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X point effects on the tokamak stability and confinement in the description of dual-poloidal-region safety factor

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Synopsis

Nowadays, the H mode confinement has become the standard operating scheme for tokamaks. However, the H mode confinement is often tied with the so-called edge localized modes (ELMs), that can discharge the pedestal heat to divertors. Such a discharge may severely damage the divertors. Both H mode confinement and ELMs are related to the X point physics. This motivates us to further examine the X point effects on the tokamak stability and confinement.

We pointed out that the local safety factor q_{local} tends to infinity only at the X points while remaining finite elsewhere. It is the surface average in the definition of q in the conventional flux coordinates that makes q tend to infinity everywhere on a surface as approaching the plasma edge. Mathematically, it is known that the singular point should be isolated with a subtle treatment, instead of being averaged to spread it. From a physical point of view, one can see that a flux tube with finite size shrinks sharply from the poloidally core region to the X point or, the other way around, expands dramatically from the X point to the poloidally core region. These indicate the necessity of the dual-poloidal-regional q description of tokamak edge stability and confinement [1].

Using the dual q coordinates, we found that there are two types of modes at the tokamak plasma edge: the conventional peeling or peeling-ballooning modes and the axisymmetric modes localized in the vicinity of X points. The X points are shown to contribute to a stabilizing effect on the conventionally treated peeling-ballooning modes with the surface-averaged q and with the tokamak edge portion truncated. The axisymmetric modes at X points are shown to be universally unstable on the low field side in the resistive MHD description. The existence of axisymmetric modes points to the possibility of applying a toroidally axisymmetric resonant magnetic perturbation (RMP) in the X-point area for mitigating ELMs. The dual q description also helps to understand why the RMP suppression of ELMs is difficult to achieve in the double-null tokamak configurations and points to the possibility of further improving the current RMP concept by considering the alignment to the local safety factor q_{local}.

To develop the dual q coordinate system, the maximum Jacobian $\mathcal{J}_{eq}^{\max}$ is imposed to define the poloidally core regions (Θ_{core}) and the vicinity of X point (Θ_X) with $\hat{J}_{eq} = J_{eq}$ in Θ_{core} and $\hat{J}_{eq} = \mathcal{J}_{eq}^{\max}$ in Θ_X. The magnetic field is then expressed as

$$\boldsymbol{B} = \boldsymbol{\nabla}\phi \times \boldsymbol{\nabla}\chi + f(\chi)\boldsymbol{\nabla}\phi = \boldsymbol{\nabla}\phi \times \boldsymbol{\nabla}\chi + q_c(\chi)h(\chi,\theta_c)\boldsymbol{\nabla}\chi \times \boldsymbol{\nabla}\theta_c,$$

where

$$\theta_c = \frac{f}{q_c}\int_0^{\theta_{eq}} d\theta_{eq}\frac{\hat{J}_{eq}}{X^2}, \quad q_c(\chi) = \frac{f}{2\pi}\int_0^{2\pi} d\theta\frac{\hat{J}_{eq}}{X^2}, \text{ and } h(\chi,\theta_{eq}) = \begin{cases} 1, & \theta_{eq} \in \Theta_{core} \\ \frac{\mathcal{J}_{eq}}{J_{eq}^{\max}}, & \theta_{eq} \in \Theta_X \end{cases}.$$

This coordinate system isolates the singularity of q_{local} in the vicinity of X points Θ_X, while the remaining core region Θ_{core} is described by the finite averaged q_c with the q singularity excluded. This helps to subtly examine the X point effects on edge physics.

For peeling type of modes, it is found that the total plasma energy becomes

$$\delta W = \delta W_c + \delta W_X,$$

where δW_c is the peeling mode energy in the conventional treatment with the surfaced averaged q_c and with the plasma edge truncated and δW_X is the X point contribution

$$\delta W_X = \frac{n^2}{2\mu_0} \int_{\Theta_X} \left| \frac{B\boldsymbol{B} \cdot \boldsymbol{B}_c}{B_c^2 |\boldsymbol{\nabla} Z|} \frac{f}{R} \frac{\boldsymbol{\xi} \cdot \boldsymbol{B}_c \times \boldsymbol{\nabla} Z}{|\boldsymbol{\nabla} Z|^2} \right|^2 d\boldsymbol{r}.$$

Here, $\boldsymbol{B}_c = \boldsymbol{\nabla}\phi \times \boldsymbol{\nabla}\chi + q_c(\chi)\boldsymbol{\nabla}\chi \times \boldsymbol{\nabla}\theta_c$ and Z labels the magnetic flux surface. δW_X describes the energy resulting from the field line bending effects in Θ_X due to the localized perturbation following the magnetic field $\boldsymbol{B}_c$. Since δW_X is positive definite, the X point effects are found to contribute a stabilizing effect. This stabilization effect is also demonstrated numerically for the global external kink modes [1]. The dual q description also paves the way for nonideal MHD description of X point physics in Θ_X, which otherwise spreads over the whole magnetic surface in the conventional treatment.

It is found that there is an axisymmetric mode localized at the X points with its stability determined by the Mercier index

$$D_I \approx \frac{f\mu_0 P' \kappa_n}{(\boldsymbol{B} \cdot \nabla\Lambda_s)^2} - \frac{1}{4} < 0 \tag{1}$$

with the magnetic shear parameter $\boldsymbol{B} \cdot \nabla\Lambda_s = (\boldsymbol{B} \cdot \boldsymbol{\nabla}\theta_c/|\boldsymbol{\nabla}\chi|^2)\nabla\chi \cdot \boldsymbol{\nabla}(q_c h)$. Eq. (1) represents the existence condition of the localized axisymmetric modes. Note that the term $\mu_0 P' \kappa_n$ denotes the well-known magnetic well effect. The term $1/4$ is related to the magnetic shear square $(\boldsymbol{B} \cdot \nabla\Lambda_s)^2$ as relatively compared with the first term on the right-hand side of Eq. (1). The appearance of axisymmetric modes in Θ_X where $q \to \infty$ resembles the tokamak scenario without a toroidal current to generate the poloidal magnetic field for field line rotational transform $1/q$. The interchange modes tend to develop on the low field side in this case. The difference for the localized axisymmetric modes is that there is a large magnetic shear in the vicinity of X points. However, the resistivity can suppress the shear stabilization effects, $1/4$ term in Eq. (1). Therefore, the localized axisymmetric modes become generally unstable in resistive MHD on the low field side in the vicinity of the X point.

Matching the local field line pitch determines the efficiency of the RMPs to generate magnetic island and stochastic field structure to mitigate ELMs. This explains why the RMP ELM suppression is usually observed only in the single null configurations and not in the double null configurations. The double null configurations may lead to a larger mismatch for finite m/n RMPs. Noting that $q \to \infty$ in the vicinity of X points, one can imagine applying the toroidal axisymmetric RMPs in the region of X point to resonantly perturb the local magnetic field and thus mitigate ELMs. The dual q feature shows the importance of including the $n = 0$ or high m/n Fourier components in the RMP design.

In conclusion, we point out that in the presence of X points the dual-poloidal-regional q becomes an important feature of tokamak plasma edge. Respecting this fact helps to further understand the X point physics and more effectively introduce the RMP field for mitigating ELMs. This research is supported by the Department of Energy Grant DE-FG02-04ER54742.

References

[1] Linjin Zheng, M. T. Kotschenreuther, F. L. Waelbroeck, and M. E. Austin, “X-point effects on the ideal MHD modes in tokamaks in the description of dual-poloidal-region safety factor”, *Phys. Plasmas* **32**, 012501 (2025).