

Development of an eigenvalue solver for electrostatic drift-wave instabilities in tokamaks

Jie Wang , Shaojie Wang*
University of Science and Technology of China
Email: wangj19@mail.ustc.edu.cn

Abstract

The magnetic mirror effects are included in the local Fourier-space gyrokinetic eigenvalue solver, ESR, to study the ion-temperature-gradient (ITG) mode in toroidal geometry. It is found that the magnetic mirror effects are dominated by trapped ions' destabilizing effects in the low $k_{\perp}$ regime and dominated by the passing ions' stabilizing effects in the high $k_{\perp}$ regime. The magnetic mirror effects modify the ITG linear growth rate through inducing the additional toroidal coupling between the neighboring poloidal harmonics.

1. INTRODUCTION

Drift-wave (DW) instabilities are considered to be the main source of plasma turbulence responsible for anomalous transport in tokamaks. To estimate the level of DW-induced transport, it is important to know the local structure of the eigenmodes. For the electrostatic drift wave (DW) instabilities in tokamaks, many local eigenvalue codes(e.g., FULL [1, 2] and HD-7 [3]) have been developed in the ballooning space by using the ballooning mode transformation. By using the translational invariance[3], we developed a new local electrostatic gyrokinetic eigenvalue code, ESR, in the Fourier transformed space for the drift-wave modes (ITG, TEM, ETG) based on the Vlasov-Poisson description, which benchmarks well with HD-7 and FULL. However, in HD-7 and ESR, with the trapped ions treated as passing ions, the assumption of well-circulating particles is adopted. This assumption neglects the magnetic mirror effects by approximating $B \approx B_0$. In contrast, the FULL code retains the magnetic mirror effects.

2. METHODS AND IMPLEMENTATION

The passing-ion magnetic mirror (PIMM) effects are included by the poloidal Fourier decomposition of the magnetic-mirror-effect-related terms in the linear operators of eigenvalue equations, while the trapped-ion effects are handled by solving the orbit-averaged Vlasov equation.

We apply the following poloidal Fourier decomposition to perturbations,

$$\delta\phi_n(r, \theta) = \sum_m e^{im\theta} \delta\bar{\phi}_n(z, m). \quad (1)$$

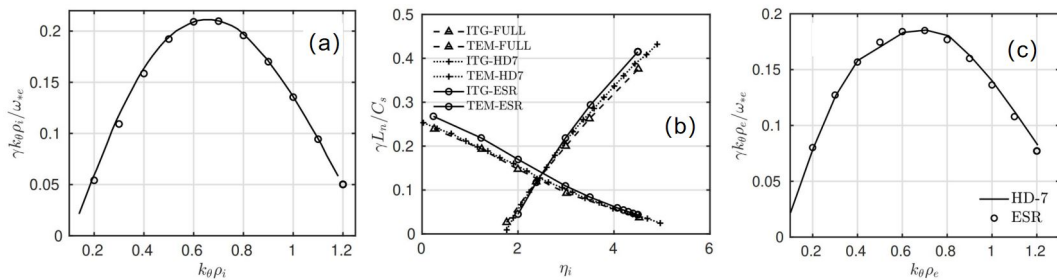


FIG. 1. Benchmark results [4] of linear modes: (a) ITG , (b) TEM and (c) ETG.

Here, $z \equiv nq(r) - m$ is related to the radial distance from the mode rational surface r_m , with m the poloidal mode number. Since $n \gg 1$, the neighboring harmonics can be treated by using the translational invariance[5, 4]

$$\delta\bar{\phi}_n(z, m \pm 1) = \delta\bar{\phi}_n(z, m). \quad (2)$$

The coupled 1D ordinary differential equations for the poloidal harmonics in the (r, θ) space transformed from the 2D partial differential equation in the (z, m) space are decoupled by using the translational invariance.

If magnetic-mirror effects are included, the parallel ion transit term ω_{ti0} , the magnetic drift term ω_{di0} , and the diamagnetic drift term ω_{*Ti0} in the gyrokinetic Vlasov equation depend on θ . Specifically, $\omega_{ti} = \frac{v_{\parallel}}{qR}$, $\omega_{di} = -k_{\theta} \frac{2E - \mu B}{e_i B R}$, and $\omega_{*Ti} = \frac{k_{\theta} T_i}{e_i B L_n} [1 + \eta_i (-\frac{3}{2} + \frac{E}{T_i})]$. We can represent the three magnetic-mirror-related terms in the linear operator as Fourier series

$$\omega_{ti} = \sum_{p=0}^2 \omega_p^{\parallel}(r, E, \mu, \sigma) \cos p\theta, \quad (3)$$

$$\omega_{di} = \sum_{p=0}^3 \omega_p^d(r, E, \mu) \cos p\theta, \quad (4)$$

$$\omega_{*Ti} = \sum_{p=0}^1 \omega_p^T(r, E) \cos p\theta. \quad (5)$$

The Fourier coefficients $\omega_p^{\parallel}$, ω_p^d and ω_p^T are computed via Discrete Fourier Transform (DFT), with truncation orders determined by numerical convergence tests.

The orbital average along the unperturbed orbit[6] is defined as

$$\langle \delta\phi_n \rangle_{orb}(r, \theta, E, \Lambda, \sigma) = \oint \frac{d\tau}{\tau_b} \delta\phi_n(\tilde{r}, \tilde{\theta}), \quad (6)$$

with $\tau_b = \oint d\tau$; the integral is taken over the unperturbed orbit determined by $H_0 = E$; $\tilde{\theta}(\tau; r, \theta, E, \Lambda, \sigma, \tau_0)$ and $\tilde{r}(\tau; r, \theta, E, \Lambda, \sigma, \tau_0)$ are the spatial points at $t = \tau$ on the orbit of the gyro-center launched from (r, θ) at $t = \tau_0$; $\tilde{\theta}(\tau_0; r, \theta, E, \Lambda, \sigma, \tau_0) = \theta$, $\tilde{r}(\tau_0; r, \theta, E, \Lambda, \sigma, \tau_0) = r$, $\dot{\tilde{\theta}} = \{\tilde{\theta}, H_0\}$, $\dot{\tilde{r}} = \{\tilde{r}, H_0\}$.

3. MAIN RESULTS

3.1. Passing ion

The modified GKE for the passing ions, which is written as

$$\begin{aligned} & \left\{ -\frac{\omega_1^{\parallel}}{2} (z+1) + \frac{1}{2} [(\omega_0^d + \frac{\omega_2^d}{2}) \frac{m-1}{m} - (\omega_0^d - \frac{\omega_2^d}{2}) \frac{nq\hat{s}}{m} \partial_z] \right\} \hat{g}(z+1) \\ & + \left\{ -\frac{\omega_1^{\parallel}}{2} (z-1) + \frac{1}{2} [(\omega_0^d + \frac{\omega_2^d}{2}) \frac{m+1}{m} + (\omega_0^d - \frac{\omega_2^d}{2}) \frac{nq\hat{s}}{m} \partial_z] \right\} \hat{g}(z-1) \\ & + \left[-\frac{\omega_2^{\parallel}}{2} (z+2) + \frac{1}{2} (\frac{\omega_1^d + \omega_3^d}{2} \frac{m-2}{m} - \frac{\omega_1^d - \omega_3^d}{2} \frac{nq\hat{s}}{m} \partial_z) \right] \hat{g}(z+2) \\ & + \left[-\frac{\omega_2^{\parallel}}{2} (z-2) + \frac{1}{2} (\frac{\omega_1^d + \omega_3^d}{2} \frac{m+2}{m} + \frac{\omega_1^d - \omega_3^d}{2} \frac{nq\hat{s}}{m} \partial_z) \right] \hat{g}(z-2) \\ & + [-\omega - \omega_0^{\parallel} z + \frac{1}{2} \omega_1^d] \hat{g}(z) = -\frac{e_i F_M}{T_i} \left\{ (\omega + \omega_0^T) J_0(\frac{k_{\perp} v_{\perp}}{\Omega_{ci}}) \delta\hat{\phi}(z) \right. \\ & \left. + \frac{\omega_1^T}{2} [\frac{m-1}{m} J_0(\frac{k_{\perp} v_{\perp}}{\Omega_{ci}}) \delta\hat{\phi}(z+1) + \frac{m+1}{m} J_0(\frac{k_{\perp} v_{\perp}}{\Omega_{ci}}) \delta\hat{\phi}(z-1)] \right\}. \end{aligned} \quad (7)$$

Here, the side-bands $p = \pm 1, \pm 2$ are retained. The additional toroidal coupling between the neighboring poloidal harmonics due to the PIMM effects is introduced into the GKE, and the additional toroidal coupling is from the poloidal variation of the parallel ion transit term ω_{ti} , the magnetic drift term ω_{di} , and the diamagnetic drift term ω_{*Ti} .

3.2. Trapped ion

Following the treatment of trapped electrons in Refs. [7] and [4], we derive the orbit-averaged GKE to obtain the nonadiabatic perturbed distribution function of the trapped ions in the $(r, \theta, \zeta, E, \Lambda, \sigma)$ coordinates:

$$g_n(r, \theta, E, \Lambda, \sigma) = e^{inq\theta} \frac{\omega + \bar{\omega}_{*Ti}}{\omega - \bar{\omega}_{di}} \frac{e_i F_M(r, E)}{T_i(r)} \langle e^{-inq\theta} J_0 \delta \phi_n \rangle_{orb}. \quad (8)$$

3.3. Poisson Equation

The non-adiabatic perturbed density of passing ions with the magnetic mirror effects in the Fourier space is written as

$$\begin{aligned} \delta \hat{n}_i^{(p)}(z) = & \sqrt{2\pi} m_i^{-\frac{3}{2}} \sum_{\sigma, l} \int_0^\infty \sqrt{E} dE \left[C_0^l J_0 \left(\frac{k_\perp v_\perp}{\Omega_{ci}} \right) \hat{g}(z, \mu_l, E, \sigma) \right. \\ & + \frac{C_1^l}{2} J_0 \left(\frac{k_\perp v_\perp}{\Omega_{ci}} \right) \hat{g}(z-1, \mu_l, E, \sigma) + \frac{C_1^l}{2} J_0 \left(\frac{k_\perp v_\perp}{\Omega_{ci}} \right) \hat{g}(z+1, \mu_l, E, \sigma) \\ & \left. + \frac{C_2^l}{2} J_0 \left(\frac{k_\perp v_\perp}{\Omega_{ci}} \right) \hat{g}(z-2, \mu_l, E, \sigma) + \frac{C_2^l}{2} J_0 \left(\frac{k_\perp v_\perp}{\Omega_{ci}} \right) \hat{g}(z+2, \mu_l, E, \sigma) \right]. \end{aligned} \quad (9)$$

The non-adiabatic perturbed density of the trapped ions in the Fourier space is written as

$$\begin{aligned} \delta \hat{n}_i^{(t)}(z) = & \iint \frac{\sqrt{E} dE d\Lambda}{\sqrt{2m_i^3 \epsilon_r}} \frac{\omega + \bar{\omega}_{*Ti}}{\omega - \bar{\omega}_{di}} \frac{e_i F_M(r, E)}{T_i(r)} \sum_{\sigma=\pm 1} \int_{-\theta_b}^{\theta_b} \frac{e^{-im\theta} d\theta / \sqrt{1 + \epsilon_r \cos \theta}}{\sqrt{\cos \theta - \cos \theta_b}} \\ & \times J_0 e^{inq\theta} \sum_p \langle e^{-i(\tilde{z}-p)\tilde{\theta}} J_0 \delta \hat{\phi}(\tilde{z}-p) \rangle_{orb}. \end{aligned} \quad (10)$$

The quasi-neutrality equation is written as

$$-\left(\frac{e_i^2}{T_i} + \frac{e_e^2}{T_e}\right) n_0 \delta \hat{\phi}(z) + e_i \left[\delta \hat{n}_i^{(p)}(z) + \delta \hat{n}_i^{(t)}(z) \right] = 0. \quad (11)$$

Here, the non-adiabatic part of the ion perturbed density is divided into passing and trapped parts: $\delta \hat{n}_i^{(p)}(z)$ and $\delta \hat{n}_i^{(t)}(z)$, which are determined by Eqs. (9) and (10), respectively. The boundary conditions are $\hat{g}_i(z, E, \Lambda, \sigma)|_{z \rightarrow \pm \infty} = 0$ and $\delta \hat{\phi}(z)|_{z \rightarrow \pm \infty} = 0$. The algebraic eigenvalue problem is solved numerically by combining the inverse power method and Newton's method.

3.4. numerical results

The upgraded Fourier-space eigenvalue code ESR shows good agreement with the well known ballooning-space eigenvalue code FULL.

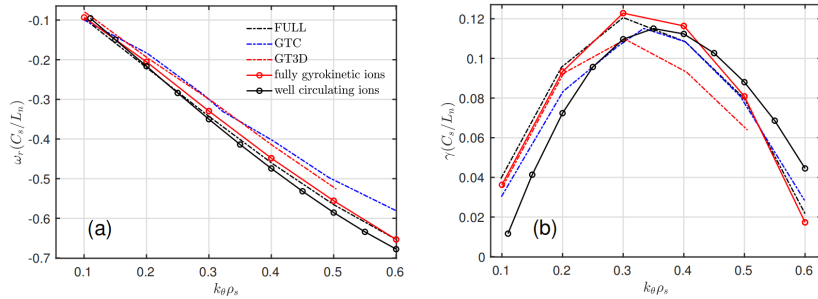


FIG. 2. The ITG linear (a) real frequency and (b) growth rate spectra with adiabatic electron response. The GTC, GT3D, and FULL data are from Ref. [8].

Instability analysis of the ITG mode with adiabatic electrons and full kinetic ions by the upgraded ESR code yields two primary characteristics: (i) In the low $k_\theta \rho_i$ (< 0.2) regime, the trapped-ion effects are dominate and enhance the growth rates; (ii) In the high $k_\theta \rho_i$ (> 0.5) regime, the PIMM effects are significant and reduce the growth rates.

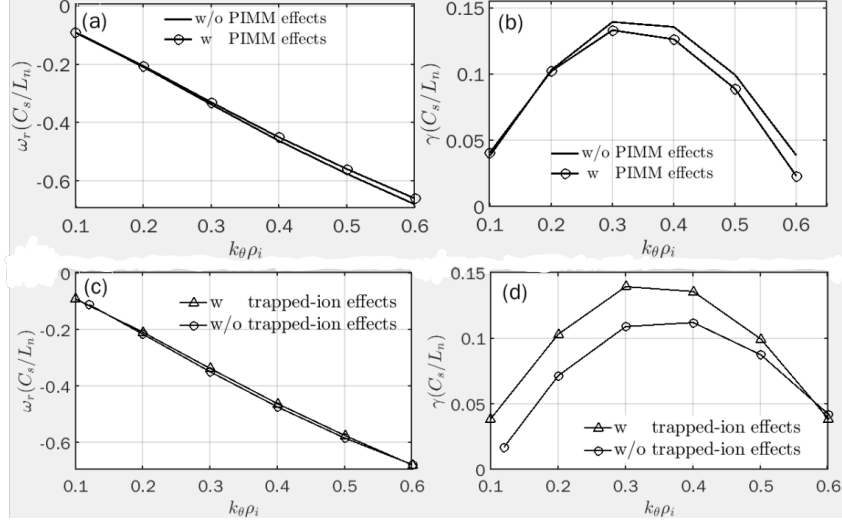


FIG. 3. Stabilizing effects of PIMM in the high $k_\theta \rho_i$ regime; destabilizing effects of trapped ions in the low $k_\theta \rho_i$ regime.

4. CONCLUSION

The instability analysis of the adiabatic electron ITG mode with magnetic mirror effects suggests that the PIMM effects stabilize the ITG mode mainly in the high $k_\theta \rho_i$ (> 0.5) regime, whereas the trapped-ion effects exhibit a strong destabilizing effect mainly in the low $k_\theta \rho_i$ (< 0.2) regime. This work provides a linear eigenmode analysis tool for investigating ITG instability in toroidal geometry with magnetic mirror effects in Fourier space.

REFERENCES

- [1] G Rewoldt, W. M Tang, and M. S Chance. In: Phys. Fluids 25.3 (1982), p. 480.
- [2] G Rewoldt, W. M Tang, and R. J Hastie. In: Phys. Fluids 30.3 (1987), p. 807.
- [3] J. Q Dong, W Horton, and J. Y Kim. In: Phys. Fluids B 4.7 (1992), p. 1867.
- [4] Jie Wang, Yuefeng Qiu, and Shaojie Wang. In: Phys. Plasmas 31.5 (2024), p. 052115.
- [5] Fulvio Zonca and Liu Chen. In: Phys. Fluids B 5.10 (1993), p. 3668.
- [6] Shaojie Wang, Zihao Wang, and Tiannan Wu. In: Phys. Rev. Lett. 132.6 (2024), p. 065106.
- [7] Shaojie Wang. In: Phys. Rev. E 64.5 (2001), p. 056404.
- [8] G Rewoldt, Z Lin, and Yasuhiro Idomura. In: Comput. Phys. Comm. 177.10 (2007), p. 775.