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TURBULENCE, ZONAL FLOWS, AND GLOBAL MODES IN BURNING PLASMAS: CODE DEVELOPMENT AND SIMULATIONS

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Abstract

Fusion-born alpha particles are expected to significantly affect the plasma dynamics in coming experiments such as ITER or BEST whereas DEMO will operate close to ignition with auxiliary power employed only for control. This makes understanding the burning plasma physics through theory and modeling an urgent task especially since a significant rate of alpha particle generation is difficult to attain in present experiments. The paper describes the application of gyro-kinetic codes and hybrid fluid-kinetic models which are coordinated within an EUROfusion E-TASC project dedicated to burning plasma physics. The results include global gyrokinetic simulations of chirping Alfvénic waves in tokamak plasmas, electromagnetic micro-instabilities, turbulence and MHD modes in tokamak and stellarator plasmas, beat-wave-driven zonal flows in JET and their effect on turbulence, and nonlinear generation of Alfvénic modes by the ambient turbulence in Wendelstein 7-X.

1. INTRODUCTION

Turbulence plays an important role in all fusion plasmas. It leads to energy transport and is therefore crucial for the plasma confinement conditions. While electrostatic ion temperature gradient driven (ITG) turbulence is thought to be the main player for low plasma beta conditions, the situation will change in the future when the heating power will strongly increase. It can be safely assumed that fusion reactor plasmas will be electromagnetic. In such plasmas, fine-scale microturbulence and zonal flows cannot be considered separately from Alfvénic modes of various types. Global MHD-type instabilities will also couple to and affect the plasma dynamics, turbulence, and transport, which, for the reactor conditions, will develop coherent structures in phase space. That means that they are appearing and evolving kinetically not only in the real but also in the velocity space. Under such conditions, the plasma profile evolution will become a multi-scale highly nonlinear problem with global and kinetic contributions. Global nonlinear gyrokinetics is a minimal description which includes all relevant components on the same footing self-consistently. The paper shows characteristic examples of global gyrokinetic simulations addressing the multiscale plasma dynamics of fusion plasmas in tokamak and stellarator geometry.

First, we describe effects of the bulk-plasma beta on the turbulent transport in tokamaks and in the W7-X stellarator. Increasing beta can stabilize ITG modes and suppress turbulent transport. However, when the values of the bulk-plasma beta reach a threshold, the Kinetic Ballooning Mode (KBM) branch is destabilized, which may lead to a sharp increase in the heat and particle fluxes.

In a burning plasma, the picture gets even more complex as 3.5MeV α-particles or hydrogen ions heated by Neutral Beam Injection (NBI) appear as fast particle species and may drive instabilities. By resonant interaction, their energy can be transferred to e.g. Alfvén eigenmodes. In principle, fast particles may interact with any type of mode or instability in the plasma. Fortunately, their behavior can also be described by gyro-kinetic theory. Illustrating such a process, we present an example of a non-perturbative non-linear interaction of fast particles and an Alfvénic mode that leads to frequency chirping in Sec. 4.

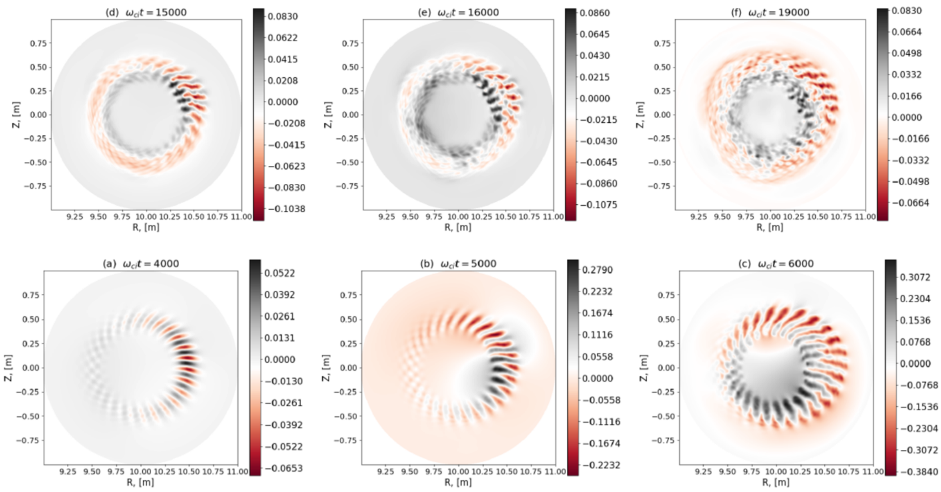


FIG. 1. upper: Time evolution of the the poloidal mode structure of ITG turbulence ($\beta = 0.1\%$) in a circular tokamak. The developing zonal flow sheares the tubulent eddies what leads to saturation.
lower: The same but for KBM turbulence at $\beta = 2.08\%$. The turbulence develops fingers which extend radially. The mode saturation is here reached by a much stronger relaxation of the ion temperature profile. (The figure is taken from Mishchenko et al. [6])

As has been stated above, turbulence and other unstable modes develop not independently. Most obviously, for instance, both may excite and react on zonal flows. In Sec. 5, ORB5 simulations of collisionless tearing modes are presented showing a variety of non-linear phenomena including self-healing and the drive of zonal flows. Following this paradigm, in Sec. 6 the zonal flow driven by a TAE is calculated and subsequently, via an numerical antenna, applied to unstable ITG modes. The observed stabilizing effect may be helpful explaining the observation of a better confinement in the afterglow experiments in JET [1]. Observations of broadband Alfvénic modes in W7-X suggested their excitation by the abundant ITG turbulence which is confirmed by simulations with EUTERPE in Sec. 7. Finally, in Sec. 8 EUTERPE simulations of MHD instabilities in a four-periodic Mercier-unstable stellarator show that the gyro-kinetic theory, and even more important, their numerical implementation, is robust enough to obtain growth rates agreeing with ideal MHD and successful non-linear simulations on the full torus using realistic parameters including the electron mass.

2. THEORETICAL AND NUMERICAL MODELS

The gyro-kinetic theory is the common basis used for all calculations in this paper. The simulations are done with either the ORB5 [2] or the EUTERPE [3] code. Both codes solve the gyro-kinetic Vlasov-Poisson system of equations with a particle-in-cell method. The field equations are discretized with B-splines. The implementation in ORB5 is suitable for tokamaks, while that for the EUTERPE code can use magnetic equilibria of stellarators generated with the VMEC code. The codes use the same advanced algorithms for calculations as eg. the mixed variables scheme for electro-magnetic calculations [4] or advanced diagnostics as the DMUSIC algorithm [5].

3. ELECTROMAGNETIC TURBULENCE IN TOKAMAKS AND IN W7-X

In global simulations using the ORB5 code, the transition from ITG to KBM has been reproduced [6]. In Fig. 1, the mode evolution is shown in real space plotted for a poloidal cross-section. The ITG turbulence develops sheared-flow eddy-breaking structures which lead to modes saturation. Instead, in the KBM case, finger-like structures develop in the electrostatic potential. These structures are nonlinearly expelled out of the linearly-unstable domain. This results in the relaxation of the density profile.

Also, the temperature profile nonlinearly evolves which contributes to the saturation of the turbulence at a reasonable level. This saturation mechanism is missing in the local description.

The nonlinear heat flux shown in the KBM regime is quite large but it saturates at a reasonable value. This simulation has been carried out for a very long time with the heat flux remaining saturated.

The EUTERPE code has been used to perform the simulations at $\beta = 4\%$ for a Mercier-unstable high-mirror configuration of W7-X. Simulated was the outer part of the plasma radius. The turbulence is driven by the temperature profile. Also in this case, the heat flux is quite large but limited and saturates at a finite value. First, the linear instability evolves, spreads, generates

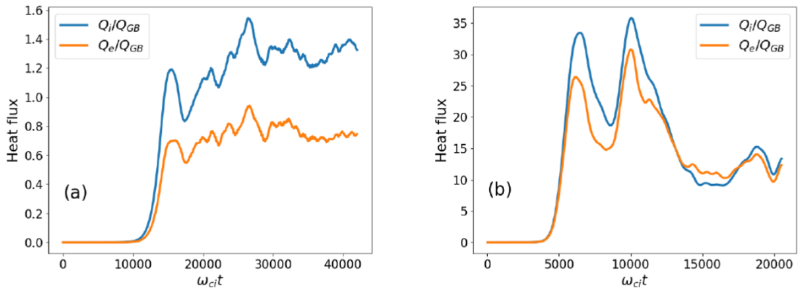


FIG. 2. Comparison of the energy fluxes for the same case as before. The heat flux for the ITG cas (left) is much smaller that for the KBM case (right). Nevertheless, in contrast to local calculations, it remains finite. (The figure is taken from Mishchenko et al. [6])

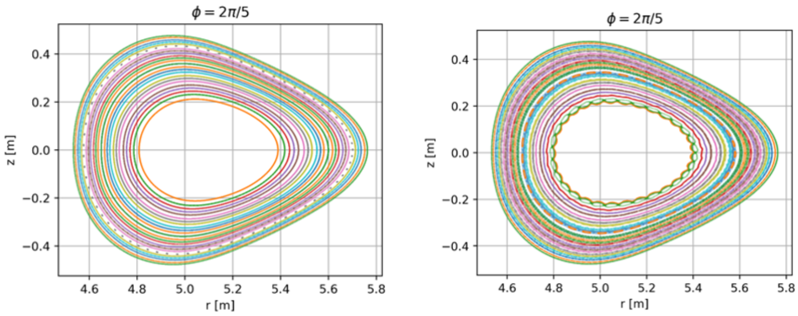


FIG. 3. Flux surfaces as the electromagnetic turbulence develops in the very high mirror configuration of W7-X - from the linear stage (left) to the highly nonlinear stage (right). The field lines get partially ergodized.

lower mode-number components and zonal flows which finally leads to a saturation of the mode and an ergodization of the flux surfaces.

4. CHIRPING OF FAST PARTICLE DRIVEN MODES

Energetic particles in burning or strongly heated fusion plasma may drive Alfvénic modes unstable. The particle trajectories will finally be changed if the amplitude of the perturbed fields is large enough such that the particles will be redistributed or expelled from the device. Therefore, the investigation of this type of particle wave interaction is of particular interest. This chapter focuses on the case where the drive is strong, i.e. the excited mode is far from marginality and the time scale of the wave particle interaction and the non-linear wave evolution are of the same order. Modes of this type are usually called energetic particle modes (EPM). Resonant particles may be trapped and de-trapped in the resonant structures what leads to change of the frequency (frequency chirping) even when the mode is already saturated. Note, that this is in contrast to the more often considered energetic particle excitation close to marginality. The relevant physics is then equivalent to a bump-on-tail instability (the so-called Berk-Breizman model is often used) and the saturated mode amplitude is determined by particles trapped in the wave. It then scales linearly or quadratically with the linear growth rate in the case of radial decoupling or resonance detuning, respectively.

To shed light on the mode dynamics in the strongly driven case, the ORB5 code is used to simulate an EPM mode in a tokamak using an anisotropic slowing down distribution. To speed up calculations heavy electrons $m_e/m_p = 0.005$ have been used.

The mode structure shows a typical boomerang-like shape. Scaling gradients and other parameters (for details see Wang et al. [7]) lead to a linear scaling of the frequency chirping range with the mode saturation amplitude (see Fig. 4) in agreement with earlier theoretical predictions [8].

The calculation of chirping modes with ORB5 have also been validated against the experiment: The chirping amplitude was found to be in agreement with experimental results at ASDEX-Upgrade, where it turned out to be crucial to use the correct distribution function (isotropic slowing down) in the simulation[9].

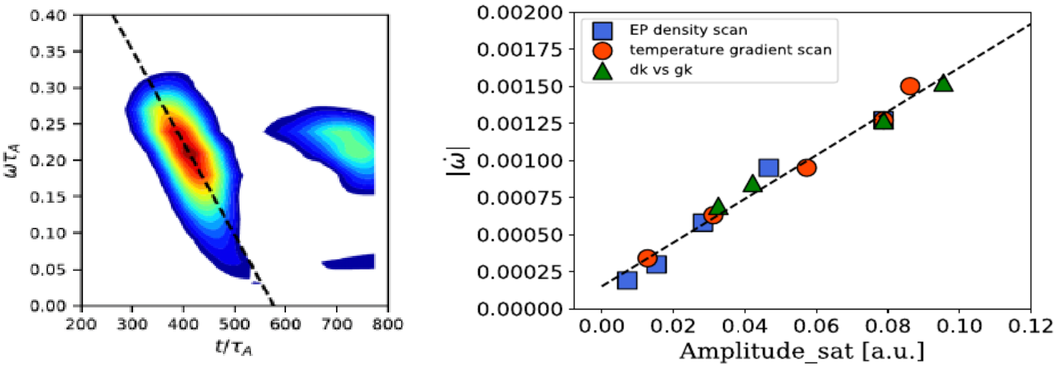
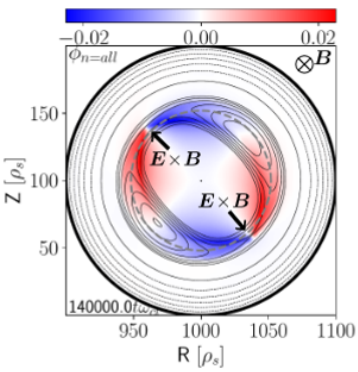


FIG. 4. Left: Chirping frequency spectrum vs. time of an $m = 7/8$ EPM in a circular tokamak equilibrium. The slope of the dashed line is the chirping rate. Right: Linear scaling of the chirping frequency rate with the mode saturation amplitude. This scaling has been obtained varying different parameters as the energetic particle density, the temperature gradient and using drift-kinetic fast ions (dk) instead of gyrokinetic ones (gk). The figures are taken from Wang et al. [7].

5. GYRO-KINETIC SIMULATION OF TEARING MODES IN A TOKAMAK

In tokamak geometry, gyrokinetic simulations of nonlinear tearing instabilities are performed using the ORB5 code. A circular tokamak equilibrium is chosen which exhibits a $q = 2$ surface close to the mid-radius position. It was found that the nonlinear growth of the magnetic island leads to a self-induced turbulence around the island structure and, in some cases, to perturbed zonal currents and modifications of the safety factor [10] (see Fig. 6). Both effects can result in an "island healing" reducing the strength of the tearing instability in the nonlinear regime. In contrast, ambient turbulence may lead to island generation even in the regimes which otherwise, in the absence of the turbulence, are stable with respect to the tearing instability. This happens via the nonlinear coalescence of small-scale high-mode-number island chains generated by the nonlinear perturbations with the tearing parity.



(a) Electrostatic potential ϕ.   (b) Electron density n_e.

FIG. 5. Electrostatic potential and the associated electron density perturbation for a tearing mode at a $q = 2$ surface in a circular tokamak. The direction of the $E \times B$ flow is indicated at the x-points. (Figure taken from Widmer et al. [10])

6. MITIGATION OF ITG GROWTH RATES BY TAE GENERATED ZONAL FLOWS

The ORB5 code was used to investigate the interaction of energetic particles with turbulence in a JET afterglow experiment. It had been suggested earlier [11] that the beat-driven zonal flow generated by a fast particle driven Alfvén eigenmode may influence plasma turbulence.

To prove the hypothesis, an $n = 5$ TAE was calculated driven by a species of fast ions with a Maxwellian distribution adapted to the parallel and perpendicular NBI beam energy. 128 million electron markers (heavy electrons wich $m = 0.005$

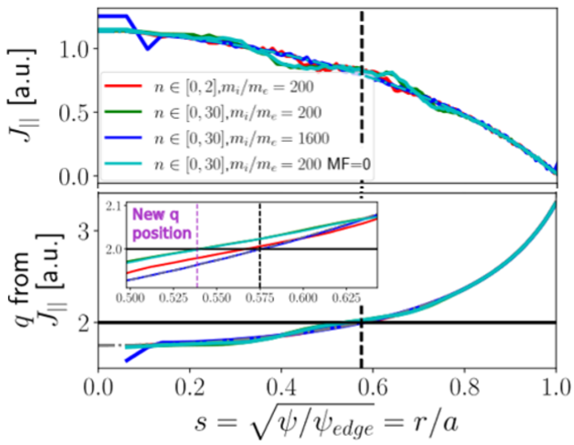


FIG. 6. It is shown that the evolution of the parallel current leads to a weak local flattening of the current profile (upper figure), a change in the q-profile and a shift of the position of the resonant surface. (Figure taken from Widmer et al. [10])

to speed up computation) and four times less ion and fast particle markers have been used along with experimental parameters from JET 92416. The unstable TAE mode drove a zonal component by self-beating with twice its own growth rate.

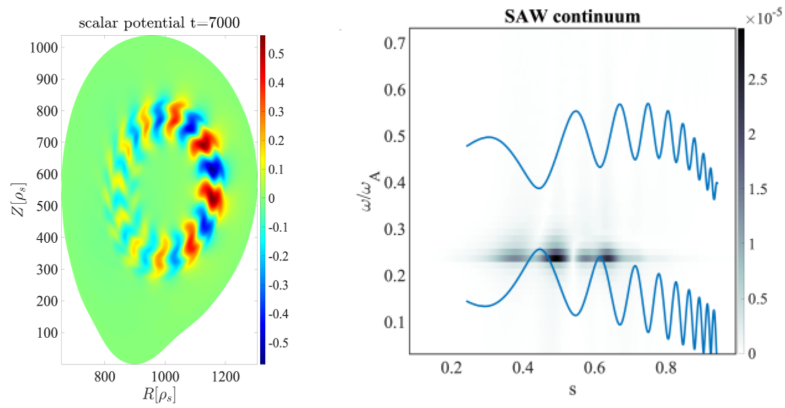


FIG. 7. (a): Mode structure of the TAE as calculated by ORB5. (b): The frequency of the calculated mode obtained by a Fourier transform is plotted into the Alfvén continuum as calculated with the FALCON code [12] (Figures taken from Sama et al. [13])

In the form of a ”numerical expriment” [13] the zonal flow was fed to an ”antenna” implemented in ORB5 while an ITG mode was active. The $\vec{E} \times \vec{B}$ shear lead to a scattering of the ITG to higher wave numbers and a lowering of its growth rate (Fig. 8. The scattering pattern followed that of the zonal flow.

So, the interplay between beat driven zonal flows and the plasma turbulence could be demonstrated. It might be possible that also turbulence and the connected heat transport can be diminished this way. However, it might be possible that the heat and particle transport caused by strongly driven Alfvén eigenmodes lead to a detrimental net effect.

7. EXCITATION OF ALFVÉNIC MODES BY ITG TURBULENCE IN W7-X

In many discharges of W7-X, broad-band activity of Alfvénic modes has been observed without a driving energetic particle population. It could be shown, however, that this activity was correlated with turbulent density fluctuations [14].

With the EUTERPE code, non-linear electromagnetic simulations of ITG turbulence have been performed [15] allowing a coupling also to lower mode numbers. The simulation was done on the outer $2/3$ of the plasma radius and the temperature was scaled up to decrease the mode numbers of the ITG, while the density was scaled up so as to maintain the plasma β. It was found that the ITG turbulence could excite mode activity at lower mode numbers. It could be further shown that when the ITG was filtered away, no excitation took place.

Interestingly, a cascade of growth rates ($\gamma_1 : \gamma_2 : \gamma_3 = 1 : 2 : 3$) could be found for the growth of the lower mode numbers which resemble the beat-driven growth of the zonal flow with twice the growth rate found for fast particle driven TAE [16]

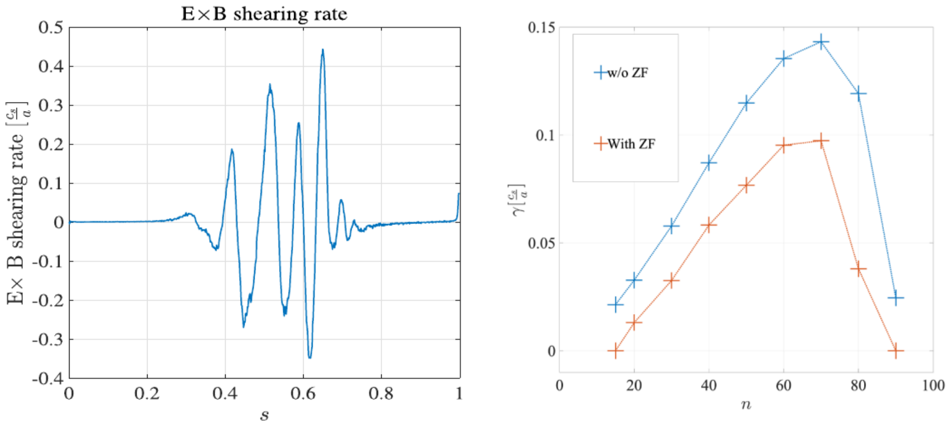


FIG. 8. (a): $\vec{E} \times \vec{B}$ shearing rate stemming from the beat driven zonal flow (by the $n = 5$ TAE) (b): Diminishing of the ITG growth rate if the zonal flow is applied to the ITG calculation with the antenna version of ORB5 (orange) in comparison with the original growth rate (blue). (Figures are taken from Sama et al. [13])

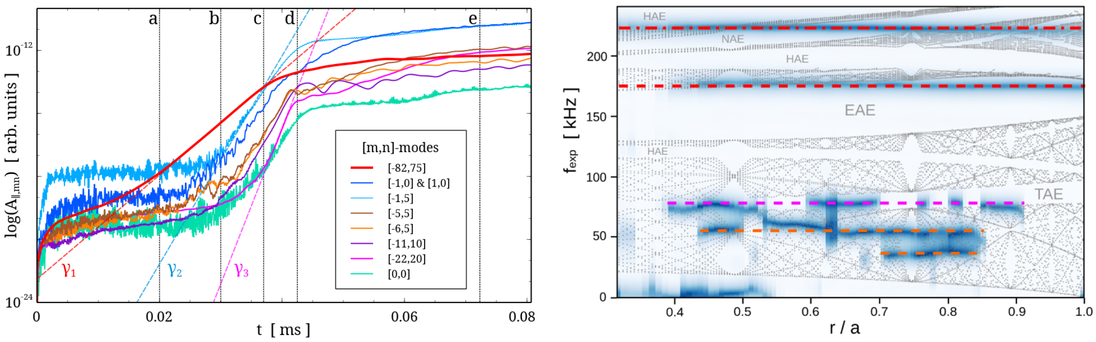


FIG. 9. (a): Part of the simulated mode spectrum for $A_{\|}$. For the sake of clarity, one high mode number Fourier harmonic (red) is singled out to demonstrate the growth of the ITG instability and its non-linear behavior. Modes with smaller mode numbers are excited by the ITG in a cascade of growth rates with $\gamma_1 : \gamma_2 : \gamma_3 = 1 : 2 : 3$.
(b): The frequencies of the excited modes lay in the gaps of the Alfvén continuum and match the experimental frequencies. (Figure taken from Riemann et al. [15])

(Fig. 9a). The zonal flow grows indeed with twice the growth rate, a satisfying theoretical explanation for the found cascade is not yet known.

The frequencies of the simulated modes lay mostly in the gaps of the Alfvén continuum and thus support the Alfvénic character of these modes. Furthermore, they match partly the experimental frequencies (Fig. 9b).

8. GYRO-KINETIC CALCULATION OF MHD IN STELLARATOR PLASMAS

A sequence of four-periodic MHD (Mercier) unstable stellarators was used to compare the growth rates of ideal MHD instabilities with those calculated by the gyro-kinetic EUTERPE code [17]. The calculations have been performed on the full torus (to allow also symmetry breaking perturbations in the non-linear regime). The resolution was 300 x 300 x 2048 grid points, while 300 million electron markers and 150 million ion markers have been used. For the linear growth rates a very good agreement between the ideal MHD and gyro-kinetic theory has been found.

In the non-linear regime, when the unstable modes start to saturate, magnetic islands appear which rotate in the electron-diamagnetic direction and distort the flux surfaces [17]. These island grow in the course of the simulation what leads to an increasing ergodization of the flux surfaces (see Fig. 10.

This phenomenon is in agreement with experimental findings of a rather smooth transition into the MHD-unstable regime: It leads to a degradation of the plasma confinement rather than to violent crashes. Furthermore, it is in agreement with recent non-linear MHD calculations [18].

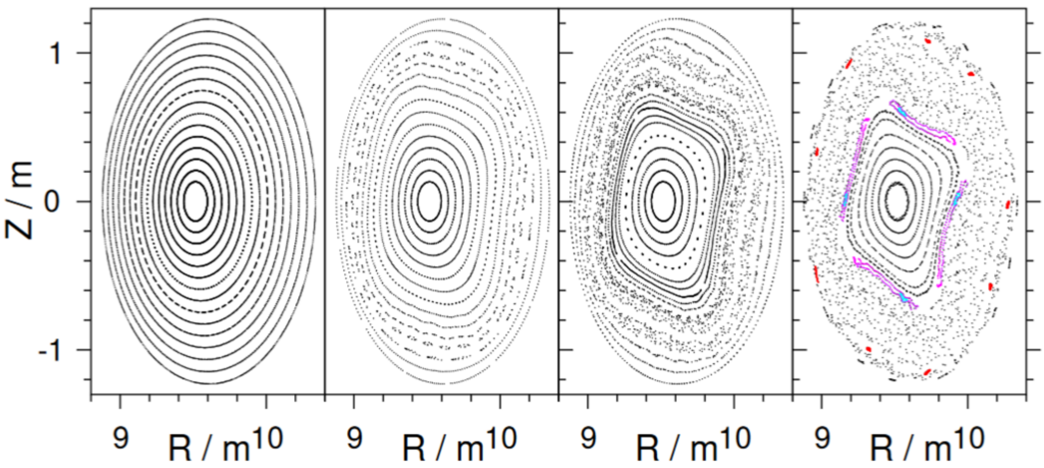


FIG. 10. Field-line tracing for the gyrokinetic perturbed magnetic field in a helical-axis stellarator with turning-ellipse cross-sections shown at different (increasing) times for the toroidal angle $\phi = 0°$. Global nonlinear gyrokinetic simulations using the EUTERPE code [3]. (Figure taken from Nührenberg et al. [17])

9. CONCLUSION

Using gyro-kinetic particle-in-cell codes ORB5 and EUTERPE which are applicable to 2D and 3D magnetic equilibria, respectively, we have addressed the interplay of small and large scale instabilities and the zonal flow. We could demonstrate that a variety of plasma instabilities can be described using the same theoretical and numerical background even when their nonlinear evolution is addressed. As the phenomena range from electromagnetic turbulence and heat transport, tearing modes, fast particle generated zonal flows and their interaction with ITG instabilities, chirping fast particle modes, the interaction of turbulence and Alfvén modes to finally the gyro-kinetic high-β stability of stellarator plasmas, gyro-kinetic codes have successfully proved that they can be applied to burning plasmas. The latter the more as experimental findings could be reproduced.

While in some cases the parameters had to be amended to make the calculations feasible, the calculations for stellarators in the high-β regime could be done on the full torus and with realistic parameters. Numerical and algorithmic developments need to be implemented which will allow to push also the turbulence calculations to the burning plasma regime.

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