

CONFERENCE PRE-PRINT**A NOVEL METHOD TO OPTIMIZE OMNIGENITY
LIKE QUASISYMMETRY FOR STELLARATORS**

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Abstract

Stellarators confine plasmas with fully 3D, externally generated magnetic fields, but the loss of axisymmetry can elevate neoclassical transport. Omnigenous fields, those with vanishing bounce-averaged radial drift for all trapped orbits, offer a route to improved confinement, yet non-quasisymmetric (non-QS) omnigenity has lacked a general optimization strategy. The paper introduces OOPS (Omnigenity OPTimization like quasi-Symmetry), a simple and versatile method that extracts the key idea of homeomorphic coordinate mappings to pose omnigenity constraints with complexity comparable to conventional quasisymmetry (QS) optimization. OOPS unifies QS and non-QS omnigenity in a single formulation and achieves precise omnigenity along multiple symmetry directions. This method is further generalized to optimize configurations beyond omnigenity, enabling multi-objective trade-offs between physics performance and engineering cost. Using OOPS, precisely omnigenous, compact configurations with reduced effective ripple and favorable fast-ion behavior have been realized. Novel classes including pseudosymmetry and piecewise omnigenity (pwO) can be directly optimized. These advances substantially broaden the stellarator design space and provide a practical pathway to high-confinement, engineering-feasible configurations.

1. INTRODUCTION

Stellarators generally rely on externally generated three-dimensional magnetic fields to confine plasmas, which are less prone to current-driven instabilities and can inherently achieve steady-state operation. However, the three-dimensional nature of stellarator magnetic fields, in contrast to the axisymmetry of tokamaks, generally leads to poorer particle confinement. Consequently, unoptimized three-dimensional magnetic fields typically lead to unacceptably high levels of neoclassical transport.

The challenge in confinement is typically associated with trapped particles. In the collisionless limit, unlike passing particles that remain confined to nested toroidal flux surfaces, trapped particles execute bounce motion along field lines while experiencing guiding-center drifts with radial components. To suppress these drifts, the 3D field must be designed so that the time-averaged radial drift vanishes for every trapped orbit, a property known as omnigenity[1]. Omnigenity is closely connected to the second adiabatic invariant $\mathcal{J}$. In an omnigenous field, the surface distribution of $\mathcal{J}$ is independent of the field-line label α , so the condition $\partial\mathcal{J}/\partial\alpha = 0$ provides a powerful metric for assessing omnigenity. A notable subset of omnigenity is quasisymmetry (QS), in which the magnetic-field strength B depends on a single angular coordinate in Boozer coordinates. This symmetry in B ensures conservation of the canonical angular momentum and renders QS optimization relatively simple and robust.

Therefore, a standard strategy is to minimize the asymmetric Fourier components of B in Boozer coordinates—a simplicity that has led to the practical success of QS. By contrast, non-QS omnigenity, although less restrictive, remains underexplored because there is no comparably simple optimization method. In generic omnigenous fields, the B contours are not straight in either equilibrium coordinates or Boozer coordinates. However, when $\mathcal{J}$ is plotted in the field-line coordinates (η, α) , a perfectly omnigenous (or QS) configuration displays straight contours of $\mathcal{J}$ that are closed in α . This observation motivates the approach developed below.

Omnigenous fields satisfy several stringent geometric criteria, including: contours of constant magnetic-field strength $|\mathbf{B}|$ that close poloidally, toroidally, or helically; straight contours of $B_{\max}$ in Boozer coordinates; and a bounce distance along a field line is independent of the field line label. Optimization studies have demonstrated configurations that closely satisfy omnigenity (and the stricter subclass of quasisymmetry), with correspondingly improved particle confinement [2, 3, 4, 5, 6].

By further relaxing the magnetic structure requirements for particle confinement, a novel family of stellarator magnetic fields, known as piecewise omnigenous (pwO) fields, has been proposed [7]. These configurations can effectively reduce neoclassical transport [8] and achieve good particle confinement even when the magnetic field distribution does not strictly satisfy the conditions for omnigenity. This theory offers novel insights into explaining the record-breaking experimental results achieved in the standard configuration of Wendelstein 7-X (W7X) and the inward-shifted configuration of the Large Helical Device (LHD) [9]. Moreover, recent work incorporating a physical model for the bootstrap current has proven the existence of piecewise omnigenous fields that yield zero bootstrap current at low collisionality for any density and temperature profiles [10]. The pwO concept expands the range of configurations that can be candidates for fusion reactors, some of which might be easier to design, build, or operate. Comprehensive research is required to search new configurations that meet all favorable criteria.

A further class of configurations, termed pseudosymmetry (PS) [11], does not contain locally trapped particles. Two conditions characterize PS: (i) the magnetic-field strength possesses no locally closed contours; and (ii) the slope of field lines exceeds the slope of the B -contours. Omnigenity is a subset of PS. Although PS does not guarantee that particles remain confined to a single flux surface, the absence of locally trapped particles will help improve confinement. It warrants systematic exploration of PS under multi-objective trade-offs to assess its overall performance for differing design targets.

We introduce a novel approach for optimizing omnigenity. With simplicity comparable to QS optimization, the new method unifies both QS and non-QS omnigenity optimization and can be further generalized to optimize configurations beyond omnigenity, especially pwO and PS. This approach has led to the realization of precisely omnigenous configurations with exceptional confinement and unprecedented compactness.

2. METHODOLOGY

Inspired by the success of QS optimization, we introduce a new approach for optimizing omnigenity and OOPS (Omnigenity OPTimization like quasiSymmetry) [6]. For magnetic fields with *no* locally closed B contours, such as QS, omnigenity, and pseudosymmetry, one can define a homeomorphic coordinate system (α, η) in which the B contours become straight along the α direction. The homeomorphism between (α, η) and (θ_B, ζ_B) is yet undetermined and differs for QS, omnigenity, and other configurations. In the case of QS, the homeomorphism is relatively straightforward, since B contours are straight in Boozer or Hamada coordinates. A simple choice is $\theta_B = \alpha, \zeta_B = \eta$.

For omnigenity, the homeomorphism must be carefully constructed to ensure that the required omnigenity conditions are satisfied. Previous works by Cary & Shasharina [1], Landreman & Catto [12], and Dudt *et al.* [4] offer different strategies to construct ideal omnigenous fields. They can all be understood as different homeomorphisms for omnigenity.

Once the coordinate mapping from (α, η) to (θ_B, ζ_B) has been established, omnigenity can be optimized by minimizing the asymmetric modes,

$$f_{\text{omni}} = \sum_{m \neq 0} (B_{m,n}/B_{0,0})^2, \quad (1)$$

where $B(\alpha, \eta) = \sum B_{m,n} \cos(m\alpha - n\eta)$ and $B_{m,n}$ are computed with Fourier decomposition (stellarator symmetry is imposed). This is similar to the standard QS cost function. f_{omni} can be used in any stellarator optimization codes, *e.g.* STELOPT [13], SIMSOPT [14], or DESC [15].

In this work, we present a parametric formulation, similar to the Cary–Shasharina mapping [1]. Hereafter, we refer to it as the OOPS mapping. We choose $\tilde{\theta} = \alpha$ and define $\tilde{\zeta}$ as

$$\tilde{\zeta}(\alpha, \eta) = \eta - D(\eta) \cdot S(\alpha, \eta), \quad (2)$$

where $\alpha \in [0, 2\pi)$ labels field lines and $\eta \in [-\pi, \pi)$ labels B contours ($\eta = 0$ for minimum B and $\eta = \pm\pi$ for maximum B). Here, $S(\alpha, \eta)$ controls the contour shape and $D(\eta)$ determines the bounce distance between two

equal magnetic field contours along field lines. To preserve stellarator symmetry, $S(\alpha, \eta)$ is chosen to be an odd function and $D(\eta)$ is an even function, expressed using Fourier series as

$$\begin{cases} S(\alpha, \eta) = \sum_m s_m \sin[m y(\alpha, \eta)], \\ D(\eta) = \pi - |\eta| + \sum_n d_n \cos\left[\left(n + \frac{1}{2}\right)\eta\right], \end{cases} \quad (3)$$

where s_m and d_n are Fourier coefficients.

Field strength contours in omnigenous fields can close toroidally, poloidally, or helically, corresponding to *toroidal omnigenity* (TO), *poloidal omnigenity* (PO), and *helical omnigenity* (HO), respectively. PO, like W7-X, is more commonly referred to as “quasi-isodynamicity (QI)”. The variable $y(\alpha, \eta) = Y[\alpha \ \eta]^T$ is used to control the helicity direction. We choose $Y = [n_{fp} \ -n_{fp}/\iota]$ for TO, $Y = [1 \ -\iota/n_{fp}]$ for PO, and $Y = [1 \ \iota/(\iota + n_{fp})]$ for HO, where ι is the rotational transform and n_{fp} is the number of field periodicity. The actual Boozer coordinates with field periodicity are then derived with

$$[\theta_B \ \zeta_B]^T = M [\tilde{\theta} \ \tilde{\zeta}]^T, \quad (4)$$

with the matrix M given by

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{n_{fp}} \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ -\frac{1}{n_{fp}} & -\frac{1}{n_{fp}} \end{bmatrix},$$

for TO, PO, and HO, respectively. Note that n_{fp} is divided in PO and HO, not TO. This avoids TO having a rotational transform comparable to n_{fp} , which becomes impractical for configurations with large n_{fp} .

3. OPTIMIZED CONFIGURATIONS

We demonstrate that precisely omnigenous stellarators with excellent confinement and low aspect ratio can be realized using the new method. Specifically, we optimize three vacuum equilibria—representing toroidal(TO), poloidal(PO), and helical(HO) omnigenity—on the last closed flux magnetic surface, subject to constraints on the aspect ratio (A_p) and the rotational transform (ι). For comparison, all equilibria are rescaled to a volume-averaged field strength $\langle B \rangle = 1$ T and a major radius $R_0 = 1$ m (Fig. 1). The TO case has $N_{fp} = 2$, $A_p = 6$, and $\iota = 0.70$; the PO case has $N_{fp} = 3$, $A_p = 6.5$, and $\iota \in [0.76, 0.87]$; and the HO case has $N_{fp} = 4$, $A_p = 8$, and $\iota \in [1.19, 1.30]$. In all three cases, the resulting equilibria are verified to possess continuous nested flux surfaces throughout the volume.

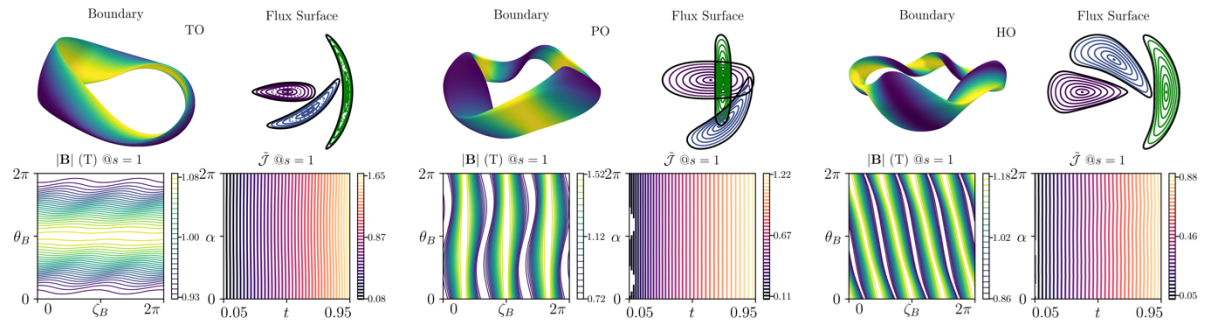


FIG. 1. Boundary shapes, flux surfaces, B contours, and J distributions of TO (left), PO (mid), and HO (right).

All three configurations demonstrate precise omnigenity, leading to exceptional confinement in both collisional transport and collisionless alpha particle losses. As shown in Fig. 2, the effective ripple for the three configurations is orders lower than W7-X. When scaled to the reactor size, the loss fractions of alpha particles in TO and HO are 0.50% and 0.16%, respectively, while PO has no losses before the slowing-down time, achieving the best performance among the proposed designs (comparable to the precise QS [5]). In particular, the aspect ratios are lower than existing omnigenous configurations, which will substantially reduce the construction cost.

We have also obtained attractive PS and pwO configurations. The pwO case is achieved by choosing a strongly-shaped B contours and bounding $\eta \in [-0.255\pi, 0.255\pi]$. By doing so, the B contours will close locally and form pwO-like parallelograms in the $B_{\max}$ region. The omnigenity mapping in the $B_{\min}$ region imposes some particular values for the rotational transform. Since the magnetic field is smooth, the rotational transform meets

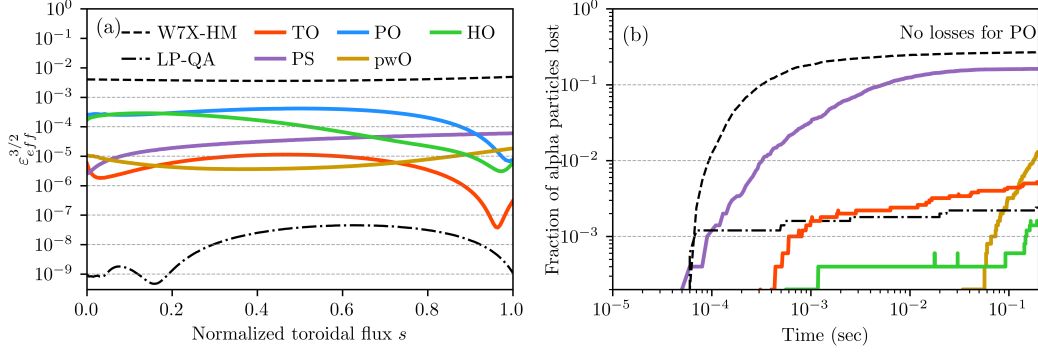


FIG. 2. The neoclassical transport coefficient (left), and the alpha particle losses (right)

the requirement of pwO, which is connecting corners of the B_{max} parallelograms using two field lines. A new configuration combining omnigenity and pwO has been obtained and is shown in Fig. 3. It has $N_{\text{fp}} = 3$, $A_p = 6$, and $\iota = 0.62$. Despite relaxing the omnigenity, the new QI+pwO configuration demonstrates good confinement properties. As shown in Fig. 2, it has a considerably low $\epsilon_{\text{eff}}^{3/2}$ and the alpha-particle loss is about 1%.

Fig. 3 shows a PS configuration with $N_{\text{fp}} = 2$, $A_p = 5$, and a constant $\iota = 0.55$. The B contours close toroidally, resembling TO. With the elimination of locally trapped particles, PS has a considerably low $\epsilon_{\text{eff}}^{3/2}$, comparable to TO, as shown in Fig. 2 (a). More interestingly, PS is less elongated than an equivalent TO implying potentially simpler coils and it has a lower aspect ratio which is generally favorable for compact reactors. Fig. 2 (b) confirms that PS has poor alpha-particle confinement with all trapped particles lost.

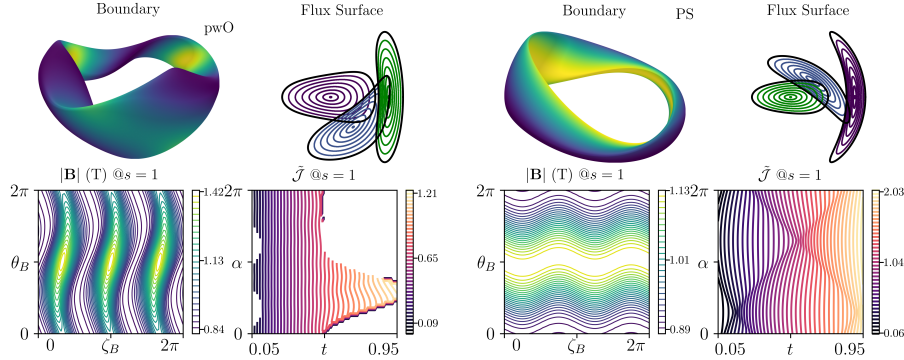


FIG. 3. Boundary shapes, flux surfaces, B contours, and J distributions of pwO (left), PS (right).

4. SUMMARY AND DISCUSSION

We have introduced a unified optimization framework to optimize quasisymmetry (QS), omnigenity, pseudosymmetry (PS), and piecewise omnigenity (pwO). By selecting suitable homeomorphic coordinate transformations, advanced configurations can be realized by reducing symmetry-breaking modes with a conceptual simplicity comparable to standard QS optimization. Using this approach, we have obtained precisely omnigenous configurations outside the QS subset that exhibit excellent confinement in both neoclassical transport and collisionless alpha-particle dynamics. In particular, the toroidally omnigenous (TO) and helically omnigenous (HO) examples achieve substantially lower aspect ratios than previously reported, identifying promising candidates for future stellarators. The method is robust and successful optimizations were obtained from initial guesses of a circular torus without any “warm” starts. Moreover, we directly optimized PS and pwO configurations for the first time, finding favorable properties—such as low neoclassical transport and reduced elongation—that may simplify coil design. These results open new pathways for practical stellarator development.

Although the present study focuses on vacuum equilibria, the same methodology readily produces finite-beta solutions. The omnigenity objective can be enforced on multiple flux surfaces to approach global omnigenity, and it can be combined with additional figures of merit—e.g., ideal-MHD stability and turbulence surrogates—even when trade-offs arise. Engineering-feasible coil sets can be pursued with existing tools [16]. As generalized

forms of QA and QH, TO and HO provide extra degrees of freedom that may improve engineering practicality. With weaker global constraints, PS and pwO demonstrate strong confinement while potentially reducing coil complexity; pwO, in particular, shows excellent alpha-particle confinement and merits further study. Finally, “less-constrained” omnigenity with distinct local extrema [17] is a natural extension within this framework.

ACKNOWLEDGEMENTS

This work was supported by the Strategic Priority Research Program of the Chinese Academy of Sciences under Grant No. XDB0790302 and the Anhui Provincial Key Research and Development Project under Grant No. 2023a05020008.

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