

CONFERENCE PRE-PRINT**NONLINEAR SPECTRUM EVOLUTION OF LOWER HYBRID WAVES AND DENSITY LIMIT OF LOWER HYBRID CURRENT DRIVE**

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Abstract

Noninductive current drive is essential for steady-state operation on tokamaks. The density limit, i. e. strong efficiency degradation of lower hybrid current drive (LHCD) at high density, makes the effectiveness of LHCD questionable on ITER and future fusion reactors. This paper provides a self-consistent modelling and simulation of LHWs in the SOL plasma by coupling the propagation of waves to the power transfer among waves by parametric instabilities (PIs) in a 3D model. LHW spectrum broadening both in wave frequency and wave number was obtained self-consistently, and, correspondingly, the power flux of the pump was converted to sideband waves. When the density increases exceeding a critical value, stronger PIs transfer most of the pump power flux, resulting in the density limit of LHCD. For the first time, the density limit observed in LHCD experiments, as well as various efforts to overcome/increase the density limit of LHCD, can be successfully reproduced by the simulation. Based on simulation results and simplified theoretical analysis, a scaling relation of the density limit of LHCD was obtained. According to this scaling, higher LHW frequency, stronger magnetic field and higher periphery plasma temperature makes LHCD remain a promising method of driving plasma current for ITER and future fusion reactors.

1. INTRODUCTION

Noninductive current drive is essential for steady-state operation on tokamaks. In the past few decades, lower hybrid current drive (LHCD) has recorded the highest efficiency of driving plasma current among all available methods. However, LHCD faces serious challenge of density limit, an anomalous efficiency loss at a plasma density much lower than the limit of wave accessibility. The density limits of LHCD observed on various tokamaks including JET [1], EAST [2], Alcator C-Mod [3] and Tore Supra [4] are significantly lower than the required level for ITER, which makes the effectiveness of LHCD questionable on ITER and future fusion reactors. It might be one of the main reasons that the lower hybrid wave (LHW) system was removed from the second stage upgrade option of heating and current driving (H&CD) systems on ITER [5].

Various efforts have been made to understand and overcome the density limit of LHCD. A common understanding is that it can be attributed to wave-plasma interactions in the scrape-off layer (SOL) plasma, typically the parametric decay instabilities (PDI or PI) process. PDI is a common and important nonlinear wave-wave interaction. It can be understood as that a pump wave is scattered by the low frequency oscillation or wave in plasma and decays into the sideband. The instability means this process is in positive feedback. PDIs can arise a degradation of efficiency of H&CD and/or unexpected power deposition. On FTU [6], EAST [7] and C-Mod [8], better LHCD efficiencies can be obtained when PDIs were suppressed by different approaches. However, there is still an absence of a comprehensive theoretical framework capable of unifying the experimental findings. Also, the lack of quantitative results makes still uncertain in predicting LHCD efficiency under ITER parameters.

The earlier theories [9–14] of parametric instabilities established in the 1970s mainly focused on stimulated scattering in laser plasmas. There the daughter waves are propagating waves, called resonant decay, and the quasi-linear approach is enough to describe the coupling. Convective instabilities can be saturated by the mismatch of wavenumbers due to plasma inhomogeneity or finite width of the pump. However, due to strong driving and weak damping, the absolute instabilities are usually excited in laser plasma, so other nonlinearities should be involved.

However, in magnetic confinement fusion (MCF) plasmas, PDIs are typically displayed as quasi-mode decay, such as nonlinear ion Landau damping (usually called ion sound quasi-mode decay, ISQM decay) or nonlinear ion cyclotron damping (usually called ion cyclotron quasi-mode decay, ICQM decay). Since the linear dispersion relation cannot describe the low frequency wave well, early researches encountered two difficulties. The first is the insufficiency of the quasi-linear approach for coupling. The other is that the mismatch of the wavenumbers due to plasma inhomogeneity cannot be given by the linear dispersion relation. Therefore, both absolute instabilities and convective instabilities, which were studied extensively for laser plasma, are not properly calculated for MCF plasma, leading to the quantitative failure.

In this work, we will first introduce a technique to describe the nonlinear coupling during the PDI and make the local model to be valid in the scenarios where non-resonant quasi-mode decays are dominant, which is displayed in Section 2, accompanying with main results from the local model. In Section 3, the nonlocal model is introduced, in which the key issue is how to describe the plasma inhomogeneity in the quasi-mode decay. And the absolute instabilities and convective instabilities are solved from the improved nonlocal model. Based on the theories above, a simulation model is developed in section 4. And spectral evolution and the density limit in this self-consistent modelling during propagation of lower hybrid waves with parametric instabilities are presented in section 5. Section 6 is devoted to a summary.

2. NONLINEAR COUPLING AND THE LOCAL MODEL

2.1. Nonlinear coupling in PDIs

Firstly, let us examine the PDI coupling for quasi-mode decay [15]. The PDI comes from the fact that the interaction between the pump and the low frequency daughter wave excites the sideband response and the interaction between the pump and the sideband excites the low frequency response, that is,

$$\begin{aligned} f_{s,LF}^{NL} &= C(f_{s,0}, \mathbf{E}_1) + D(f_{s,1}, \mathbf{E}_0) \\ f_{s,1}^{NL} &= E(f_{s,0}, \mathbf{E}_{LF}) + F(f_{s,LF}, \mathbf{E}_0) \end{aligned} \quad (1)$$

where $f_{s,j}$ refers to the distribution function response of particle s to the electric field $\mathbf{E}$ of wave j . $j = 0, 1, LF$ represent the pump, sideband and low frequency wave respectively. The superscripts L, QL and NL mean the linear, quasi-linear and nonlinear response respectively. C , D , E and F are special operators shown in Ref. [15]. The sideband and low frequency wave are coupled with each other and difficult to solve. However, for resonant decay, the linear response is much larger than the nonlinear response, $f^L \gg f^{NL}$, then we can use the linear response to replace the total response, that is

$$\begin{aligned} f_{s,LF}^{QL} &= C(f_{s,0}^L, \mathbf{E}_1) + D(f_{s,1}^L, \mathbf{E}_0) \\ f_{s,1}^{QL} &= E(f_{s,0}^L, \mathbf{E}_{LF}) + F(f_{s,LF}^L, \mathbf{E}_0) \end{aligned} \quad (2)$$

This is the well-known quasi-linear approach. These two equations are decoupled. However, for quasi-mode decay, the sideband is still resonant $f_1^L \gg f_1^{NL}$, but the low frequency wave is far from the resonance $f_{LF}^L \sim f_{LF}^{NL}$. In this case, the linear part is not enough for describing the low frequency response coming from the interaction between the pump and the sideband. Thus, its nonlinear response should be included, which is still approximated by the quasi-linear part as follows,

$$\begin{aligned} f_{s,LF}^{NL} &\approx f_{s,LF}^{QL} = C(f_{s,0}^L, \mathbf{E}_1) + D(f_{s,1}^L, \mathbf{E}_0) \\ f_{s,1}^{NL} &\approx E(f_{s,0}^L, \mathbf{E}_{LF}) + F(f_{s,LF}^L + f_{s,LF}^{QL}, \mathbf{E}_0) \end{aligned} \quad (3)$$

Our calculation proved only one iteration can reach the convergence and get the new physics without more iteration [16]. This so-called QL-NL treatment makes the coupling more complex than the QL approach. But fortunately, the coupling is mainly electrostatic (ES) and fluid, which is showed in our recent work [15]. Neglecting the electromagnetic (EM) effect and the kinetic effect in the coupling coefficient only results in a slight change in most-unstable region and growth rate of the instability, if the kinetic effect in the linear response and the EM effect in the linear dispersion relation of the pump remained. On the discussion of NL treatment and EM effect, more details can be found in other two references [16,17]. This makes it possible to use the simplest kinetic-fluid hybrid and ES model to continue the following study.

2.2 Main results from the local model

The local model means both wave and plasma are homogeneous. With correct treatment of nonlinearity, the local model can be employed to investigate the excitation channels and parameter dependences of PDI, which are shown in previous publications [15,18]. Here are only main results introduced here. At very low density regime, the ISQM decay instability is dominant but can be usually suppressed by increasing plasma density. On the contrary, the ICQM decay instability is destabilized by increasing plasma density. Both these two kinds of PDIs can be suppressed by plasma temperature and magnetic field.

Fig.1 show the maximum linear growth rates of different decay channels around the SOL is calculated by the local model using typical plasma parameters and wave parameters of JET [19]. It is clear that As the pump wave

propagates into the plasma, the linear growth rates of the ISQM channel drop quickly due to high plasma density. And those of the ICQM channels gradually reach the maximum value where the plasma density has been high but the plasma temperature is still low, and decrease due to higher temperature.

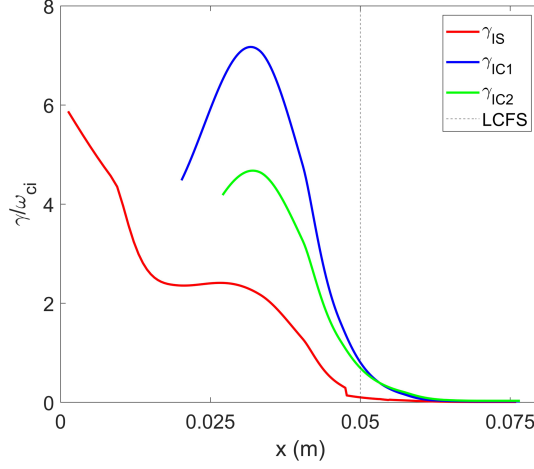


Fig. 1 The spatial distribution of the maximum linear growth rates normalized by the ion cyclotron frequency along the x -axis. The red, blue and green curves indicate the maximum linear growth rates of the ISQM channel, the 1st order and 2nd order ICQM channel respectively.

3. NONLOCAL THEORETICAL MODEL

3.1. From local to nonlocal

Now it is needed to develop the nonlocal model from the local model by introducing the plasma inhomogeneity. The coupled equations are the same,

$$\begin{cases} \varepsilon_{LF} \phi_{LF} = \alpha_{LF \leftarrow 1} \phi_0^* \phi_{LF} \\ \varepsilon_1 \phi_1 = \alpha_{1 \leftarrow LF} \phi_0 \phi_1 \end{cases} \quad (4)$$

where ε_j is the dielectric constant and ϕ_j is the potential of wave j . $\alpha_{i \leftarrow j}$ are the coupling coefficients and $*$ refers to the complex conjugation. But for the nonlocal model, ω_j and k_j in the dielectric constants are changed into $\omega_j + i\partial_t$ and $k_j - i\partial_x$, that is,

$$\varepsilon_j(\omega_j, k_j) \rightarrow \varepsilon_j(\omega_j + i\partial_t, k_j - i\partial_x) \quad (5)$$

The physics is that the wavenumber is changed at different x , which is set to be the direction of the plasma inhomogeneity, i.e.

$$\Phi_j = \phi_j(x, t) \exp[i\mathbf{k}_j \cdot \mathbf{x} - i\omega_j t + i \int \Delta k_j dt] \quad (6)$$

Then the match condition of the wavenumbers is destroyed when the waves propagate out from the resonant point where the PDI is excited. This is the saturation mechanism of convective instabilities due to plasma inhomogeneity developed by Rosenbluth [9]. However, as we mentioned before, the low frequency wave is far from the linear dispersion relation for quasi-mode decay, so it is difficult to calculate the mismatch actually. But some previous works [20, 21] just neglect this effect or use some assumptions with no clear physics.

In fact, we do not need to put the effect of plasma inhomogeneity in the wavenumber term, but put it directly to the amplitude envelop term and the dielectric constants become the function of x , that is,

$$\varepsilon_j(\omega_j, k_j, x=0) \rightarrow \varepsilon_j(\omega_j + i\partial_t, k_{jx} - i\partial_x, x) \quad (7)$$

When the wave is propagating out from the resonant point ($x=0$), a differential term of the dielectric constants appears to characterize the inhomogeneity, that is,

$$\left[\varepsilon_j(\omega_j, k_j, x) + \frac{\partial \varepsilon_j}{\partial \omega_j} \cdot i \partial_t - \frac{\partial \varepsilon_j}{\partial k_j} \cdot i \partial_x + \frac{\partial \varepsilon_j}{\partial x} \cdot x \right] \phi_j(x, t) = \alpha_{j \leftarrow i} \phi_0(x, t) \phi_i(x, t) \quad (8)$$

Then the calculation of wavenumber is avoided. The cost is that the coupled differential equations become more complex. It is easy to verify that it can reduce to the Rosenbluth form in cases of resonant decay where the real parts of the dielectric constants are always zero.

Eliminating time by Laplace transform $t \rightarrow p$, the two nonlocal coupling equations turn to a Schrödinger form,

$$\left[\frac{\partial^2}{\partial X^2} - \left(\frac{1}{4} ((iK_1 + K_2)X + \sigma)^2 - \lambda - \frac{iK_1}{2} \right) \right] A_i(X, p) = A_{iF}(X) |_{t=0} \quad (9)$$

The dimensionless parameters are as follows, coordinate X , amplitude A , plasma inhomogeneity K_1 , finite pump profile K_2 , growth and/or damping rate σ , and the coupling coefficient λ .

This seems to be the initial value of low frequency wave possibly from the thermal noise. However, in fact, it is an initial value problem only for the convective instabilities. For the absolute instabilities, it is an eigenmode problem of the Schrödinger form. The difficulty is that the Hamiltonian is non-Hermitian, but a complex function.

3.2. Absolute instabilities and convective instabilities

If the absolute instabilities exit, the PDI can not be saturated by plasma inhomogeneity. In this case, the solutions are the superposition of temporal eigenmodes. For the absolute instabilities, accurate solution of the eigenmode problem is almost impossible. We just try to estimate the threshold of a quasi-mode PDI with a WKB analysis on the complex plane for finite pump profile. The mathematics is very complex, and details are in recent paper [23]. Here the necessary condition of the absolute instabilities is directly given by,

$$\lambda > \exp \left[\frac{1}{4} \text{Re}(\sigma^2) \right] \quad (10)$$

That is, for resonant decay, it reduces to previous result by White [11],

$$\lambda > 1 + \frac{1}{4} \sigma^2 \quad (11)$$

When the PDI coupling is stronger than the damping, there will be absolute instabilities. However, for quasi-mode decay, the damping is strong and, in the exponentials, so there is usually no absolute instabilities in a quasi-mode decay in MCF plasma.

Then we focus on the convective instabilities. The solutions are different in different scattering directions. For resonant decay, similar results obtained as in Rosenbluth with a tiny correction. For quasi-mode decay, no opposite scattering due to strong damping. For same-direction scattering, the convective amplification factor is as follows,

$$A = \int \frac{dx}{v_{1gx}} \gamma_0 = \int dt \gamma_0 \quad (12)$$

Here v_{1gx} is the group velocity in x-direction, and the PDI growth rate is calculated with the linear damping ν_1 by

$$\gamma_0 = \text{Im} \left[\frac{\alpha_{LF \leftarrow i} \alpha_{i \leftarrow LF} |\phi_0|^2}{\varepsilon_{LF} (\partial \varepsilon_i / \partial \omega_i)} \right] - \nu_1 \quad (13)$$

which varies spatially due to plasma inhomogeneity and the integral domain is decided by finite pump width, actually by finite trajectory length. It is a typical saturation of convective instabilities. Details of physics and mathematics can be found in recent NF paper [24].

4. SIMULATION MODEL

Now we try to employ the theory to develop a nonlocal simulation around the SOL in tokamak. Typically, around the SOL, from the antenna mouse inward the density increases more quickly than the temperature. Near the

antenna, both density and temperature are low, the ISQM type PDI is excited and in the cool and dense SOL plasma, the ICQM PDI is excited. Then a two-dimensional configuration is employed, in which plasma inhomogeneity exists in x-direction (radial) and the finite pump profile both in toroidal and poloidal, but mainly in toroidal z-direction.

We have developed a ray tracing code with the PDI effect included. The power transferred to the amplified sideband is exhausted because they have relatively large parallel refraction index. And the dissipation due to pump damping and low frequency wave terms are neglected. Therefore, it is given the energy conservation equations constrained by PDIs,

$$\begin{cases} \nabla \cdot \mathbf{P}_0 + \int_{\omega_1, k_1} (\nabla \cdot \mathbf{P}_1 - 2\gamma_{1L} U_1) = 0 \\ U_1 = U_{th} \exp[2A] \end{cases} \quad (14)$$

where $\mathbf{P}_0$ is the power flux of the pump wave $\mathbf{P}_1 = U_1 \mathbf{v}_{lg}$ is the power flux of the sideband with its linear damping rate γ_{1L} and the energy density U_1 . The initial value of PDIs U_{th} is the electrostatic thermal noise and the convective amplification factor A is integrated along the trajectories of the sideband waves around the SOL from Eq.(12). About this simulation code, more details might be found in the proceeding of the RFPPC 2025 conference [25].

5. SIMULATION RESULTS OF LOWER HYBRID WAVE PROPAGATION WITH PDIS

5.1. Spectrum evolution at different SOL densities

A simulation is performed using typical plasma parameters and wave parameters of JET [25]. These are the plots of spectral evolution in Fig.2, (a) for the frequency spectrum and (b) for the parallel wavenumber spectrum, respectively. The y-axis is the power density compared to the pump. The direction from the screen outside is the distance from the antenna. We can see that frequency sidebands and wavenumber broadening appear. But with these parameters, no significant power transfer occurs since PDI is weak. Therefore, significant current drive can be achieved.

However, we can increase the SOL density artificially. Fig. 3 shows the case of $n_\alpha = 1.41$, which means the SOL density increases by 1.41 times of original density. It is clear that frequency sidebands and wavenumber broadening become clearer, which implies the PDI becomes stronger and stronger. At this case, the power transferring from the pump to the sideband increases dramatically to about 90%. It is clear that, when the density increases above a limit (in this case, the limit is the 1.41 times of JET density parameter), significant power is transferred to sidebands and most of pump power is exhausted. This means the density limit is observed in simulation, which is clear shown in Fig. 4.

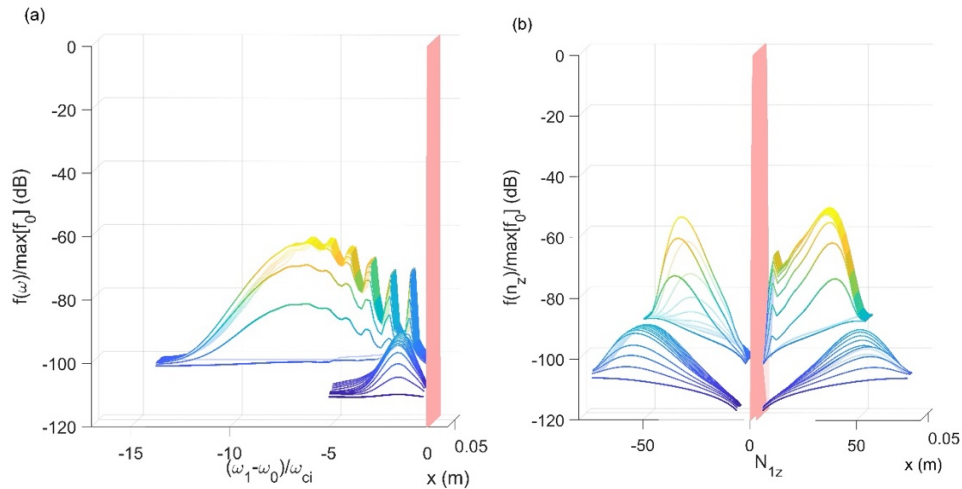


Fig. 2. Evolution of LHW frequency spectrum (a) and parallel wavenumber spectrum (b) at typical plasma parameters and wave parameters of JET, where there is no significant power transfer occurs.

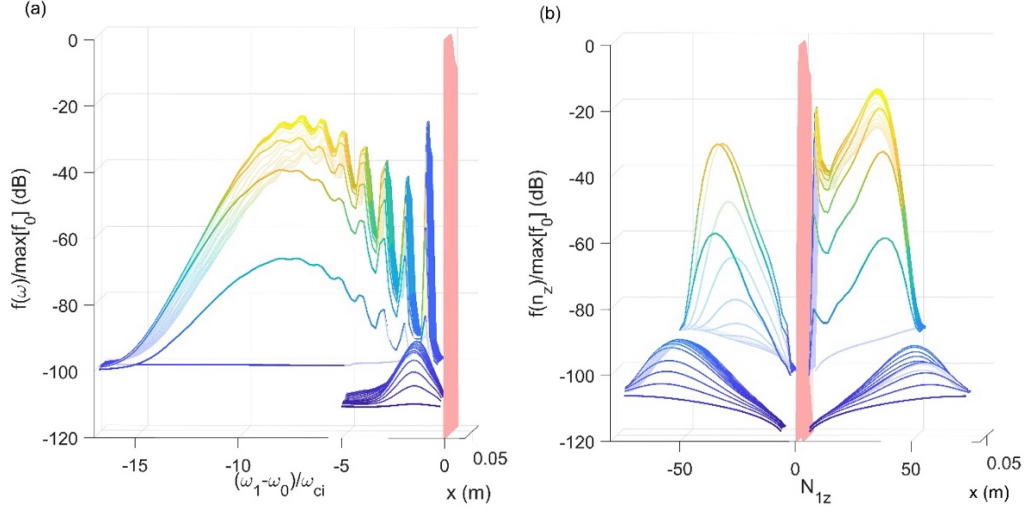


Fig. 3. Evolution of LHW frequency spectrum (a) and parallel wavenumber spectrum (b) at $n_\alpha = 1.41$, where n_α is the multiply factor for JET-like reference SOL density profile, where almost all pump power is transferred to the sideband.

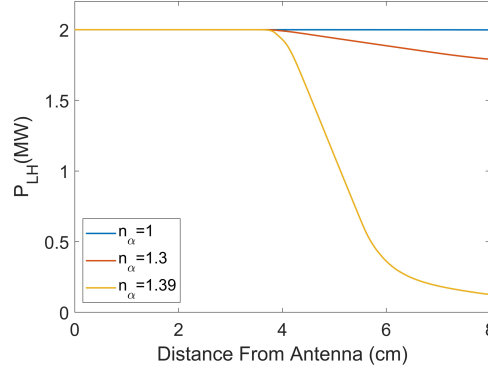


Fig 4. Evolution of LHW pump power for $n_\alpha = 1, 1.31$ and 1.41

5.2. Identification of the density limit of LHCD

If we assume that when the convective amplification factor is above a certain value, the PDI become strong enough to exhaust the pump. Then we can also derive a theoretical scaling relation of the density limit, that is,

$$n_{\text{PDI}} \propto P_0^{-2/3} L_y^{2/3} \omega_0^2 B_0^{4/3} T_e \quad (14)$$

At the same time, we can change the parameters to get the parameter dependences of the density limit from the simulation. The simulation results agree quite well with the theoretical scaling relation [26], which is clear shown in Fig. 5.

Also, from previous high density LHCD experiments, a critical convective amplification factor, about 12, can be found, although more targeted experiments with precise diagnostics are still required to confirm this point. On the density limit, more details might be found in the proceeding of the RFPPC 2025 conference [27].

Finally, we may use the ITER parameters to perform a simulation, it is found that the working density in the ITER is far from the density limit of LHCD. That's because, although ITER works on higher density than present devices, the SOL temperature, magnetic field and LHW frequency are also higher than those in present devices. It may indicate that the LHCD remains a promising method of driving plasma current for ITER and future fusion reactors.

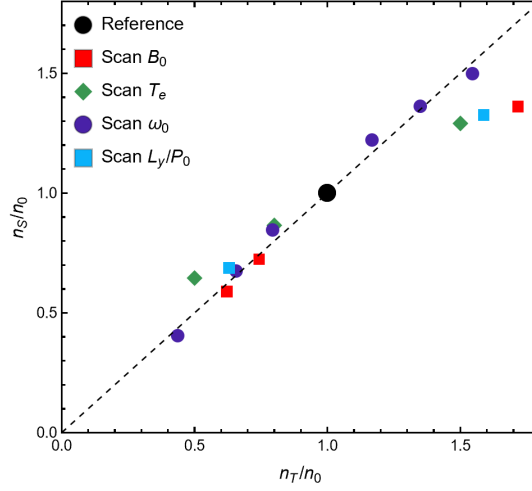


Fig 5. Comparison between density limit n_S from simulation results and n_T from the scaling law, Eq. (15).

6. SUMMARY

Firstly, the PDI theories, including the local theory and nonlocal saturation theory, are extended to adapt to the common scenarios in MCF plasmas where non-resonant quasi-mode decays are dominant.

This work provides a self-consistent modelling and simulation of LHWs in the SOL plasma by coupling the propagation of waves to the power transfer among waves by parametric decay instabilities in a 3D model based on the ray-tracing method. Frequency sidebands and wavenumber broadening appear due to PDIs and a cool and dense SOL leads to considerable PDI growth rate and pump convective loss. When the plasma density increases, stronger PDIs transfer most of pump power, resulting in the density limit of LHCD. For the first time, the density limit of LHCD observed in experiments can be successfully reproduced through theories and simulation.

Finally, a theoretical scaling relation of the density limit, which shows agreement with simulation and experimental results, indicates that LHCD remains a promising method of driving plasma current for ITER and future fusion reactors.

The theory and the code developed in this work can be extend to apply in other frequency-domain such as electron cyclotron waves [28] and helicon waves[29].

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