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FAST ION TRANSPORT INDUCED BY EDGE LOCALIZED MODES

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Abstract

Fast ions exhibit a notable acceleration during edge localized modes (ELMs) in tokamak devices. This paper presents an analytical investigation into the phase-space transport of fast ions driven by ELMs. Contrary to previous simulation results, it is shown that ELMs with low-frequency characteristics are inefficient at accelerating fast ions. Instead, the transport of fast ions is dominated by radial particle transport, resulting from the exchange of canonical toroidal angular momentum. The associated diffusivity increases sharply for high-energy particles, making fast-ion loss measurements in velocity space appear as an acceleration process. These theoretical findings are consistent with recent experimental observations and carry practical implications for the performance of future tokamak reactors.

1. INTRODUCTION

The high confinement mode (H-mode) [1] is the designated operational scenario for next-generation tokamak fusion devices such as ITER [2]. A key feature of H-mode is the formation of a narrow transport barrier at the plasma edge, which creates a pressure pedestal. Due to the steep pressure gradient in this pedestal region, H-mode is typically accompanied by burst-like edge perturbations known as edge localized modes (ELMs) [1, 3, 4]. These recurring bursts can rapidly expel plasma-stored energy toward plasma-facing components, causing undesirable material erosion and surface melting [5]. Understanding the interaction between ELMs and charged particles, as well as related transport processes, is therefore crucial for optimizing fusion performance. On the other hand, confining fast ions is a critical challenge for self-sustaining tokamak reactors, as ignition relies on self-heating by fusion-produced alpha particles. While the role of Alfvénic fluctuations in fast-ion transport has been extensively studied [6], ELM fluctuations, from a physical point of view, may also significantly impact fast-ion transport. However, the key features and underlying physical mechanisms of ELM-induced fast-ion transport remain unclear.

Recent experiments on the ASDEX-Upgrade (AUG) tokamak have reported that fast ions exhibit unusual apparent acceleration during type-I ELMs [7, 8]. Direct velocity-space measurements using fast-ion loss detectors (FILD) identified a population of ions with energies (~160 keV) well above the main energy of neutral beam-injected (NBI) deuterium ions ($E_0 = 82$ keV) during ELM crashes. This phenomenon is reproducible and strongly correlated with NBI heating and ELM occurrence. It occurs on a timescale of 10–100 μs during a single ELM crash and disappears when ELMs are suppressed. The high-energy tail is observed for both circulating and trapped fast ions, though the FILD signal for trapped particles is significantly weaker than for circulating ones. Additionally, the high-energy component of circulating particles shows fine pitch-angle structures (so-called "spikes" in FILD signals) that depend on the safety factor q_{95} (the safety factor at 95% of the plasma minor radius). In contrast, the high-energy component of trapped particles exhibits only a single spike, independent of q_{95}. Based on simulation results, it was hypothesized that parallel electric fields might account for the observed fast-ion energy gain, assuming magnetic reconnection occurs during ELMs [7, 8]. However, this mechanism is ineffective for accelerating fast ions for three reasons: (i) it relies on large-amplitude parallel electric fields, whose strength in previous simulations depends on artificial hyper-resistivity [9]; (ii) it accelerates charged particles only in the parallel direction, whereas experimental observations show little change in fast-ion pitch angles; (iii) any parallel electric field would be highly localized near a thin current sheet, with a width much smaller than the fast-ion gyroradius [10], and such small-scale fluctuations would be averaged out by the finite Larmor radius (FLR) effect.

In this study, we conduct a gyrokinetic analysis of ELM-induced phase-space transport of fast ions. We demonstrate that ELM fluctuations primarily drive radial transport of fast ions via magnetic perturbations, and the high-energy fast-ion component observed in experiments cannot be attributed to ELMs. Unlike fast-ion transport caused by microturbulence [11], ELM-induced diffusivity increases sharply with fast-ion energy. As a result, higher-energy fast ions are expelled outward more rapidly by ELMs, creating a high-energy tail in FILD signals that mimics an acceleration process. Both quantitative estimates of cross-field diffusion time and qualitative transport features are consistent with experimental observations.

The remainder of the paper is structured as follows: Section 2 presents the theoretical model. Section 3 discusses fast-ion phase-space dynamics, and Section 4 outlines the quasilinear theory. Finally, Section 5 summarizes the conclusions.

2. THEORETICAL MODEL

We use gyrokinetic theory [12] to analyze fast-ion dynamics during type-I ELM cycles [4]. Following [13], the gyrocenter motion of a charged particle with mass m and charge e in a background magnetic field B is described by the Hamiltonian:

$$H(X, \mu, w, t) = \mu B + \frac{w^2}{2} + \frac{e}{m}\left(< \delta\phi > - \frac{w}{c} < \delta A_\parallel > + \frac{v_\perp \rho}{c} < \delta B_\parallel >_*\right) \quad (1)$$

Here, X, μ, and w denote the gyrocenter position, gyrocenter magnetic moment, and parallel velocity, respectively. $v_\perp = \sqrt{2\mu B}$is the perpendicular velocity. The terms $< (\cdots) \geq J_0(\cdots)$ and $< (\cdots) >_* = J_1/(i\rho\nabla_\perp)(\cdots)$ represent gyroaverages of field variables, where J_n is a Bessel function with argument $i\rho\nabla_\perp$, accounting for FLR effects. Notably, H is an adiabatic invariant in the presence of short-wavelength perturbations [12, 14, 15], so gyrokinetic theory can describe phase-space transport of fast ions induced by low-frequency, short-wavelength fluctuations [6]. Fast-ion acceleration due to μ breaking by low-frequency, short-wavelength electric fields [16] only occurs if the perturbed electric potential energy is comparable to the fast-ion kinetic energy [15].

Consistent with gyrokinetic orderings [12], we describe electromagnetic fluctuations using three scalar field variables in the Coulomb gauge: the scalar potential perturbation δφ, the parallel vector potential fluctuation $\delta A_\parallel$, and the parallel magnetic field perturbation $\delta B_\parallel$. In some cases, it is more convenient to use the scalar induced potential δψ [17], defined by $-c\nabla_\parallel \delta\psi = \partial_t \delta A_\parallel$.

The gyrocenter motion in phase space can be written as:

$$\dot{X} \equiv \dot{X_0} + \delta\dot{X} \equiv \left(\frac{B}{\Omega B_\parallel^*} \times \nabla + \frac{B^*}{B_\parallel^*}\frac{\partial}{\partial w}\right)(H_0 + \delta H) \quad (2)$$

where Ω is the cyclotron frequency, $\delta H = (e/m)[< \delta\phi > -(w/c) < \delta A_\parallel > + \cdots]$ is the perturbed Hamiltonian, and B^* is the modified magnetic field with a unit vector in the parallel direction. For phase-space transport in axisymmetric toroidal systems, we consider a tokamak with concentric, circular magnetic surfaces and use standard coordinates: minor radius (r), poloidal angle (θ), and toroidal angle (ζ). The unperturbed motion of charged particles in equilibrium is described by three pairs of action-angle coordinates: (ξ, μ) with ξ as the gyrophase; (ζ, P_ζ) with P_ζ as the canonical toroidal angular momentum [13]; and (t, E) with E as energy.

To clarify the fundamental physics of ELM-induced fast-ion transport, we focus on perturbation spatiotemporal scales relevant to the aforementioned experiments, avoiding broader discussions of dominant instabilities in ELMs (an open issue [4]). Specifically, in typical AUG discharges, type-I ELM-related edge fluctuations can be divided into two components [7, 18, 19, 8, 20, 21]. During ELM crashes, fluctuations span a broad frequency range, but low-frequency components dominate, with low mode numbers (n ~ 5) and perpendicular magnetic perturbations $\delta B_\perp/B \sim 10^{-3}$. Between ELM cycles, inter-ELM modes emerge in the high-frequency range, with n ~ 20–30 and $\delta B_\perp/B \sim 10^{-4}$.

Noting that ELM-related divertor current signals show no significant change before and after NBI injection [8], ELM perturbations in AUG are primarily driven by thermal particles and can be described by ideal magnetohydrodynamics (MHD) to the lowest order [18, 19]. Thus, the parallel electric field is negligible, leading to $\delta\phi \approx 0$ as a first approximation. Meanwhile, in the low-β limit, perpendicular pressure balance [17, 6] gives the compressional component of magnetic field fluctuations as:

$$\frac{\delta B_\parallel}{B} \simeq \frac{4\pi e n_i}{B^2}\frac{k_\theta c T_i}{eBr_p}\frac{\delta\phi}{\omega} \quad (3)$$

Here, e is the particle charge, c is the speed of light, $n_i$and T_i are the equilibrium ion density and temperature, $r_p^{-1} = |\partial_r \ln P|$ with P as the equilibrium pressure, and $k_\theta$is the poloidal wavenumber. Relative fluctuation levels can be estimated using the ordering parameter $\epsilon_B = \delta B_\perp/B$, measured directly in AUG [8, 21]:

$$\frac{e\delta\psi}{T_i} \sim \frac{2\omega\epsilon_B}{k_\perp\rho_i k_\parallel v_i}, \quad \frac{\delta B_\parallel}{B} \sim \frac{\beta\epsilon_B}{2r_p k_\parallel} \quad (4)$$

where $k_\parallel$ is the parallel wavenumber, and v_i and ρ_i are the velocity and gyroradius of thermal ions, respectively. For typical AUG edge parameters [7, 8] (B ≈ 2.5 T, $I_P \approx 0.8$ MA, $n_i \approx 5 \times 10^{19}$ m^{-3}, $T_i \approx 0.5$ keV, $r_p = 0.05$ m), Eq. (4) shows $|\delta B_\parallel| \ll |\delta B_\perp|$. Substituting these results into the fast-ion Hamiltonian (Eq. 1) gives:

$$\frac{w\delta A_\parallel}{c\delta\phi} \sim \frac{k_\parallel w}{\omega} \gg 1, \quad \frac{v_\perp\rho\delta B_\parallel}{w\delta A_\parallel} \sim \frac{v_\perp k_\perp \rho}{w}\frac{\delta B_\parallel}{\delta B_\perp} \ll 1 \quad (5)$$

Thus, the perpendicular magnetic perturbation term dominates the perturbed Hamiltonian of fast ions.

From Hamiltonian theory, particle acceleration is governed by:

$$\dot{H} = \partial_t \delta H \tag{6}$$

This gives an order-of-magnitude estimate for the acceleration time: $\Delta t \sim \Delta H/(\omega\delta H)$. Given that the observed energy gain is comparable to the unperturbed energy ($\Delta H \sim H_0 = v_0^2/2$ [7, 8]) and the perturbed Hamiltonian is dominated by perpendicular magnetic fluctuations ($\delta H \approx -ew < \delta A_\parallel >/(mc) \sim -(\Omega w/k_\perp) < \delta B_\perp >/B$), we find:

$$\Delta t \sim \frac{H_0}{\omega\delta H} \sim \frac{k_\perp \rho_0}{\omega}\frac{B}{<\delta B_\perp>} \tag{7}$$

Equations (3)–(7) are consistent with standard nonlinear gyrokinetic orderings [12] and apply to general low-frequency (compared to the ion cyclotron frequency) electromagnetic fluctuations. Using parameters for low-n modes ($|\delta B_\perp/B| \sim \mathcal{O}(10^{-3} - 10^{-2}), k_\perp\rho_0 \lesssim 1$ and $\omega_{low} \sim \mathcal{O}(10)kHz$) and inter-ELM modes ($|\delta B_\perp/B| \sim \mathcal{O}(10^{-5} - 10^{-4}), k_\perp\rho_0 \sim \mathcal{O}(1)$, $\omega_{int} \sim \mathcal{O}(100)kHz$), the estimated acceleration times are:

$$\Delta t_{\text{low}} \sim \mathcal{O}(10^2 - 10^3)/\omega_{\text{low}} \sim 0.1 - 1\ \text{s},$$

$$\Delta t_{\text{int}} \sim \mathcal{O}(10^4 - 10^5)/\omega_{\text{int}} \sim 1 - 10\ \text{s} \tag{8}$$

Thus, due to their low frequency and amplitude, neither inter-ELM modes nor dominant low-n modes can explain the observed rapid fast-ion acceleration on a timescale of $\Delta t \sim 10 - 100\ \mu s$. In other words, ELMs cannot effectively accelerate fast ions.

3. PHASE SPACE DYNAMICS

To understand the observed "acceleration" phenomenon, we analyze phase-space particle dynamics during the ELM cycle from a Lagrangian perspective. An ELM cycle includes a single ELM burst and the inter-ELM period [22], typically on a millisecond timescale in AUG discharges—much longer than the fast-ion bounce/transit period [8, 21].

Physically, gyrocenter motion in electromagnetic fluctuations involves two distinct time scales [23]. On short timescales, the gyrocenter undergoes unperturbed bounce/transit motion in the equilibrium axisymmetric magnetic field; small-amplitude fluctuations do not significantly affect trajectories. On long timescales, however, cumulative fluctuations over many bounce/transit periods can drive significant phase-space transport. For typical AUG parameters, cumulative effects become noticeable on a timescale between the fast-ion bounce/transit period and the duration of fast-ion filaments observed by FILD ($\sim \mathcal{O}(100)\mu s$ [8]). To account for this time-scale disparity, we extend the time variable from t to two variables \(t_0\) and \(t_1\), where:

$$\frac{dt_1}{dt} \ll \frac{dt_0}{dt} = 1 \tag{9}$$

Following [23], particle trajectories can be expressed in terms of unperturbed motion constants as:

$$\theta(t_0,t_1) = \delta_c\left[\omega_b t_0 + \partial_{P_\zeta}\omega_{b\int_0^{t_1}\delta P_\zeta dt'} + \partial_E\omega_{b\int_0^{t_1}\delta E}dt'\right] + \tilde{\theta}(t_0,t_1) + \int_0^{t_1}\delta\dot{\theta}\,dt',$$

$$\zeta(t_0,t_1) = \omega_\zeta t_0 + \partial_{P_\zeta}\omega_{\zeta\int_0^{t_1}\delta}\quad_\zeta t' + \partial_E\omega_{\zeta\int_0^{t_1}\delta}\ t' + \tilde{\zeta}(t_0,\quad_1) + \int_0^{t_1}\delta\dot{\zeta}\,dt',$$

$$r(t_0,t_1) = \bar{r} + \tilde{r}(t_0,t_1) + \int_0^{t_1}\delta\dot{r}\,dt'. \tag{10}$$

Here, ω_b and ω_t denote the bounce and transit frequencies for trapped ($\delta_c = 1$) and circulating ($\delta_c = 0$) particles, respectively. $\omega_\zeta = \dot{\zeta} = q\dot{\theta} + \omega_d$, where q is the safety factor and $\overline{\omega_d} = \overline{\dot{\zeta} - \dot{q}\theta - q\dot{\theta}}$ is the toroidal precession frequency. The average $\overline{(\cdots)} \equiv (\omega_b/2\pi)\oint(\cdots)d\theta/\dot{\theta}$ is over the equilibrium particle orbit. The functions $\tilde{\theta}, \tilde{\zeta}$, and $\tilde{r}$ describe oscillations in the equilibrium trajectory, parameterized by actions (μ, P_ζ, E) and periodic in t_0 with zero average. For finite-amplitude fluctuations, these functions may lose periodicity on the nonlinear timescale t_1, corresponding to secular transport. The terms $\delta\dot{r} \equiv \nabla r \cdot \delta\dot{X}$, $\delta\dot{\theta} \equiv \nabla\theta \cdot \delta\dot{X}$, and $\delta\dot{\zeta} \equiv \nabla\zeta \cdot \delta\dot{X}$ represent orbit modifications due to field perturbations.

An arbitrary single-particle scalar field δQ_n experienced along the trajectory can be expressed as:

$$\delta Q_n = \sum_m e^{i(m\theta - n\zeta + \omega_n t)}A_{n,m}(r) = e^{i\omega_n t_0 - in\left[\omega_\zeta t_0 + \partial_{P_\zeta}\omega_\zeta\int_0^{t_1}\delta P_\zeta dt' + \partial_E\omega_\zeta\int_0^{t_1}\delta E dt'\right] - in\int_0^{t_1}\delta\dot{\zeta}dt'}$$

$$\times \sum_{m,l} e^{i(\delta_c m + l)\left[\omega_b t_0 + \partial_{P_\zeta}\omega_b\int_0^{t_1}\delta P_\zeta dt' + \partial_E\omega_b\int_0^{t_1}\delta E dt'\right] + im\int_0^{t_1}\delta\dot{\theta}dt'}c_{m,l} \tag{11}$$

where

$$c_{m,l} = \oint \frac{\omega_b dt_0}{2\pi} e^{-il\omega_b t_0 - in\tilde{\zeta} + im\tilde{\theta}} A_{n,m,l}\left(\bar{r} + \underbrace{\tilde{r} + \int_0^{t_1} \delta\dot{r}\, dt'}_{FDOW}\right) \tag{12}$$

Equation (11) corresponds to a Lagrangian decomposition including nonlinear orbit distortion from electromagnetic fluctuations. Equation (12) shows that, due to the finite drift orbit width (FDOW) effect, orbit-averaged fields for trapped particles are typically smaller than for circulating particles, leading to weaker cross-field transport—consistent with experimental observations of weaker FILD signals for trapped particles [7, 8].

The nonlinear resonance condition is expressed via the time variation of the wave-particle phase Θ: $\dot{\Theta} = n\left[\omega_\zeta + \partial_{P_\zeta}\omega_\zeta \delta P_\zeta + \partial_E \omega_\zeta \delta E\right] - (\delta_c m + l)\left[\omega_b + \partial_{P_\zeta}\omega_b \delta P_\zeta + \partial_E \omega_b \delta E\right] + n\delta\dot{\zeta} - m\delta\dot{\theta} - \omega_n = 0$, where m and l are integers. Since ω_ζ and ω_b depend on motion constants, we derive:

$$\ddot{\Theta} = \left[n\,\partial_{P_\zeta}\omega_\zeta - (\delta_c m + l)\,\partial_{P_\zeta}\omega_b\right]\delta\dot{P}_\zeta + \left[n\,\partial_E\omega_\zeta - (\delta_c m + l)\,\partial_E\omega_b\right]\delta\dot{E} + n\delta\ddot{\zeta} - m\delta\ddot{\theta} \tag{13}$$

where $\delta\dot{P}_\zeta$ and $\delta\dot{\zeta}$ account for nonlinear wave-particle resonance [24]. For weak turbulence near a single phase-space island, we neglect $\delta\ddot{\theta}$ and $\delta\ddot{\zeta}$ and use Hamiltonian equations $\delta\dot{P}_\zeta = -\partial_\zeta \delta H$ and $\delta\dot{E} = \partial_t \delta\bar{H}$. This reduces Eq. (13) to a nonlinear pendulum equation:

$$\ddot{\Theta} + \omega_B^2 \sin\Theta = 0 \tag{14}$$

with initial conditions $\left(\Theta_0, \dot{\Theta}_0 = n\omega_\zeta - (\delta_c m + l)\omega_b - \omega_n\right)$. The wave trapping frequency satisfies $\omega_B^2 = \delta H_{m,l}\{n\left[n\,\partial_{P_\zeta}\omega_\zeta - (\delta_c m + l)\,\partial_{P_\zeta}\omega_b\right] + \omega_n\left[n\,\partial_E\omega_\zeta - (\delta_c m + l)\,\partial_E\omega_b\right]\}$. Equation (14) can be derived from the Hamiltonian $H_p = \dot{\Theta}^2/2 - \omega_B^2\cos\Theta$ describes nonlinear wave-particle dynamics around a finite-size phase-space island and can be solved exactly using Jacobi elliptic functions [25]. Particle motion has two forms: wave-circulating ($H_p > \omega_B^2$) and wave-trapped ($H_p < \omega_B^2$). Focusing on the separatrix ($H_p = \omega_B^2$), the island full width at Θ = 0 is:

$$\Delta\dot{\Theta} = 4\omega_B \tag{15}$$

From Hamiltonian trajectory properties:

$$\frac{d\delta P_\zeta}{d\Theta} = -\frac{nH_{m,l}\sin\Theta}{\dot{\Theta}}, \tag{16}$$

$$\frac{d\delta E}{d\Theta} = -\frac{\omega_n H_{m,l}\sin\Theta}{\dot{\Theta}}, \tag{17}$$

with $\dot{\Theta}$ along the separatrix given by energy conservation:

$$\dot{\Theta}^2 = 2\omega_B^2(1 + \cos\Theta) \tag{18}$$

Solving Eq. (17) gives the total energy change during half a bounce along the separatrix:

$$\delta E = \frac{\sqrt{2}\omega_n H_{m,l}}{\omega_B}\left[\sqrt{1 + \cos\Theta_0} - \sqrt{1 - \cos\Theta_0}\right] \tag{19}$$

The maximum energy exchange in a single phase-space island is:

$$\delta E = \frac{2\omega_n \delta H_{m,l}}{\omega_B} \tag{20}$$

Equation (20) shows efficient acceleration requires $|\omega_n| \gg |\omega_B|$, consistent with the weak turbulence limit. The maximum canonical momentum exchange is:

$$\delta P_\zeta = \frac{2n\delta H_{m,l}}{\omega_B} \tag{21}$$

Notably, $n\delta E - \omega_n \delta P_\zeta = 0$ [26]. For radial displacement, projecting $\delta\dot{r} \approx -\partial_\theta \delta H/(r\,\Omega)$ into the island orbit gives:

$$\delta\dot{r} = \frac{k_\theta}{\Omega}\delta H_{m,l} \sin\Theta \tag{22}$$

The maximum radial displacement is:

$$\delta r = \frac{2k_\theta H_{m,l}}{\Omega\,\omega_B} \tag{23}$$

For typical AUG edge parameters [7, 8], both low-n dominant modes and high-n inter-ELM modes satisfy:

$$\frac{\delta E}{H_0}\frac{R_0}{\delta r} = \frac{\omega_n R_0}{v_0 k_\theta \rho_0} \ll 1 \tag{24}$$

Thus, canonical momentum exchange dominates phase-space transport. This analysis clarifies that low-frequency ELM-related fluctuations primarily drive radial transport of fast ions rather than altering their energy. It also provides a basis for applying quasilinear theory to multi-island scenarios.

4. QUASILINEAR THEORY

To model phase-space transport involving multiple resonance islands, we shift to an Eulerian perspective and conduct a quasilinear analysis based on gyrokinetic theory. First, we validate quasilinear theory for ELM-cycle fluctuations in AUG using the Chirikov island overlap criterion [27, 28, 29]. Quasilinear theory applies best when modes induce strong orbit stochasticity via island overlap. In tokamaks, phase-space island fixed points lie near mode rational surfaces [30]. The island overlap condition is estimated by the ratio of δr (Eq. 23) to the distance between neighboring rational surfaces $\Delta r \sim 1/(nq')$:

$$\frac{\delta r}{\Delta r} \simeq 2\sqrt{k_\theta \rho_0}\sqrt{\frac{nq^2 s}{\epsilon}\frac{J_0(k_\perp \rho_0)\delta B_\perp}{B}} \tag{25}$$

For the dominant low-n modes with typical parameters $n \simeq 2, q \simeq, s \simeq, \epsilon \equiv a/R_0 =, \rho_0/a \sim 0$ and $|\delta B_\perp/B| \sim \mathcal{O}(10^{-3} - 10^{-2})$, it is found that $k_\theta \rho \simeq 1$, and $\delta r/\Delta \simeq \mathcal{O}(1-10) > 1$. Conversely, for the inter-ELM modes with $n \simeq 10$ and $|\delta B_\perp/B| \sim \mathcal{O}(10^{-5} - 10^{-4})$, one obtains $k_\theta \rho \simeq 5$ and $\delta r/\Delta \simeq \mathcal{O}(10^{-2} - 10^{-1}) < 1$. Therefore, only low-n ELM fluctuations drive global transport.

We derive the quasilinear transport equation for low-n modes. Using canonical phase-space coordinates (r, θ, ζ, μ, P_ζ, E), the gyrocenter distribution is described by the full-F gyrokinetic equation [13]:

$$\left[\partial_t + \dot{\theta}\,\partial_\theta + \dot{\zeta}\,\partial_\zeta\right] F = iQF\delta H \tag{26}$$

where the operator Q is:

$$QF\delta H = i\left[\frac{\partial F}{\partial E}\partial_t - \frac{\partial F}{\partial P_\zeta}\partial_\zeta\right]\delta H \tag{27}$$

Separating the convective term from kinetic compression K via $F = i(e/m)QF\,\partial_{t_0}^{-1} J_0 \delta\psi + K$, the gyrokinetic equation becomes:

$$\left[\partial_t + \dot{\theta}\,\partial_\theta + \dot{\zeta}\,\partial_\zeta\right] K = iQF\delta\Phi \tag{28}$$

where

$$\delta\Phi = \frac{e}{m}\left[J_0(\delta\phi - \delta\psi) - J_0 \omega_d\, \partial_t^{-1}\, \partial_\zeta \delta\psi + \frac{i v_\perp J_1}{c}\nabla_\perp^{-1}\delta B_\parallel\right] \tag{29}$$

is an effective potential from three field-aligned forces [31]: $\delta\phi - \delta\psi$ (parallel electric field, negligible in ideal MHD), $\omega_d\, \partial_t^{-1}\, \partial_\zeta \delta\psi$ (dominant $v_d \times \delta B_\perp$ force in ELM-fast ion interactions), and $\delta B_\parallel$ (mirror force). The magnetic drift frequency is $\omega_d \equiv \dot{\zeta} - \theta\dot{q} - q\dot{\theta} \approx q/(\Omega R_0 r)(w^2 + \mu B)(\cos\theta + s\theta\sin\theta)$ with $s = rq'/q$ (magnetic shear).

Using Reynolds decomposition $K = K_0 + \delta K,\ K_0 =< K >_s$with spatial averaging $< \cdots > s$, the background distribution equation is:

$$\partial_{t_1} F_0 = i < Q\delta K\delta\Phi >_s - \frac{e}{m} < Q\left[\left(QF_0\,\partial_{t_0}^{-1} J_0 \delta\psi\right)\delta\Phi\right] >_s \tag{30}$$

Here, t_0 and t_1 distinguish transport and turbulence timescales. The second term (convective contribution) vanishes for $\delta\psi$-dominated fluctuations. The perturbed distribution satisfies:

$$\left[\partial_t + \dot{\theta}\,\partial_\theta + \dot{\zeta}\,\partial_\zeta\right]\delta K = iQF_0\delta\Phi \tag{31}$$

Combining Eqs. (30) and (31) defines the quasilinear transport problem.

For Fourier modes$\delta\Phi = \sum_n \exp(i\omega t - in\zeta)\,\delta\Phi(r,\theta)$, Eq. (31) is solved by integrating along unperturbed orbits. For circulating particles ($\theta \in [-\pi, +\pi)$), transforming to drift orbit centers using $\exp -i\int^\theta (n\omega_d/\omega_t + nq)d\theta'$ and approximating the integral (odd in θ) with its lowest Fourier component gives:

$$\int^\theta \frac{n\omega_d}{\omega_t} d\theta' \approx \frac{qk_\theta}{\Omega w}(w^2 + \mu B)(1 + 3s/2)\sin\theta \equiv k_\theta \rho_d \sin\theta \tag{32}$$

Equation (31) becomes:

$$\left[\omega - i\dot{\theta}\,\partial_\theta\right] e^{-ik_\theta\rho_d\sin\theta - inq\theta}\delta K = e^{-ik_\theta\rho_d\sin\theta - inq\theta} QF_0\delta\Phi \tag{33}$$

Using

$$e^{-ik_\theta\rho_d\sin\theta - inq\theta} = \sum_{p=-\infty}^{+\infty} J_p(k_\theta\rho_d) e^{-i(p+nq)\theta}, \tag{34}$$

the perturbed distribution is:

$$\delta K = \sum_{n,m,p,p'=-\infty}^{+\infty} \frac{J_p(k_\theta\rho_d) J_{p'}(k_\theta\rho_d) QF_0 \delta\Phi_{m-p'+p} e^{-in\zeta + im\theta}}{\omega + (m - nq - p')\omega_t} \tag{35}$$

For trapped particles ($\theta \in [-\theta_0, +\theta_0]$), defining the Fourier transform as:

$$\mathcal{F}_{\mathscr{b}}[g]_m = \frac{1}{T}\oint g(\theta) e^{-im\omega_b\int^\theta \frac{d\theta'}{\dot{\theta}'}} d\int^\theta \frac{d\theta'}{\dot{\theta}'} \tag{36}$$

where $T = \oint d\theta/\dot{\theta} = 2\pi/\omega_b$ is the bounce period, and $\theta_c = \omega_b \int^\theta d\theta'/\dot{\theta}'$ is the canonical angle conjugate to the second invariant J [6]), the gyrokinetic equation becomes:

$$\left[\omega - i\dot{\theta}\,\partial_\theta - \omega_d - nq\dot{\theta}\right]\delta K = QF_0\delta\Phi \tag{37}$$

Decomposing $\omega_d$as:

$$\omega_d \approx \overline{\omega}_d + \widehat{\omega_d}\cos\left(\omega_b \int^\theta \frac{d\theta'}{\dot{\theta}'}\right) \tag{38}$$

where $\overline{\omega}_d$ is the precession frequency, and transforming to banana centers gives:

$$\left[\omega - i\dot{\theta}\,\partial_\theta - \overline{\omega}_d\right] e^{-ik_\theta\rho_d\sin\left(\omega_b\int^\theta \frac{d\theta'}{\dot{\theta}'}\right) - inq\theta}\delta K = e^{-ik_\theta\rho_d\sin\left(\omega_b\int^\theta \frac{d\theta'}{\dot{\theta}'}\right) - inq\theta} QF_0\delta\Phi \tag{39}$$

The solution is:

$$< e^{-inq\theta}\delta K >_m = \sum_{p,l} \frac{J_p(k_\theta\rho_d) J_{m-l}(k_\theta\rho_d)}{[\omega + l\omega_b - \overline{\omega}_d]} QF_0 < \delta\Phi\, e^{-inq\theta} >_{b,p+l} \tag{40}$$

where

$$< \delta\Phi\, e^{-inq\theta} >_{b,m} = \frac{1}{T}\oint \frac{d\theta''}{\dot{\theta}''}\delta\Phi\, e^{-inq\theta''} e^{-im\omega_b\int^{\theta''}\frac{d\theta'}{\dot{\theta}'}} \tag{41}$$

Substituting the linear response into Eq. (30) yields the phase-space transport equation:

$$\partial_{t_1} F_0 = \frac{\partial}{\partial J} \cdot \left[D \cdot \frac{\partial F_0}{\partial J} \right] \tag{42}$$

where the transport coefficient matrix is:

$$D(\lambda) = \begin{pmatrix} D_{P_\zeta P_\zeta} & D_{P_\zeta E} \\ D_{EP_\zeta} & D_{EE} \end{pmatrix}$$

with elements depending on wave-particle resonance. Equation (42) confines interactions to a 2D manifold (P_ζ, E) in 5D phase space. Low-frequency fluctuations drive transport primarily in the canonical momentum channel, leading to cross-field transport.

For circulating particles, the canonical momentum diffusion coefficient is:

$$D_{P_\zeta P_\zeta} = -Im \left[\sum_{n,m,p,p'} \frac{n^2 J_p(k_\theta \rho_d) J_{p'}(k_\theta \rho_d) \left| \delta\Phi_{m-p'+p} \right|^2}{\omega + (m - nq - p')\omega_t + i\delta\omega} \right] \tag{43}$$

where ρ_d is the magnetic drift orbit width, and $\delta\omega$ accounts for resonance broadening. For low-n ELMs ($k_\theta \rho \ll k_\theta \rho_d \lesssim 1$), FLR and FDOW effects are negligible. Since $\delta\Phi \propto \omega_d(w^2 + \mu B) \propto v^2$, diffusivity scales as $D_{P_\zeta P_\zeta} \propto v^3$—higher-energy fast ions are transported more rapidly, creating a high-energy tail in FILD signals that mimics acceleration. This contrasts with microturbulence-driven transport [11], where $D_{P_\zeta P_\zeta} \propto v^{-3}$ due to electrostatic potential dominance and Bessel function scaling. For trapped particles, the diffusion coefficient is:

$$D_{P_\zeta P_\zeta} = -Im \left[\sum_{n,m,p,l} \frac{n^2 J_p(k_\theta \rho_d) J_{m-l}(k_\theta \rho_d) \left| <\delta\Phi e^{-inq\theta}>_{b,p+l} \right|^2}{[\omega + l\omega_b - n\overline{\omega}_d + i\delta\omega]} \right] \tag{44}$$

with $D_{P_\zeta P_\zeta} \propto v^2$ due to precessional resonance. This explains weaker FILD signals for trapped particles [7, 8].

Using the line-broadened quasilinear model [32, 33] ($\delta\omega \approx 2\omega_B$), the fast-ion diffusion time out of the plasma is:

$$\Delta t = \frac{(\Delta r)^2}{D_{rr}} \simeq \sum_{n,m} \frac{2\omega_B v^2 (\Delta r)^2}{k_\theta^2 \rho^2 J_0^2(k_\theta \rho_d) \left| \delta\Phi_{n,m} \right|^2} \tag{45}$$

For AUG low-n modes and $\Delta r \approx 0.1a$, $\Delta t \approx 10 - 100\,\mu\text{s}$, consistent with experiments [7, 8]. This framework explains FILD pitch-angle structures [7, 8]. Resonance conditions for circulating($\omega + (m - n\bar{q} + l)\omega_b - n\overline{\omega_d} = 0$) and trapped ($\omega + l\omega_b - n\overline{\omega_d} = 0$) particles show only circulating particles depend on q_{95}. As q_{95} evolves, circulating particles encounter different resonant islands, creating multiple spikes, while trapped particles remain with one island, showing a single spike. ELM fluctuations localize to the pedestal, limiting resonant island number and width, making spikes measurable.

5. CONCLUSIONS

We analytically investigated ELM-induced phase-space transport of fast ions using gyrokinetic theory. Contrary to previous simulations [7, 8], the high-energy fast-ion population observed in AUG during ELMs is not accelerated by low-frequency ELM fluctuations. Effective acceleration requires high-frequency/high-amplitude fluctuations, beyond this gyrokinetic study's scope. Instead, low-frequency ELMs drive radial transport via magnetic perturbations. Unlike microturbulence-driven transport, ELM-induced diffusivity scales with v^3 (circulating particles) and v^2 (trapped particles), causing faster outward transport of higher-energy fast ions and a high-energy tail in FILD signals. Cross-field diffusion time estimates agree with experiments. Fine pitch-angle spikes in FILD signals correspond to multiple phase-space islands of circulating particles, detuned by the safety factor via resonance conditions. Trapped particles show a single spike due to q-independent resonance. Weaker trapped-particle signals result from FDOW effects and precessional resonance. These findings have implications for future tokamaks: ELMs may detrimentally transport high-energy alpha particles, threatening fusion self-sustainment, while associated heat loads could challenge plasma-facing component materials.

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