NONLINEAR SELF-CONSISTENT DYNAMICS OF GEODESIC ACOUSTIC MODES AND ZONAL FLOWS IN TOROIDALLY ROTATING TOKAMAK PLASMAS

[1]V.I. ILGISONIS, [2]E.A. SOROKINA

[1]State Atomic Energy Corporation ROSATOM, Moscow, Russian Federation
[2]National Research Centre "Kurchatov Institute", Moscow, Russian Federation

E-mail: Sorokina_EA@nrcki.ru

Shear plasma flows, which are almost uniform along the magnetic surfaces, are observed on many tokamaks and stellarators [1]. The flows characterized by the oscillations of the electric potential at a frequency about ~ 20 kHz are known as geodesic acoustic modes (GAMs) while the zonal flows (ZFs) have frequencies about or less than several kHz. The flows of both types are thought to regulate drift-wave turbulence and to play an important role in having H-mode regimes. That is why the study of their mutual dynamics and interaction looks very relevant.

Usually, GAM is stable and has a finite frequency [2]. Stability of ZF depends on the distribution of the entropy on the magnetic surfaces. If the entropy is constant on the magnetic surface, the ZF is stationary; if not – ZF can be either oscillatory or linearly unstable. The "unfreezing" of the entropy from magnetic surfaces can be provided by different factors, e.g., by the stationary toroidal plasma rotation [3].

This paper presents the results of the modeling of joint dynamics of GAM and ZF in a toroidally rotating tokamak plasma. The model is based on the magnetohydrodynamics and takes into account quadratic nonlinearities (the right-hand sides of the equations):

$$\frac{\partial \rho}{\partial t}+\frac{1}{q}\frac{\partial v}{\partial \theta}+A\sin\theta\left(2+\frac{\gamma}{\alpha}M^2\right)=\frac{A}{q}\frac{\partial \rho}{\partial \theta},$$

$$\frac{\partial p}{\partial t}+\frac{1}{q}\frac{\partial v}{\partial \theta}+A\sin\theta(2+M^2)=\frac{A}{q}\frac{\partial p}{\partial \theta},$$

$$\frac{\partial v}{\partial t}+\frac{1}{q}\frac{\partial p}{\partial \theta}+2AM\sin\theta=\frac{A}{q}\frac{\partial v}{\partial \theta},$$

$$\frac{\partial A}{\partial t}-2\oint \sin\theta\left(p+Mv+\frac{M^2}{2}\rho\right)d\theta=0.$$

The first two equations correspond to the equations of continuity and adiabaticity, the third equation is the longitudinal projection of the equation of motion, the fourth one is the condition for quasi–neutrality of perturbations – see the detailed derivation in Ref. [4]. The values $\rho, p, v=\rho, p, v(\Psi,\theta,t)$ are the normalized perturbations of density, pressure, and longitudinal plasma velocity, respectively; the value $A(\Psi,t)$ is proportional to the perturbation of the radial electric field, t is the time normalized on the sound frequency ω_s, θ is the poloidal angle, Ψ is the label of the magnetic surface, $q(\Psi)$ is the safety factor, $M(\Psi)$ is the Mach number of the stationary toroidal plasma rotation, γ is the adiabatic index, the constant α determines the thermodynamic function, which is constant on the magnetic surface in the equilibrium, so that $p_0/\rho_0^\alpha=\Pi(\Psi)$.

Without the nonlinear terms in the right-hand sides of the equations, the system reduces to the well-known dispersion law in a toroidally rotating plasma [5]:

$$\omega^4-\omega^2\omega_s^2\left(2+\frac{1}{q^2}+4M^2+\frac{\gamma M^4}{2\alpha}\right)+\omega_s^4\frac{\gamma-\alpha}{\alpha}\frac{M^4}{2q^2}=0.$$

The higher root of the dispersion equation corresponds to the square of the GAM frequency, and the lower one – to ZF; ZF is unstable ($\omega^2<0$) at $\alpha>\gamma$.

Due to the nonlinearity, the obtained set of equations describes a variety of oscillation regimes, the onset of which can depend on the equilibrium parameters and starting conditions. Below the results of the typical regime calculations are presented. Linearly unstable ZF appears to be stabilized due to nonlinear interaction with GAM.

In a linear calculation (Fig.1 on the left), the amplitude of the initial perturbation increases indefinitely with time: $A(t)\sim\exp\frac{M^2}{q\sqrt{2+1/q^2}}\sqrt{\frac{\alpha-\gamma}{2\alpha}}t$. The perturbations under nonlinear modeling (Fig.1 on the right), initially growing, then reach their maxima and transform in low-frequency oscillations of finite amplitudes. The low-frequency ZF mode appears to be modulated by high-frequency oscillations at the frequency of GAM.

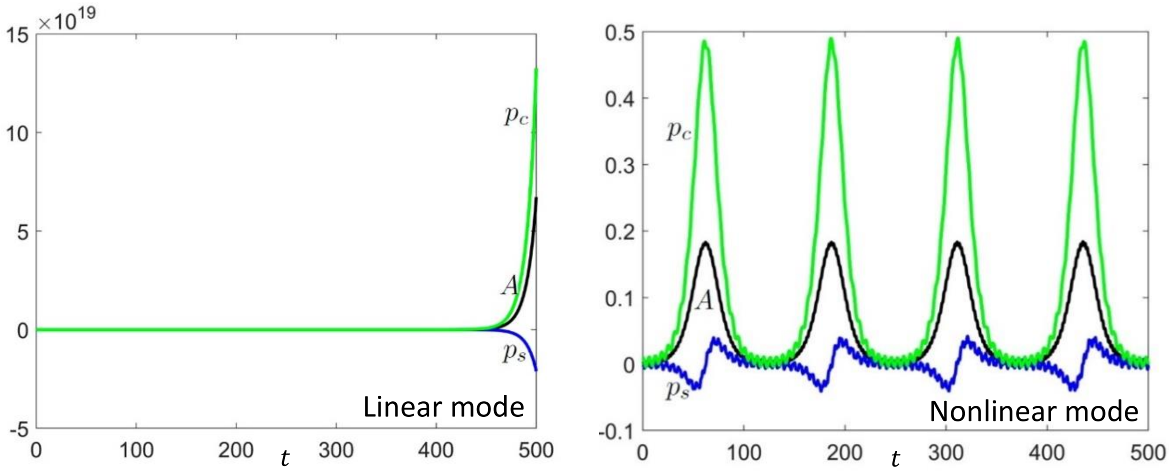

Fig.1. Temporal dynamics (linear on the left, nonlinear on the right) of ZF oscillations. The subscripts c and s correspond to the cos- and sin-poloidal harmonics of the pressure.

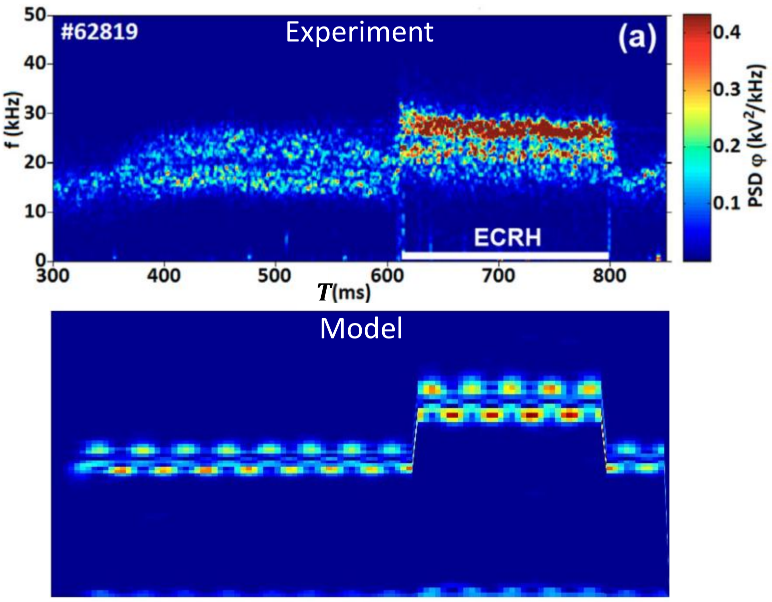

Fig.2. Spectrogram of potential perturbations in the discharge with Ohmic and auxiliary ECRH of tokamak T-10 [6] (top), the result of nonlinear modeling (bottom).

In the spectrum of GAM oscillations, the observed dynamics manifests itself in the form of bursts. Figure 2 shows the evolution of the GAM power spectrum calculated within the framework of the presented model (down graph) which correlates evidently with the typical experimental pattern of perturbations (top) in the tokamak T-10 [6].

The demonstrated nonlinear interaction of GAM and ZF is a possible reason for the observed intermittency and periodic modulation of GAM [7]. The interaction with unstable ZF serves as an excitation mechanism for the GAM, which clarifies the regular observation of this linearly stable mode in the experiments.

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