

Nonlinear saturation of toroidal Alfvén eigenmodes via ion induced scattering in nonuniform plasmas

Zhiyong Qiu¹, Liu Chen^{2,3}, Fulvio Zonca^{4,2}, Zhiwen Cheng² and Kexun Sheng²

¹CAS Key Laboratory of Frontier Physics in Controlled Nuclear Fusion and Institutes of Plasma Physics, Chinese Academy of Sciences, Hefei 230031, China

²Institute for Fusion Theory and Simulation, Zhejiang University, Hangzhou, P.R.C.

³Department of Physics and Astronomy, University of California, Irvine CA, U.S.A.

⁴Center for Nonlinear Plasma Science and ENEA, Frascati (Roma), Italy

Corresponding Author: zqiu@ipp.ac.cn

Abstract:

Motivated by recent work on kinetic Alfvén wave's (KAW's) parametric decay in nonuniform plasmas, nonlinear saturation of toroidal Alfvén eigenmode (TAE) in nonuniform plasmas is investigated. This saturation is achieved via the ion induced scattering process, in which a drift sound wave (DSW) quasi-mode is generated by the nonlinear coupling between TAEs and then heavily Landau damped. In the case of three-wave coupling, the parametric decay process of TAE is analysed, which shows that plasma nonuniformity can both quantitatively enhance the scattering cross-section and qualitatively change the decay behaviour. In the continuum limit with multiple TAEs co-exist, the three wave parametric dispersion relation is generalized into a spectral evolution equation describing the TAE nonlinear evolution. By numerically solving it under a reactor-relevant parameter regime, the nonlinear evolution and saturation process of TAE spectrum is reproduced. An estimation of the TAE saturation spectrum induced magnetic perturbation is also provided.

1 Introduction

In burning plasmas of next generation fusion devices, energetic particle physics are of crucial importance due to their ability of destabilizing shear Alfvén wave (SAW) instabilities. These SAW instabilities can further induce EPs anomalous transport across the magnetic surfaces, resulting in significant plasma performance degradation and even serious damage to the plasma facing components [1, 2, 3]. The EP transport rate is mainly determined by the saturation amplitude and spectrum of SAW instabilities, it is thus, of particular interest to investigate the nonlinear evolution and saturation process of SAW instabilities. In the past decades, taking toroidal Alfvén eigenmode (TAE) as a paradigm, nonlinear saturation mechanisms for SAW instabilities have been investigated [4, 5, 6]. One of the

crucial mode coupling channel for TAE to saturate is the ion induced scattering process, in which a TAE decays into another counter-propagating TAE and a heavily ion Landau damped ion sound wave (ISW) or drift sound wave (DSW) quasi-mode. While previous works on ion induced scattering focused on uniform plasmas [7, 8], a recent work found that plasma nonuniformity can both quantitatively and qualitatively change the parametric decay process of kinetic Alfvén waves (KAWs) [9]. Motivated by it, in this work, the nonlinear saturation of TAE via ion induced scattering is investigated in the burning plasma relevant short wavelength ($k_{\perp}^2 \rho_i^2 \gg \omega/\Omega_{ci}$) regime, with particular attention to the role of plasma nonuniformities in determining both qualitatively and quantitatively the evolution process of TAE. Here, $k_{\perp}$ is the perpendicular wavenumber, ρ_i and Ω_{ci} correspond to the gyroradius and gyrofrequency of ion. Specifically, our theory includes (1) nonlinear parametric decay of TAE via ion induced scattering, where the conditions for spontaneous decay is discussed; and (2) nonlinear equation describing the resulting TAE spectral energy evolution is derived and solved, yielding the saturated spectrum of TAE in nonuniform plasmas. The nonlinear saturation levels of TAEs induced electromagnetic perturbation are derived from first-principle-based theory, allowing us to estimate the consequences on EP transport and potentially induced intrinsic rotation.

In the first part of this work, we investigate the parametric decay process of TAE via nonlinear ion induced scattering in nonuniform plasmas. This nonlinear coupling process was originally investigated in Ref. [7] in the long wavelength MHD limit, and was extended to the burning plasma relevant short wavelength regime [8]. With the new ingredients associated with plasma nonuniformity included here, it is found that, the nonuniformity associated with bulk plasma density gradient will significantly enhance the ion induced scattering, and changing the condition for spontaneous decay from decaying into lower frequency TAE sideband, to decaying into TAE sidebands with higher toroidal mode number [10].

In the second part, by summation over all the background TAEs within strong interaction region, the equation describing a test TAE nonlinear evolution due to the interaction with the background TAEs is derived, which is then applied to deriving the equation for the TAE spectrum evolution in the k_{θ} space [11]. The spectrum evolution equation is then numerically solved as an initial value problem to obtain the saturation spectrum of TAE, yielding an overall fluctuation level much lower than that predicted by Refs. [7, 8] as a consequence of the further enhanced nonlinear couplings due to plasma nonuniformity. The implication on plasma intrinsic rotation is also discussed.

2 Parametric decay of TAE in nonuniform plasma

In the first part, the TAE ion-induced scattering process is investigated in nonuniform plasmas using a three-wave parametric decay model [10]. In this system, we consider the process in which a pump TAE decays into a sideband TAE and a DSW quasi-mode, which generally suffers significant ion Landau damping and contributes to the nonlinear saturation of the original TAE. This process was first investigated in uniform plasmas with the sideband being an ISW quasi-mode [7]. Based on drift kinetic theory, the governing equa-

tion describing TAE spectrum energy transfer was derived in the long-wavelength regime with $k_{\perp}^2 \rho_i^2 \ll \omega/\Omega_{ci}$. Further analysis found that this process could spontaneously occur when the sideband TAE frequency is lower than that of the pump TAE, which implies a spontaneous energy transfer to the lower-frequency component of the TAE spectrum and the final saturation due to enhanced coupling to the lower part of the SAW continuum. An investigation on the potential fuel ion heating was also provided [12]. A later work extended this mechanism into the short-wavelength regime with $k_{\perp}^2 \rho_i^2 \gg \omega/\Omega_{ci}$, which is more relevant to next-generation reactors [8]. Using gyrokinetic theory, the parametric dispersion relation describing this ion-induced scattering process was derived, which showed an enhanced nonlinear coupling coefficient due to the inclusion of perpendicular nonlinearities, i.e., Reynolds and Maxwell stresses. The TAE spectrum evolution equation was then obtained by summing over background TAEs and adopting the continuum limit, assuming many background TAEs within the strong interaction range. Using quasi-linear transport theory [8, 3], the resultant EP anomalous transport rate induced by the saturated electromagnetic perturbation was also derived.

The inclusion of plasma nonuniformity is motivated by a recent work on KAW parametric decay process [9]. Here, plasma nonuniformity refers particularly to the plasma density or temperature profile, which contributes to the ion diamagnetic drift frequency ω_{*i}^t . Considering the process of a pump KAW decaying into a counter-propagating KAW and a DSW, it was demonstrated that the plasma nonuniformity, not only quantitatively significantly enhance the nonlinear coupling cross-section, but also qualitatively change the parity of the resultant KAW spectrum.

Similarly, to check effects of plasma nonuniformity on TAE's nonlinear decay, consider the interaction between a test TAE $\Omega_0(\omega_0, \mathbf{k}_0)$ and a counter-propagating background TAE $\Omega_1(\omega_1, \mathbf{k}_1)$ and a DSW $\Omega_s(\omega_s, \mathbf{k}_s)$, with the matching condition $\Omega_0 = \Omega_1 + \Omega_s$ assumed. Following the standard procedure of nonlinear gyrokinetic theory [13], the nonlinear equations describing DSW generation, as well as feedback to Ω_0 by nonlinear coupling between Ω_1 and Ω_s , are derived, which then yield the parametric decay dispersion relation.

The nonlinear equation for the predominantly electrostatic DSW Ω_s generation can be derived, by substituting the nonlinear particle responses to Ω_s into the Q.N. condition, and one obtains

$$\varepsilon_{s*} \delta\phi_s = -i \frac{\Lambda_0^1}{\omega_0} \alpha_{s*} \delta\phi_0 \delta\phi_{1*}. \quad (1)$$

Here, $\varepsilon_{s*} \equiv 1 + \tau + \tau \Gamma_s \xi_s Z(\xi_s) (1 - \omega_{*i,s}/\omega_s)$ is the linear dispersion relation of Ω_s , $\alpha_{s*} \equiv \tau F_1 [1 + \xi_s Z(\xi_s) (1 - \omega_{*i,s}/\omega_s)] + \sigma_{0*} \sigma_{1*}$ is the nonlinear coupling coefficient, $\tau \equiv T_e/T_i$, $\Gamma_k \equiv \langle J_k^2 F_M / N_0 \rangle$, $F_1 \equiv \langle J_0 J_1 J_s F_M / N_0 \rangle$ with $\langle \cdots \rangle$ accounting for velocity space integration, $Z(\xi_s)$ is the well-known plasma dispersion function with $\xi_s \equiv \omega/|k_{\parallel s} v_i|$, and $\sigma_{k*} \equiv [1 + \tau - \tau \Gamma_k (1 - \omega_{*i,k}/\omega_k)] / (1 - \omega_{*e,k}/\omega_k)$ is the ratio between $\delta\psi$ and $\delta\phi$.

The nonlinear equation for test TAE including the feedback of DSW Ω_s and background TAE Ω_1 , can be obtained by the coupling between quasi-neutrality condition and

nonlinear gyrokinetic vorticity equation,

$$\left[\varepsilon_{A0} + \left(\frac{\Lambda_0^1}{\omega_0} \right)^2 \varepsilon_{A0}^{\text{NL}} |\delta\phi_1|^2 \right] \delta\phi_0 = i \frac{\Lambda_0^1}{\omega_0} \alpha_{0*} \delta\phi_1 \delta\phi_s. \quad (2)$$

Here, $\varepsilon_{A0} \equiv (1 - \Gamma_0)(1 - \omega_{*i,0}/\omega_0)/b_0 - k_{\parallel 0}^2 v_A^2 \sigma_{0*}/\omega_0^2$ is the linear dispersion relation of the test TAE, $\varepsilon_{A0}^{\text{NL}} \equiv [(F_2 - F_1)/b_0 - k_{\parallel 0}^2 v_A^2 \tau F_2/\omega_0^2][1 + \xi_s Z(\xi_s)(1 - \omega_{*i,s}/\omega_s)] + k_{\parallel 0}^2 v_A^2 \sigma_{0*} \sigma_{1*}^2/\omega_0^2$, and $\alpha_{0*} \equiv (1/b_0 - \tau k_{\parallel 0}^2 v_A^2/\omega_0^2)[1 + \xi_s Z(\xi_s)(1 - \omega_{*i,s}/\omega_s)]F_1 + (\sigma_{1*} - \varepsilon_{s*})/(\tau b_0)$ are the nonlinear coupling coefficients between test TAE and DSW, with $b_0 \equiv k_{\perp}^2 \rho_i^2/2$ and $F_2 \equiv \langle J_0^2 J_1^2 F_M/N_0 \rangle$. Note that in the derivation of equation (2), both the linear and nonlinear particle responses to DSW Ω_s are included, since it is generally a heavily damped quasi-mode.

Combining eqs. (1) and (2), the parametric dispersion relation can be derived as,

$$\varepsilon_{A0} |\delta\phi_0|^2 = - \left(\Delta + \chi_0 \varepsilon_{s*} - \frac{C_0}{\varepsilon_{s*}} \right) |\delta\phi_0|^2 |\delta\phi_1|^2. \quad (3)$$

Here, $\Delta \equiv (\Lambda_0^1/\omega_0)^2 [\sigma_{0*}^2 \sigma_{1*} - 2(F_1/\Gamma_s)(\sigma_{0*} \sigma_{1*} - F_1 \sigma_s/\Gamma_s) - F_2 \sigma_s/\Gamma_s]/(\tau b_0 \sigma_{0*})$, $\chi_0 \equiv (\Lambda_0^1/\omega_0)^2 [F_2/\Gamma_s - (F_1/\Gamma_s)^2]/(\tau b_0 \sigma_{0*})$ and $C_0 \equiv (\Lambda_0^1/\omega_0)^2 (\sigma_{0*} \sigma_{1*} - F_1 \sigma_s/\Gamma_s)^2/(\tau b_0 \sigma_{0*})$ stand for nonlinear frequency shift, ion Compton scattering and shielded-ion scattering, respectively. Equation (3) describes the evolution of test TAE Ω_0 due to interacting with the background TAE Ω_1 . By assuming Ω_0 as an eigenmode, one can expand $\varepsilon_{A0} \simeq i(\gamma + \gamma_{A0})\partial_{\omega_0} \varepsilon_{A1,R}$ and take the imaginary component of equation (3) as,

$$\gamma + \gamma_{A0} = \frac{|\delta\phi_0|^2}{\partial \varepsilon_{A0,R}/\partial \omega_0} \left(\chi_0 + \frac{C_1}{|\varepsilon_{s*}|^2} \right) \varepsilon_{s*,I}. \quad (4)$$

Here, γ represents the parametric growth rate, $\gamma_{A0} \equiv \varepsilon_{A0,I}/\partial_{\omega_0} \varepsilon_{A1,R}$ is the linear damping rate of Ω_1 . Recalling that $\varepsilon_{s*,I} \equiv \tau \Gamma_s \xi_s \text{Im}(Z_s)(1 - \omega_{*i,s}/\omega_s)$, equation (4) has similar expression as the case of uniform plasma [8], with the main difference coming from the $\omega_{*i,s}/\omega_s$ term of linear DSW dispersion relation. Noting that the nonlinear drive term in equation (4) is proportional to $\varepsilon_{s*,I}$, the scattering cross-section is expected to be greatly enhanced by $|\omega_{*i,s}/\omega_s| \gg 1$ in nonuniform plasmas. The dependence of nonlinear drive on frequency is plotted in figure 1, with the horizontal axis ξ_s representing normalized DSW frequency and the vertical axis ε_{s*} standing for the strength of nonlinear drive. Comparison between uniform and nonuniform plasmas shows that, (1) in uniform plasmas, the nonlinear drive is maximized when test and background TAEs have a frequency difference about the ion transit frequency ω_{tr} , (2) in nonuniform plasmas, nonlinear drive is maximized when test and background TAEs have similar frequencies. Apart from the different optimized conditions for scattering cross-section, the difference in spontaneous decay condition can be shown by letting $\gamma > 0$, which is equivalent to $\varepsilon_{s*,I} > 0$. (1) In uniform plasmas with $\omega_{*i} = 0$, this requires $\omega_s > 0$ thus $\omega_0 > \omega_1$, resulting in a frequency downward decay process. (2) In nonuniform plasmas, this requires $(1 - \omega_{*i,s}/\omega_s)\xi_s > 0$ and approximately, $\omega_{*i,s} \propto k_{\theta 1} - k_{\theta 0} > 0$, which implies a normal cascading in toroidal mode number space, i.e., $|n_1| > |n_0|$.

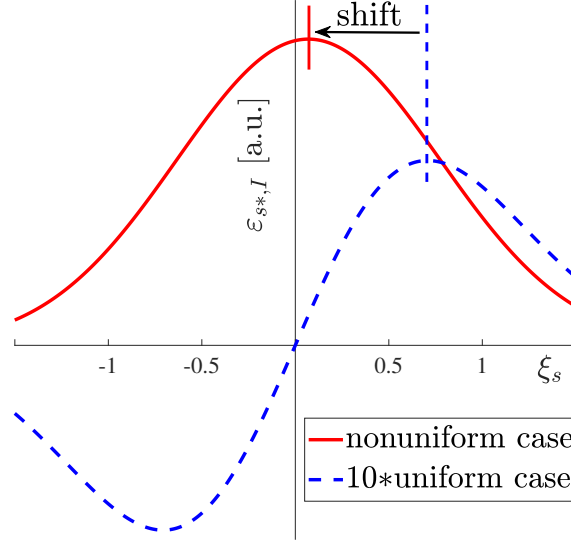


FIG. 1: $\varepsilon_{s*,I}$ in arbitrary units v.s. ξ_s , with the blue dashed curve standing for uniform case multiplied by factor 10, and the red solid curve for nonuniform case.

So far, the TAE's ion induced scattering has been fully investigated in the three wave parametric decay model, which shows completely different behaviours due to plasma nonuniformity. Considering burning plasmas in ITER-like tokamaks where a rich spectrum of TAEs exist, the nonlinear evolution behavior of TAE spectrum could be significantly influenced due to this ion induced scattering process in nonuniform plasmas. Thus, in the next section, based on this three wave parametric decay model, we will consider multiple wave case and investigate the nonlinear evolution process of TAE spectrum in nonuniform plasmas.

3 TAE spectrum evolution and saturated e.m. perturbation

In the second part, we extend the three wave parametric decay process to multiple wave case [11]. The equation describing nonlinear evolution of TAE spectral intensity is derived based on the original parametric dispersion relation, and then transformed into k_θ space, in correspondence with TAEs' cascading behaviour in k_θ in nonuniform plasmas.

Considering the process of a test TAE Ω_0 interacting with many background TAEs, the nonlinear equation can be written as,

$$\varepsilon_{A0}|\delta\phi_0|^2 = - \sum_{k_1} \left(\Delta + \chi_0\varepsilon_{s*} - \frac{C_0}{\varepsilon_{s*}} \right) |\delta\phi_0|^2 |\delta\phi_1|^2. \quad (5)$$

Equation (5) is consistent with the parametric dispersion relation derived in last section, with an additional summation of the nonlinear terms over background TAEs. From the three wave analysis, background TAEs with lower n contribute to a nonlinear drive effect while those with high n contribute to a nonlinear damping effect to test TAE.

For the analytical treatment of equation (5), the linear dispersion relation of test TAE is expanded as $\varepsilon_{A0} \simeq 2i(\gamma - \gamma_L)/\omega_0$. The radial-mode-width-averaged spectral intensity is also assumed for both test and background TAEs by letting $|(e/T_e)^2|\delta\phi_k|^2 = I(k_\theta) \exp[-(x_k \pm \Delta_k/2)^2/\Delta_k^2]$ with $I(k_\theta)$ being the TAE spectral intensity, $\Delta_k \equiv -1/(k_\theta S)$ being the distance between the neighbouring rational surfaces, S being the magnetic shear and $x_k \equiv (nq - m)/(nq')$. Also, to perform the mode summation over background TAEs, following [14], the continuum approximation is adopted, which transfers the summation over n_1 and m_1 into integration over $k_{\theta 1}$ and x_1 . On the other hand, certain simplifications are employed, particularly to the scattering terms, i.e., neglecting the shielded-ion scattering term, expanding the ion Compton scattering term at $b \equiv k_\perp^2 \rho_i^2/2 \lesssim 1$, and transforming frequency terms into k_θ space by $\omega_s = \omega_0 - \omega_1 \simeq (k_{\theta 1} - k_{\theta 0})\partial\omega/\partial k_{\theta 0}$. Given all these assumptions and approximations, the imaginary part of the above equation can be reduced to a dimensionless form,

$$\begin{aligned} \frac{\partial_t - 2\gamma_L}{\omega_{0R}} I(\hat{k}, t) A B^6 \hat{k}^4 = & - \left[\frac{\sqrt{\pi}}{2} y(3 + 2y^2)(1 + \text{Erf}(y)) + e^{-y^2}(1 + y^2) \right] I^2(\hat{k}) \\ & - \frac{1}{16B\hat{k}} \left[\sqrt{\pi}(15 + 4y^2(9 + y^2))(1 + \text{Erf}(y)) + 2e^{-y^2}y(17 + 2y^2) \right] I \frac{\partial I}{\partial \hat{k}}. \end{aligned} \quad (6)$$

Here, $\hat{k} = |k_{\theta 0} \rho_i|$, $A \equiv 32(\omega_0/\Omega_{ci})^2 L_N \rho_i / (\pi \tau^2 S^2 r R)$, $B \equiv (2qR/v_i)\partial\omega/\partial \hat{k}^2 \simeq (3/4 + \tau)/(2\beta_i^{1/2})$ with the linear dispersion relation $\omega = k_\parallel v_A \sqrt{1 + (3/4 + \tau)k_\perp^2 \rho_i^2}$ containing the lowest order FLR effects used and β_i contributed by only thermal ions. $y \equiv B\hat{k}^2$, and $\text{Erf}(y) \equiv 2/\sqrt{\pi} \int_0^y \exp(-\eta^2) d\eta$ is the error function. Equation (6) is the desired form of the nonlinear spectral evolution equation, which is generally a first order partial differential equation containing nonlinear terms and integration terms, thus is difficult to solve analytically.

However, equation (6) can be solved numerically as an initial value problem with the explicit form of $\gamma_L(\hat{k})$ available. Consider a reactor-relevant case, where TAEs are located at $r/a \simeq 0.2$ and linearly unstable at the range of $n = 10 - 50$, with the most unstable mode being $n = 30$. Other parameters are taken as ITER relevant values, i.e., $B_0 = 5.3$ T, $R = 6.2$ m, $a = 2$ m, $\beta_i = 0.01$, $q = 1$, $S = 0.1$, and $R/L_N = 5$ with L_N being the scale length for density nonuniformity. With the corresponding $\gamma_L(\hat{k})$ distribution shown as figure 2(a) and a random initial perturbation shown as figure 2(b), the nonlinear evolution process of TAE spectrum in k_θ space is reproduced by numerically solving the wave-kinetic equation (6). In figure 2(c), the TAE spectrum intensity is excited by γ_L , which then drive the TAE spectrum to a maximized intensity after hundreds of Alfvén times, as in figure 2(d). Meanwhile, the nonlinear interaction term also takes effect by shifting the most unstable mode to a larger k_θ . Finally, in figure 2(e), the linear stable region is also nonlinearly excited by lower n TAEs and the TAE spectrum reaches a steady state. Figure 2 (f) shows a good agreement of TAE saturation spectrum between

the initial value solution and fixed point solution.

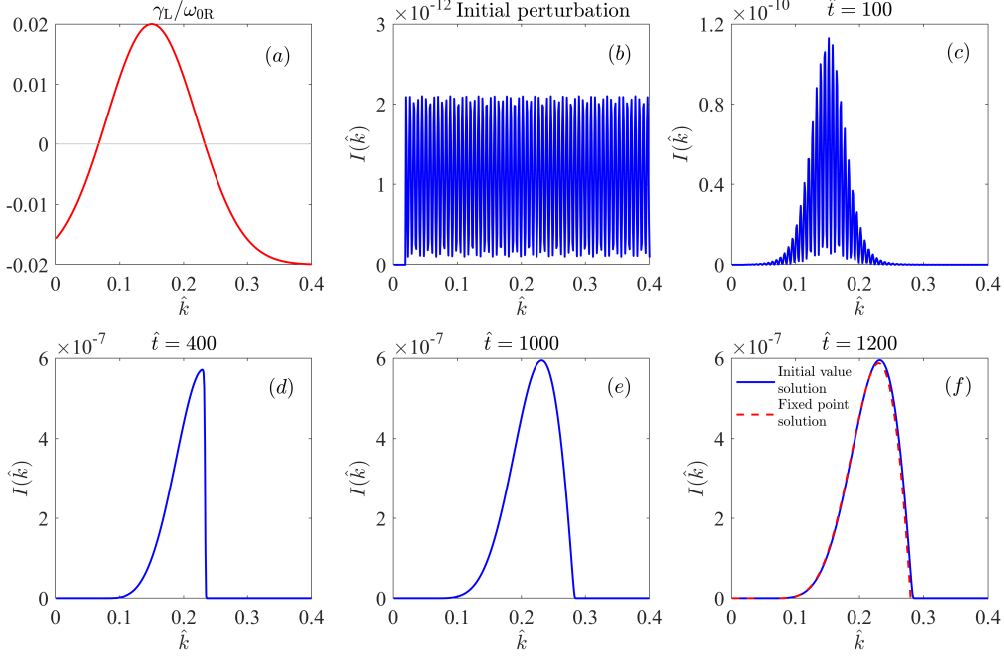


FIG. 2: Plots of TAE spectrum evolution and saturation process by initial value solution and comparison with fixed point solution. Here, $\hat{t} \equiv \omega_0 t$, figures (a) and (b) are the provided linear growth rate distribution and random initial perturbation respectively. The evolution and saturation process of TAE spectrum is shown in figures (c), (d) and (e). A comparison of the final saturation spectrum with fixed point solution is given in figure (f).

On the other hand, with the TAE saturation spectrum provided, one can estimate the magnetic perturbation amplitude associated with it. Using $|\delta B_r|^2 = |k_\theta \delta A_\parallel|^2$ and noting $\delta\phi \sim \delta\psi$ in ideal MHD condition, the saturation spectrum induced magnetic perturbation can be estimated as,

$$\left| \frac{\delta B_r}{B_0} \right|^2 = \sum_k \frac{\tau^2}{4} \frac{\Omega_{ci}^2}{\omega^2} \frac{e^2}{T_e^2} k_\parallel^2 k_\theta^2 \rho_i^4 |\delta\phi_k|^2 = \frac{3\sqrt{\pi}}{16} \frac{\tau^2 r \rho_i}{q^3 R^2} \frac{\Omega_{ci}^2}{\omega^2} \int d\hat{k} \hat{k}^2 I_k \sim \mathcal{O}(10^{-8}).$$

Remarkably, in figure 2 the TAE spectrum exhibits an obvious energy transfer towards larger k_θ region. This evolution process implies a potential momentum transport process, which could contribute to the plasma intrinsic rotation and be beneficial to the plasma confinement [15]. Also, the saturation mechanism presented in this work requires verification by large-scale simulations. A simulation of three wave parametric decay process with the pump TAE driven by an antenna is in progress, which may provide a qualitatively picture of the mode coupling process. However, these works are beyond the present scope and will be reported in a future publication.

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