

# THE DEVELOPMENT OF A 3D MHD CODE IN COMSOL MULTIPHYSICS AND ITS APPLICATION TO MHD FLOW IN A RIPPLED MAGNETIC FIELD

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## Motivation and background

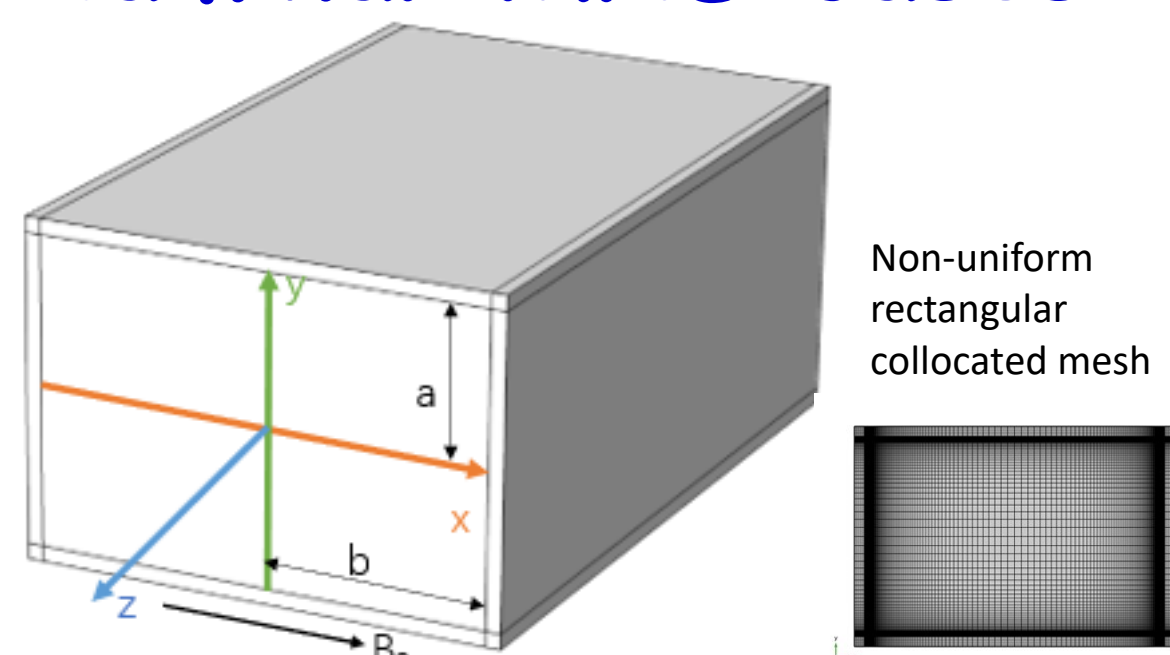
- Critical Role of MHD in Fusion Liquid Metal Blankets:** Liquid metal blankets are core candidates for fusion technology, but liquid metal flow in plasma-confining magnetic fields induces MHD effects—these effects directly shape flow profiles and cause significant pressure drops, which are decisive for blanket performance and safety. Accurate MHD simulation is thus essential for reliable blanket design.
- Limitations of Existing MHD Codes:** Existing 3D MHD codes for fusion fall into two categories: magnetic induction formulation (e.g., HIMAG, with intrinsic current conservation) and electric potential formulation (e.g., MHD-UCAS, requiring extra consistency conservation schemes). Additionally, codes are not shared in the fusion community, forcing laboratories to develop proprietary tools, increasing redundant efforts.
- Opportunity & Challenge of Commercial Software:** COMSOL Multiphysics is ideal for user-friendly 3D MHD code development due to its equation customization module and diverse solvers. However, existing COMSOL-based MHD codes (relying on electric potential formulation) lack consistency conservation schemes—modifying COMSOL's source code to add such schemes is technically challenging for most users, creating a barrier to reliable code development.
- Unaddressed Need in Rippled Magnetic Field Scenarios:** Fusion reactor toroidal magnetic fields have “toroidal field ripple” (periodic inhomogeneity) in inter-coil gaps, where liquid lithium-lead blanket inlet/outlet pipes pass. These pipes’ pressure drops dominate total blanket pressure drops, but the impact of rippled magnetic field gradients on MHD pressure drops remains unclarified—there is an urgent need for validated codes to analyze this scenario and guide blanket design.

## Mathematical formulation

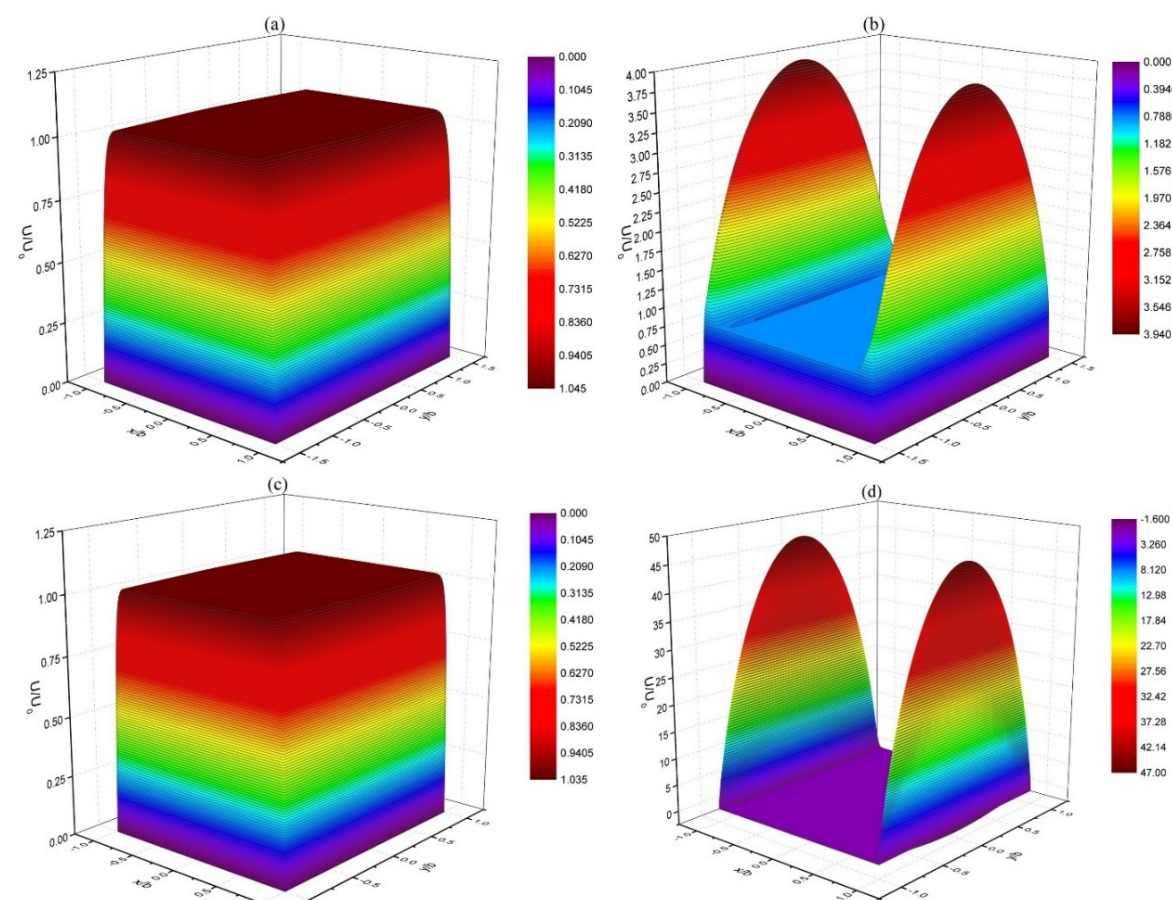
- The modified N-S equations:**
$$\frac{\partial \rho}{\partial t} + \rho \nabla \cdot \mathbf{V} = 0$$
$$\rho \left[ \frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla) \mathbf{V} \right] = -\nabla P + \mu \nabla^2 \mathbf{V} + \mathbf{J} \times \mathbf{B} + \mathbf{f}$$
- Magnetic diffusion equation:**
$$\mathbf{J} = \frac{1}{\mu_0} \nabla \times \mathbf{B} \quad \eta = \frac{1}{\mu_0 \sigma}$$
$$\frac{\partial \mathbf{B}}{\partial t} = \eta \nabla^2 \mathbf{B} + (\mathbf{B} \cdot \nabla) \mathbf{V} - (\mathbf{V} \cdot \nabla) \mathbf{B}$$
- Heat convective-diffusion equation:**
$$\rho C_p \frac{\partial T}{\partial t} + \rho C_p \mathbf{V} \cdot \nabla T = \kappa \nabla^2 T + S$$
- Tritium transport equations:**
$$\frac{\partial C_s}{\partial t} = D_s \nabla^2 C_s$$
$$\frac{\partial C_L}{\partial t} + \mathbf{V} \cdot \nabla C_L = D_L \nabla^2 C_L + Q \quad C_s/C_L = S_s/S_L$$

## Validation of 3D fully developed laminar MHD cases

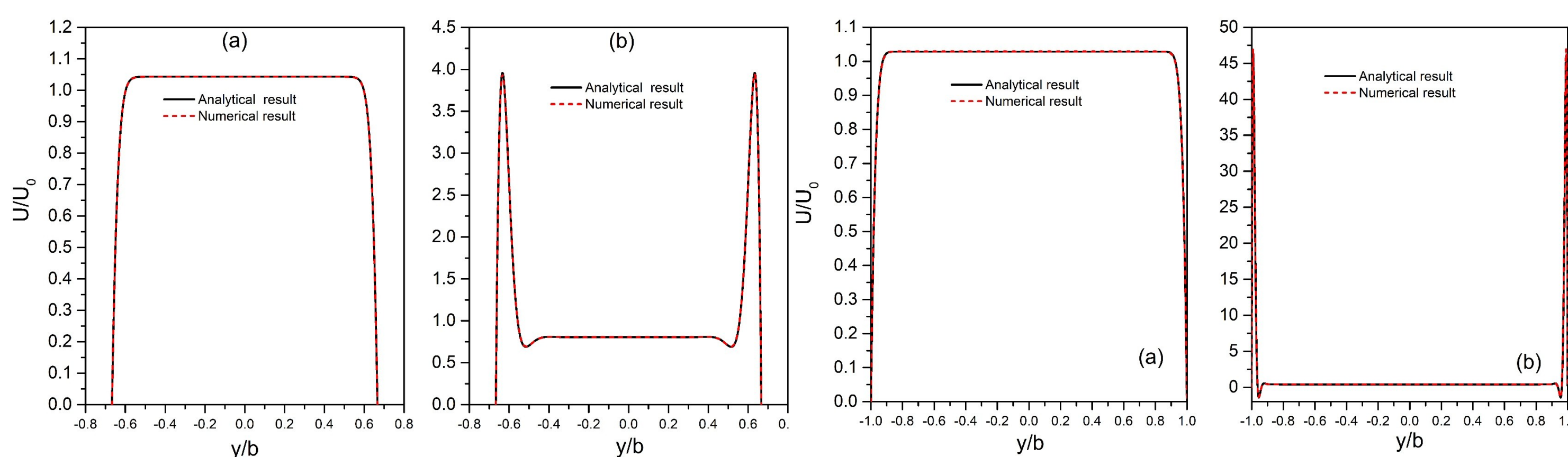
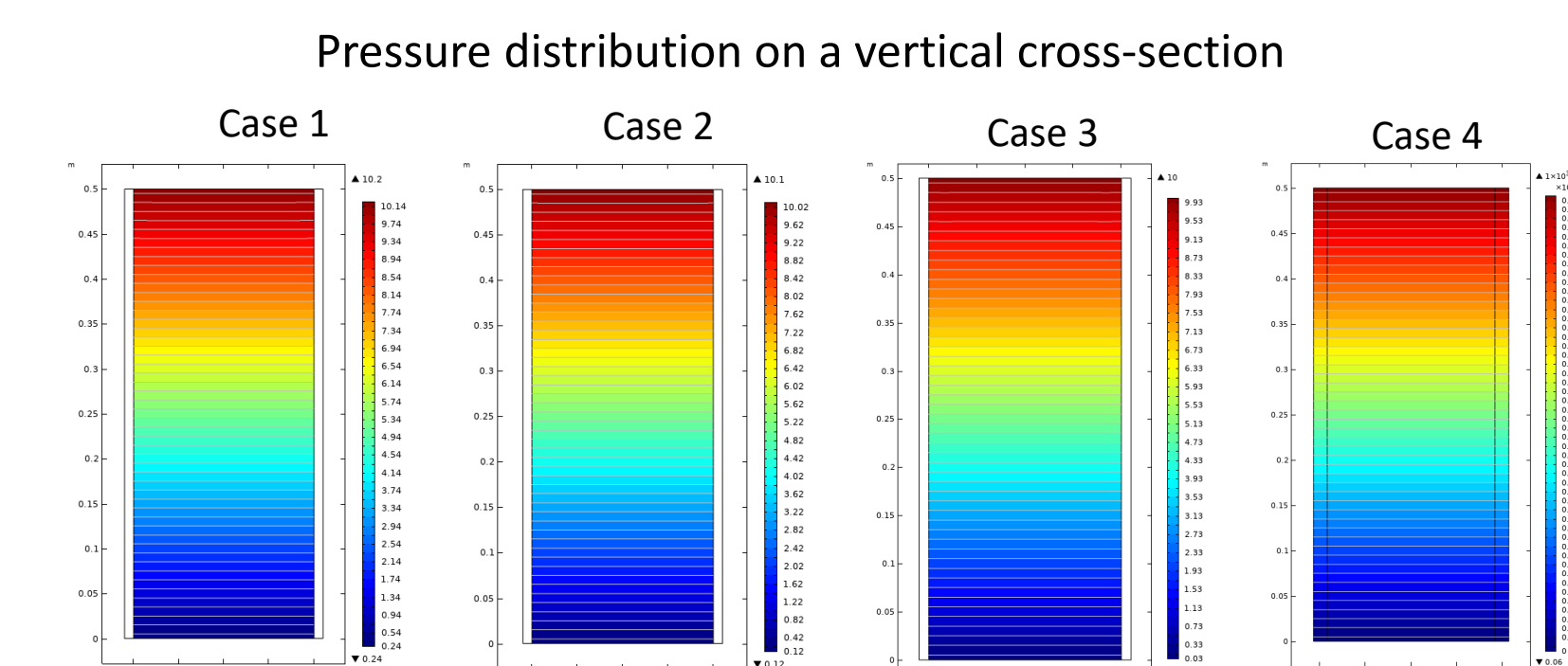
|                                                         | Shercliff Cases            |                            | Hunt Cases                     |                                |
|---------------------------------------------------------|----------------------------|----------------------------|--------------------------------|--------------------------------|
| Case number                                             | Case 1                     | Case 2                     | Case 3                         | Case 4                         |
| Dimensionless parameters                                | Ha=1000, $\chi=1.5$ , Cw=0 | Ha=1000, $\chi=1.0$ , Cw=0 | Ha=1000, $\chi=1.5$ , Cw=0.016 | Ha=10000, $\chi=1.0$ , Cw=0.05 |
| Pressure gradient, $-dp/dz$ [Pa/m]                      | 20                         | 20                         | 20                             | 1000                           |
| Analytical average velocity, $V_0$ [m/s]                | $4.29 \times 10^{-2}$      | $1.94 \times 10^{-2}$      | $0.32 \times 10^{-3}$          | $1.01 \times 10^{-3}$          |
| Analytical volumetric velocity, $Q$ [m <sup>3</sup> /s] | $2.60 \times 10^{-2}$      | $7.77 \times 10^{-4}$      | $1.92 \times 10^{-4}$          | $4.05 \times 10^{-5}$          |
| Numerical average velocity, $V_0$ [m/s]                 | $4.25 \times 10^{-2}$      | $1.92 \times 10^{-2}$      | $3.17 \times 10^{-3}$          | $1.02 \times 10^{-3}$          |
| Numerical volumetric velocity, $Q$ [m <sup>3</sup> /s]  | $2.58 \times 10^{-2}$      | $7.68 \times 10^{-4}$      | $1.98 \times 10^{-4}$          | $4.08 \times 10^{-5}$          |
| $V_0$ error (%)                                         | 0.93                       | 1.03                       | 0.94                           | 1.96                           |



Sketch of straight rectangular pipe with imposed uniform background magnetic field in X direction.



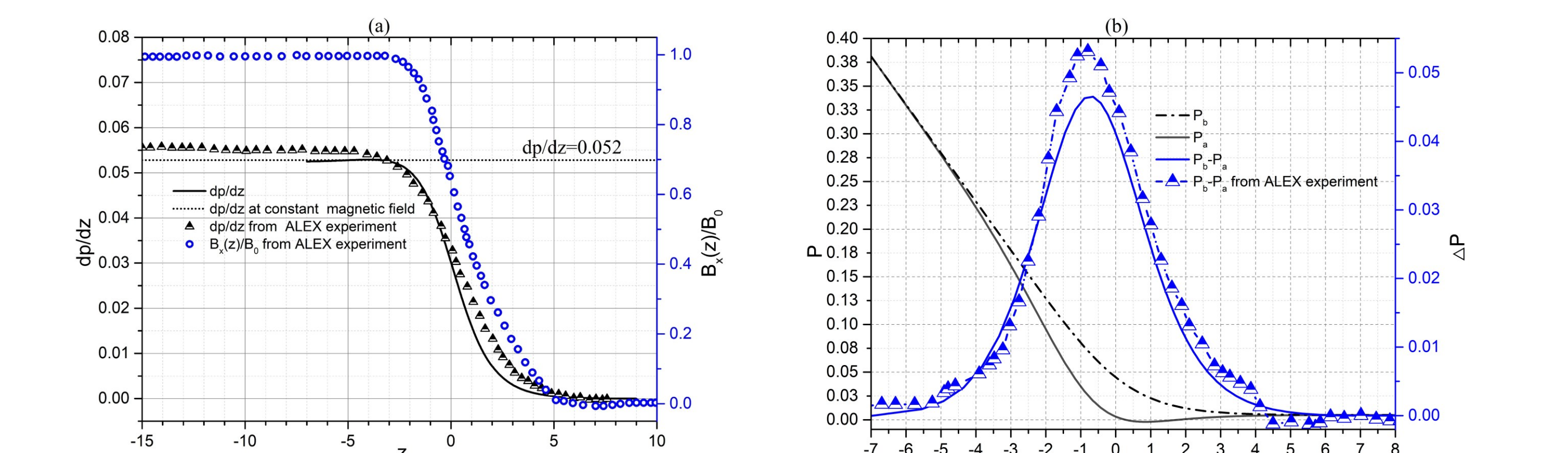
Fully developed velocity profile on the rectangular pipe cross-section of case 1(a), case 2(b), case 3(c) and case 4(d)



Comparison of the calculated velocity profiles along the non-dimensional y coordinate with analytical solutions (shown in solid line)

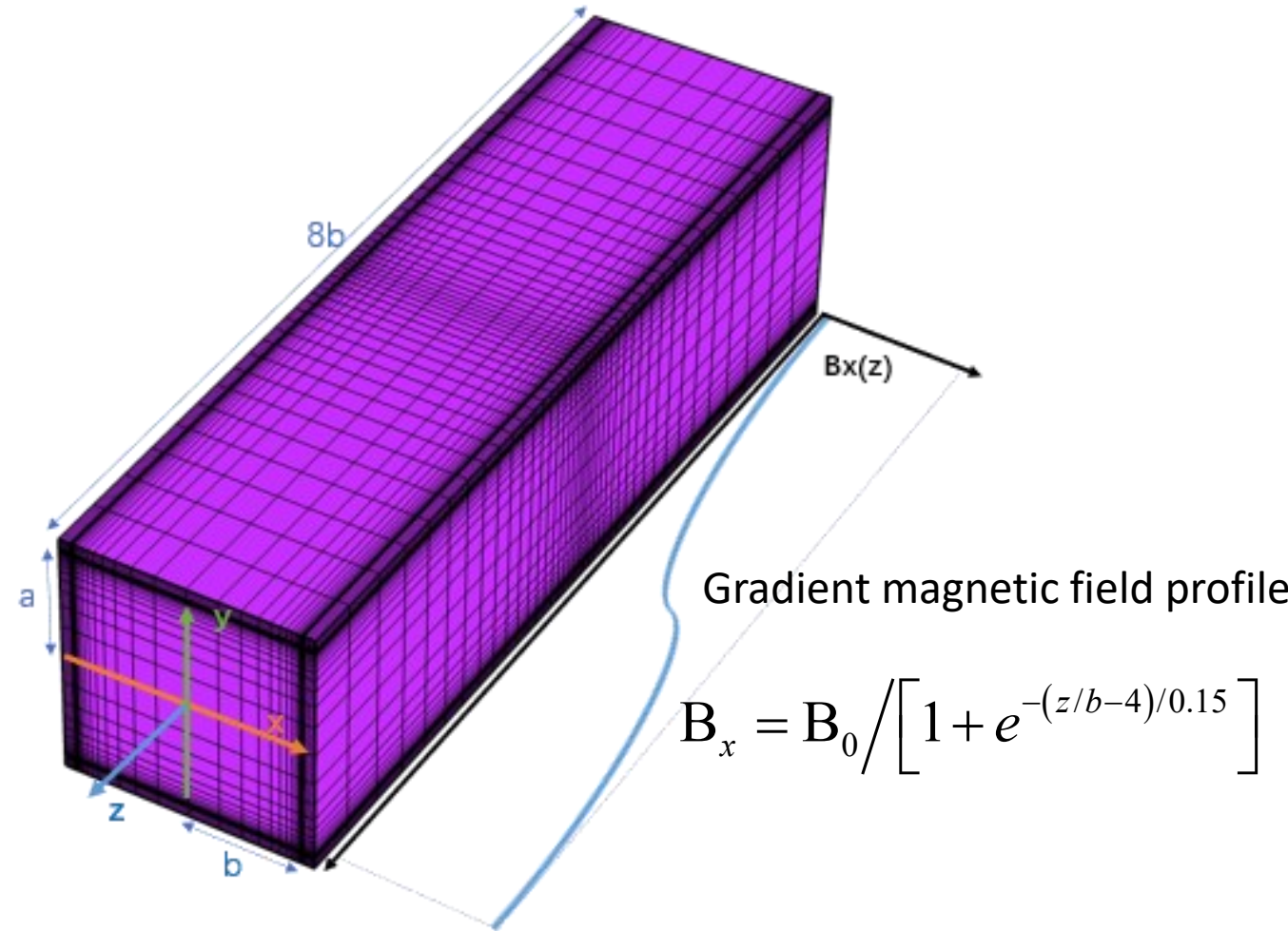
## Verification of 3D rectangular pipe MHD flow cases in longitudinally varying magnetic field

- ALEX Experiment:** Simulated MHD flow in a square duct with longitudinally reduced magnetic field (working fluid: <sup>22</sup>Na<sup>78</sup>K, Ha = 2900, N = 540, C<sub>w</sub> = 0.07). Numerical pressure gradient trends match experimental data; deviations in uniform magnetic field regions are attributed to experimental measurement errors.

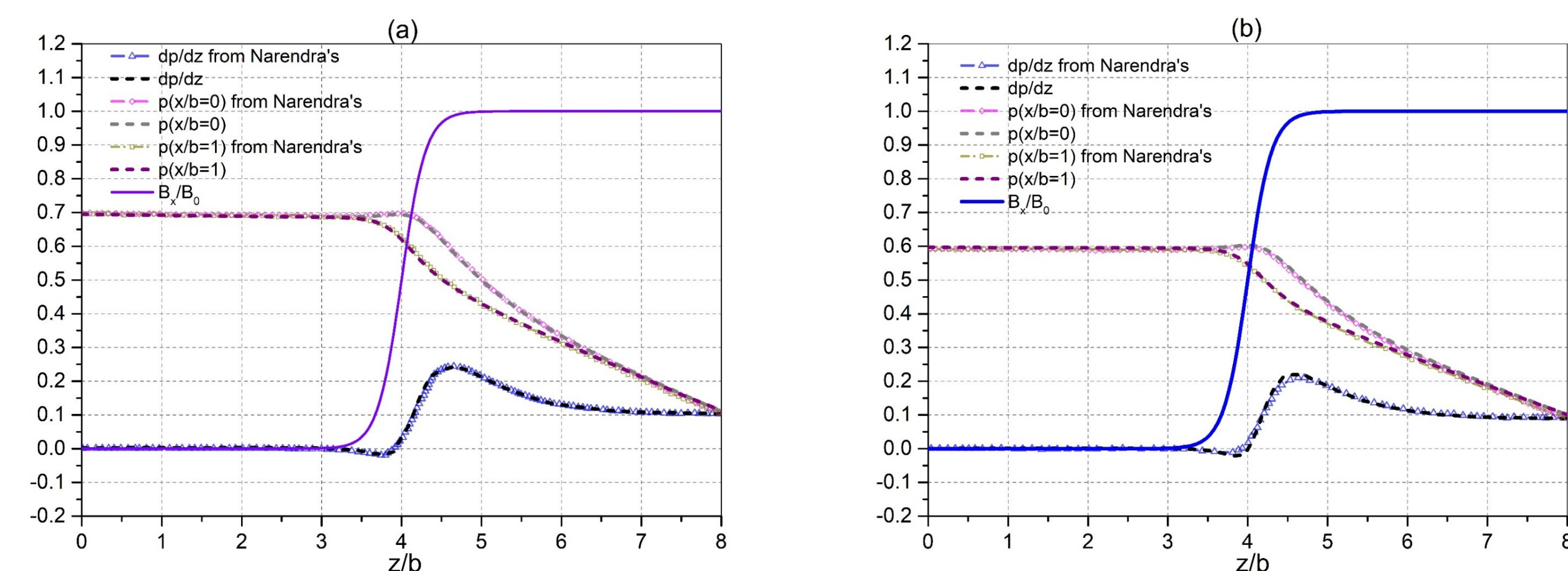


(a) Comparison between numerical pressure gradient in axial direction and corresponding experiment data, normalized background magnetic field marked with circles, (b) Pressure on the central line of Hartmann wall ( $P_b$ ) and pressure on the duct central line ( $P_a$ ) and their difference ( $P_b - P_a$ ).

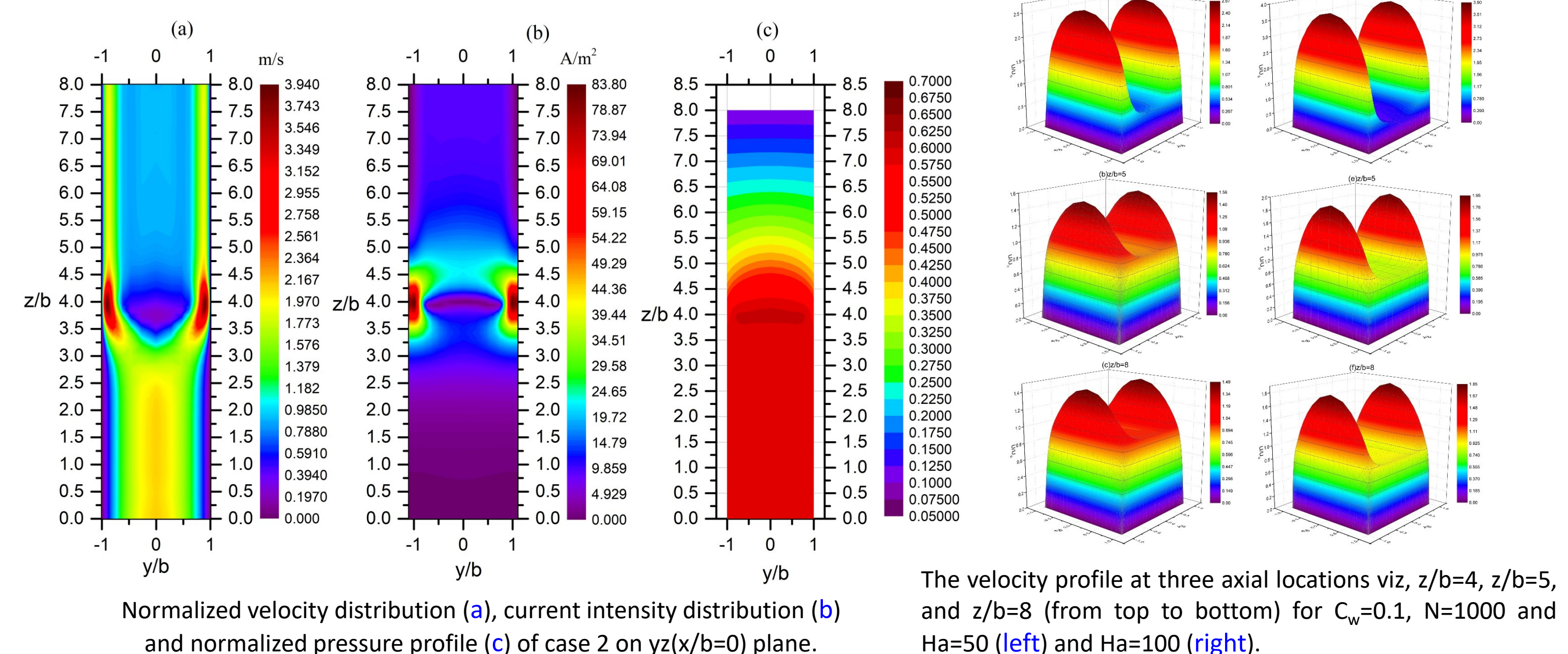
- Narendra Gajbhiya's Cases:** Simulated MHD flow in rectangular ducts with transverse non-uniform magnetic fields (Ha = 50/100, N = 1000, C<sub>w</sub> = 0.1).



Schematic of rectangular duct with imposed non-uniform background magnetic field in X direction.



Comparison of the profiles of the axial(z=0) pressure and pressure gradient in the longitudinal direction obtained in present work with Narendra for case 1 (a) and case 2(b)



The velocity profile at three axial locations viz, z/b=4, z/b=5, and z/b=8 (from top to bottom) for C<sub>w</sub>=0.1, N=1000 and Ha=50 (left) and Ha=100 (right).

## APPLICATION: MHD FLOW IN RIPPLED MAGNETIC FIELD

- Rippled Magnetic Field Context**

- Fusion reactor toroidal magnetic fields (e.g., ITER's 18 coils) have periodic “toroidal field ripple”—weaker in inter-coil gaps, with large radial gradients. Liquid lithium-lead blanket inlet/outlet pipes pass through these gaps, and their pressure drops dominate total blanket pressure drops.

- Simulation Setup**

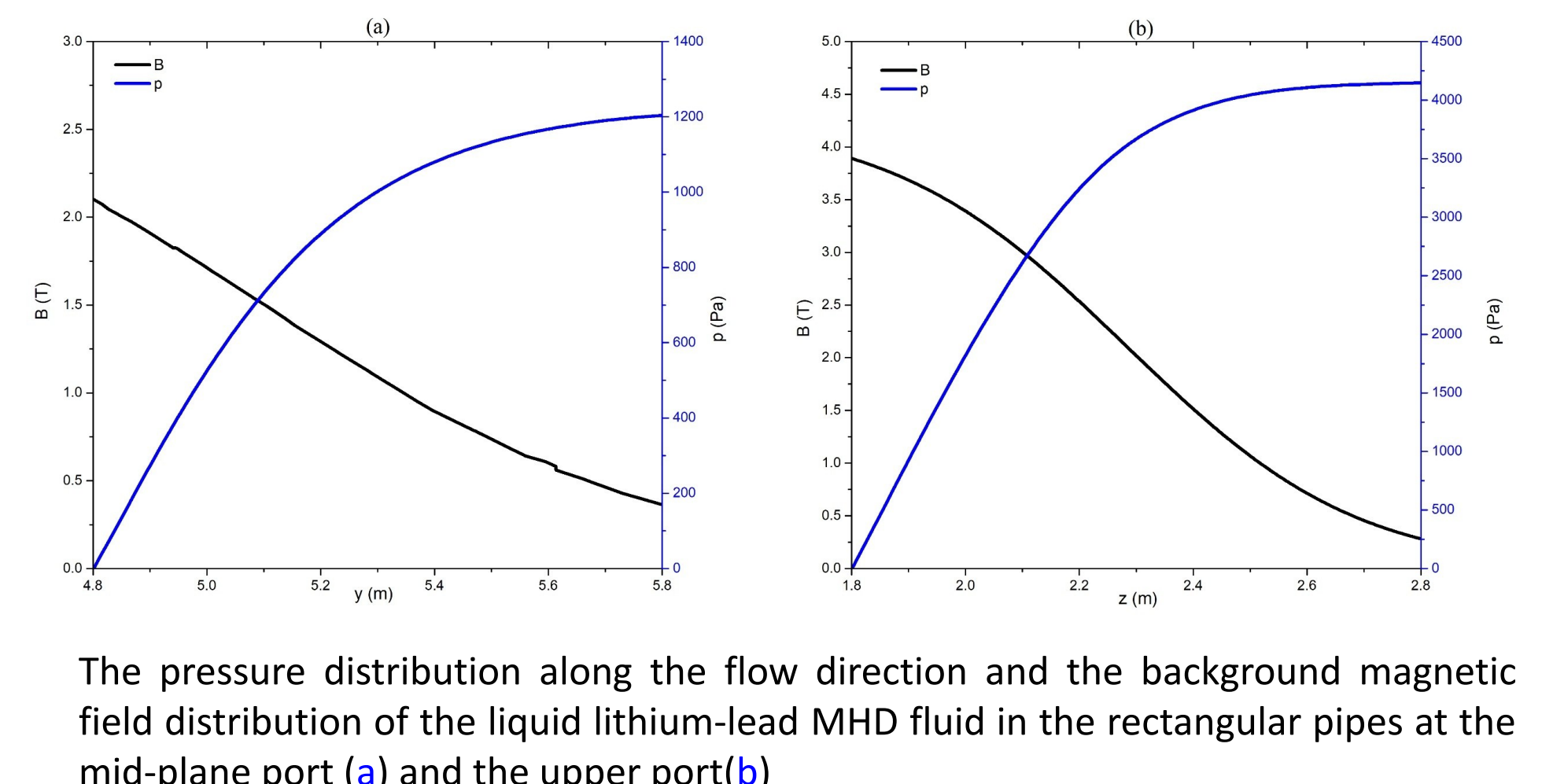
- Coil & Pipe Geometry:** Toroidally arranged circular coils (major radius = 3.0 m, minor radius = 2.5 m, B<sub>0</sub> = 5 T at magnetic axis); vertical rectangular pipes (Hartmann half-wall length = 0.1 m, wall thickness = 0.01 m, length = 1 m) at mid-plane port (inlet y=5.8 m, outlet y=4.8 m) and upper port (inlet z=1.8 m, outlet z=2.8 m).
- Fluid & Wall Properties:** Liquid lithium-lead ( $\mu = 1.93 \times 10^{-3}$  Pa·s,  $\sigma_f = 7.82 \times 10^6$  S/m); reduced-activation steel wall ( $\sigma_w = 1.145 \times 10^6$  S/m).

- Key Results**

- Numerical pressure drops (considering magnetic field gradients) are comparable to 2D fully developed MHD pressure drops with discrepancies < 9%. This indicates gradient-induced 3D pressure drops are negligible for the studied scenarios.

Pressure Drop Comparison (3D Numerical vs 2D Fully Developed)

| Case                                                                          | Inlet MHD flow in mid-plane port | Inlet MHD flow in upper port |
|-------------------------------------------------------------------------------|----------------------------------|------------------------------|
| Pressure drops from numerical calculation considering magnetic field gradient | 1200 Pa                          | 4147Pa                       |
| Pressure drops from 2D fully developed MHD calculations                       | 1246 Pa                          | 4200Pa                       |
| Discrepancy                                                                   | 3.8%                             | 8.9%                         |



The pressure distribution along the flow direction and the background magnetic field distribution of the liquid lithium-lead MHD fluid in the rectangular pipes at the mid-plane port (a) and the upper port(b)

## Conclusion

- The 3D MHD code (COMSOL + magnetic induction/modified Navier-Stokes equations) is reliable: Validated for fully developed laminar MHD flow (uniform magnetic fields) and flow transitions (gradient magnetic fields), with numerical results matching analytical/experimental/reference data.
- For liquid lithium-lead blanket pipes in inter-coil rippled magnetic fields, 2D fully developed MHD calculations adequately approximate pressure drops—supporting 2D approximations in blanket pipe design to simplify engineering workflows.

## Discussion topics

- How to extend the code for thermal MHD and tritium transport simulations?
- What gradient thresholds define “negligible 3D drops” for advanced reactor designs?
- Can this COMSOL development approach be standardized to reduce redundant work?

## Reference

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- [2] Muller et al. (2002). Springer Berlin, 55, 1.
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