

APPLICATION AND ANALYSIS OF THE REVISED ACCURATE WEIGHT METHOD FOR FUSION FACILITIES

Do Hyun Kim, Hyoseong Gwon, and Mu-Young Ahn
Korea Institute of Fusion Energy (KFE)
Daejeon, Republic of Korea
Email: dhkim@kfe.re.kr

Abstract

Monte Carlo (MC) methods are widely used for nuclear analysis in fusion facilities. However, their computational cost is high. Hybrid MC methods such as CADIS and FW-CADIS, which are implemented in ADVANTG, address this issue by generating variance-reduction parameters deterministically and then running MC simulations. In our previous work, the Revising Accurate Weight (RAW) method was proposed, which corrects weight windows using relative errors (REs) obtained from MC simulations for problems involving multiple tallies. In this study, the RAW method was applied to a fusion experimental facility at the conceptual design stage and compared with FW-CADIS. In the IBTF test cell model, the average figure of merit increased by about 3.5 times, while the maximum figure of merit decreased by 65 percent. REs at the outer boundary, which were high under the FW-CADIS method, were reduced when the RAW method was applied. On the other hand, REs in near pipes increased. To investigate this behavior, a penetration model was analyzed. The model consisted of a concrete block with a square-shaped penetration modeled as a void, with a point source placed at the center. The RAW method showed a more uniform RE distribution in regions without penetration. However, the RE increased at the bends and at the end of the penetration. This was attributed to weight increases near the bends, which reduced the number of MC particles, and to larger variations in particle weights reaching the terminal region. These results indicate that in geometries with gaps or penetrations, it is necessary to consider not only the high-RE regions but also the neighboring regions that influence them.

1. INTRODUCTION

Monte Carlo (MC) methods are widely used for nuclear analysis in fusion-related facilities such as the ITER Test Blanket Module (TBM) [1]. However, it is well known that MC simulations are computationally expensive. Various variance reduction (VR) techniques [2] have been developed to enhance computational efficiency. One of the most widely used approaches in recent years is the hybrid MC method, which employs deterministic methods to generate variance reduction parameters, followed by MC simulations for accurate results. Among these methods, the Consistent Adjoint Driven Importance Sampling (CADIS) [3] and Forward Weighted CADIS (FW-CADIS) [4] have been successfully applied in nuclear analyses. The CADIS method is designed to accelerate calculations for a single tally, such as a detector. In contrast, FW-CADIS is applied to multiple tallies, such as dose maps. These methods are implemented in the ADVANTG [5] code. In our previous study, the Revising Accurate Weight (RAW) [6] method was proposed to refine variance reduction parameters using relative errors (RE) for problems involving multiple tallies. In this study, the RAW method was applied to a fusion experimental facility in the conceptual design phase to identify potential issues. After identifying these issues, representative model was constructed to reproduce the problems, and the underlying causes were analysed.

2. BACKGROUND

2.1. Hybrid MC Method

2.1.1. Single Response

The term Single Response refers to an analysis conducted to obtain a single physical quantity, such as evaluating the response of one detector. In earlier approaches, methods were sometimes used to maintain a roughly equal number of particles along the paths leading to the point of interest. However, with the development of the CADIS method, a significant improvement in computational speed was achieved, and the method has since been widely adopted. The weights in the CADIS method are determined using the following equation:

$$w(\vec{r}, E, \hat{\Omega}) = \frac{R}{\psi^+(\vec{r}, E, \hat{\Omega})} \quad (1)$$

where ψ^+ represents the adjoint flux, calculated using a single response as the adjoint source, and R denotes the response. Since calculating Eq. (1) using the MC method requires a long computational time, it is instead solved by a deterministic method. The resulting weights are then applied in the MC simulation. This approach is referred to as a hybrid MC method, as it combines both deterministic and MC techniques.

2.1.2. Global Problem

A global problem refers to obtaining results for the entire system, such as calculating a dose map. To obtain reliable results in this problem, the RE across the entire system needs to remain uniformly low. Cooper and Larsen [7] suggested that a uniform distribution of MC particles can lead to uniformly low statistical uncertainties. The density of MC particle can be expressed by the following relationship:

$$m(\vec{r}) \approx n(\vec{r})/\bar{w}(\vec{r}) \quad (2)$$

where $n(\vec{r})$ and $\bar{w}(\vec{r})$ are the density of analog particles and average weight at position $\vec{r}$, respectively. From uniform distribution of MC particle, $m(\vec{r})$ becomes constant. Accordingly, the average weight can be expressed as follows:

$$\bar{w}(\vec{r}) \propto n(\vec{r}) \text{ or } \phi(\vec{r}) \quad (3)$$

where $\phi(\vec{r}) = n(\vec{r}) \times v$ and $\phi(\vec{r})$ represents the forward scalar flux at position $\vec{r}$. With the position of the highest flux taken as the reference value, the weight proposed by Cooper and Larsen can be expressed by the following equation:

$$\bar{w}(\vec{r}) = \phi(\vec{r})/\max(\phi(\vec{r})). \quad (4)$$

The FW-CADIS method, recognized as one of the most efficient approaches for global problems, defines the adjoint source as follows, based on the concept introduced by Cooper and Larsen:

$$q^+(\vec{r}, E) = \sigma_d(\vec{r}, E)/R(\vec{r}) \quad (5)$$

where R is a response at position $\vec{r}$ and $\sigma_d(\vec{r}, E)$ is the objective function. The weight values in the FW-CADIS method [4] are obtained by substituting the adjoint fluxes from the adjoint sources in Eq. (5) into Eq. (1).

2.2. RAW Method

2.2.1. Theory

For global problems, Van Wijk et al. [8] proposed a method that uses the RE instead of the flux, as follows:

$$\bar{w}(\vec{r}) = \text{Min}(RE)/RE(\vec{r}) \quad (6)$$

where $\text{Min}(RE)$ denotes the minimum RE in the mesh space, and $RE(\vec{r})$ denotes the RE in the mesh cell at position $\vec{r}$, respectively. In hybrid MC simulations, high RE values can arise due to the methodologies or assumptions adopted in deterministic calculations. Therefore, the weight values can be corrected using the RE obtained from MC simulations. In analog MC simulations, the RE is inversely proportional to the square root of the analog particle density or flux [9], and can be expressed by the following relationship:

$$RE(\vec{r}) \propto 1/\sqrt{n(\vec{r})} \text{ or } 1/\sqrt{\phi(\vec{r})}. \quad (7)$$

By substituting Eq. (7) into Eq. (3) and rearranging, the following relationship is obtained:

$$\bar{w}(\vec{r}) \propto 1/RE(\vec{r})^2. \quad (8)$$

With the position of the minimum RE taken as the reference, the weight values of the hybrid MC method can be revised using the RE from the MC simulation, as given by the following equation:

$$\bar{w}_{\text{raw}}(\vec{r}, E) = \bar{w}_{\text{hy}}(\vec{r}, E) \times \text{Min}[RE]^2/RE^2(\vec{r}) \quad (9)$$

where $\bar{w}_{\text{raw}}$ and $\bar{w}_{\text{hy}}$ represent the mean weights of the RAW method and the hybrid MC method, respectively.

2.2.2. Procedure for RAW Method

Fig.1 illustrates the procedure for applying the RAW method, which is broadly divided into two steps. The first step involves using the conventional hybrid method: if an MC model is available, the ADVANTG code is employed to generate VR parameters such as the weight window, and the MC simulation is then performed. The second step involves correcting the weight window using Eq. (9) with the RE values obtained from the same mesh as the weight window, after which the MC simulation is performed again. All MC simulation were performed by MCNP6 [10] code.

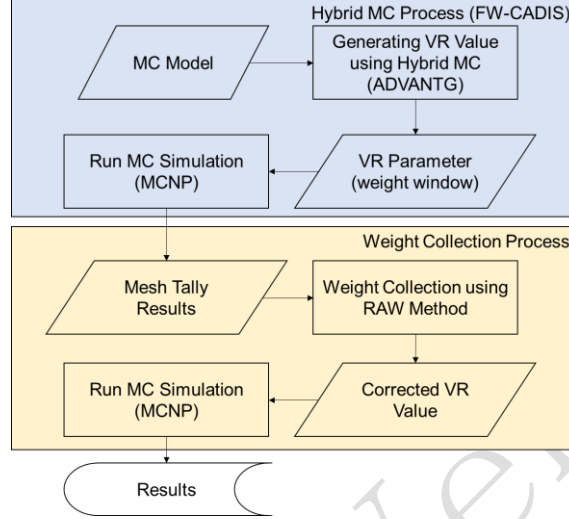


FIG. 1. Schematic of process for RAW method.

3. APPLICATION AND CONPARISON

3.1. Estimation of Efficiency

The Figure of Merit (FOM) is an indicator of computational efficiency. For a single response, it is calculated using the following equation:

$$FOM = \frac{1}{RE \times T}, \quad (10)$$

where T is the MC calculation time. Since global problems involve multiple responses, this equation cannot be applied directly. Therefore, the average FOM [4] and the maximum FOM [11] have been defined as follows and were used in this study.

$$FOM_{ave} = \frac{1}{\frac{1}{N} \sum_{i=1}^N RE_i \times T}, \quad (16)$$

and

$$FOM_{max} = \frac{1}{Max[RE_i] \times T}, \quad (17)$$

Where N indicate the number of response, and RE_i is the RE for the i^{th} response.

3.2. Fusion facility

The model used for application and comparison is the test cell model of the Integrated Breeding Test Facility (IBTF) [12], which is currently at the pre-conceptual design stage, as shown in Fig. 2. At the center is the tritium breeding unit, which consists of Li_2TiO_3 pebbles, Be_{12}Ti beryllide, and structural materials, and is surrounded by a carbon reflector. For radiation shielding, the outer region is enclosed by concrete. On the right-hand side, incident deuterons strike the Be target located in front of the breeding unit, producing neutrons. Behind the breeding unit, water pipes for target cooling as well as pipes for breeding unit cooling and tritium extraction are located. Except for the water pipes, the other pipes were assumed to be filled with void, as they are used for gas.

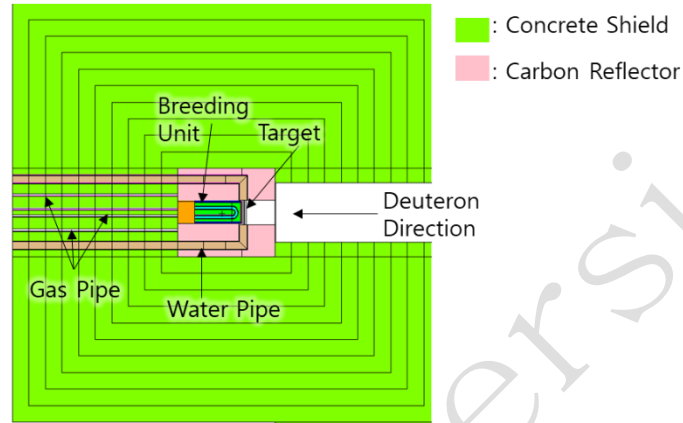


FIG. 2. Nuclear analysis model for application and comparison

Table 1 summarizes the computational efficiency. For the RAW method, the total MC time includes both the time required to obtain RE values and the time required to produce the final results. While the value of FOM_{ave} increased by approximately 3.5 times, the value of FOM_{max} decreased by 65% Fig. 3 shows the RE maps obtained using the RAW method and the FW-CADIS method. As indicated by the white rectangles, the REs at the outer boundary calculated using the FW-CADIS method were reduced when the RAW method was applied. However, in the pipe regions where REs were already high in the FW-CADIS method, the RAW method resulted in even higher RE.

Table 1 Results from RAW and FW-CADIS method

Method	Total MC time [Min]	FOM_{ave}	FOM_{max}
RAW Method	32634	0.565	4.48×10^{-5}
FW-CADIS	29985	0.163	6.90×10^{-5}

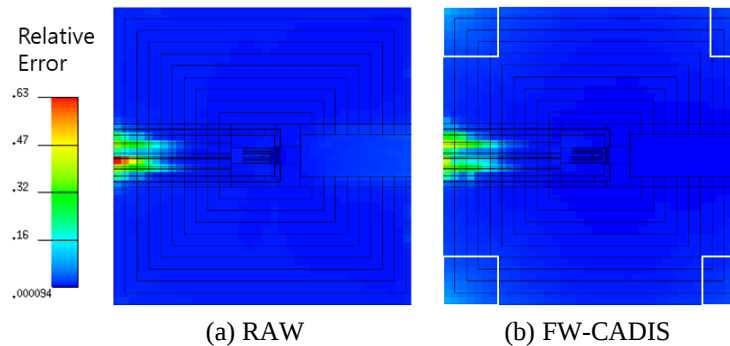


FIG. 3. RE map for test cell of IBTF

3.3. Analysis for RE near penetration area

In the results of Section 3.2, it was observed that the RAW method yielded higher REs for models containing void regions such as pipes. To investigate the underlying cause of this behavior, the model shown in Fig. 4 was selected. The geometry consists of a concrete block with dimensions of $250 \times 250 \times 250 \text{ cm}^3$, containing a square-shaped penetration of $10 \times 10 \text{ cm}^2$ modeled as a void. A 1 MeV neutron source is placed at the center of the geometry.

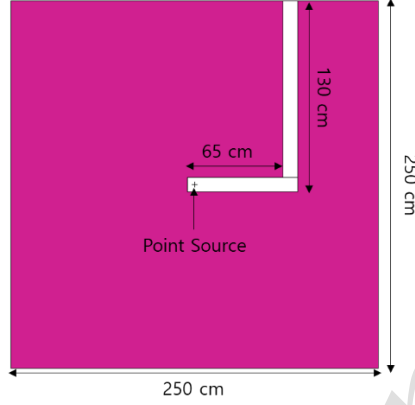


FIG. 4. Calculation model for penetration area analysis

Figure 5 shows the RE maps obtained using the FW-CADIS and RAW methods after 1237 minutes of computation time. In this calculation, all REs were constrained to within 5% to ensure reliable results. It was confirmed that the RAW method provides a more uniform RE distribution in regions without penetration. However, at the end of the penetration and at the bending region, the RE increases. At the bending location, where particles can reach relatively easily, the RE is originally low. Nevertheless, since the RAW method increases the weight in such regions, the number of particles decreases, leading to a slight increase in RE. At the end of the penetration, the elevated RE is attributed to the larger weight assigned at the bending part compared to FW-CADIS, which results in greater variation in particle weights arriving at the final penetration region. Therefore, to further reduce RE in geometries with gaps or penetrations, it is necessary to consider not only the high RE locations but also the regions that influence those locations.

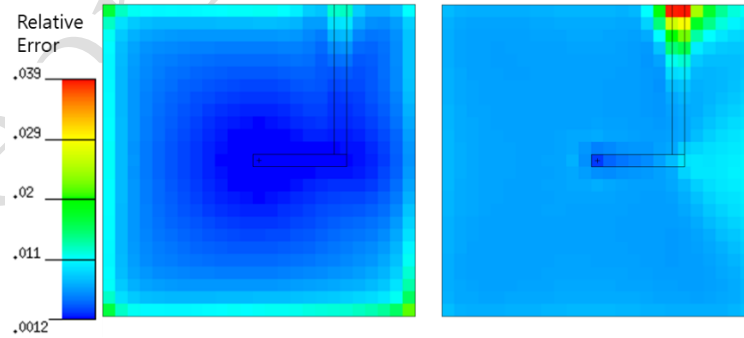


FIG. 5. RE map for penetration area analysis model

4. CONCLUSION

In this study, the RAW method, which adjusts variance reduction parameters using REs from Monte Carlo simulations, was applied to a conceptual fusion facility model and compared with FW-CADIS. In the IBTF test cell model, the RAW method reduced the REs at the outer boundary, which were pronounced under FW-CADIS. The average FOM improved by about a factor of 3.5, while the maximum FOM decreased by 65 percent. To investigate the cause of this behavior, a penetration model was analyzed. The RAW method provided more

uniform REs in regions without penetration. However, due to the redistribution of MC particle weights and the resulting variation in particle transport, REs increased at the bends and at the end of the penetration. These results suggest that in geometries containing gaps or penetrations, attention must be paid not only to the locations with high REs but also to the regions associated with them. In future research, methods will be developed to reduce the REs in regions associated with locations where eREs are large.

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