A novel computation of the linear plasma response to a resonant error field in single-fluid visco-resistive MHD and application to the RFX-mod2 tokamak

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ABSTRACT

- Dynamical simulations of error field (EF) penetration in a tokamak plasma are performed in cylindrical geometry.
- This is accomplished by the RFX-locking code, already used to simulate the tearing mode dynamic in tokamaks [1].
- The code is enriched by recently developed specific physics describing the linear plasma response to a resonant EF [2].
- Application to cylindrical proxy of JET and RFX-mod2, the revamped RFX-mod experiment, are discussed.

BACKGROUND

- Linear MHD models [3, 4] are commonly adopted to interpret the EF penetration.
- However, the generally adopted Fourier transform based analysis method is not adequately justified.
- Moreover, the comparison between their predictions of the EF penetration threshold and the experiment is not satisfactory: the density dependence is too weak, and the major radius dependence has a wrong sign.
- We present a new formulation of the single-fluid linear approach, which is able to fill these gaps

CHALLENGES / METHODS / IMPLEMENTATION

'OUTER REGION' ANALYSIS

Motion equations:

$$\rho\frac{\partial}{\partial t}\Omega_\phi = \frac{\rho a^2}{\tau_V}\frac{1}{r}\frac{\partial}{\partial r}\left(r\,\frac{\partial}{\partial r}\Omega_\phi\right) + S_\phi + \frac{1}{4\pi^2 r R_0^3}T_{EM,\phi}\,\delta(r-r_{m,n})$$

$$\rho\frac{\partial}{\partial t}\Omega_\theta = \frac{\rho a^2}{\tau_V}\frac{1}{r^3}\frac{\partial}{\partial r}\left(r^3\,\frac{\partial}{\partial r}\Omega_\theta\right) - \frac{\rho}{\tau_\theta}\Omega_\theta + S_\theta + \frac{1}{4\pi^2 r^3 R_0}T_{EM,\theta}\,\delta(r-r_{m,n})$$

In place of the customary $\tau_V = \tau_E$, we take $\tau_V = \tau'_{Ei}$ [5] . Moreover:

$$\tau_\theta = 0.507\tau_i/ln(2)\ [6],\quad T_{EM,} \propto |\Psi_s|^2 Im(\Delta'),\quad \Delta' = \frac{1}{\Psi_s}\frac{d}{dr}\psi^{m,n}\Big|_{r_{m,n}-}^{r_{m,n}+}$$

The perturbation at the resonant surface Ψ_s is expressed in terms of the EF at the vessel Ψ_v : $\Psi_s = \frac{E_{vs}\Psi_v}{r_{mn}\Delta' - E_s}$

'INNER REGION' ANALYSIS

The purpose is computing Δ'. Reduction of single-fluids visco-resistive MHD equations gives

$$\frac{d^2\tilde{\psi}}{dX^2} = iQ\,\tilde{\psi} - iX\,\tilde{\phi},\quad X\frac{d^2\tilde{\psi}}{dX^2} = Q\frac{d^2\tilde{\phi}}{dX^2} + iP\frac{d^4}{dX^4}\tilde{\phi},$$

$$X = S^{1/3}(r-r_{m,n})/r_{m,n}\,,\quad S = \tau_R/\tau_A,\quad P = \tau_R/\tau_V,\quad Q = S^{1/3}\tau_A\omega,\quad \omega = m\Omega_\theta(r_{m,n}) - n\Omega_\phi(r_{m,n})$$

Asymptotic behavior of $\tilde{\psi}, \tilde{\phi}$ at $X \to \infty$ gives Δ'.

<u>Solution method 1</u>: order reduction by $Y = X\frac{d}{dX}\tilde{\psi} - \tilde{\psi}$ giving

$$16Pz\frac{d^4}{dz^4}Y + \left(\frac{12P}{z} - \lambda\right)\frac{d^2}{dz^2}Y - \left(\frac{12P}{z^2} - \frac{\nu}{z}\right)\frac{d}{dz}Y + \left(1 - \frac{\mu}{z^2} - \frac{Q^2}{z}\right)Y + 1 = 0$$

$$\lambda = 4iQ(P+1),\quad \nu = 2iQ(3P+1),\quad \mu = 6iQP,\quad \Delta' = 2\int_0^{+\infty}\frac{1}{\sqrt{z}}\frac{dY}{dz}dz$$

<u>Solution method 2</u>: combination of Fourier transformed equations gives

$$\frac{d}{dk}\left[\frac{k^2}{k^2+iQ}\,\frac{d}{dk}\bar{\phi}\right] = k^2(iQ + Pk^2)\bar{\phi}$$

Then, use of Riccati transform $w(k) = \frac{k^2}{k^2+iQ}\frac{d\bar{\phi}/dk}{\bar{\phi}}$ to get

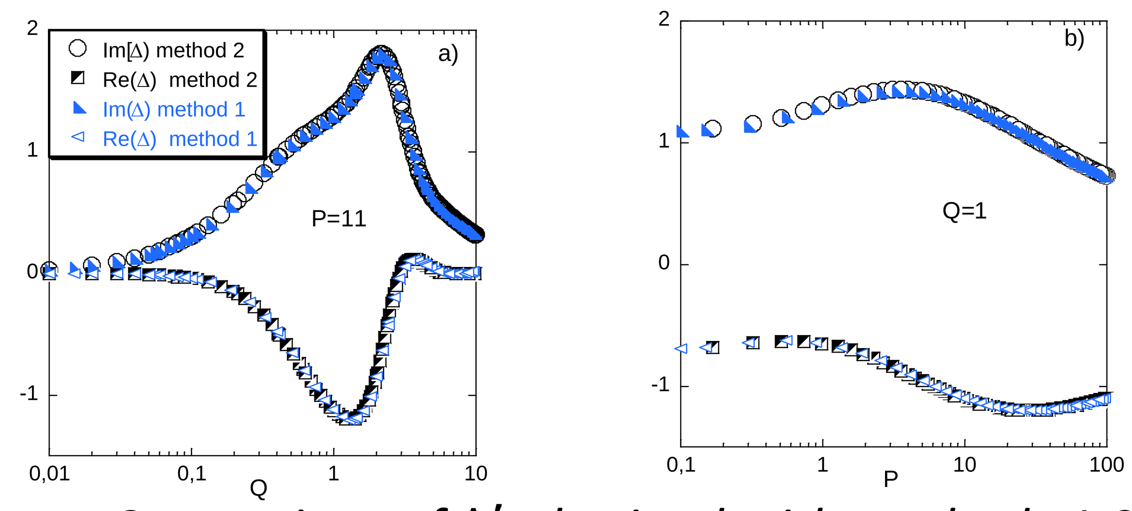
$$\frac{d}{dk}w + \frac{k^2+iQ}{k^2}\,w^2 = k^2(iQ + Pk^2)$$

$$w(k) = \frac{i\,k}{Q}\left[1 - \frac{\pi}{\Delta'}\,k\,sgn(k)\right] + o(k^2),\ k \to 0$$

OUTCOME

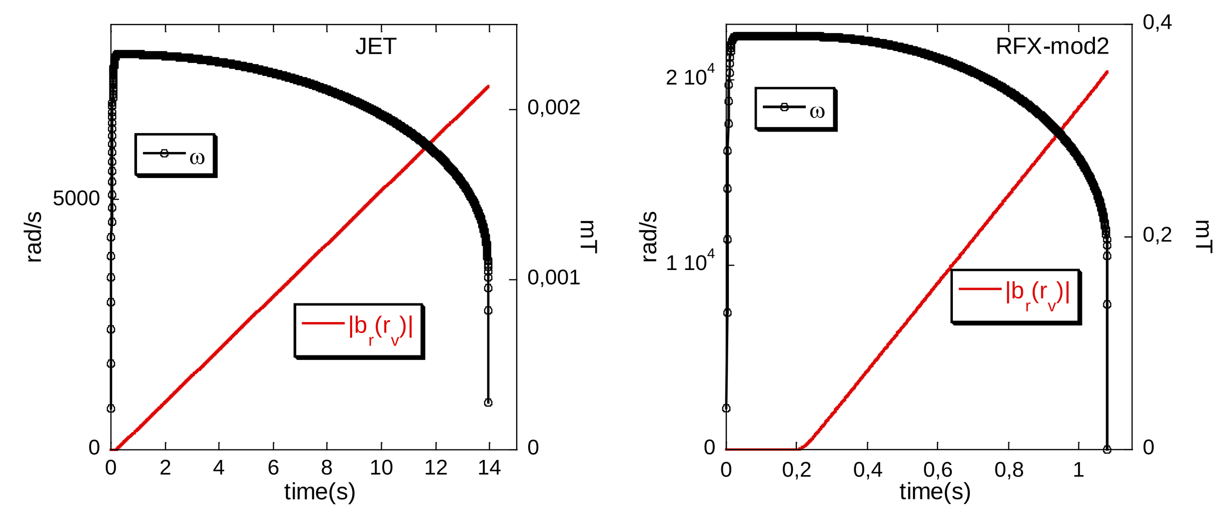
COMPARISON OF THE TWO METHODS

Agreement between methods 1,2: Fourier transform is then justified



Comparison of Δ' obtained with methods 1,2

DYNAMICAL SIMULATION OF EF PENETRATION



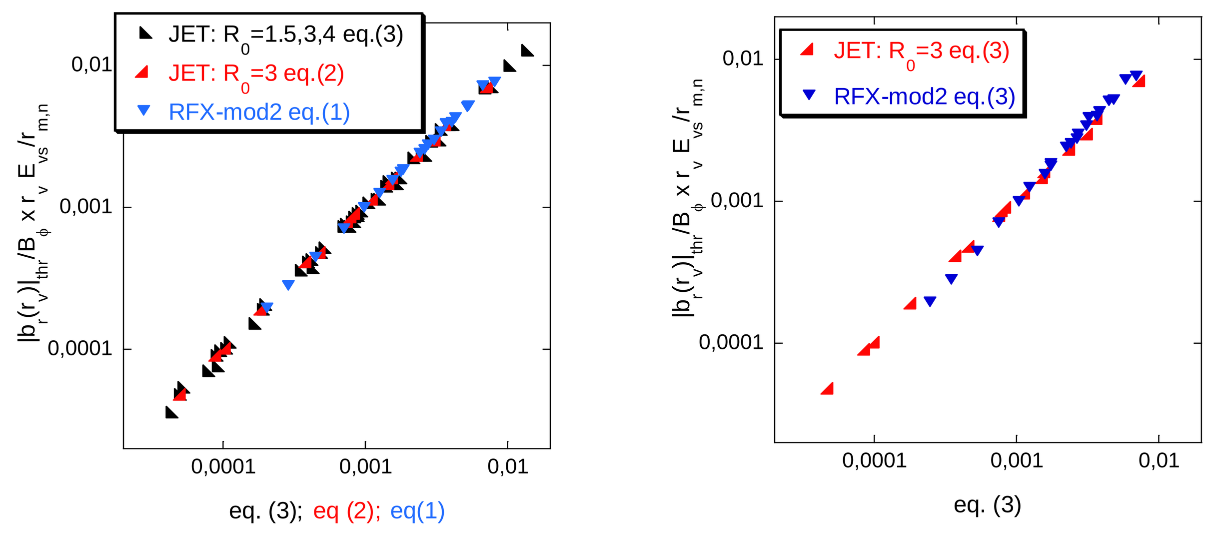
JET: n=10^{20} m^{-3},B_ϕ=2.2T. RFX-mod2 n_=1.3×10^{19} m^{-3},B_ϕ=0.5T.
EF penetration corresponds to the instant where ω drops to zero.

SCALING LAWS

1) $RFXmod2 \rightarrow |b_r(r_v)|_{thr}/B_\phi \times r_v\,E_{vs}/r_{m,n} = 9.5\times10^{-4}n_e^{1.13\pm0.01}B_\phi^{-1.54\pm0.03}$

2) $JET(R_0=3) \rightarrow |b_r(r_v)|_{thr}/B_\phi \times r_v\,E_{vs}/r_{m,n} = 9.6\times10^{-4}n_e^{0.98\pm0.01}B_\phi^{-1.48\pm0.02}$

3) $JET(R_0=1.5,3,4) \rightarrow |b_r(r_v)|_{thr}/B_\phi \times r_v\,E_{vs}/r_{m,n} = 7.7\times 10^{-4}n_e^{1.00\pm0.01}B_\phi^{-1.49\pm0.02}R_0^{0.19\pm0.02}$



Left: simulated data vs regressions (1), (2), (3)
Right: simulated JET and RFX-mod2 vs scaling law (3)

Scaling (3) unifies JET and RFX-mod2 (see right plot)

CONCLUSION

- We have presented a new formulation of the single-fluid linear approach.
- We have given a justification of the Fourier transform based technique commonly used to compute the Δ' parameter.
- We have provided EF penetration threshold scaling law compatible with experiments [7, 8]: the identification of the momentum confinement time with the ion energy replacement time is crucial.

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