartheta . \tag{16}$$

The additive value φ_b in the cosine argument does not affect the value of the integral, since the result of integrating a periodic function is the same over any interval of the length of a period. The value of the polar angle of emission of the fusion product under consideration, i.e. the particle of type "3", in the laboratory frame of reference ϑ_3 is a prescribed parameter in these calculations.

Integration over the variable ξ in (9), that is, over the polar angle of emission in the center of mass frame, corresponds to integration over the kinetic energy of the fusion reaction product of type "3", since, as can be seen in Fig. 1, there is a relation

$$E_3 = \frac{m_3}{2}\left(\mathrm{V}^2 + \mathscr{L}^2(\upsilon) + 2\mathrm{V}\mathscr{L}(\upsilon)\cos\xi\right). \tag{17}$$

Since, according to formula (5), the value of the angle Θ is determined by the values of the variables ξ, υ, and V, to calculate the angular distribution of the reaction product "3" we must abandon integration over $d\Theta$. The factor of 2π in formula (9) is related to azimuthal symmetry and thus must also be omitted to formally obtain the properly normalized value $\dfrac{d\mathcal{R}_{12}}{d\Omega_3}$.

Finally, we arrive at the expression

$$\frac{d\mathcal{R}_{12}}{d\Omega_3} = \int_0^\infty \upsilon^3 d\upsilon \int_0^\pi \frac{d\sigma(\upsilon,\xi)}{d\Omega_{C.M.}} \sin\xi d\xi \int_0^\infty \mathrm{V}^2 d\mathrm{V} \int_0^\pi \sin\vartheta d\vartheta \int_0^{2\pi} d\varphi \int_0^{2\pi} d\Phi \; f_1(\mathrm{v}_1,\vartheta_1)\, f_2(\mathrm{v}_2,\vartheta_2). \tag{18}$$

This six-fold integral is the overall angular distribution of the local source of nuclear fusion products of type "3" in plasma.

3. TOTAL ENERGETIC DISTRIBUTION

Along with the use of the obtained formula (18) for verification of the total angular distribution considered above, it is also possible to carry out an independent verification of the total, i.e., integral over all emission angles, energy distribution of fusion products.

To obtain the total energy distribution of fusion products “3” $\frac{d\mathcal{R}_{12}}{dE_3}$, let us start again from formula (7) for the reaction rate $\mathcal{R}_{12}$. Let us change the integration variables from the components of the velocity vectors of fuel nuclei “1” and “2” in the laboratory frame $\mathbf{v}_1$ and $\mathbf{v}_2$ to the components of the velocity vector of the center of mass $\mathbf{V}$ and the relative velocity vector υ, and then introduce spherical polar coordinates in velocity space with the polar axis along the direction of the magnetic field as shown in Fig. 2.

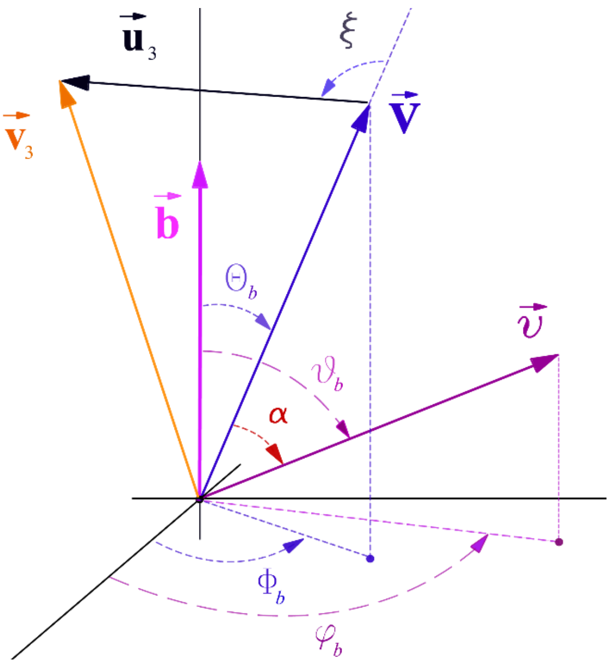


FIG. 2. Calculation of the total energy distribution of fusion products using the geometry of integration in velocity space in the variables of the center of mass frame.

We need to differentiate $\mathcal{R}_{12}$ with respect to the variable E_3. To do so, we apply the chain rule as in [2]

$$\frac{d\sigma}{dE_3} = \frac{d\sigma}{d\cos\xi}\frac{d\cos\xi}{dE_3} \tag{19}$$

and bear in mind that with azimuthal symmetry

$$\frac{d\sigma}{d\cos\xi} = 2\pi\frac{d\sigma}{d\Omega_{C.M.}}. \tag{20}$$

Taking into account the relation (17), as explained in [2], we have

$$\cos\xi = \frac{\mathrm{v}_3^2 - \mathcal{L}^2(\upsilon) - \mathrm{V}^2}{2\mathcal{L}(\upsilon)\mathrm{V}}, \tag{21}$$

$$\frac{d\cos\xi}{dE_3} = \begin{cases} \dfrac{1}{m_3 \mathscr{L}(\upsilon)\mathrm{V}}, & |\cos\xi| \le 1 \\ 0, & \text{otherwise} \end{cases}. \tag{22}$$

For the absolute values of the velocities of the fuel nuclei “1” and “2” and for the cosines of the polar angles of these velocities expressions (10), (11) and (13), (14) hold, however, Θ_b and ϑ_b are now the independent integration variables themselves, so formulae (15) and (16) are not needed in this case. For the cosine of the angle α between the vector of the center of mass velocity $\mathbf{V}$ and the vector of the relative velocity υ, instead of formula (12), in accordance with Fig. 2 we now have

$$\cos\alpha = \sin\vartheta_b \sin\Theta_b \cos(\varphi_b - \Phi_b) + \cos\vartheta_b \cos\Theta_b. \tag{23}$$

It is possible to replace $\cos(\varphi_b - \Phi_b)$ by $\cos\varphi_b$ because the integral of a periodic function is the same over any interval of the length of a period, i.e. integration over Φ_b is trivial and results in a multiplier of 2π.

We thus arrive at the expression

$$\frac{d\mathscr{R}_{12}}{dE_3} = \frac{4\pi^2}{m_3}\int_0^\infty \mathrm{V}d\mathrm{V}\int_0^\infty \frac{\upsilon^3 J(\mathrm{V},\upsilon)}{\mathscr{L}(\upsilon)}\frac{d\sigma(\upsilon,\xi)}{d\Omega_{C.M.}}d\upsilon\int_0^\pi \sin\Theta_b d\Theta_b \int_0^\pi \sin\vartheta_b d\vartheta_b \int_0^{2\pi} f_1(\mathrm{v}_1,\vartheta_1)\, f_2(\mathrm{v}_2,\vartheta_2)\, d\varphi_b, \tag{24}$$

where the cosine of the polar angle ξ in the center of mass frame, as depicted in Fig. 2, is determined by (21), and the dimensionless factor

$$J(\mathrm{V},\upsilon) = \begin{cases} 1, & |\cos\xi| \le 1 \\ 0, & \text{otherwise} \end{cases} \tag{25}$$

in the integrand is and indicator necessary to take into account only the contribution of regions in velocity space permitted by the laws of conservation of energy and momentum.

The five-fold integral (24) is the overall energetic distribution of the local source of nuclear fusion products of type “3” in plasma.

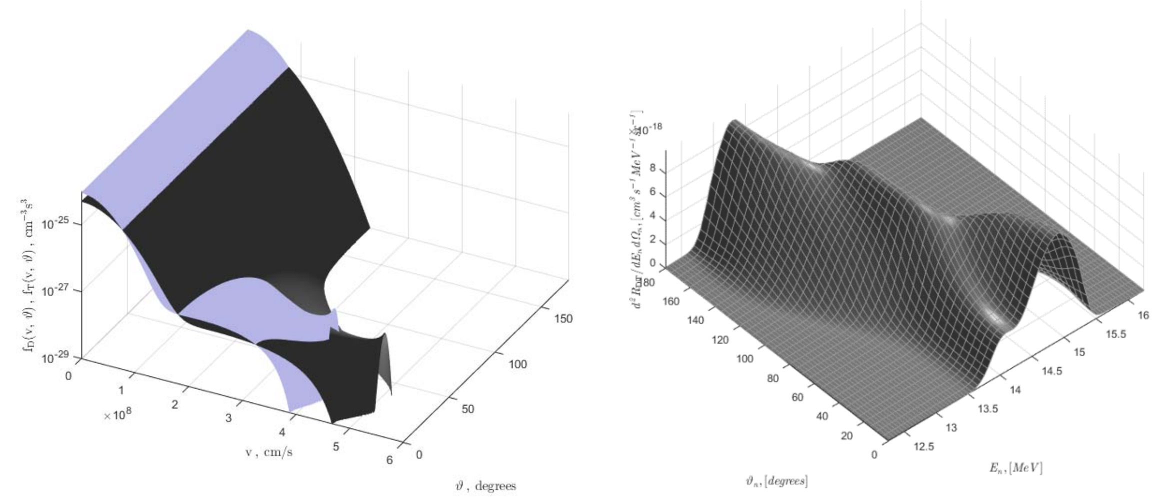
4. RESULTS OF CALCULATIONS

Fig. 3 a) shows the anisotropic velocity distribution functions of deuterons (dark surface) and tritons (light surface) in plasma, calculated for the case of injection of fast deuterium and tritium beams with the energy of 275 keV into a deuterium-tritium Maxwellian background plasma with the temperature of 5 keV and the electron density of 10^{14} cm^{-3}. These distributions were obtained using a simplified model that takes into account slowing down and angular scattering, but neglects the speed diffusion.

Fig. 3 b) shows the double differential ^{3}H(^{2}H,n)^{4}He reaction rate coefficient $\frac{d^2\mathscr{R}_{12}}{dE_3 d\Omega_3}$ obtained using the S-formula [2]. Alternatively, the same result can be obtained using the L-formula [2].

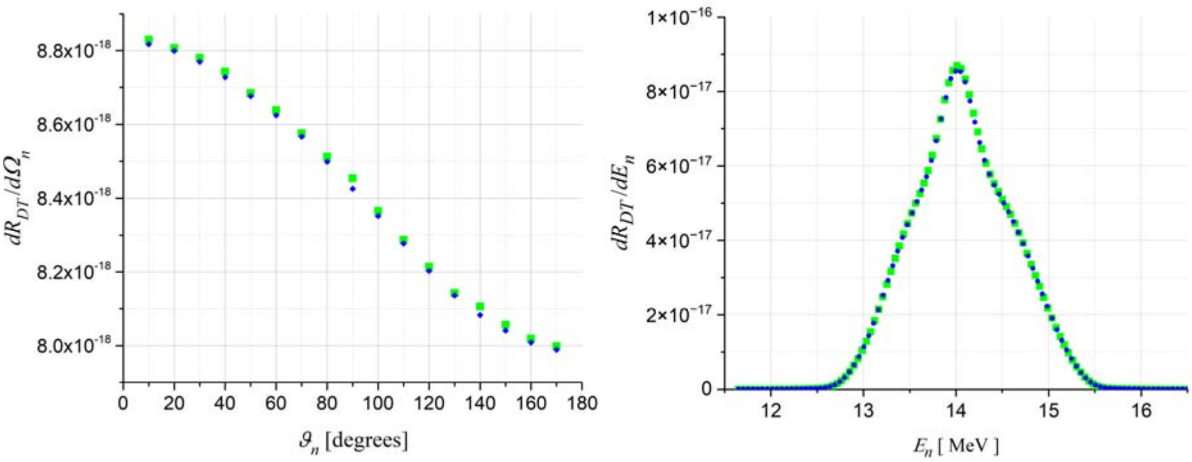
An example of verification of the full angular distribution $\frac{d\mathscr{R}_{12}}{d\Omega_3}$ of the local source of neutrons produced in the ^{3}H(^{2}H,n)^{4}He reactions in plasma with the specified model parameters is illustrated in Fig. 4 a). The green colour shows the total angular distribution calculated by numerical integration of the double differential rate coefficient shown in Fig. 3 b) over the entire range of kinetic energies. The blue colour shows the total angular distribution calculated independently using the obtained formula (18).

An example of verification of the full energetic distribution $\frac{d\mathcal{R}_{12}}{dE_3}$ of the local source of neutrons produced in the ^{3}H(^{2}H,n)^{4}He reactions in plasma with the specified model parameters is illustrated in Fig. 4 b). The green colour shows the total energetic distribution calculated by numerical integration of the double differential rate coefficient shown in Fig. 3 b) over the entire range of emission angles. The blue colour shows the total energetic distribution calculated independently using the obtained formula (24).



a) b)

FIG. 3. a) velocity distributions of deuterons (dark) and tritons (light) used to calculate distributions of nuclear fusion products; b) double differential rate coefficient of the ^{3}H(^{2}H,n)^{4}He reaction.



a) b)

FIG. 4. a) verification of the integral angular distribution of neutrons $\frac{d\mathcal{R}_{12}}{d\Omega_3}$; b) verification of the integral energy spectrum of neutrons $\frac{d\mathcal{R}_{12}}{dE_3}$.

5. CONCLUSIONS

Independent explicit general analytical formulae have been obtained for the integral angular distribution as well as for the integral energetic distribution of nuclear fusion products in plasma. Verifications of the double differential nuclear fusion reaction rate coefficients have been performed.

In fusion neutron sources based on tokamaks the use of fast neutral beam injection is being considered as the main method for the production of the population of high-energy particles enabling non-inductive generation of the electric current in the plasma and making the predominant contribution to the production of primary neutrons. In the recent bibliography pertaining to the neutron emission of toroidal devices with magnetic plasma confinement, various experimental and theoretical works are described, being carried out on prospective and operating tokamaks such as [5,6] as well as on stellarator/heliotron devices, e.g. [7].

The obtained general results primarily described for the case of neutral beam heated tokamak plasmas are readily applicable for the modelling of fusion products in other configurations including stellarator/heliotron plasmas and inertially confined plasmas.

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