

CONFERENCE PRE-PRINT**ENERGETIC PARTICLE DISTRIBUTIONS FOR QUANTITATIVE CALCULATIONS OF BURNING PLASMA STABILITY***The EPCoM steady-state distribution transformation tool*

S.D. PINCHES
 ITER Organization
 Saint-Paul-lez-Durance
 Email: Simon.Pinches@iter.org

G. BROCHARD
 CEA, IRFM
 Saint-Paul-lez-Durance, France

W.W. HEIDBRINK
 University of California
 Irvine, United States of America

Z. LIN
 University of California
 Irvine, United States of America

Abstract

Accurate characterization of energetic particle (EP) distributions is essential for quantitative predictions of burning plasma stability, as gradients in EP phase space govern the drive and damping of instabilities that impact confinement and fusion performance. The EPCoM tool enables transformation of EP distributions between common numerical representations and a Constants-of-the-Motion (CoM) form, utilizing the invariants energy, magnetic moment, and toroidal canonical angular momentum in axisymmetric plasmas, with pitch-angle sign included to resolve orbit degeneracy. Representation in CoM space ensures true steady-state solutions of the Fokker–Planck equation, reduces dimensionality, and suppresses numerical noise. EPCoM constructs a three-dimensional CoM grid and applies high-order projection and spline techniques to produce strictly continuous distribution functions, with singularities at the trapped–passing boundary eliminated by splitting into co- and counter-going components. Orbit frequencies facilitate direct visualization of resonances and, combined with distribution gradients, allow identification of phase-space regions responsible for observed instabilities. Validation on distributions for DIII-D, JET, and ITER demonstrates accurate transformations and low mean errors, with discrepancies attributed to non-stationary initial conditions. Integration with the Integrated Modelling & Analysis Suite (IMAS) provides physically consistent inputs for stability and transport workflows, supporting both δf and full- f approaches in gyrokinetic and kinetic-magnetohydrodynamic (MHD) simulations. Experimental application in DIII-D plasmas shows that changes in neutral beam geometry alter resonance overlap and mode drive, consistent with observations of Beta-induced Alfvén Eigenmodes (BAE). EPCoM offers a robust, portable, and IMAS-compliant solution for quantitative EP stability and transport analysis, and serves as a foundation for multi-code benchmarking activities within the International Tokamak Physics Activity (ITPA) energetic particle physics community.

1. INTRODUCTION**1.1. Sources of energetic particles**

Energetic ions in fusion plasmas originate from both external heating systems and the fusion process itself. Auxiliary heating methods such as neutral beam injection (NBI) and ion cyclotron resonance heating (ICRH) are primary sources of fast ions. NBI operates by injecting high-energy neutral atoms into the plasma, where they are ionized and subsequently slow down through collisions—first with electrons and then with ions—resulting in a characteristic slowing-down energy distribution. In contrast, ICRH accelerates ions via resonant energy transfer at the ion cyclotron frequency, producing highly anisotropic populations of trapped fast ions. Additionally, deuterium-tritium (D-T) fusion reactions in the plasma core generate alpha particles with an initial energy of 3.52 MeV, forming a nearly isotropic distribution that also slows down as these particles interact with the bulk plasma. Together, these mechanisms create diverse energetic particle populations that play a critical role in plasma behaviour and stability.

1.2. Influence of energetic particles

Energetic particles (EP) can drive a variety of instabilities in tokamak plasmas which can affect fusion performance. Consequences of these instabilities include a performance-degrading outward transport of high-energy fusion-born alpha particles before they thermalize with the bulk plasma [1] and the beneficial moderation of turbulent transport [2]. In all cases it is the gradients in the distribution of energetic particles, in both real and velocity space, that determines the strength of the power transfer from these particles to modes in the plasma. In order to be able to make quantitative predictions of EP stability, it is thus necessary to accurately represent the 6-dimensional distribution energetic particles (in real and velocity space) which arises as the solution of the Fokker-Planck equation [1,2]:

$$\frac{df}{dt} = C(f) + S$$

The distributions obtained are typically far from Maxwellian in nature.

In axisymmetric plasmas, in addition to the conservation of energy, E , and magnetic moment, μ , toroidal canonical angular momentum, P_ϕ , is also conserved and allows steady-state distribution functions to be written in terms of these three invariants or constants-of-the-motion, $f(E, \mu, P_\phi)$. In this work, the development of a tool to transform between common numerical representations of energetic particle distributions and a CoM form is introduced.

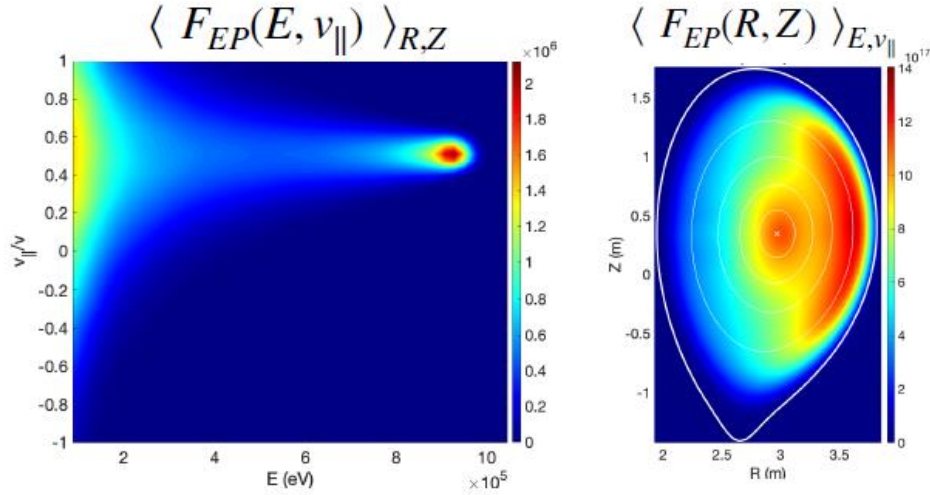


FIG. 1. Example distribution of beam heated plasma showing distribution in velocity space and real space.

The linear growth rate, γ , of an instability that resonates with energetic particles in the plasma can be expressed as

$$\gamma \sim \omega \frac{\partial f}{\partial E} + n \frac{\partial f}{\partial P_\phi} \sim \omega \frac{\partial f}{\partial E} - n \frac{\partial f}{\partial \psi}$$

Indicating that negative gradients in the radial direction, ψ , drive instabilities with positive toroidal mode numbers ($n > 0$) in this plasma (i.e., waves propagating in the fast ion diamagnetic direction), whilst negative gradients with respect to energy (as arises from slowing down) damps them. This is a universal phenomenon that means unstable EP-driven modes such as Alfvén Eigenmodes (AE) and Energetic Particle Modes (EPMs), generally extract energy from the radial gradient of the EP distribution, relaxing it and transporting particles outwards.

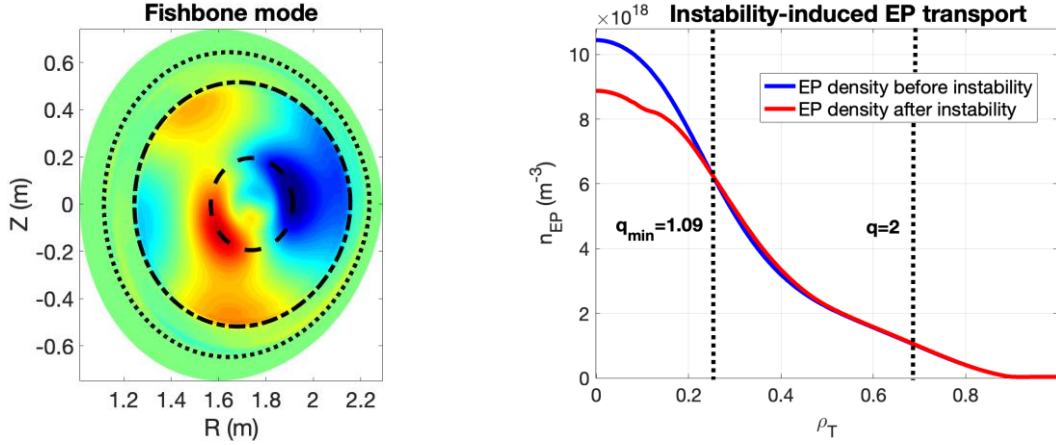


FIG. 2. Fishbone simulation performed with the gyrokinetic GTC code [3] showing impact on radial energetic particle profile

The effects, however, are not all detrimental and the interaction of energetic particles can also lead to the generation of zonal flows which are well-known to influence thermal transport mechanisms [2].

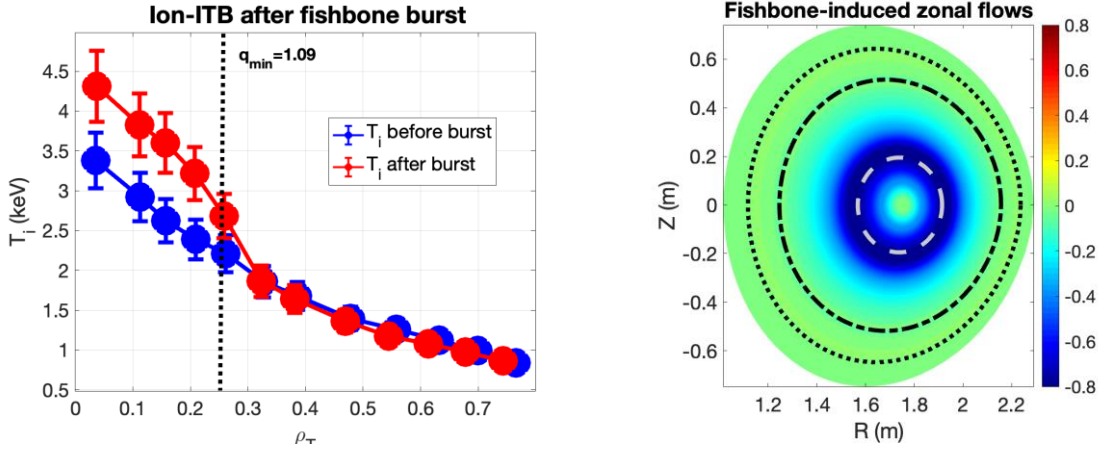


FIG. 3: Ion temperature profiles before and after fishbone bursts from CXRS measurements in DIII-D shot 178631 and associated $m = 0$ zonal flow simulated by the GTC code

2. REPRESENTING & TRANSFORMING ENERGETIC PARTICLE DISTRIBUTIONS

The examples of energetic particle behaviour in the previous section have revealed the role that energetic particles play in understanding experimental observations and the importance of gradients in their distribution. Being able to go beyond qualitative simulations to perform quantitative simulations requires an accurate presentation that captures the nature of the energetic particle distribution function reflecting the details of its creation (sources/sinks and collisions) as well as being a true steady-state solution of the Fokker-Planck equation.

Numerically obtained solutions to the Fokker-Planck equation are typically too noisy to directly use for time-dependent stability calculations. Indeed, distributions represented in the typical $(E, v_{\parallel}, R, Z)$ space are not steady-state by construction and often lead to unphysical results if they are used directly.

Describing energetic particle distributions in terms of the Constants-of-the-Motion (CoM) naturally ensures the distributions are true equilibrium steady-state solutions in axisymmetric plasmas and eliminates any further unphysical relaxation of the profiles. It also reduces the dimensionality from 4 to 3 which has been found to greatly reduce the noise in the numerically generated EP distributions. Once an EP distribution is expressed in terms of the CoM, and the EP orbits are traced, the EP distribution can be transformed to alternate representations for use within different stability and transport codes.

The initial process of representing a numerical distribution in CoM space for a specific magnetic equilibrium configuration starts with defining a 3-dimensional grid in (E, Λ, P_ϕ) CoM space (actually, two grids since $\sigma = v_\parallel/|v| = \pm 1$ is additionally needed to remove orbit degeneracy) where $\Lambda = \mu B_0/E$. For each grid point the corresponding energetic particle trajectory is computed from which the poloidal, ω_θ , and toroidal, ω_ϕ , periodic motion frequencies can be calculated which allows the CoM Jacobian to be determined. These orbit frequencies also allow resonances to be directly visualised for a mode with a given n and ω by using the resonance condition $\omega = n\omega_\phi - p\omega_\theta$ where p is the bounce harmonic.

The initial numerical distribution arising from the solution of the Fokker-Planck equation is projected onto a 4-dimensional (E, λ, R, Z) grid with high-order interpolation from which a 4D $\mathcal{C}^2$ B-spline is created as shown in Fig. 4. This spline, together with the Jacobian determined above, allows the distribution to be represented in CoM space with arbitrary resolution.

The distribution in CoM space is represented by 3D $\mathcal{C}^2$ B-splines, to ensure strictly continuous gradients for stability and nonlinear simulations. The intrinsic singularity that arises at the trapped-passing boundary [4] is eliminated by the splitting of the distribution into co-going and counter-going components, $\sigma = v_\parallel/|v| = \pm 1$, for both trapped and passing particles, which enables enforcement of the $\mathcal{C}^2$ condition.

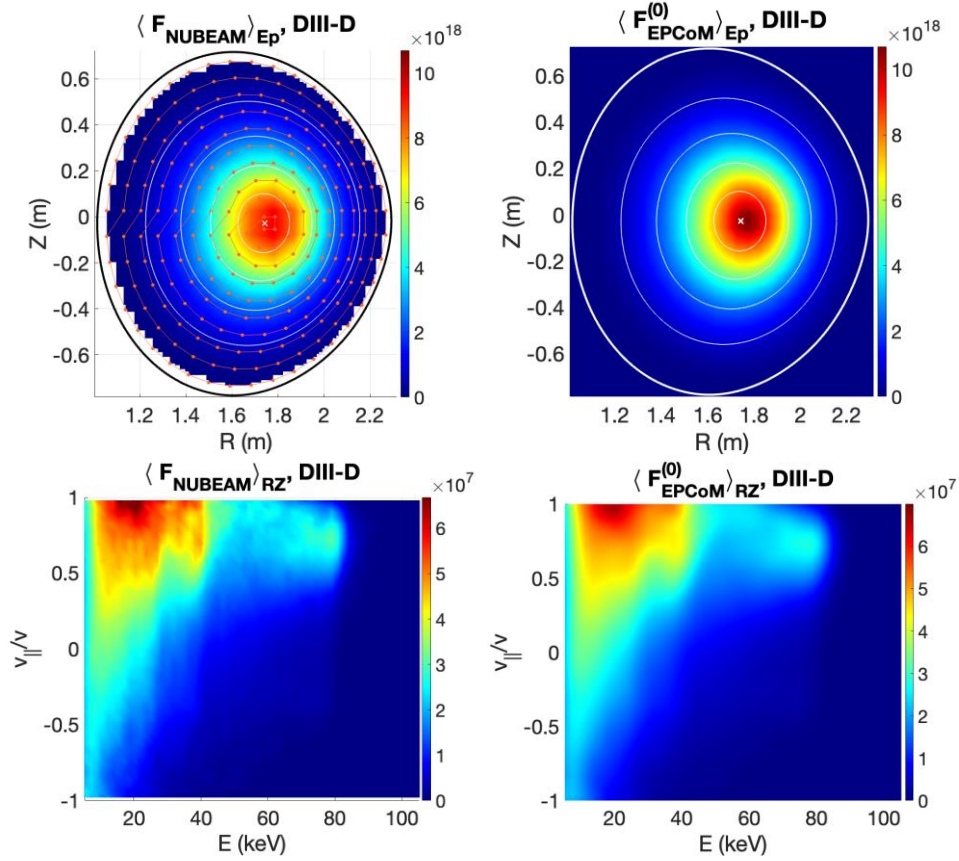


FIG. 4: Transformation of NBI distribution calculated by NUBEAM for DIII-D into a 4D $\mathcal{C}^2$ spline on an (E, λ, R, Z) grid.

The consistency of the transformation between representations can be successfully demonstrated by performing multiple transformations between spaces for distributions obtained from Fokker-Planck calculations for DIII-D, JET and ITER plasmas.

Fig. 5 shows the convergence with respect to CoM grid resolution for distributions representing NBI injection in DIII-D, JET and ITER. The accuracy scales well with discrepancies attributed to non-steady-state initial conditions (so a perfectly matching CoM representation is strictly not possible) and the repeated interpolation / splining process.

Extending IMAS to include a method for deriving, representing and transforming energetic particle distributions via a CoM representation, thereby intrinsically ensuring stationarity, is a valuable asset in its capabilities. Stability calculations, such as those performed with the IMAS EP Stability Workflow, rely upon physically realistic EP distributions that properly describe the experimental configuration to correctly interpret and predict stability behaviour. Physics codes that use the δf approach can directly obtain the gradients they need from the $\mathcal{C}^2$ B-spline representation. In full- f codes the EP distribution must be initialised around entire orbits (points in CoM space represent entire orbits and don't distinguish between points on the same orbit), this can be done by directly sampling markers in the domain of interest of their 4D position-velocity space, computing the associated 3D CoM invariants, and then using f_{CoM} to weight the markers and represent the distribution of interest. These schemes have been demonstrated using the δf gyrokinetic code GTC [3] and the full- f kinetic-MHD code XTOR-K [6].

4. EXPERIMENTAL APPLICATION

As indicated earlier, the calculation of orbit properties including the poloidal and toroidal transit frequencies, ω_θ and ω_ϕ , allows the identification of the surfaces in phase-space where a resonant exchange of energy between the energetic particles and a plasma mode with a give frequency, ω , and toroidal mode number, n , can occur. Namely where $\omega - n\omega_\phi + p\omega_\theta = 0$ with p an integer [7]. This information, when combined with the gradient that drives instabilities, $n \frac{\partial f}{\partial P_\phi} + \omega \frac{\partial f}{\partial E}$, allows the regions of phase space responsible for observed instabilities to be identified.

The following is an example in which the fast ion distribution in a DIII-D plasma is changed by switching between NBI sources and helps validate the approach. Fig. 8 shows the regions of CoM phase space where an $n = 5$ core-localised BAE mode interacts with neutral beam ions in a DIII-D plasma [8].

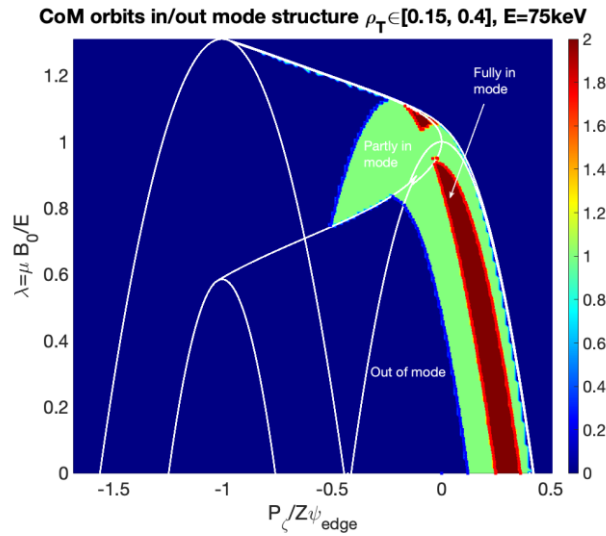


FIG. 8: Red region shows particle orbits that wholly overlap with BAE region. Green region shows particle orbits that partly overlap with BAE region.

The red area indicates the region of CoM space where fast ions fully overlap with the BAE, and the green area where they only partly overlap. Figs. 9 and 10 shows the resonance lines for the observed mode as black lines overlaid upon a coloured contour plot showing the drive that the fast ion distribution provides to the mode. In Fig. 9 tangential NBI is used while in Fig. 10, perpendicular NBI heating, which leads to the disappearance of the BAE in the experiment [8]. A slight change in the q -profile between the time points means the resonance lines move slightly whilst the change in injection geometry means the distribution in Fig. 10 contains more trapped ions. The red and blue lines in Figs. 9 and 10 indicate the boundaries of the regions in Fig 8. It can be seen that the fast ions that lie along the $p = 6$ resonance line (a necessary but not sufficient condition for energy exchange) and that wholly overlap with the mode (inside the red boundary) are in a region of stronger drive in Fig. 9 compared to Fig. 10. This is consistent with the experimental observation of the mode existence in Fig. 9 but its disappearance in Fig. 10.

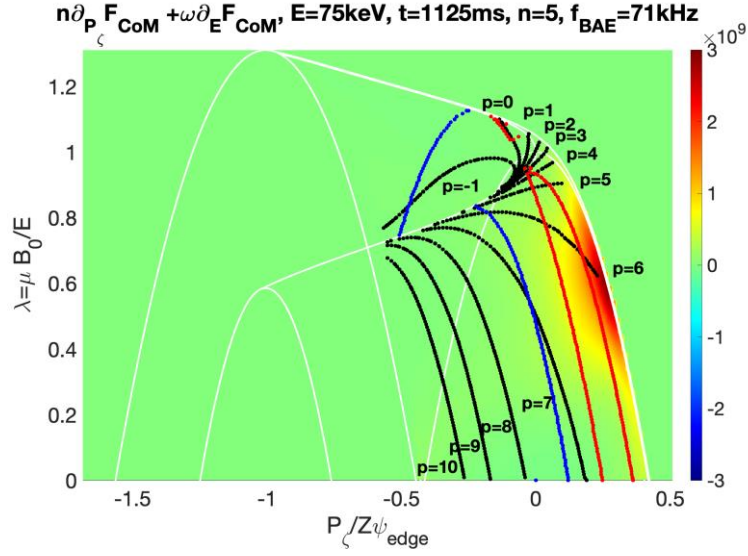


FIG. 9: Black lines show resonances for $n = 5$ BAE overlaid on colour map showing strength of distribution gradient (drive for mode). Red lines outline the region where orbits wholly overlap with mode localization.

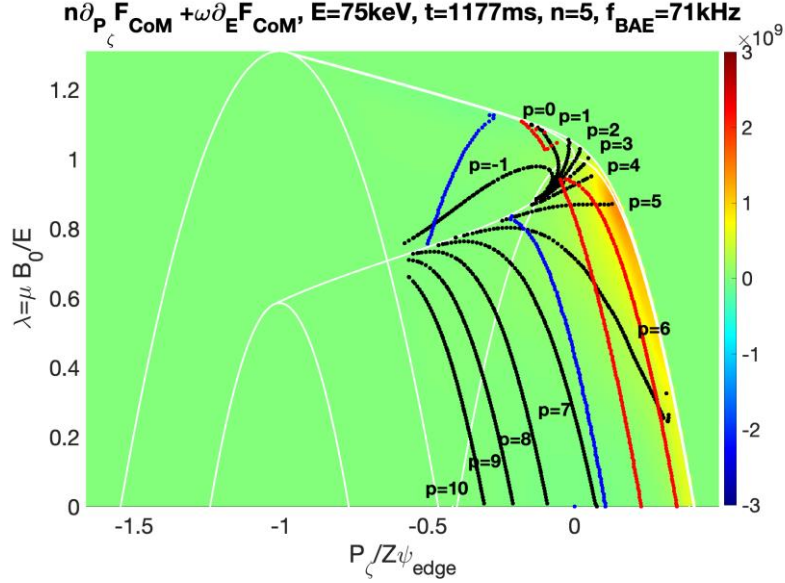


FIG. 10: Black lines show resonances for $n = 5$ BAE overlaid on colour map showing strength of distribution gradient (drive for mode). Red lines outline the region where orbits wholly overlap with mode localization.

These experimental observations in DIII-D are the subject of further work and extensive gyrokinetic and kinetic-MHD simulations are being undertaken as part of an on-going ITPA joint activity within the energetic particle physics topical group to validate the tools.

5. SUMMARY AND OUTLOOK

A numerical tool, EPCoM, has been developed to transform numerical representations of realistic energetic particle distributions into Constants-of-the-Motion space. A 3-dimensional (E, Λ, P_ϕ) CoM grid (plus $\sigma = \pm 1$) is used to generate a $\mathcal{C}^2$ CoM distribution function without discontinuities. The accuracy of the transformation has been demonstrated for a realistic DIII-D case by transforming back-and-forwards.

It is expected that EPCoM will become a key tool when undertaking quantitative assessments of the role of energetic particles in fusion plasmas. The code is adapted to produce energetic particle distributions following

the IMAS data model [5] and is expected to become open source in the near future together with many other IMAS physics components.

EPCoM is also the cornerstone of a new ITPA joint activity within the energetic particle physics topical group where multiple linear/nonlinear kinetic-MHD/gyrokinetic codes will be benchmarked.

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