EFFECTS OF FINITE ION TEMPERATURE AND ITS GRADIENT ON HASEGAWA-MIMA EQUATION AND ZONAL FLOW GENERATION

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The drift-wave (DW) turbulence and associated anomalous transport, which can be regulated by zonal flow (ZF) are important topics for the achievement of ignition conditions in future tokamak fusion reactor. The Hasegawa-Mima (H-M) equation, renowned for its simplicity and robust conservation properties [1], has been widely adopted to study DW-ZF systems. However, its fundamental assumption of cold ions becomes invalid in high-temperature core plasmas of magnetic confinement fusion devices, necessitating modifications to incorporate effects of finite ion temperature and its gradient. This work reports the conservation properties related to modified H-M equation, such as potential vorticity (PV), potential enstrophy (PE) with effects of ion diamagnetic drift and FLR. The momentum theorem and generation of ZF from modified H-M equation are discussed as well.

We start from the gyro-fluid equations in slab geometry [2]

$$\frac{\partial}{\partial t}\tilde{q} + \tilde{\boldsymbol{v}}_E \cdot \nabla\tilde{q} - \tilde{\boldsymbol{v}}_* \cdot \nabla\left(\nabla_\perp^2\tilde{\phi}\right) - \hat{\boldsymbol{b}} \times (\nabla\nabla_{\perp i}\tilde{p}_\perp)\cdot\left(\nabla\nabla_{\perp i}\tilde{\phi}\right) = \omega_{*e}(1+\tau_{ie}(1+\eta_i)\nabla_\perp^2)\tilde{\phi}. \qquad (1)$$

Here, $\tilde{q} = \tilde{\phi} - \nabla_\perp^2\tilde{\phi}$ is the PV fluctuation. The standard normalizations are used as follows: the electric potential fluctuation $\tilde{\phi} \equiv \frac{e\delta\phi}{T_{e0}}$, ion perpendicular pressure fluctuation $\tilde{p}_\perp \equiv \frac{\delta P_{\perp i}}{P_{e0}}$ with $P_{i0} = n_0 T_{e0}$, time scale by Ω_i with $\Omega_i = eB/(m_i c)$ being the ion gyrofrequency, spatial scale by ρ_s with $\rho_s = \sqrt{\frac{T_{e0}}{m_i}}/\Omega_i$ being the ion gyroradius at acoustic velocity, $\tilde{\boldsymbol{v}}_E = \hat{\boldsymbol{b}} \times \nabla\tilde{\phi}$ and $\tilde{\boldsymbol{v}}_* = \hat{\boldsymbol{b}} \times \nabla\tilde{p}_\perp$ are the fluctuating $\boldsymbol{E}\times\boldsymbol{B}$ and ion diamagnetic velocities, respectively, $\omega_{*e} = -k_y\left(\frac{\partial}{\partial r}\ln n_0\right)$ is the electron diamagnetic frequency, and $\tau_{ie} = \frac{T_{i0}}{T_{e0}}$. For uniform plasmas, using $\{f,g\} = \left(\hat{b}\times\nabla f\right)\cdot\nabla g$, Eq. (1) can be rewritten as

$$\frac{\partial}{\partial t}\tilde{q} - \nabla_{\perp i}\{\tilde{\phi} + \tilde{p}_\perp, \nabla_{\perp i}\tilde{\phi}\} = 0. \qquad (2)$$

It can be seen that the PV is locally conserved rather than globally conserved.

Taking flux average of Eq. (2), integrating in *x* direction and adding the damping, ZF evolution equation can be obtained as

$$\frac{\partial}{\partial t}\langle v_y\rangle + \frac{\partial}{\partial x}\langle(\tilde{v}_{Ex} + \tilde{v}_{*x})\tilde{v}_{Ey}\rangle = -\nu\langle v_y\rangle. \qquad (3)$$

In addition to the usual Reynolds stress force, the ion diamagnetic Reynolds stress. Multiplying Eq. (1) by $\tilde{q}$, we can obtain the evolution equation of PE

$$\frac{\partial}{\partial t}\langle\frac{\tilde{q}^2}{2}\rangle + \frac{\partial}{\partial x}\langle\tilde{v}_{Ex}\frac{\tilde{q}^2}{2}\rangle + \frac{\partial}{\partial x}\langle\tilde{v}_{*x}\frac{1}{2}\left[\left(\nabla_\perp^2\tilde{\phi}\right)^2 + \left(\nabla_\perp\tilde{\phi}\right)^2\right]\rangle - \frac{\partial}{\partial x}\langle\tilde{\phi}\left(\tilde{v}_{*y}\frac{\partial}{\partial y} + \tilde{v}_{*x}\frac{\partial}{\partial x}\right)\frac{\partial}{\partial x}\tilde{\phi}\rangle$$

$$+\langle\nabla_\perp^2\tilde{\phi}\{\nabla_{\perp i}\tilde{p}_\perp, \nabla_{\perp i}\tilde{\phi}\}\rangle = -\left(\frac{\partial}{\partial r}\ln n_0\right)[1+\tau_{ie}(1+\eta_i)]\langle\tilde{v}_{Ex}\tilde{q}\rangle. \qquad (4)$$

The PE is not conserved either due to last term on the LHS contributed from the ion pressure fluctuation even for uniform plasmas.

Then, dividing Eq. (4) by a factor $\alpha = \left(\frac{\partial}{\partial r}\ln n_0\right)[1+\tau_{ie}(1+\eta_i)]$ and adding the ZF evolution equation, we can obtain the total zonal momentum equation [3, 4] modified by the effects of FLR and the fluctuating ion diamagnetic drift velocity

$$\frac{\partial}{\partial t}\left[\langle v_y\rangle + \langle\frac{\tilde{q}^2}{2\alpha}\rangle\right] = -\nu\langle v_y\rangle - \frac{\partial}{\partial x}\langle\tilde{v}_{*x}\,\tilde{v}_{Ey}\rangle - \frac{\partial}{\partial x}\langle\tilde{v}_{Ex}\frac{\tilde{q}^2}{2\alpha}\rangle + \frac{\partial}{\partial x}\langle\frac{\tilde{v}_{Ex}\tilde{v}_{*y}\tilde{v}_{Ey}}{\alpha}\rangle - \frac{\partial}{\partial x}\langle\tilde{v}_{*x}\frac{\left(\nabla_\perp^2\tilde{\phi}\right)^2 + \left(\nabla_\perp\tilde{\phi}\right)^2 + 2\tilde{v}_{*y}^2}{2\alpha}\rangle + \frac{\partial^2}{\partial x^2}\langle\tilde{v}_{*x}\frac{\partial^2}{\partial x^2}\left(\frac{\tilde{\phi}^2}{2\alpha}\right)\rangle \quad (5)$$

It is found that the ion diamagnetic drift contributes additional zonal flow drive via its effects on Reynolds stress [5–7] and turbulence spreading. Moreover, under fixed PE intensity, finite ion temperature and its gradient reduce the allocation of turbulent pseudo-momentum in the total zonal momentum, thereby may enhance the zonal flow generation. These theoretical insights advance the understanding of DW-ZF interactions in magnetically confined fusion plasmas.

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